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An Empirical Study on Explosive Volatility Test with Possibly Nonstationary GARCH(1, 1) Models

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Abstract

In this paper, we implement an empirical study to test whether the time series of daily returns in stock and Won/USD exchange markets is strictly stationary or explosive. The results indicate that only a few series show nonstationary volatility when dramatic events erupted; in addition, this nonstationary behavior occurs more often in the Won/USD exchange market than in the stock market.

Keywords: GARCH model, Lyapunov exponent, strict stationarity testing.

1. Introduction

Time varying volatility is an important feature of financial time series. To capture this phenomenon, Engle (1982) introduced the idea of conditional heteroscedasticity. Since then, generalized autoregressive conditional heteroscedasticity(GARCH) models have drawn significant attention from researchers. The stationarity is a basic assumption for GARCH models; however, the stationarity test has not been intensively studied yet: see Francq and Zakoïan (2012). Some articles consider weakly stationary GARCH models (Bollerslev, 1986; Weiss, 1986; Pantula, 1988); however, in practice, the parameter estimation result often appears to violate the weak stationarity condition since the underlying process turns out to be integrated GARCH(IGARCH). It is well known that the IGARCH process is not weakly stationary but strictly stationary: see Nelson (1990). To cover the IGARCH case, Lee and Hansen (1994) and Lumsdaine (1996) derived asymptotic results for GARCH(1, 1) models without the weak stationarity assumption; in addition, Bougerol and Picard (1992) verified a necessary and sufficient condition for GARCH(p, q) process to be strictly stationary by applying the theory of products of random matrices and the top Lyapunov exponent. It is noteworthy that the region of parameters (to allow the strict stationarity) is larger than that for the weak stationarity. Berkes *et al.* (2003), Francq and Zakoïan (2004), and Straumann and Mikosch (2006) established the asymptotic properties of quasi-maximum likelihood estimators(QMLE) for GARCH-type models under the strict stationarity assumption: see also Li *et al.* (2002).

According to Nelson (1990), the conditional variance of GARCH(1, 1) process explodes to the infinity if the process is not strictly stationary and the intercept is positive. Recently, Jensen and Rahbek (2004) and Francq and Zakoïan (2012) studied the asymptotic properties of Gaussian-QMLE for nonstationary GARCH(1, 1) processes. Francq and Zakoïan (2012) also proposed a strict stationarity

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test based on the estimator of the Lyapunov exponent and showed that their test has a robust feature against the model misspecification.

In this paper, we analyze 11 series of daily asset returns ranging from 1996 to 2012. We perform a strict stationarity test in each year to investigate how often the time series show nonstationary behavior such as explosive volatility. In Section 2, we review the conditions for the strict stationarity of GARCH processes. Further, the asymptotic theory for the nonstationary GARCH models and the strict stationarity test are summarized. In Section 3 provides, an empirical study is provided.

2. Strict Stationarity of GARCH Processes

This section reviews the strict stationarity of GARCH processes and introduces a recent method for the stationarity test.

2.1. Lyapunov exponent

The GARCH(1, 1) model is defined by

$$\varepsilon_t = \sqrt{h_t}\eta_t, \quad h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{2.1}$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\{\eta_t\}$ is a sequence of i.i.d. random variables with $E\eta_t = 0$ and $E\eta_t^2 = 1$. The Equation (2.1) can be interpreted as the one defining a first order homogeneous Markov process $\{(\varepsilon_t, h_t) : t = 0, 1, 2, \ldots\}$, of which the state space is $\mathbb{R} \times (0, \infty)$ and the initial state is (ε_0, h_0). Bollerslev (1986) verified that the Equation (2.1) admits a weakly stationary solution if and only if $\alpha + \beta < 1$. Theorem 2 of Nelson (1990) shows that the Equation (2.1) has a unique strictly stationary solution if and only if

$$E\left[\log\left(\beta + \alpha\eta_t^2\right)\right] < 0. \tag{2.2}$$

It is noteworthy that Theorem 2 of Nelson (1990) does not require the restrictions $E\eta_t = 0$ and $E\eta_t^2 = 1$. It is only assumed that $E[\log(\beta + \alpha\eta_t^2)]$ exists, for which $E\log^+ \eta_t^2 < \infty$ is sufficient where $\log^+ x = \max\{\log x, 0\}$: see Lemma 2.2 of Straumann and Mikosch (2006).

