A Single Feedback Based Interference Alignment for Three-User MIMO Interference Channels with Limited Feedback

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Abstract

Conventional interference alignment (IA) for a MIMO interference channel (IFC) requires *global* and *perfect* channel state information at transmitter (CSIT) to achieve the optimal degrees of freedom (DoF), which prohibits practical implementation. In order to alleviate the global CSIT requirement caused by the coupled relation among all of IA equations, we propose an IA scheme with a single feedback link of each receiver in a limited feedback environment for a three-user MIMO IFC. The main feature of the proposed scheme is that one of users takes out a fraction of its maximum number of data streams to decouple IA equations for three-user MIMO IFC, which results in a single link feedback structure at each receiver. While for the conventional IA each receiver has to feed back to all transmitters for transmitting the maximum number of data streams. With the assumption of a random codebook, we analyze the upper bound of the average throughput loss caused by quantized channel knowledge as a function of feedback bits. Analytic results show that the proposed scheme outperforms the conventional IA scheme in term of the feedback overhead and the sum rate as well.

Keywords: Interference channel, MIMO, limited feedback, degrees of freedom

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1. Introduction

Recently, there has been much interest in studying interference management which leads to exploring the potential for interference alignment (IA) to achieve the optimal degrees of freedom (DoF) in wireless network. However, the conventional IA in [1] requires *perfect* and *global* channel state information at transmitters (CSIT), which makes it difficult to implement IA in the most of practical systems. Specifically, the *perfect* CSIT takes a long training phase to estimate channel information accurately. Besides, the *global* CSIT assumption requires the CSIT of other communication pairs, which leads to much feedback overhead. If the total number of feedback bits for each receiver is limited, the quantization quality of feedback links is degraded.

To alleviate the *global* CSIT requirement, finding precoders algorithmically using a reciprocity assumption with local CSI is proposed in [2] and [3]. However, these algorithms cannot be applied to frequency division duplexed (FDD) systems and require a great number of iteration. Furthermore, they require an accurate calibration for the channel variation resulting from the iterating procedure. The works of [4] and [5] propose the modified precoder finding algorithms to achieve better sum rate performance than the conventional IA. However, these algorithms still require several iterations and multiple feedback links for each receiver.

In FDD systems, the effect of quantized CSIT is analyzed in [6] and [7], in which the authors use Grassmannian codebooks to quantize the channel direction information and feedback to transmitters with frequency domain symbol extension. Consequently, the DoF is preserved by scaling the number of feedback bits with SNR. However, a large Grassmannian codebook is required, which is prohibitive for the practical systems. Additionally, its analysis is applicable for a long size symbol-extension based IA scheme with the global feedback assumption. In [10], an efficient feedback-feedforward CSI exchange algorithm to alleviate global CSIT requirement in FDD systems was proposed. However, this algorithm is only based on the closed form solution for $M = N = K\text{-}1$ antenna configuration, where M, N and K denote the number of transmit antennas, receive antennas, and users, respectively and the channel should be fixed when the feedback-feedforward CSI exchange takes place.

Our focus of this paper is a limited feedback based MIMO interference channel (IFC). In these environments, we propose a sub-optimal DoF achieving IA with only single feedback link and single time feedback (no CSI exchange), meaning that each receiver only sends CSI back to a single transmitter not multiple ones at only one time. With the constraint of the total feedback bits in practical systems, the number of feedback links becomes a crucial factor of the system performance. Since the quantization quality of an individual CSI is degrade as the number of feedback links and channel coefficients increase.

The main feature of the proposed scheme is to reduce a number of data streams at the first transmitter in order to decouple equations of the conventional IA. The decoupled IA solution results in a single feedback structure. In particular, some of precoder beams can be lied in its pre-designed interference direction without feedback and the rest of precoder beams are determined by feedback. We analyze the upper bound of the average throughput loss as a function of feedback bits using a random codebook. Compared to [7], our analysis can be applied to the constant MIMO channels. We show that the required feedback overhead of the proposed IA is much less than that of the conventional IA achieving the same sum-rate performance. Note that we do not propose any channel quantization method for IA with limited feedback. Several advanced quantization methods as proposed in [12],[13] could be applied to our proposed IA scheme.

The reminder of this paper is organized as follows: In section 2, the system model is provided. In section 3, we present the conventional IA and feedback methods for the conventional IA. The proposed IA algorithm is provided in section 4. In section 5, we introduce a rigorous analysis for residual interference for an IA system with quantized CSIT. Numerical results are given in section 6. Finally, concluding remarks are presented in section 7.

The following notations are used throughout the paper. $\mathbf{A}$ is a matrix, a^m is a mth column vector of $\mathbf{A}$, $\mathbf{A}^H$ denotes the conjugate transpose of $\mathbf{A}$, and $\|\mathbf{A}\|_F$ is its Frobenius norm; $\mathrm{tr}(\mathbf{A})$ is its trace, $\mathrm{span}(\mathbf{A})$ is its column space, $\mathrm{null}(\mathbf{A})$ returns nullspace vectors of column space of $\mathbf{A}$, $\mathrm{eig}(\mathbf{A})$ is any eigenvector of $\mathbf{A}$, $\mathrm{O}(\mathbf{A})$ consists of the orthonormal basis vectors that span the column space of $\mathbf{A}$; $\mathbf{I}_M$ is the $M \times M$ identity matrix; $\mathbb{C}^M$ is the M-dimensional complex vector space.

