Journal of the Korea Institute for Structural Maintenance and Inspection
Vol. 17, No. 3, May 2013, pp.020-027
http://dx.doi.org/10.11112/jksmi.2013.17.3.020

pISSN 2234-6937
eISSN 2287-6979

복합신소재구조물의 고유진동수에 대한 하중크기의 영향

The Influence of the Loading Sizes on Natural Frequency of the Advanced Composite Material Structures

한 봉 구[1)*]
Han, Bong Koo

Abstract

Simple Iteration Method for calculating the natural frequency is presented in this paper. This method is simple but exact method of calculating natural frequencies corresponding to the modes of vibration of beams and tower structures with irregular cross sections and arbitrary boundary conditions. This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. Finite difference method is used for this purpose. The influence of the D_{22} stiffness on the natural frequency is rigorously investigated. In this paper, the influence of the loading sizes, different cross section on the natural frequency of vibration of some structural elements is presented. This method extends to two dimensional problems including advanced composite material structures.

Keywords : Simple Iteration Method, Natural frequencies, Loading sizes, Finite difference method, Advanced composite material

1. INTRODUCTION

The advanced composite materials can be used economically and efficiently in broad civil engineering applications when standards and processes for analysis, design, fabrication, construction and quality control are established. The problem of deteriorating infrastructures is very serious in our country.

The advanced composite materials can be effectively used for repairing such structures. Because of the advantages of these materials, such repair job can fulfill two purposes :

(1) Repairing of existing damage caused by corrosion, impact, earthquake, and others.
(2) Reinforcing the structure against the possible future situation which will require the increase of the load beyond the design parameters used for this structure.

Before making any decision on the repair, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a [0, 90, 0]r type specially orthotropic plate as a close approximation, assuming that the influences on the stiffness B_{16}, B_{26}, D_{16} and D_{26} are negligible. Many of the bridge and building floor systems, including the girders and cross beams, also work as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator

1) 정회원, 서울과학기술대학교 건설시스템디자인공학과 교수, 교신저자

* Corresponding author : bkhan@seoultech.ac.kr

• 본 논문에 대한 토의를 2013년 6월 30일까지 학회로 보내주시면 2013년 7월호에 토론결과를 게재하겠습니다.

in addition to their own masses. Analysis of such problems is usually very difficult.

Most of the design engineers for construction have bachelors level of academic background. Theories for advanced composite structures are too difficult for such engineers, so simpler but still accurate methods are necessary.

Most of the civil structures are large in sizes and the numbers of laminae are large, even though the thickness to length ratios are small enough to allow neglecting the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above work as specially orthotropic plates and simple formulas developed by the reference can be used (Kim 1995, Han & Kim, 2001, 2003, 2009).

Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms (M_x) on the relevant partial differential equations of equilibrium (Han & Kim, 2001). This paper presents the result of the study on the subject problem. Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved in seeking solutions (Ashton, Pagano, Whitney, 1970, Timoshenko, 1989).

2. METHOD OF ANALYSIS

2.1 Finite Difference Method

The equilibrium equation for the specially orthotropic plate is :

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = q(x,y) \qquad (1)$$

where $D_1 = D_{11}, D_2 = D_{22}, D_3 = D_{12} + 2D_{66}$

The assumptions needed for this equation are :

(1) The transverse shear deformation is neglected.

(2) Specially orthotropic layers are arranged so that no coupling terms exist, i.e. $B_{ij} = 0, (\ )_{16} = (\ )_{26} = 0$.

(3) No temperature or hygrothermal terms exist.

The purpose of this paper is to demonstrate, to practicing engineers, how to apply this equation to the slab systems made of plate girders and cross beams.

In case of an orthotropic plate with boundary conditions other than Navier or Levy solution type, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved in seeking such a solution. Finite difference method is used in this paper. The resulting linear algebraic equations can be used for any cases with minor modifications at the boundaries, and so on.