Let us recall the linkage of (2.1) and (2.2). Model (2.1) involves the following stochastic recurrence equation(SRE): for $t \geq 1$,

$$h_t = \omega + \left(\beta + \alpha\eta_{t-1}^2\right) h_{t-1}. \tag{2.3}$$

Subsequent substitution yields that

$$h_t = \omega\left\{1 + \sum_{k=1}^{t-1}\left(\beta + \alpha\eta_{t-1}^2\right)\cdots\left(\beta + \alpha\eta_{t-k}^2\right)\right\} + \left(\beta + \alpha\eta_{t-1}^2\right)\cdots\left(\beta + \alpha\eta_0^2\right) h_0.$$

By the law of large numbers, the condition (2.2) implies that there exists $\delta > 0$ such that $\prod_{i=1}^{k}(\beta + \alpha\eta_{t-i}^2) = O(e^{-\delta k})$ with probability one, so $\sum_{k=1}^{\infty}\prod_{i=1}^{k}(\beta + \alpha\eta_{t-i}^2)$ converges a.s. Theorem 2 of Nelson (1990) verified that (2.2) is a necessary and sufficient condition for the convergence of the series. Then, we can see that $h_{t,\infty} := \omega\left\{1 + \sum_{k=1}^{\infty}(\beta + \alpha\eta_{t-1}^2)\cdots(\beta + \alpha\eta_{t-k}^2)\right\}$ is well-defined and is a strictly stationary solution to the SRE (2.3). Assuming $h_0 = h_{0,\infty}$, the bivariate Markov process $\{(\varepsilon_t, h_t) : t = 0, 1, 2, \ldots\}$ is strictly stationary, so its time domain can be extended to $\mathbb{Z}$ by Kolmogorov's extension theorem; see Billingsley (1995). Further, the stationary solution to the Equation (2.1) is explicitly expressed as $\varepsilon_t = h_{t,\infty}^{1/2}\eta_t$ and $\{(\varepsilon_t, h_t) : t \in \mathbb{Z}\}$ is ergodic by Theorem 36.4 of Billingsley (1995).

Bougerol and Picard (1992) studied the strict stationarity of the GARCH(p, q) model:

$$\varepsilon_t = \sqrt{h_t}\eta_t, \quad h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{2.4}$$

where $\omega > 0$, $\alpha_i \geq 0$ $(1 \leq i \leq q)$, $\beta_j \geq 0$ $(1 \leq j \leq p)$, and $\{\eta_t\}$ is defined as in Model (2.1). Following Bougerol and Picard (1992), let $Y_t = (h_t, \ldots, h_{t-p+1}, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q+1}^2)^T$. Similarly to (2.3), the GARCH(p, q) equation involves the following vector SRE:

$$Y_t = b + A_{t-1} Y_{t-1}, \tag{2.5}$$

where $b = (\omega, 0, \ldots, 0)^T \in \mathbb{R}^{p+q-1}$ and A_t is a $(p + q - 1) \times (p + q - 1)$ random matrix written by

$$A_t = \begin{bmatrix} \tau_t & \beta_p & \alpha & \alpha_q \\ I_{p-1} & 0 & 0 & 0 \\ \xi_t & 0 & 0 & 0 \\ 0 & 0 & I_{q-2} & 0 \end{bmatrix},$$

with $\tau_t = (\beta_1 + \alpha_1\eta_t^2, \beta_2, \ldots, \beta_{p-1}) \in \mathbb{R}^{p-1}$, $\alpha = (\alpha_2, \ldots, \alpha_{q-1}) \in \mathbb{R}^{q-2}$, and $\xi_t = (\eta_t^2, 0, \ldots, 0) \in \mathbb{R}^{p-1}$. I_d denotes the identity matrix of size d. Let $\|A\|$ be any norm of a square matrix A. As in the case of GARCH$(1, 1)$, the SRE (2.5) has a strictly stationary solution if and only if $\sum_{k=1}^{\infty} \prod_{i=1}^{k} A_{t-i}$ converges a.s. It is well known that the series converges if and only if