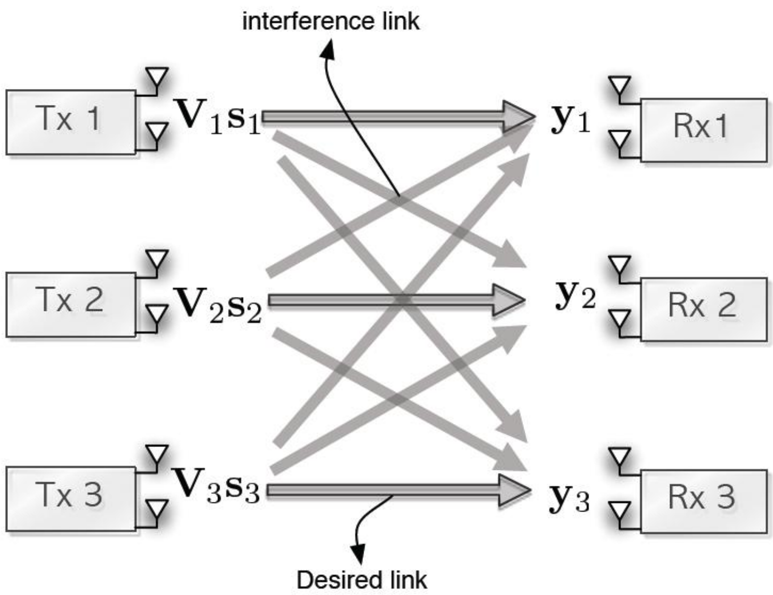

Fig. 1. Schematic of three-user MIMO IFC

2. System Model

Fig. 1 shows a description of the three-user IFC where there are 3 transmitter (Tx)-receiver (Rx) pairs and all nodes are equipped with M antennas. We consider only a three-user MIMO IFC since it is easy to compute the closed form of IA [1] and it can be often assumed that the number of dominant interferences in practical system is not so many. For three-user MIMO IFC, it is known that 2M/3 is the optimal network DoF in [14]. The received signal at Rx i can be written as

$$\mathbf{y}_i = \sqrt{\frac{P}{d_i}}\mathbf{H}_{i,i}\mathbf{V}_i\mathbf{s}_i + \sum_{j\neq i}\sqrt{\frac{\mu P}{d_j}}\mathbf{H}_{i,j}\mathbf{V}_j\mathbf{s}_j + \mathbf{n}_i \tag{1}$$

where $\mathbf{y}_i$ is the $M\times 1$ received signal vector, P is transmit power, factor $\mu \leq 1$ quantifies the path-loss difference between the data and interference links, $\mathbf{H}_{i,j}$ is the $M\times M$ channel matrix from Tx j to Rx i, $\mathbf{V}_i$ is the $M\times d_i$ precoding matrix used at Tx i, $\mathbf{s}_i$ is the $d_i\times 1$ transmitted symbol vector at Tx i, with unit norm element, i.e, $\mathrm{E}\left(\mathbf{s}_i^H\mathbf{s}_i\right) = d_i$, and $\mathbf{n}_i$ is a complex Gaussian noise vector at Rx i with covariance matrix $\sigma^2\mathbf{I}_M$.

At Rx i, the receive filter $\mathbf{U}_i \in \mathbb{C}^{d_i\times M}$ is post multiplied to $\mathbf{y}_i$ to obtain $\hat{\mathbf{s}}_i = \mathbf{U}_i^k\mathbf{y}_i^k$. Under these assumptions, the achievable sum-rate with a single user detection is given by

$$R_{sum} = \sum_{i=1}^{3}\sum_{m=1}^{d_i}\log_2\left(1+\frac{\frac{P}{d_i}\left|\left(\mathbf{u}_i^m\right)^H\mathbf{H}_{i,i}\mathbf{v}_i^m\right|^2}{A+B+\sigma^2}\right) \tag{2}$$

where $A=\sum_{l\neq m}\frac{P}{d_i}\left|\left(\mathbf{u}_i^m\right)^H H_{i,i}v_i^l\right|^2$ and $B=\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\left|\left(\mathbf{u}_i^m\right)^H H_{i,k}v_k^l\right|^2$ denote the intracell interference and the intercell interference respectively. We assume that each Rx has perfect CSIR but each Tx has only quantized CSIT. Even if imperfect CSIT is applied, the first term of denominator if logarithm function in (2) can be zero by using two stage decoding process (i.e. $\mathbf{U}_i = \mathbf{U}_{MU,i}\mathbf{U}_{SU,i}\mathbf{\Lambda}_i$, where $\mathbf{U}_{MU,i}$ denotes a multiuser decoder, $\mathbf{U}_{SU,i} = \left(\mathbf{U}_{MU,i}^H\mathbf{H}_{i,i}\mathbf{V}_i\right)^{-1}$ is a single user decoder to cancel inter-stream interference, and $\mathbf{\Lambda}_i$ is a power normalization diagonal matrix subjects to $\left\|\mathbf{u}_i^m\right\|^2 = 1$). If CSIT is perfect, under the IA constraint, i.e., $\left\|\mathbf{U}_j^H\mathbf{H}_{j,i}\mathbf{V}_i\right\|_F^2 = 0, i\neq j$ in [1], the second term of denominator of logarithm function in (2) will be zero. However, imperfect CSIT causes residual interference. According to precoding method or feedback quality, the amount of the resultant residual interference can be changed. In the next section, we describe two different IA precoding schemes.

3. Conventional Interference Alignment for Three-User MIMO Interference Channels

In this section, we briefly review the conventional IA scheme in [1] and discuss feedback methods for the conventional IA.

3.1 Review of Conventional IA

The interference signal space should have at most $M/2$ dimension and be linearly independent with the desired signal space. Thus, each precoder has to be satisfied with following three interference alignment constraints

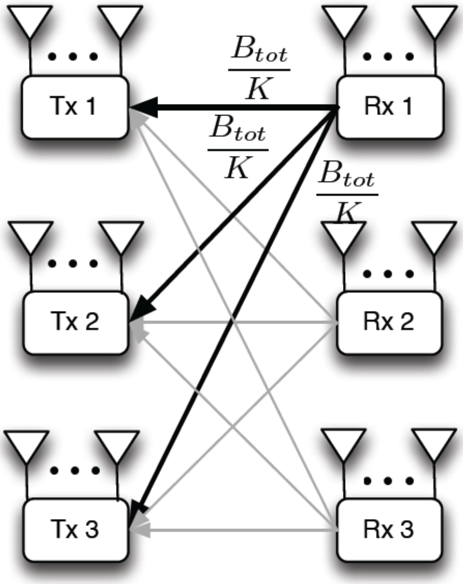

Fig. 2. Feedback method 1 of conventional IA - without backhaul

$$
\begin{aligned}
\operatorname{span}(\mathbf{H}_{3,1}\mathbf{V}_1) &= \operatorname{span}(\mathbf{H}_{3,2}\mathbf{V}_2) \\
\operatorname{span}(\mathbf{H}_{2,1}\mathbf{V}_1) &= \operatorname{span}(\mathbf{H}_{2,3}\mathbf{V}_3) \\
\operatorname{span}(\mathbf{H}_{1,2}\mathbf{V}_2) &= \operatorname{span}(\mathbf{H}_{1,3}\mathbf{V}_3)
\end{aligned} \tag{3}
$$