The problem of deteriorating infrastructures is very serious all over the world. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The basic concept of the Rayleigh method, the most popular analytical method for vibration analysis of a single degree of freedom system, is the principle of conservation of energy ; the energy in a free vibrating system must remain constant if no damping forces act to absorb it. In case of a beam, which has an infinite number of degree of freedom, it is necessary to assume a shape function in order to reduce the beam to a single degree of freedom system (Clough, 1995). The frequency of vibration can be found by equating the maximum strain energy developed during the motion to the maximum kinetic energy. This method, however, yields the solution either equal to or larger than the real one. Recall that Rayleigh's quotient $\geq$ 1 (Kim, 1995). For a complex beam, assuming a correct shape function is not possible. In such cases, the solution obtained is larger than the real one.

2.2 A proposal of the Simple Iteration Method

Structural engineers need to calculate the natural frequencies of such element, but obtaining exact solution to such problems is very difficult. Pretlove reported a method of analysis of beams with attached masses using the concept of effective mass. This method, however, is useful only for certain simple types of beams. Such problems can be easily solved by the method presented in this paper.

Simple Iteration Method for calculating the natural frequency is presented in this paper. it is a simple but exact method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures with irregular cross sections and attached mass/masses is presented. This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. Beginning with initially "guessed" mode shape, "exact" mode shape is obtained by the process similar to iteration. Recently, this method was extended to two dimensional problems including composite laminates, and has been applied to composite plates with various boundary conditions with/without shear deformation effects. This method is used for vibration analysis in this paper.

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections (maximum) of both the converged and the previous one should remain unchanged under the inertia force related with this natural frequency. Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function. Considering only the first mode as a start, the deflection shape of a structural member can be expressed as

$$w = W(x,y)F(t) = W(x,y)\sin\omega t \tag{2}$$

where

W : maximum amplitude

ω : circular frequency of vibration

t : time

By Newton's second law, the dynamic force of the vibrating mass, m, is

$$F = m\frac{\partial^2 w}{\partial t^2} \tag{3}$$

Substituting (2) into this,

$$F = -m\,(\omega)^2\,W(x,y)\,\sin\omega t \tag{4}$$

In this expression, ω and W are unknowns. In order to obtain the natural circular frequency ω, the following process is taken.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w(i,j)^{(1)} = W(i,j)^{(1)} \tag{5}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accel erating convergence. The dynamic force corresponding to this (maximum) amplitude is

$$F(i,j)^{(1)} = \mathrm{m}(i,j)\,[\omega(i,j)^{(1)}]^2\,\mathrm{w}(i,j)^{(1)} \tag{6}$$

The "new" deflection caused by this force is a function of F and can be expressed as

$$\mathrm{w}(i,j)^{(2)} = f[\mathrm{m}(k,l)[\omega(i,j)^{(1)}]^2\mathrm{w}(k,l)^{(1)}]$$
$$\sum^{k,l}\triangle(i,j,k,l)\mathrm{m}(k,l)[\omega(i,j)^{(1)}]^2\mathrm{w}(k,l)^{(1)} \tag{7}$$

where $\triangle$ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $\mathrm{w}(i,j)^{(1)}$

and $w(i,j)^{(2)}$, have to remain unchanged and the following condition has to be held :

$$w(i,j)^{(1)} / w(i,j)^{(2)} = 1 \tag{8}$$

From this equation, $w(i,j)^{(1)}$ at each point of (i,j) can be obtained, but they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e., $w(i,j)$ should be equal for all (i,j), this step is repeated until sufficient equal magnitude of $w(i,j)$ is obtained at all (i,j) points.