$$0 > \gamma_L := \inf_{t\in\mathbb{N}} \frac{1}{t} E[\log \|A_t A_{t-1} \cdots A_1\|] = \lim_{t\to\infty} \frac{1}{t} \log \|A_t A_{t-1} \cdots A_1\| \quad \text{a.s.}, \tag{2.6}$$

where γ_L is called the top Lyapunov exponent of a sequence of i.i.d. random matrices $\{A_t : t \in \mathbb{Z}\}$ and the second equality follows from Kingman (1973). Hence, the condition (2.6) is a necessary and sufficient condition for the existence of a strictly stationary solution to the GARCH(p, q) Equation (2.4). For instance, if we set $p = 1$, $q = 2$, and $\alpha_2 = 0$, then

$$\frac{1}{t} \log \|A_t A_{t-1} \cdots A_1\| = \frac{1}{t} \sum_{i=1}^{t-1} \log\left(\beta_1 + \alpha_1 \eta_{t-i}^2\right) + \frac{1}{t} \log \|A_t\|.$$

Thus, the Lyapunov exponent, γ_L, associated with GARCH$(1, 1)$ model is $E[\log(\beta_1 + \alpha_1\eta_t^2)]$. An alternative vector representation instead of (2.5) can be found in Francq and Zakoïan (2004). Further, the stationarity conditions for various GARCH-type models can be stated by using the concept of Lyapunov exponent; see Lee and Lee (2012) and Medeiros and Veiga (2009).

Let us turn our attention back to GARCH$(1, 1)$ model (2.1). If the condition (2.2) is violated, that is, $\gamma_L \geq 0$, then the GARCH process becomes nonstationary and $h_t \to \infty$ a.s., as shown by Nelson (1990). To check the stationarity assumption, we need to test whether $\gamma_L < 0$ or not. This test will expose whether the volatility of a given heteroscedastic series is explosive or stationary. Construction of the test obviously requires the inference of $\gamma_L = E[\log(\beta + \alpha\eta_t^2)]$ for nonstationary GARCH$(1, 1)$ process, which was recently studied by Francq and Zakoïan (2012).

2.2. Estimation in possibly nonstationary GARCH$(1, 1)$ models

Let $\{\varepsilon_t : 1 \leq t \leq n\}$ be a sample from the GARCH$(1, 1)$ model (2.1) with the true parameter $\theta_0 = (\omega_0, \alpha_0, \beta_0)^T$. The Gaussian-QMLE is defined as

$$\hat{\theta}_n = \left(\hat{\omega}_n, \hat{\alpha}_n, \hat{\beta}_n\right)^T = \underset{\theta\in\Theta}{\operatorname{argmin}} \frac{1}{n} \sum_{t=1}^{n} \left\{ \frac{\varepsilon_t^2}{\sigma_t^2(\theta)} + \log \sigma_t^2(\theta) \right\}, \tag{2.7}$$

where $\sigma_t^2(\theta) = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2(\theta)$ for $t \geq 1$ with some initial values for ε_0^2 and $\sigma_0^2(\theta)$. Since we consider both the stationary and nonstationary cases, we allow the parameter space to be Θ be any compact subset of $(0,\infty) \times [0,\infty)^2$. Denote by $\gamma_0 = E[\log(\beta_0 + \alpha_0\eta_1^2)]$ the Lyapunov exponent associated with the GARCH(1, 1) model. In the case of $\gamma_0 < 0$, Berkes *et al.* (2003) and Francq and Zakoïan (2004) studied the $\sqrt{n}$-consistency and asymptotic normality of the QMLE, $\{\hat{\theta}_n\}$ under mild conditions. In the nonstationary case of $\gamma_0 \geq 0$, Francq and Zakoïan (2012) verified that if θ_0 is an interior point of Θ, $\kappa_\eta = E\eta_1^4 \in (1,\infty)$ and $E|\log\eta_1^2| < \infty$, $\{(\hat{\alpha}_n, \hat{\beta}_n)\}$ is consistent and asymptotically normal, while $\{\hat{\omega}_n\}$ is inconsistent. Though the technical assumptions put certain restrictions on Θ, they appear to be unimportant in view of the numerical study in Section 3.