Then, these equations can be equivalently expressed as

$$
\begin{aligned}
\mathbf{V}_1 &= \mathrm{O}\left(\operatorname{eig}\left(\mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,2}\mathbf{H}_{1,2}^{-1}\mathbf{H}_{1,3}\mathbf{H}_{2,3}^{-1}\mathbf{H}_{2,1}\right)\right) \\
\mathbf{V}_2 &= \mathrm{O}\left(\mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,1}\mathbf{V}_1\right) \\
\mathbf{V}_3 &= \mathrm{O}\left(\mathbf{H}_{2,3}^{-1}\mathbf{H}_{2,1}\mathbf{V}_1\right)
\end{aligned} \tag{4}
$$

All precoders require $\mathbf{E} = \mathbf{H}_{3,1}^{-1}\mathbf{H}_{3,2}\mathbf{H}_{1,2}^{-1}\mathbf{H}_{1,3}\mathbf{H}_{2,3}^{-1}\mathbf{H}_{2,1}$ matrix which results in feeding back all interference links.

3.2 Feedback Methods of Conventional IA

The conventional IA requires all cross-link channel matrices since all precoders are related to each other in (3). We consider two feedback methods for conventional IA; with backhaul and without backhaul. **Fig. 2**. shows feedback method of conventional IA without a backhaul network. If there is no backhaul between Txs, Rx i should feed back two cross link channel matrices i.e. $\mathbf{H}_{i,j}, \forall j \neq i$ to the three Txs and each feedback link uses $B_{tot}/3$ bits where B_{tot} denotes the total feedback overhead constraint of each Rx. Therefore, the total number of feedback links is 9, and each link conveys two channel matrices with $B_{tot}/6$ bits quantization resolution. Thus the conventional IA may give rise to huge feedback overhead and quantization resolution degradation of each channel matrix.

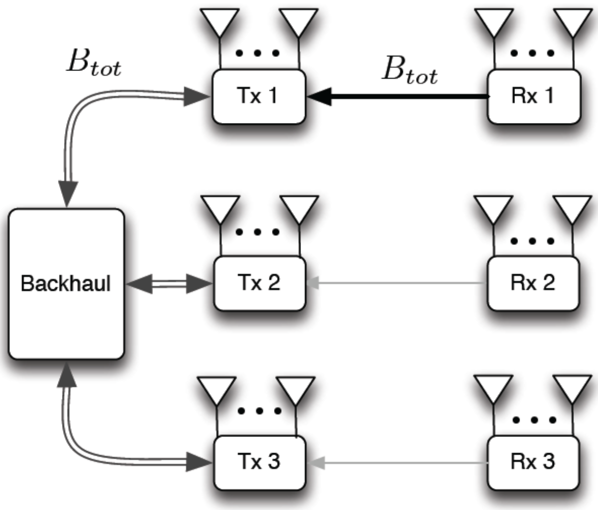

Fig. 3. Feedback method 2 of conventional IA - with backhaul

If there exists a backhaul network at Tx side, a better feedback could be possible. **Fig. 3** shows a feedback method of conventional IA with backhaul network. The *i*th Rx feeds back two channel matrices; $\mathbf{H}_{i,j}, \forall j \neq i$ to the desired Tx and each Tx shares it each other. Total number of feedback links reduces to 3 and each link conveys two channel matrices with $B_{tot}/2$ bits quantization resolution. However, there is still quantization resolution degradation of each channel matrix and the requirement of a high speed backhaul link. Note that we do not assume a broadcast nature of wireless feedback channels where a feedback information of a Rx-Tx pair could be hearable to the unintended Tx since recent standarization such as LTE/LTE-A considers only dedicated feedback channels, but assume that each feedback channel between a Tx and a Rx is dedicated for the Tx-Rx pair. If we assume the broadcast nature of the feedback links, the quantization resoulution of each cross-link channel matrix for the conventional IA regardless of the existence of the backhaul will be same as $B_{tot}/2$ bits.

4. A Single Feedback Based Interference Alignment for Three-User MIMO Interference Channels

In this section, we introduce a single feedback based IA scheme.

4.1 Proposed IA

In this section, we propose a single link feedback based IA without backhaul network requirement, where it uses all feedback bits at each receiver for only one CSI. Main idea of the proposed scheme begins with the reduction of the number of beams at an arbitrary Tx. For convenience, we assume that first Tx is called as a DoF sacrificing node, which transmits $M/2-\alpha$ streams. The transmit beams can be divide as follows,

$$\begin{aligned} \mathbf{V}_1 &= [\mathbf{V}_1^{A1}, \mathbf{0}_{M\times\alpha}] \\ \mathbf{V}_2 &= \left[\mathbf{V}_2^{B1}, \mathbf{V}_2^{B2}\right] \\ \mathbf{V}_3 &= \left[\mathbf{V}_3^{C1}, \mathbf{V}_3^{C2}\right] \end{aligned} \quad , \tag{5}$$

where $\mathbf{V}_1^{A1}$ is a $M \times (M/2-\alpha)$ precoding matrix at Tx 1, $\mathbf{0}_{M\times\alpha}$ is $M´\ a$ matrix with all zero element, $\mathbf{V}_2^{B1}$ and $\mathbf{V}_3^{C1}$ are $M \times (M/2-\alpha)$ matrices that have the first $M/2-\alpha$ selected beams, respectively, $\mathbf{V}_2^{B2}$ and $\mathbf{V}_3^{C2}$ are $M \times \alpha$ matrices that have the remaining a beams at Tx 2 and Tx 3.