However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $w(i,j)^{(2)}$ where the deflection is the maximum. For the second cycle, $w(i,j)^{(3)}$ in the absolute numerics of $w(i,j)^{(2)}$ can be used for convenience.

$$w(i,j)^{(3)} = f[m(i,j)[\omega(i,j)^{(2)}]^2 w(i,j)^{(2)}] \tag{9}$$

In case of a structural member with irregular section including composite one, and non-uniformly distributed mass, regardless of the boundary conditions, it is convenient to consider the member as divided by finite number of elements. The accuracy of the result is proportional to the accuracy of the deflection calculation.

For practical design purposes, it is desirable to simplify the vibration analysis procedure. One of the methods is to neglect the weight of the structural element. The effect of neglecting the weight (thus mass) of the plate is studied as follow. If a weightless plate is acted upon by a concentrated load, $P = N \cdot q \cdot a \cdot b$, the critical circular frequency of this plate is

$$w_n = \sqrt{\frac{g}{\delta_{st}}} \tag{10}$$

where δ_{st} is the static deflection.

Similar result can be obtained by the use of Eqs. (7) and (8).

$$[\omega(i,j)]^2 = \frac{1}{[\triangle(i,j,i,j) \cdot \frac{P(i,j)}{g}]} \tag{11}$$

where,

$$P(i,j) = N \cdot q \cdot a \cdot b \tag{12}$$

In case of the plate with more than one concentrated loads,

$$[\omega(i,j)]^2 = \frac{1}{[\sum^{k.l} \triangle(i,j,k,l) \cdot \frac{P(k,l)}{g}]} \tag{13}$$

If we consider the mass of the plate as well as the concentrated loads,

$$\begin{aligned} & w(i,j)^{(1)} = w(i,j)^{(2)} \\ & = \{\sum^{k.l} \triangle(i,j,k,l) \cdot m(k,l) \cdot w(k,l)^{(1)} \\ & + \sum^{m.n} \triangle(i,j,m,n) \cdot \frac{P(m,n)}{g} \cdot w(m,n)^{(1)}\} \times [\omega(i,j)^{(1)}]^2 \end{aligned} \tag{14}$$

where (m,n) is the location of the concentrated loads. The effect of neglecting the weight of the plate can be found by simply comparing Eq. (13) and Eq. (14).

Since no reliable analytical method is available for the subject problem, F.D.M. is applied to the governing equation of the special orthotropic plates.

The number of the pivotal points required in the case of the order of error $\triangle^2$, where $\triangle$ is the mesh size, is five for the central differences of the fourth order single derivative terms. This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, Mx, and My, are used instead of Eq.(1) for the bending of the specially orthotropic plate.

$$D_{11}\frac{\partial^2 Mx}{\partial x^2}+4D_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^2 My}{\partial y^2}$$
$$=-q(x,y)+kw(x,y) \quad (15)$$

$$M_x=-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2} \quad (16)$$

$$M_y=-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2} \quad (17)$$

If F.D.M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by author (Kim, 1965, 1967) is very efficient to solve such equations. In order to confirm the accuracy of the Simple Iteration Method, [A/B/A]r type laminate with aspect ratio of a/b=1m/1m=1 is considered. The material properties are :

$E_1=67.36\ GPa,\ E_2=8.12\ GPa,$
$G_{12}=3.0217\ GPa,$
$\nu_{12}=0.272,\ \nu_{21}=0.0328,$

The thickness of a ply is 0.005m. As the r increases, B_{16}, B_{26}, D_{16}, and D_{26} decrease and the equations for special orthotropic plates can be used. For simplicity, it is assumed that $A=0^o$, $B=90^o$ and r=1. Then D_{22}=18492 N-m.

Since one of the few efficient analytical solutions of the special orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, Simple Iteration Method is used to solve this problem and the result is compared with the Navier solution.

The mesh size is △x=a/10=0.1m, △y=b/10=0.1m. The deflection at (x, y), under the uniform load of 100N/m^2, the origin of the coordinates being at the corner of the plate, is obtained, and the ratio of the Navier solution to the Simple Iteration Method solution is 1.005~1.00028.