A natural estimator of γ_0 is

$$\hat{\gamma}_n = \frac{1}{n}\sum_{t=1}^{n}\log\left(\hat{\beta}_n + \hat{\alpha}_n\hat{\eta}_t^2\right),$$

where $\{\hat{\eta}_t\}$ are scaled residuals obtained from $\hat{\eta}_t = \varepsilon_t/\sigma_t(\hat{\theta}_n)$. Theorem 3.1 of Francq and Zakoïan (2012) states that if the above-mentioned assumptions hold and $E|\log\eta_1^2|^2 < \infty$, then

$$\sqrt{n}\,(\hat{\gamma}_n - \gamma_0) \xrightarrow{d} \mathcal{N}\left(0, \sigma_\gamma^2\right), \tag{2.8}$$

where

$$\sigma_\gamma^2 = \begin{cases} \sigma_u^2 + \Delta, & \text{when } \gamma_0 < 0, \\ \sigma_u^2, & \text{when } \gamma_0 \geq 0, \end{cases}$$

$\sigma_u^2 = \mathrm{Var}[\log(\beta_0 + \alpha_0\eta_1^2)]$, and Δ is a positive number defined in Theorem 3.1 of Francq and Zakoïan (2012).

2.3. Testing for strict stationarity

The validity of the stationarity assumption for the GARCH(1, 1) model (2.1) can be tested by using the asymptotic result (2.8). We set

$$\begin{aligned} &H_0 : \gamma_0 \geq 0 \text{ (The GARCH process is nonstationary)},\\ &H_1 : \gamma_0 < 0 \text{ (The GARCH process is strictly stationary)}. \end{aligned} \tag{2.9}$$

The test statistic, proposed by Francq and Zakoïan (2012), is simply given by $T_n = \sqrt{n}\hat{\gamma}_n/\hat{\sigma}_u$ where $\hat{\sigma}_u^2 = n^{-1}\sum_{t=1}^{n}\{\log(\hat{\beta}_n + \hat{\alpha}_n\hat{\eta}_t^2)\}^2 - \hat{\gamma}_n^2$. It is notable that $\hat{\sigma}_u^2$ converges to σ_u^2 in probability even when the GARCH process is nonstationary. If $\gamma_0 = 0$, T_n follows asymptotically $\mathcal{N}(0,1)$. Thus, H_0 is rejected at α significance level when $T_n < \Phi^{-1}(\alpha)$ ($\Phi(\cdot)$ is a c.d.f. of $\mathcal{N}(0,1)$). In other words, under the hypothesis of nonstationarity, small values of the test statistic support that the underlying GARCH(1, 1) process and the volatility process are strictly stationary. If $T_n > \Phi^{-1}(\alpha)$, the process is determined to be nonstationary, and the estimate $\hat{\omega}_n$ is not reliable while the others are still consistent as mentioned in Subsection 2.2.

The test has a robust feature against misspecification. When the true model belongs to a wide class of nonlinear GARCH-type model, it also holds that $P(T_n < \Phi^{-1}(\alpha)) \to 0$ if the corresponding Lyapunov exponent is positive, and $P(T_n < \Phi^{-1}(\alpha)) \to 1$ if the exponent is negative; see Section 4 of Francq and Zakoïan (2012).

It is worth noting that a test for $H_0 : \gamma_0 < 0$ against $H_1 : \gamma_0 \geq 0$ was also considered based on the same test statistic T_n by Francq and Zakoïan (2012). In this test, $H_0 : \gamma_0 < 0$ is rejected at level α if $T_n > \Phi^{-1}(1 - \alpha)$, thus a series is tested to be nonstationary more conservatively than in testing for (2.9).

3. Empirical Analysis

3.1. Computational aspects

The stationarity test is implemented using the QMLE defined in (2.7). Computational procedure in handling possibly nonstationary GARCH models is slightly different from the stationary case: the first is the parameter space Θ and the second is the choice of initial values for ε_0^2 and $\sigma_0^2(\theta)$. Since the weak or strict stationarity is implicitly assumed in the most packages, the domain of optimization is taken to be either $\alpha + \beta \leq 1$ or $\alpha \leq 1$ and $\beta \leq 1$. For the implication of parameter regions, see Figure 1 of Nelson (1990). In this study, we do not impose such constraints and choose Θ with $\alpha \leq 4$ and $\beta \leq 2$: normal-ARCH(1) model is strictly stationary if $\alpha < 3.5621$, see Li *et al.* (2002). When the true model is stationary, it can be seen that the Gaussian log-likelihood function for $\beta \geq 1$ has very small values. Thus, the change of Θ has no effect on the estimation for stationary GARCH models. As an initial value for ε_0^2 and $\sigma_0^2(\theta)$, the sample average of $\varepsilon_1^2, \ldots, \varepsilon_n^2$ is chosen in the stationary case; however, this can cause a serious bias in the nonstationary case since the average may diverge. In this study, the initial value is chosen as ε_1^2, which performs reasonably in both the stationary and nonstationary cases.