Based on IA conditions in (3), the first $M/2-\alpha$ beams of Tx 2 and 3 should be aligned with beams of Tx 1, where there are $M/2+\alpha$ signal domain to cancel interference signals at Rx 1. Thus, the rank of the signal space spanned by all interference at Rx 1 should be less than to $M/2+\alpha$. The constraint of the proposed IA is shown as below,

$$\begin{aligned} &\mathrm{span}\left(\mathbf{H}_{3,1}\mathbf{V}_1^{A1}\right) = \mathrm{span}\left(\mathbf{H}_{3,2}\mathbf{V}_2^{B1}\right) \\ &\mathrm{span}\left(\mathbf{H}_{2,1}\mathbf{V}_1^{A1}\right) = \mathrm{span}\left(\mathbf{H}_{2,3}\mathbf{V}_3^{C1}\right) \\ &\mathrm{rank}\left(\mathrm{span}\left(\left[\mathbf{H}_{1,2}\mathbf{V}_2, \mathbf{H}_{1,3}\mathbf{V}_3\right]\right)\right) \le \frac{M}{2} + a. \end{aligned} \tag{6}$$

Now, we show the scheme that determines the precoder with single feedback from each Rx. If $\mathbf{V}_2^{B1}$ and $\mathbf{V}_3^{C1}$ have any relation at Rx 1, multiple feedbacks are required to determine $\mathbf{V}_2^{B1}$ and $\mathbf{V}_3^{C1}$ because based on the first and second equations in (6), $\mathbf{V}_2^{B1}$ and $\mathbf{V}_3^{C1}$ are already aligned with $\mathbf{V}_1^{A1}$ at Rx 3 and Rx 2, respectively. To allow single feedback for each receiver, $\mathbf{V}_2^{B2}$ and $\mathbf{V}_2^{C2}$ should be aligned with each other at Rx 1. To satisfy the third equation of (6), the design parameter α should be satisfied with following constraint,

$$2\left(\frac{M}{2}-\alpha\right)+\alpha \le \frac{M}{2}+\alpha \ , \tag{7}$$

where $2(M/2-\alpha)$ is the rank of the signal space spanned by $\mathbf{H}_{1,2}\mathbf{V}_2^{B1}$ and $\mathbf{H}_{1,3}\mathbf{V}_3^{C1}$, α is the rank of the signal space spanned by $\mathbf{H}_{1,2}\mathbf{V}_2^{B2}$ and $\mathbf{H}_{1,3}\mathbf{V}_3^{C2}$, thus $\alpha \ge M/4$. We choose $\alpha = M/4$ to minimize the DoF loss, therefore $5M/4$ DoF can be achieved in the proposed IA.

Based on above assumptions, (6) can be equivalently rewritten as

$$\begin{aligned} \mathbf{V}_2^{B1} &= \mathrm{O}\left(\mathbf{H}_{3,2}^{-1}\mathbf{H}_{3,1}\mathbf{V}_1^{A1}\right) \\ \mathbf{V}_3^{C1} &= \mathrm{O}\left(\mathbf{H}_{2,3}^{-1}\mathbf{H}_{2,1}\mathbf{V}_1^{A1}\right) \\ \mathbf{V}_3^{C2} &= \mathrm{O}\left(\mathbf{H}_{1,3}^{-1}\mathbf{H}_{1,2}\mathbf{V}_2^{B2}\right) \end{aligned} \tag{8}$$

In (8), the first and second equations require commonly $\mathbf{V}_1^{A1}$. If $\mathbf{V}_1^{A1}$ is prefixed and known Rx2 and Rx3, $\mathbf{V}_2^{B1}$ and $\mathbf{V}_3^{C1}$ can be determined by *single* feedback from Rx 2 and Rx 3, respectively. For the third equation in (8), if $\mathbf{V}_2^{B2}$ is prefixed, $\mathbf{V}_3^{C2}$ can be determined by feedback from Rx 1, so that each constraint in the proposed IA scheme requires only single feedback link with single CSI. Total number of feedback links of the proposed scheme is 3, and each feedback link conveys a $M \times M/4$ precoding matrix. The above procedure is summarized in **Table 1** and **Fig. 4** illustrates a conceptual diagram of the conventional IA and the proposed IA.

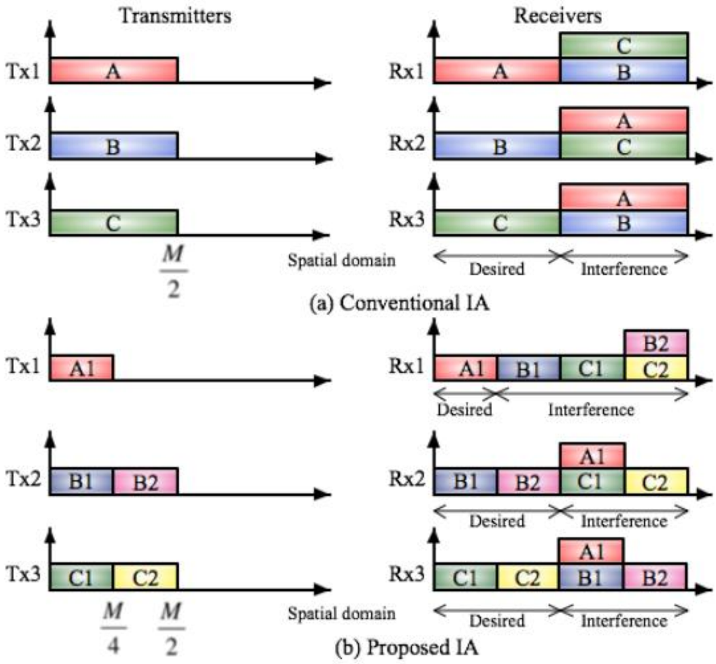

Fig. 4. Conceptual diagram of conventional IA and proposed IA

Table 1. Summary of the proposed IA

Set up: predetermine $\mathbf{V}_1^{A1}$ and $\mathbf{V}_2^{B2}$
Feedback information
Rx 1 → Tx 3 : $\mathbf{V}_3^{C2}$
Rx 2 → Tx 3 : $\mathbf{V}_3^{C1}$
Rx 3 → Tx 2 : $\mathbf{V}_2^{B1}$