3. NUMERICAL EXAMPLES

3.1 Calculation of Simple Iteration Method

As a calculation of the Simple Iteration Method, a simply supported beam with uniform flexural rigidity, EI, is considered as shown in Fig. 1.

The length of the beam is 10 m. The weight of the beam is assumed as 500kg/m. the weight acts as the mass when the beam vibrates and is treated as concentrated loads at five equally spaced points. Since a beam is one-dimensional, one subscript, i, is used. The set of influence coefficients, $\triangle_{i,j}$, where i is the point under consideration and j is the loading point (unit load), is given in Table 1.

The initially guessed maximum amplitude, $W(i)^{(1)}$, can be arbitrary and the following values are given.

$W(1)^{(1)}=W(5)^{(1)}=40$
$W(2)^{(1)}=W(4)^{(1)}=80$
$W(3)^{(1)}=100$

These values are substituted into equations 4.77 and 4.78, and from equation 4.79, the following result is

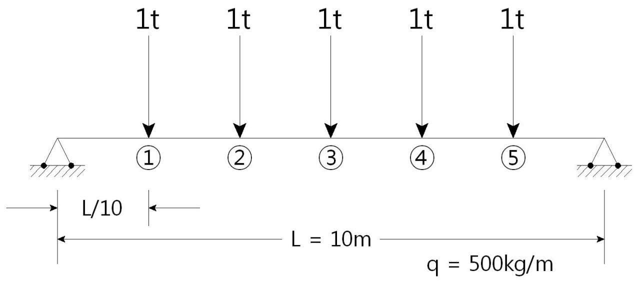

Fig. 1 Simply supported beam with uniform flexural rigidity

Table 1 Influence coefficients, $EI\triangle_{i,j}$, for simple beam

I \ j	1	2	3	4	5
1	2.7	5.8	6.2	4.5	1.6
2	5.8	14.7	16.5	12.3	4.5
3	6.2	16.5	20.8	16.5	6.2
4	4.5	12.3	16.5	14.7	5.8
5	1.6	4.5	6.2	5.8	2.7

obtained:

$$\mathrm{w}(1)^{(2)} = 1616\mathrm{m}(1)[\omega(1)^{(1)}]^2/\mathrm{EI}$$
$$\mathrm{w}(2)^{(2)} = 4222\mathrm{m}(2)[\omega(2)^{(1)}]^2/\mathrm{EI}$$
$$\mathrm{w}(3)^{(2)} = 5216\mathrm{m}(3)[\omega(3)^{(1)}]^2/\mathrm{EI}$$

Letting $\mathrm{w}(\mathrm{i})^{(1)}/\mathrm{w}(\mathrm{i})^{(2)} = 1$, we get

$$w(1)^{(1)} = 0.1573A(1)$$
$$w(2)^{(1)} = 0.1376A(1)$$
$$w(3)^{(1)} = 0.1385A(1)$$

where $A(i) = \sqrt{\dfrac{EI}{m(i)}}$

Since all $w(i)^{(1)}s$ should be equal at all i points, this process has to be repeated. For the second cycle, only the relative magnitude of the amplitude is necessary, and $W(i)^{(2)}s$ are assigned as follows:

$$W(1)^{(2)} = W(5)^{(2)} = 16.2$$
$$W(2)^{(2)} = W(4)^{(2)} = 42.2$$
$$W(3)^{(2)} = 52.2$$

The same influence coefficient for the first cycle is repeatedly used, and the 'new' amplitude, $\mathrm{w}(\mathrm{i})^{(3)}$, is obtained as

$$\mathrm{w}(1)^{(3)} = 827.76[\omega(1)^{(2)}]^2/[\mathrm{A}(1)]^2$$
$$\mathrm{w}(2)^{(3)} = 2167.08[\omega(2)^{(2)}]^2/[\mathrm{A}(2)]^2$$
$$\mathrm{w}(3)^{(3)} = 2678.64[\omega(3)^{(2)}]^2/[\mathrm{A}(3)]^2$$