In the following real data analysis, we fit each daily return series of asset prices with two models: pure GARCH(1, 1) model and AR(1)-GARCH(1, 1) model. The ordinary least squares(OLS) method is employed for the estimation of AR coefficients and then the QMLE of GARCH parameters is obtained based on the residuals. As an optimization algorithm to obtain QML estimates, `nlminb` function in `R` is utilized.

This paper examines if the strict stationarity assumption actually holds in GARCH modeling procedure and to investigate how often real series show nonstationary behavior such as explosive volatility. We study ten stock market indices and one foreign exchange rate, including the Korea Composite Stock Price Index(KOSPI), AhnLab stock price, and Korean Won/USD exchange rate. The daily log returns are computed as 100 times the difference of the log of the prices. The returns range from January 3, 1996 (if available) to December 28, 2012. Table 1 describes the category and sample period of the analyzed 11 time series of returns.

3.2. Real data analysis

The main objective of this paper is to check whether the strict stationarity assumption actually holds in GARCH modeling procedure and to investigate how often real series show nonstationary behavior such as explosive volatility. We study ten stock market indices and one foreign exchange rate, including the Korea Composite Stock Price Index (KOSPI), AhnLab stock price, and Korean Won/USD exchange rate. The daily log returns are computed as 100 times the difference of the log of the prices. The returns range from January 3, 1996 (if available) to December 28, 2012. Table 1 describes the category and sample period of the analyzed 11 time series of returns.

We examine if the volatility of returns is stationary or explosive in each year; subsequently, the strict stationarity test for the hypotheses (2.9) in Subsection 2.3 is implemented to yearly sets of returns, which totally amount to 167 sets. The result indicates that only a few series violate the stationarity assumption. Based on the pure GARCH(1, 1) model, the nonstationarity null hypothesis

Table 1: Description of the data

Name	Category	Sample period
KOSPI	Price index of Korea stock market	1996/01/03 ~ 2012/12/28
KOSPI200	Price index of Korea stock market	1996/01/03 ~ 2012/12/28
KORBANK	Price index of Korea stock market	1996/01/03 ~ 2012/12/28
KOSDAQ	Price index of Korea stock market	1997/01/04 ~ 2012/12/28
KOSDAQ IT	Price index of Korea stock market	1999/01/05 ~ 2012/12/28
Samsung Elec.	Company listed on KOSPI	1996/01/03 ~ 2012/12/28
POSCO	Company listed on KOSPI	1996/01/03 ~ 2012/12/28
AhnLab	Company listed on KOSDAQ	2002/01/02 ~ 2012/12/28
Celltrion	Company listed on KOSDAQ	2006/01/02 ~ 2012/12/28
S&P500	Price index of USA stock market	1996/01/02 ~ 2012/12/28
Won/USD	Foreign exchange rate	1996/01/03 ~ 2012/12/28

Table 2: Testing results when the nonstationarity hypothesis is not rejected based on the GARCH(1, 1) model

Name	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	p-value
KOSPI200 during 1997	0.1633(0.0645)	0.8457(0.0661)	−0.0119(0.0134)	0.1880
Won/USD during 2008	0.4585(0.1387)	0.6564(0.0699)	−0.0462(0.0300)	0.0615
S&P500 during 2009	0.0701(0.0275)	0.9277(0.0236)	−0.0056(0.0063)	0.1890
AhnLab during 2011	0.1835(0.0510)	0.8343(0.0422)	−0.0139(0.0144)	0.1667

Note: The figures in parentheses is the standard error.