5. Average Throughput Loss Analysis Due to Finite Rate Feedback

Taking approaches in [9], it can be readily shown that the proposed IA achieves the bound of the average throughput-loss (9), that is different from the bound in [9] in that it does not have any inter-stream interference, since we assume a two-stage decoding algorithm to cancel inter-stream interference. This upper bound will be used as a baseline for the following analysis,

$$\Delta R_{sum} = \mathrm{E}_{\mathbf{H}}\left(R_{perfecr-CSI}\right) - \mathrm{E}_{\mathbf{H}}\left(R_{quantized-CSI}\right)$$
$$\leq \sum_{i,m} \log_2\left(1+\frac{\mathrm{E}\left(\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\left|\left(\mathbf{u}_i^m\right)^H \mathbf{H}_{i,k}\mathbf{v}_k^l\right|^2\right)}{\sigma^2}\right) . \tag{9}$$

Based on (9), we will present the average throughput loss of the conventional IA and the proposed IA.

5.1 Residual Interference Analysis of the Conventional IA

For the conventional IA with limited feedback, Rx i has to feed back quantized channel matrices of all interference links, i.e. $\hat{\mathbf{H}}_{i,j}, \forall j \neq i$, to all transmitters. The transmit precoding and receive combining vectors are calculated to satisfy

$$\begin{aligned} &\left(\hat{\mathbf{u}}_i^m\right)^H \hat{\mathbf{H}}_{i,i}\hat{\mathbf{v}}_i^m = 0,\ \forall i, m\neq n \\ &\left(\hat{\mathbf{u}}_i^m\right)^H \hat{\mathbf{H}}_{i,j}\hat{\mathbf{v}}_j^m = 0, \forall j\neq i, \text{ and } \forall m,n . \\ &\left(\hat{\mathbf{u}}_i^m\right)^H \hat{\mathbf{H}}_{i,i}\hat{\mathbf{v}}_i^m > 0, \forall i,m \end{aligned} \tag{10}$$

However, since the channel matrix is a square matrix, the matrix quantization is impossible due to the constant chordal distance between square matrices [8]. Therefore, we consider a vectorized quantization model,

$$\hat{\mathbf{h}} = \arg\max_{\mathbf{w}\in\mathrm{F}}\left|\mathbf{w}^H \mathrm{vec}\left(\frac{\mathbf{H}}{\|\mathbf{H}\|_F}\right)\right|^2 , \tag{11}$$

where $\mathrm{vec}(\mathbf{H})$ denotes the vectorization of the matrix $\mathbf{H}$ formed by stacking the columns of $\mathbf{H}$ into a single column vector, and $\mathcal{F}$ denotes the vector codebook consisting of 2^B randomly generated $M^2\times 1$ vectors for given codebook size of B bits. Then, we can write

$$\mathbf{h} = \|\mathbf{H}\|_F\left(e^{j\varphi}\hat{\mathbf{h}} + \mathbf{e}\right) \tag{12}$$

where $e^{j\varphi} = \mathbf{h}^H\hat{\mathbf{h}} / \left|\mathbf{h}^H\hat{\mathbf{h}}\right|$ and $\mathbf{e}$ represent the error between $\mathbf{h}/\|\mathbf{H}\|_F$ and $e^{j\varphi}\hat{\mathbf{h}}$. Additionaly,

$$\mathbf{H} = \|\mathbf{H}\|_F\left(e^{j\varphi}\hat{\mathbf{H}} + \mathbf{E}\right) \tag{13}$$

The squared Frobenius norm of the error matrix $\mathbf{E}$ is bounded as

$$\mathrm{E}\left(\|\mathbf{H}\|_F^2\right) \leq 2 \cdot 2^{-\frac{B}{M^2-1}} \quad . \tag{14}$$

We omit the proof of (14). The readers can find the proof at lemma 1 in [10].

Equation (9) for the conventional IA with limited feedback can be further upper bounded by noticing that

$$\begin{aligned}
&\mathbb{E}_{\mathbf{H},\hat{\mathbf{H}}}\left(\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\left|\left(\hat{\mathbf{u}}_i^m\right)^H \mathbf{H}_{i,k}\hat{\mathbf{v}}_k^l\right|^2\right)\\
&= \mathbb{E}_{\mathbf{H},\hat{\mathbf{H}}}\left(\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\|\mathbf{H}_{i,k}\|_F^2\left|\left(\hat{\mathbf{u}}_i^m\right)^H \left(e^{j\varphi}\hat{\mathbf{H}}_{i,k}+\mathbf{E}_{i,k}\right)\hat{\mathbf{v}}_k^l\right|^2\right)\\
&= \mathbb{E}_{\mathbf{H},\hat{\mathbf{H}}}\left(\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\|\mathbf{H}_{i,k}\|_F^2\left|\left(\hat{\mathbf{u}}_i^m\right)^H \mathbf{H}_{i,k}\hat{\mathbf{v}}_k^l\right|^2\right)\\
&\leq \mathbb{E}_{\mathbf{H},\hat{\mathbf{H}}}\left(\sum_{k\neq i}\sum_{l=1}^{d_k}\frac{\mu P}{d_k}\|\mathbf{H}_{i,k}\|_F^2\|\mathbf{E}_{i,k}\|_F^2\right)\\
&\leq 2P\mu M^2 2^{-\frac{B}{M^2-1}}.
\end{aligned} \tag{15}$$

Theorem 1. *For the conventional IA, when the number of quantization bits for each channel matrix is* B*, the average throughput loss due to the quantized CSI is upper bounded by*

$$\Delta R_{conv} \leq \frac{3M}{2}\log_2\left(1+2\mu\rho M^2 2^{-\frac{B}{M^2-1}}\right) \tag{16}$$

where $\rho = P/\sigma^2$.

Proof: By plugging (15) to (9), we have the inequality (16). ■

Note that *B* denotes the codebook bit resolution for each channel matrix quantization, thus for the feedback method 1 and 2 of the conventional IA, each Rx requires 6*B* and 2*B* bits feedback overhead, respectively.