From $\mathrm{w}(\mathrm{i})^{(2)}/\mathrm{w}(\mathrm{i})^{(3)} = 1$,

$$w(1)^{(3)} = 0.1397A(1)$$
$$w(2)^{(3)} = 0.1396A(2)$$
$$w(3)^{(3)} = 0.1395A(3)$$

One more process is executed in order to obtain the better result as follows.

$$w(1)^{(3)} = 0.139575A(1)$$
$$w(2)^{(3)} = 0.139575A(2)$$
$$w(3)^{(3)} = 0.139575A(3)$$

Note that all A are the same, and $w = 0.139575A$

The result obtained by the 'exact' theory is $w = 0.139575A$. It is noted that the result of the first cycle is good enough for engineering purposes. If w at the point of the maximum deflection, $w(3)^{(1)}$, is considered, it is only 0.77% away from the "exact" result.

In the case of a variable cross section, including materials, $A(i) = E(i)(I_i)/m(i)$ should be used. Influence coefficients can be found with relative ease in any case.

Simple Iteration Method can be applied to any structural element with variable stiffnesses and loadings, and with any boundary conditions, including deep beams and thick plates for which an analytical solution is difficult to obtain. The accuracy of the result is proportional to that of deflection calculation. Calculation of the deflection influence surface is the fundamental first step in any structural analysis and design. Attention should be given to the fact that this method utilizes the deflection influence surfaces which are used at the beginning of the analysis and design.

3.2 The influence of loading sizes on natural frequencies

Simple Iteration Method is used to study the influence of loading sizes and moment of inertia on natural frequencies of simply supported beams, fixed beams and cantilever beams, and tower type structures. For a simply supported uniform beams with different loading sizes as shown in Fig. 2.

For a simply supported beams with different loading sizes as shown in Fig. 3.

For practical design purposes, it is desirable to simplify the vibration analysis procedure. One of the methods is to neglect the weight of the beam. The effect of neglecting the weight (thus mass) of the beam is studied as follows.

If a weightless beam is acted upon by a load P, the critical circular frequency of this beam is $\omega = \sqrt{g/\delta_{st}}$,