Table 3: Testing results when the nonstationarity hypothesis is not rejected based on the AR(1)-GARCH(1, 1) model

Name	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	p-value
Won/USD during 1997	0.3882(0.1208)	0.7010(0.0811)	−0.0343(0.0259)	0.0927
AhnLab during 2011	0.1785(0.0508)	0.8401(0.0430)	−0.0080(0.0135)	0.2757
Won/USD during 2012	0.0745(0.0301)	0.9241(0.0265)	−0.0034(0.0058)	0.2816

Note: The figures in parentheses is the standard error.

is not rejected at 5% significance level for 4 series among 167 sets of returns: AhnLab in 2011, KOSPI200 in 1997, S&P500 in 2009, and Won/USD in 2008. Table 2 reports the parameter estimates and p-values of test statistics for those series. A slightly different testing result is obtained based on the AR(1)-GARCH(1, 1) model. The 3 series of AhnLab in 2011 and Won/USD in 1997 and 2012 are reported to be nonstationary (Table 3). Our results confirm that most times the volatility of the considered asset returns is stationary except for the periods reported in Tables 2 and 3.

Now, we pay attention to the series in Tables 2 and 3, which show nonstationary behavior. Recall that the QMLE of ω_0 is inconsistent under $H_0 : \gamma_0 \geq 0$, while $\hat{\alpha}$ and $\hat{\beta}$ are not. Further, the asymptotic variance of $(\hat{\alpha}, \hat{\beta})$ in the nonstationary case can be estimated by the same covariance estimator as in the stationary case: see Theorems 2.3 of Francq and Zakoïan (2012). In Tables 2 and 3, we can find an interesting fact that the returns of AhnLab stock prices in 2011 exhibit nonstationary volatility based on both the two models. The time series plot of the returns in Figure 1 shows an evidence of nonstationarity, which indicates a change of unconditional variance. The change seems to be due to a political event relevant to the CEO of the company. Based on the AR(1)-GARCH(1, 1) model, Won/USD returns in 1997 are also found to be nonstationary and Figure 1 shows that the volatility is literally exploding in this period. The plot is well explained with the currency crisis erupted on November 17, 1997. The Korean exchange rate system was transferred from the market average exchange rate system to the freely floating exchange rate system on December 16, 1997.

For the whole series of AhnLab and Won/USD returns, further investigation is implemented. Based on the AR(1)-GARCH(1, 1) model, the stationarity test is implemented to every piece of con-

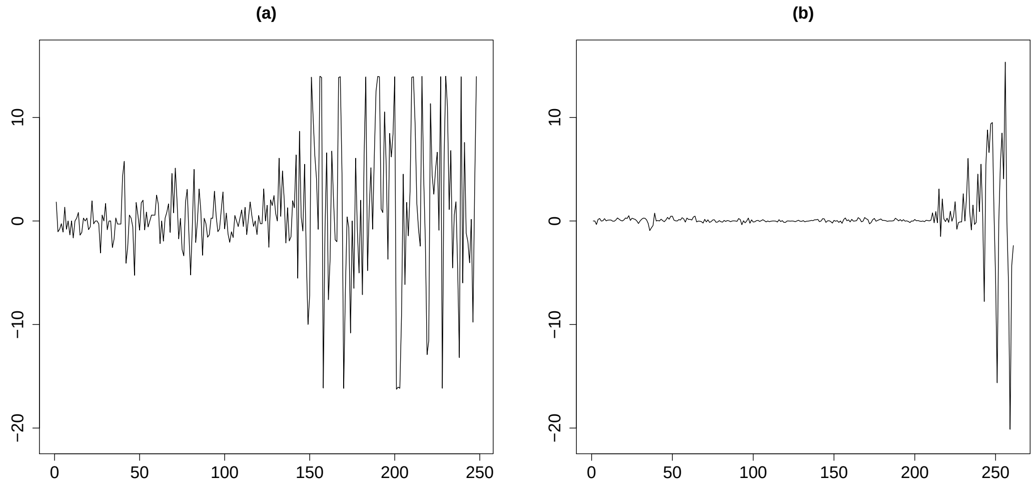

Figure 1: *Time series plots of (a) AhnLab during 2011 and (b) Won/USD during 1997.*