5.2 Residual Interference Analysis of the Proposed IA

In the proposed IA, when $\alpha = M/4$, each Rx feeds back $M' \left(M/4\right)$ beamformer matrix. The quantization codebook of each Rx is fixed beforehand and known to all transmitters. A quantization codebook $\mathcal{F}$ consists of 2^B matrices in $\mathbb{C}^{M\times\frac{M}{4}}$, where B is the codebook bit resolution for the quantization of each beamformer. The quantized beamformer $\hat{\mathbf{V}}_i$ is chosen from the codebook $\mathcal{F}$ according to the following rule

$$\hat{\mathbf{V}}_i = \arg\min_{\mathbf{W}\in\mathcal{F}} d^2\left(\mathbf{V}_i,\mathbf{W}\right) \tag{17}$$

where $d\left(\mathbf{V}_i,\mathbf{W}\right)$ is the *Chordal distance* defined as

$$d\left(\mathbf{V}_i,\mathbf{W}\right)=\sqrt{M-\mathrm{tr}\left(\mathbf{V}_i^H\mathbf{W}\mathbf{W}\mathbf{V}_i\right)}\,. \tag{18}$$

We use the following decoders,

$$\begin{aligned}
\mathbf{U}_1 &= \mathrm{null}\left(\left[\mathbf{H}_{1,2}\mathbf{V}_2^{B1},\mathbf{H}_{1,3}\mathbf{V}_3^{C1},\mathbf{H}_{1,2}\mathbf{V}_2^{B2}\right]\right)\\
\mathbf{U}_2 &= \mathrm{null}\left(\left[\mathbf{H}_{2,1}\mathbf{V}_1^{A1},\mathbf{H}_{2,3}\mathbf{V}_3^{C2}\right]\right)\\
\mathbf{U}_3 &= \mathrm{null}\left(\left[\mathbf{H}_{3,1}\mathbf{V}_1^{A1},\mathbf{H}_{3,2}\mathbf{V}_2^{B2}\right]\right)
\end{aligned} \tag{19}$$

From equation (9), the rate loss for the proposed IA with limited feedback can be upper bounded as,

$$\begin{aligned}
\Delta R_{prop} \le& \sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_1^m\right)^H\mathbf{H}_{1,3}\hat{\mathbf{V}}_3^{C2}\right\|_2^2\right)}{\sigma^2}\right)\\
&+\sum_{m=1}^{M/2}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_2^m\right)^H\mathbf{H}_{2,3}\hat{\mathbf{V}}_3^{C1}\right\|_2^2\right)}{\sigma^2}\right)\\
&+\sum_{m=1}^{M/2}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_3^m\right)^H\mathbf{H}_{2,3}\hat{\mathbf{V}}_2^{B1}\right\|_2^2\right)}{\sigma^2}\right)
\end{aligned} \tag{20}$$

in which the expectations in each logarithm function are same since the same dimensional quantization is conducted. Based on the quantization model, decoders (19), and the inequality (20), we have the following theorem.

Theorem 2. *For the proposed IA, when $\alpha = M/4$ and the number of feedback bits for each receiver is B, the average throughput loss due to the quantized CSI is upper bounded by*

$$\Delta R_{prop} \le \frac{5M}{4}\log_2\left(1+4\mu\rho D\right). \tag{21}$$

Proof: See Appendix A. ■

5.3 Analysis of Feedback Bits Scaling Law

In this subsection, we present how fast B must grow with SNR in order to maintain constant rate loss relative to a perfect CSI for the conventional IA and the proposed IA. For fair comparison, we consider rate gap between the conventional IA and the proposed IA with perfect CSIT:

$$\begin{aligned} R_g &= R_{conv-perfect-CSIT} - R_{prop-perfect-CSIT} \\ &\approx \frac{3M}{2}\log_2\left(1+\frac{2r}{M}\right) - \left(M\log_2\left(1+\frac{2r}{M}\right)+\frac{M}{4}\log_2\left(1+\frac{4r}{M}\right)\right) \\ &= \frac{M}{4}\log_2\left(\frac{(M+2r)^2}{M(M+4r)}\right). \end{aligned} \tag{22}$$

It is necessary to fix a common target rate. Basically the sum rate of the conventional IA with perfect CSI is greater than that of the proposed IA, we define new rate loss between the proposed IA with Perfect CSI and the conventional IA with limited feedback, it follows as

$$\Delta R'_{conv} = R_{\text{prop-perfect-CSIT}} - R_{\text{conv-quantized-CSIT}} = \Delta R_{conv} - R_g. \tag{23}$$

To maintain constant gap between common target rate and the conventional IA with limited feedback, we set the rate loss upper bound equal to the maximum allowable gap of $\log_2 b$:

$$\Delta R'_{conv} = \log_2 b. \tag{24}$$

Plugging (16) and (22) to (24), we have

$$B_{conv} = (M^2-1)\left[\log_2(\rho)+\log_2(2\mu M^2)-\log_2\left(2^{\frac{2}{3M}(\log_2 b+R_g)}-1\right)\right]. \tag{25}$$

In similar way, the required codebook bit resolution for the proposed IA can be found as,

$$B_{prop} = \frac{3M^2}{16}\left[\log_2(\rho)+\log_2(4\mu C')-\log_2\left(2^{\frac{4}{5M}(\log_2 b)}-1\right)\right], \tag{26}$$

where C' is given by $\frac{\Gamma\left(\frac{1}{T}\right)}{T}C^{-\frac{1}{T}}$. **Table 2** compares the required codebook bits resolution for each channel matrix (conventional IA) or precoding matrix (proposed IA) based on (25) and (26) to achieve the same target rate, i.e. 3dB away from the proposed IA with perfect CSIT when $M=4, \alpha = M/4$.