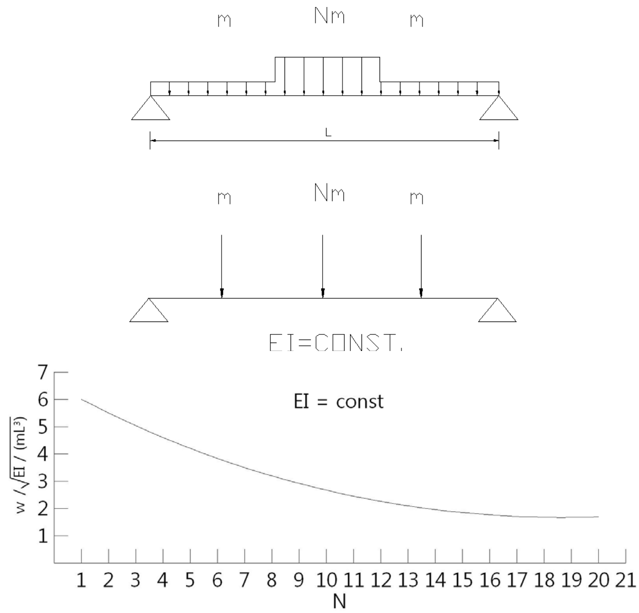

Fig. 2 Simply supported uniform beam with different loading sizes

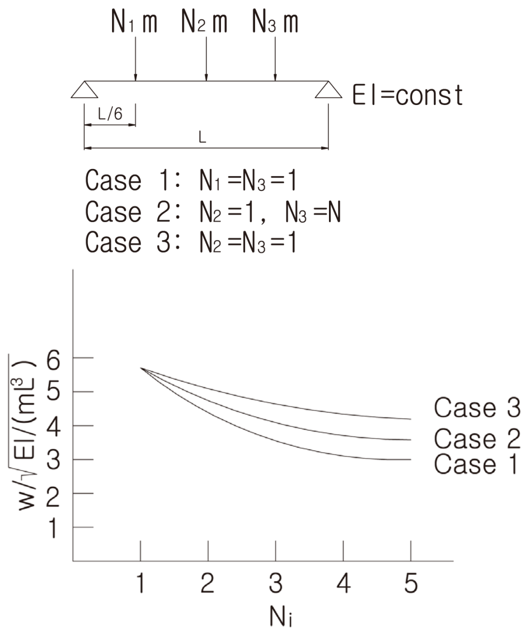

Fig. 3 Simply supported beam with different loading sizes

where δ_{st} is the static deflection.

For a massless simply supported beam with uniform EI throughout the length L, acted upon by a load P, at the center $\omega = \sqrt{48EI_g/PL^3}$.

For a fixed uniform beam with different loading sizes as shown in Fig. 4.

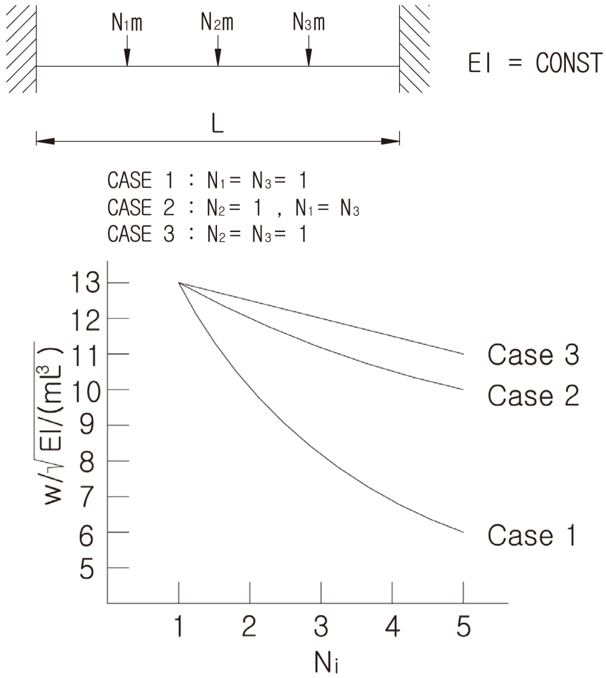

Fig. 4 Fixed uniform beam with different loading sizes

Replacing P by Nm, N is gradually increased and ω are calculated for each of N. The results are compared with those from previous study (Fig. 2~Fig. 3).

It is noted that N does not directly indicate the ratio of the weight of the concentrated load to the total weight of a uniform load. For example, N=10 indicates that the ratio is (10 - 1)/3 = 3, i.e. the weight of P is three times the total weight of the beam.

Thus, in the case of a uniform simple beam with a concentrated load at the center of the span, the weight of which is three times that of the beam, the critical frequency difference between the correct value, obtained by considering the weight of the beam, and the approximate one, obtained by neglecting this weight, is 2.30%. In the case of a fixed beam with similar condition, the difference is 0.68%.

4. CONCLUSION

This paper aims to show Simple Iteration Method for calculating the natural frequency. A simple but exact method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures with irregular cross sections and attached mass/masses is presented.

This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. This Simple Iteration Method extends to two dimensional problems including composite laminated plate.

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found and from this force, the resulting deflection can be obtained. For practical design purposes, it is desirable to simplify the vibration analysis procedure. In this paper, the relation between the applied loading sizes and the natural frequency of vibration of some structural elements is presented.

Acknowledgement

This study was partially supported by Seoul National University of Science and Technology.