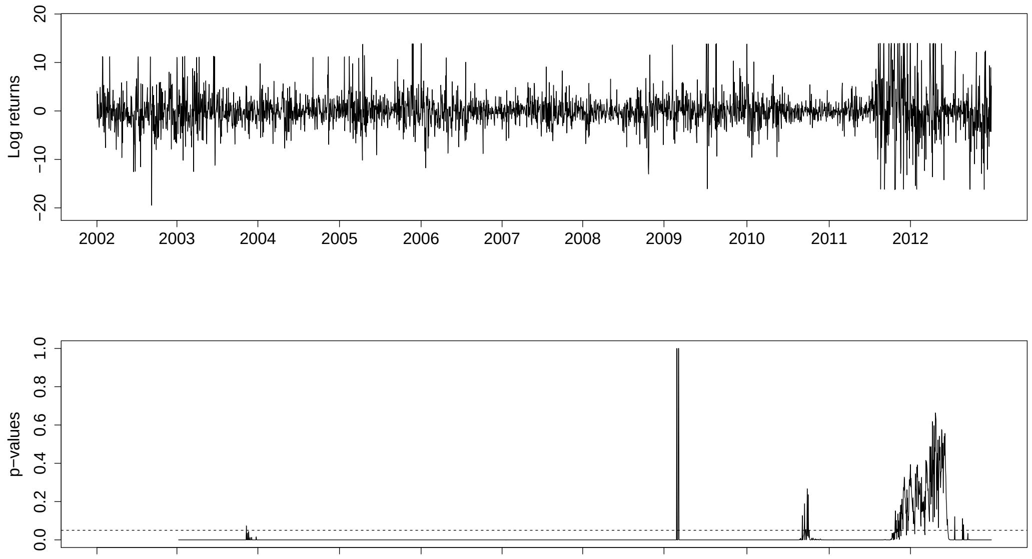

Figure 2: *Time series plots of AhnLab returns and p-values: the dotted line denotes the significance level* 0.05.

secutive 250 daily returns. Figures 2 and 3 presents the time series plots of p-values for AhnLab and Won/USD series, respectively. The results show that Won/USD returns violate the stationarity assumption more frequently than AhnLab returns.

In summary, we can conclude that the asset returns in this study show nonstationary volatility infrequently. Three yearly sets among 150 sets of stock price returns exhibit nonstationarity in either GARCH(1, 1) or AR(1)-GARCH(1, 1) fitting. Further, Won/USD returns are also reported to be nonstationary at three periods in 17 years. Explosive volatility is found to occur more often in foreign exchange markets than in Korean stock market.

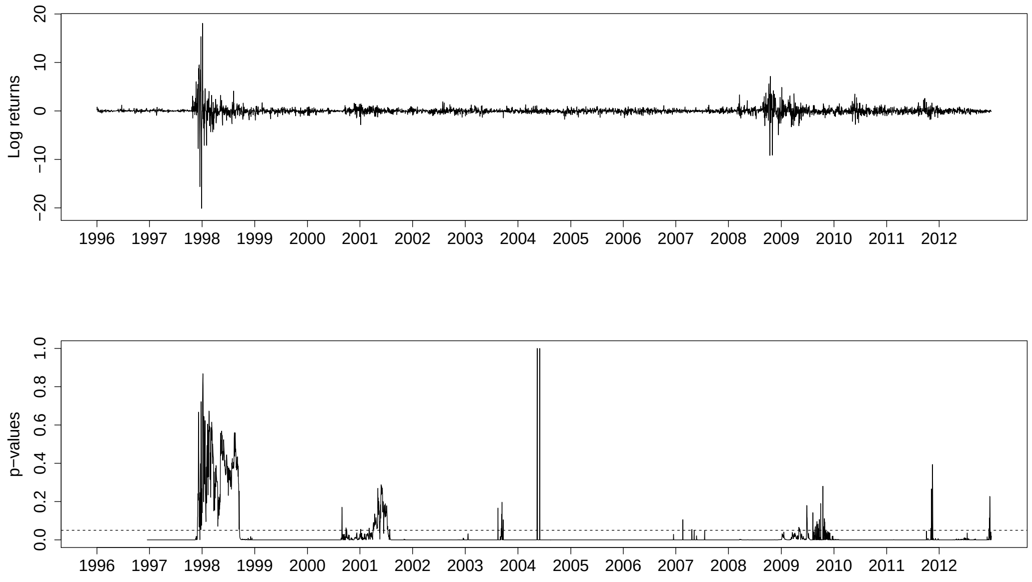

Figure 3: *Time series plots of Won/USD returns and p-values: the dotted line denotes the significance level* 0.05.

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