Table 2. Required codebook resolution (B) for Different IA Strategies when M =4

μ	**1**		**0.1**	
SNR	Conventional IA	Proposed IA	Conventional IA	Proposed IA
0 dB	75	13	25	3
5 dB	99	18	50	8
10 dB	124	23	75	13
15 dB	149	28	99	18

20 dB	174	33	124	23
25 dB	199	38	149	28
30 dB	224	43	174	33

We offer the following remarks :

1. Our analysis for the feedback bits scaling law is only based on B (codebook resolution) not B_{tot} (the number of feedback bits of each Rx). From the feedback topology perspective in [10], the feedback method 1 for the conventional IA can be considered as full feedback topology and the method 2 is considered as the star topology in terms of feedback overhead, and the number of required total feedback bits of each Rx for the feedback method 1 and 2 of the convention IA is 6B bits and 2B bits.
2. The growth rate of the required feedback resolution for the proposed IA as increasing SNR is much smaller than that of the conventional IA i.e. $3M^2/16 < M^2 - 1$.
3. To achieve the same target rate, the required number of feedback bits of the proposed IA is always smaller than that of the conventional IA. In other words, for same number of feedback bits for both IA schemes, the proposed IA outperforms the conventional IA in terms of sum rate performace.

6. Numerical Results

In this section, numerical results are provided to demonstrate results derived in the previous sections. First, we consider the sum rate comparison between the conventional IA and proposed IA when the codebook size is fixed as B. **Fig. 5** shows the sum rate performance when $M = 4$ and 8, and the codebook resolution of each CSI feedback is 2^6. The proposed IA achieves 12 bps/Hz gain for for $M = 4$ and 20bps/Hz gain for $M = 8$ at 30dB SNR, compared to the conventional IA with limited feedback. Moreover, when B=6, the total number of feedback bits of each Rx for the method 1 and 2 of the conventional IA are 36 and 12 bits, respectively, while the total number of feedback bits for the proposed IA is just 6 bits. Therfore, the proposed IA has better throughput performance as well as lower feedback overhead than those of the conventional IA.

To justify our derivation in the previous section, we run simulations with high resolutional codebook. The codebook size given by (25) and (26) can be very large and numerical simulation becomes a computationally complex task. Therefore we emulate the quantization process without having actual quantization using the numerial result generation method in [11]. We depict the sum rate for different codebook sizes in **Fig. 6**. The proposed scheme with limited feedback approaches to the sum rate with the perfect CSIT when $B = 40$, which is consistent with the analysis in the previous section. Given a low coupling factor $\mu = 0.1$, the sum rate comparison is shown in **Fig. 7**. For the lower compling factor, the performance gap between the proposed IA with limited feedback and the conventional IA with limited feedback is more reduced, but the proposed IA with limited feedback still outperforms the conventiona IA with limited feedback.