References

1. Ashton, J. E., "Anisotropic Plate Analysis- Boundary Condition", J. of Composite Materials, 1970, pp.162-171.
2. Clough, R. W. and Penzien, J., "Dynamics of Structures", McGraw-Hill, Inc., N. Y., 1995.
3. Han, B. K., Kim, D. H., "Size Effects in the Failure of Simple Supported Sandwich Slab Bridges", Journal of the Korea Institute for Structural Maintenance and Inspection, vol. 14, No. 3, 2010, pp.84-92. (in Korean)
4. Han, B. K., Kim, D. H., "Analysis of Prestressed Concrete Slab Bridge by Beam Theory", Journal of the Korea Institute for Structural Maintenance and Inspection, vol. 7, No. 2, 2003, pp.115-124. (in Korean)
5. Han, B. K., Kim, D. H., "Analysis of Reinforced Concrete Slab Bridge by Simple Method", Journal of the Korea Institute for Structural Maintenance and Inspection, vol. 7, No. 4, 2003, pp.110-118. (in Korean)
6. Han, B. K., Kim, D. H., "Simple Method of Vibration Analysis of Elastic Supported Three Span Reinforced Concrete Bridge", Journal of The Korea Society for Composite Materials, vol. 17, No. 3, 2004, pp.23-28.
7. Kim, D. H., "Composite Structures for Civil and Architectural Engineering", E&FN SPON, Chapman & Hall, London, 1995.
8. Kim, D. H., "Simple Method of Analysis for Preliminary Design of Certain Composite Laminated Primary for Civil Construction II", Journal of Material Technology, Elsevier, London, 1993.
9. Kim, D. H., Han, B. K., Lee, J. H., Hong, C. W., "Simple Methods of Vibration Analysis of Three Span Continuous Reinforced Concrete Bridge with Elastic Intermediate Supports", Proceeding of the Advances in Structural Engineering and Mechanics, Seoul, vol. 2, 1999, pp.1279-1284.
10. Kim. D. H., "A Method of Vibration Analysis of Irreqularly Shaped Structural Members", Proceedings, International Symposium on Engineering Problems in Creating Coastal Industrial Sites, Seoul, Korea, October, 1974.
11. Pagano, N. J., "Exact Solution for Rectangular Bidirectional Composites and Sandwich Plates", Journal of Composites Materials, vol. 4, No. 1, 1970, pp.20-34.
12. Stephen P. Timoshenko and S. Woinowsky-krieger, "Theory of Plates and Shells, Second Edition", Mcgraw Hill Book Co, 1989.
13. Whitney, J. M. and Leissa, A. W., "Analysis of a Simply Supported Laminated Anisotropic Rectangular Plate", J. of AIAA, vol. 8, No. 1, 1970, pp.28-33.

Received : 01/18/2013
Revised : 03/18/2013
Accepted : 03/21/2013

요 지

본 논문에서는 고유진동수를 구하기 Simple Iteration Method을 제시하였다. 이 방법은 임의의 단면과 지점을 갖고 임의의 하중을 받는 보나 탑의 진동모드와 관련된 고유진동수를 간편하면서도 정확하게 계산할 수 있는 획기적인 방법이다. 이 방법에는 공진상태에서 관성력에 기인한 부재의 처짐 모드를 구하게 된다. 진동해석을 위하여 처짐의 영향을 고려한 다양한 방법이 검토되었다. 이러한 목적으로 본 논문에서는 유한차분법을 사용하였다. 고유진동수에 대한 D_{22} 휨강성의 영향을 철저하게 검토하였다. 본 논문에서는 구조 요소의 하중 분포 또는 상이한 단면에 따른 고유진동수에 대한 영향을 연구하였으며 그 결과를 제시하였다. 이 방법은 첨단복합재료를 포함한 2차원 문제에도 적용할 수 있다.

핵심 용어 : Simple Iteration Method, 고유진동수, 하중크기, 유한차분법, 복합신소재