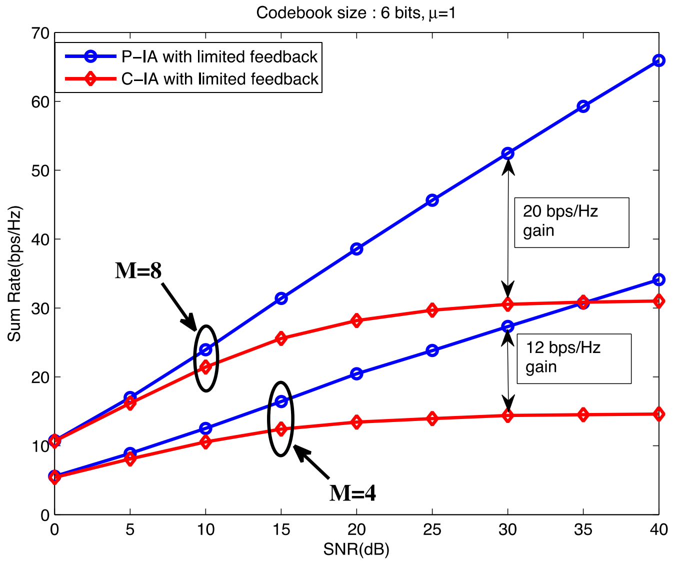

Fig. 5. Sum rate performance for different IA strategies when $B=6, \mu=1$

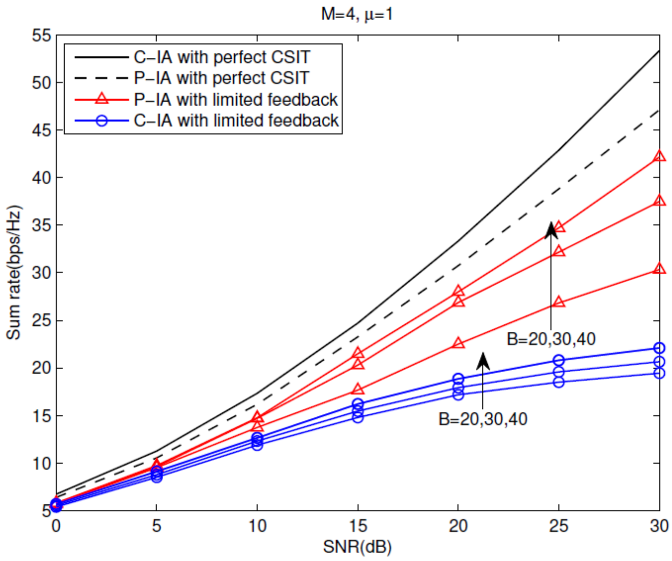

Fig. 6. Sum rate performance for different IA strategies when $M=4, \mu=1$

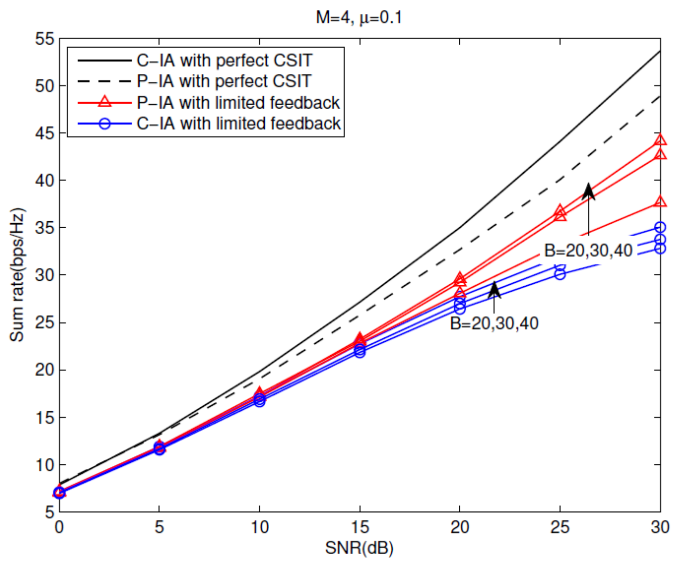

Fig. 7. Sum rate performance for different IA strategies when $M=4, \mu=0.1$

7. Conclusion

In this paper, we proposed a single feedback based IA for a three-user MIMO IFC with limited feedback, which reduces the number of data streams such that each precoder beam is aligned with only one beam of others. Compared to the conventional IA with limited feedback, the proposed IA consumes much less feedback overhead for achieving the same performance. With the same constraint of feedback bits, the proposed IA provides better performance than the conventional IA. (Even though the proposed IA does not achieve the optimal DoF, but the effectiveness of the proposed IA have been investigated theoretically and numerically.) We also investigated the effectiveness of the proposed IA for achieving the optimal DoF theoretically and numerically. However, to achieve the optimal DoF, a more efficient feedback algorithm or transmission scheme is required, which will be considered as a future research topic.

Appendix A: Proof of Theorem 2

Before proving the Theorem 2, we need following lemmas.

Lemma 1. *The quantized beamformer* $\hat{\mathbf{V}}_i$ *admits the following decomposition:*

$$\hat{\mathbf{V}}_i = \mathbf{V}_i \mathbf{A}_i \mathbf{B}_i + \mathbf{X}_i \mathbf{Y}_i \tag{A.1}$$

where $\mathbf{A}_i \in \mathbb{C}^{\frac{M}{4}\times\frac{M}{4}}$ *is a unitary matrix,* $\mathbf{Y}_i \in \mathbb{C}^{\frac{M}{4}\times\frac{M}{4}}$ *is upper triangular with positive diagonal elements and satisfied with* $\mathrm{tr}\left(\mathbf{Y}_i^H \mathbf{Y}_i\right) = d^2\left(\mathbf{V}_i, \hat{\mathbf{V}}_i\right)$, $\mathbf{B}_i \in \mathbb{C}^{\frac{M}{4}\times\frac{M}{4}}$ *is an upper triangular with positive diagonal elements and satisfies* $\mathbf{B}_i^H \mathbf{B}_i = \mathbf{I}_{M/4} - \mathbf{Y}_i^H \mathbf{Y}_i$, *and* $\mathbf{X}_i \in \mathbb{C}^{M\times\frac{M}{4}}$ *is an* *orthonormal basis for an isotropically distributed* $M/4$ *dimensional plane in the* $3M/4$ *dimensional left null space of* $\mathbf{V}_i$.

Proof: See Appendix A in [11]. ■

Also we need the following lemma,

Lemma 2. *The average distortion between* $\mathbf{V}$ and $\hat{\mathbf{V}}$ *at (17) where* $\mathbf{V}$ and $\hat{\mathbf{V}} \in \mathbb{C}^{M\times\frac{M}{4}}$ *is upper bounded as*

$$\mathrm{E}\left(d^2\left(\mathbf{V},\hat{\mathbf{V}}\right)\right)=\mathrm{E}\left(\min_{\mathbf{W}\in\mathbf{F}} d^2\left(\mathbf{V},\mathbf{W}\right)\right)\le D \tag{A.2}$$

where $D=\frac{\Gamma\left(\frac{1}{T}\right)}{T}(C)^{-\frac{1}{T}}2^{-\frac{B}{T}}+\frac{M}{4}\exp\left(-\left(2^B C\right)^{1-a}\right)$. *Here ,* $T=\frac{M}{4}\left(M-\frac{M}{4}\right)$, *and a is a real number between 0 and 1 chosen such that* $\left(C2^B\right)^{-\frac{a}{T}}\le 1$. C *is given by* $\frac{1}{T!}\prod_{i=1}^{\frac{M}{4}}\frac{(M-i)!}{\left(\frac{M}{4}-i\right)!}$.

Proof: See the section III. B in [11]. ■

$\hat{\mathbf{V}}_2^{B1}, \hat{\mathbf{V}}_3^{C1}$ and $\hat{\mathbf{V}}_3^{C2}$ are same dimensional matrices. For convenience, dropping the subscripts and superscripts, the rate loss for Rx 1 can be further upper bounded as,

$$\begin{aligned}
&\sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_1^m\right)^H\mathbf{H}_{1,3}\hat{\mathbf{V}}_3^{C2}\right\|_2^2\right)}{\sigma^2}\right)\\
&\overset{(a)}{=}\sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_i^m\right)^H\mathbf{H}\left(\mathbf{VAB}+\mathbf{XY}\right)\right\|_2^2\right)}{\sigma^2}\right)\\
&=\sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_i^m\right)^H\mathbf{H}\left(\mathbf{XY}\right)\right\|_2^2\right)}{\sigma^2}\right)\\
&\le\sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}\mathrm{E}\left(\left\|\left(\mathbf{u}_i^m\right)^H\mathbf{H}\right\|_2^2\left\|\mathbf{XY}\right\|_F^2\right)}{\sigma^2}\right)\\
&\overset{(b)}{=}\sum_{m=1}^{M/4}\log_2\left(1+\frac{\frac{4\mu P}{M}M\,\mathrm{E}\left(\mathrm{tr}\left(\mathbf{Y}^H\mathbf{Y}\right)\right)}{\sigma^2}\right)\\
&\overset{(c)}{\le}\frac{M}{4}\log_2\left(1+4\mu\rho D\right)
\end{aligned}\qquad\text{(A.3)}$$

where (a) follows from lemma 2, (b) holds since $\|\mathbf{XY}\|_F^2=\mathrm{tr}\left(\mathbf{Y}^H\mathbf{X}^H\mathbf{XY}\right)=\mathrm{tr}\left(\mathbf{Y}^H\mathbf{Y}\right)$ for the unitary matrix $\mathbf{X}$, and (c) follows from lemma 1 and 2. For $\hat{\mathbf{V}}_3^{C1}$ and $\hat{\mathbf{V}}_3^{C2}$, we can derive the upper bound as same manner.

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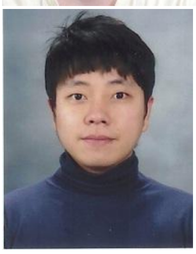

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