

A CONSTRUCTION OF COMMUTATIVE NILPOTENT SEMIGROUPS

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ABSTRACT. In this paper, we construct nilpotent semigroups S such that $S^n = \{0\}$, $S^{n-1} \neq \{0\}$ and $\Gamma(S)$ is a refinement of the star graph $K_{1,n-3}$ with center c together with finitely many or infinitely many end vertices adjacent to c , for each finite positive integer $n \geq 5$. We also give counting formulae to calculate the number of the mutually non-isomorphic nilpotent semigroups when $n = 5, 6$ and in finite cases.

1. Introduction

The present paper is motivated by works on the zero-divisor graph, which was first defined and studied for commutative rings by I. Beck in [2] and later modified and further studied by D. F. Anderson and P. S. Livingston in [1]. In 2002, DeMeyer, McKenzie and Schneider began the study of zero-divisor graph of a commutative semigroup with 0 in [6]. Since then, much work has been done and this becomes a lively branch in semigroup theory and graph theory, see e.g. [4, 5, 10, 11, 12].

For a commutative semigroup S with zero element 0 (i.e., $0S = 0$), the zero-divisor graph of S denoted by $\Gamma(S)$, is a simple undirected graph. The vertex set is $Z(S)^*$, the set of all nonzero zero-divisors of S , where for distinct $x, y \in Z(S)^*$, there is an edge connecting x and y if and only if $xy = 0$.

For notions and results about commutative semigroups, we use [7] as a basic reference. We adopt the graph theoretic notations from [3]. Especially we use $K_{1,n}$ to denote the star graph (i.e., a special complete bipartite graph). Recall that a simple graph G is called a *refinement* of a connected simple graph H if $V(G) = V(H)$ and $a - b$ in H implies $a - b$ in G for all distinct vertices of G , where $a - b$ means that $a \neq b$ and a is adjacent to b . A vertex c is called

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a *center* of a graph G if c is adjacent to every vertex of G . In [4, Theorem 3(3)], it was proved that any refinement of a star graph has a corresponding semigroup.

Throughout this paper, for any vertex c in a graph G , let G_c^* be the subgraph of G induced on the following vertex set

$$V(G) \setminus \{u \in V(G) \mid u = c \text{ or } u \text{ is an end vertex adjacent to } c\}.$$

In this paper, we construct nilpotent semigroup S such that $S^n = \{0\}$, $S^{n-1} \neq \{0\}$ and the zero-divisor graph $\Gamma(S)$ is a refinement of a star graph $K_{1,n-3}$ together with arbitrarily many end vertices adjacent to the center, where $n \geq 5$. We also prove that there is (up to isomorphism) exactly one nilpotent semigroup satisfying this condition. With the aid of the detailed structure results just mentioned, we are able to give a counting formula for calculating the number of mutually non-isomorphic nilpotent semigroups S such that $\Gamma(S) \cong K_{1,n-3}$, when $n = 5, 6$ and $\Gamma(S)$ is a finite graph.

Now we record a general result on the centers of a refinement of a star graph. Recall that an ideal I of S is called an *annihilating ideal* if $I = \text{Ann}(x)$ for some $x \in S$.

Proposition 1.1. *For any semigroup S with 0 whose zero-divisor graph $\Gamma(S)$ is a refinement of a star graph with c as one of its centers, either $c^2 = c$ or $c^2 = 0$ holds.*

Proof. Let $Z(S)$ be the set of all zero-divisors of S . Assume further that $Z(S)$ is not an annihilating ideal of S . Then $Z(S) \setminus \{c\} = \text{Ann}(c) \trianglelefteq S$ and $c^2 \neq 0$. Clearly, we can see that $\text{Ann}(c)$ is a maximal annihilating ideal of S , thus $\text{Ann}(c)$ is a prime ideal of S . We assert $c^2 = c$. In fact if $c^2 \neq c$, then $c^2 \in \text{Ann}(c)$. Since $\text{Ann}(c)$ is a prime ideal of S , we obtain $c \in \text{Ann}(c)$. Thus $c^2 = 0$, a contradiction. If $Z(S)$ is an annihilating ideal of S , then $Z(S) = \text{Ann}(c)$ and therefore $c^2 = 0$. □

Corollary 1.2. *Assume that S is a nilpotent semigroup whose zero-divisor graph $\Gamma(S)$ is a refinement of a star graph with a center c . Then $c^2 = 0$.*

2. Main result

Theorem 2.1. *For any finite integer $n \geq 5$, let $m = n - 3$. For any finite or infinite set T with $|T| \geq 1$, we fix an element $t_0 \in T$. Let $S = \{0, c\} \cup \{a_1, a_2, \dots, a_m\} \cup T$ be a disjoint union of three subsets. Define in S a binary operation by $0S = S0 = \{0\} = cS = Sc$ and the following:*

$$T^2 = \{a_m\}, \quad a_iT = \{a_{i-1}\} = Ta_i, \quad a_i a_j = a_{i-1} a_{j+1}, \quad \forall 1 \leq i, j \leq m,$$

where we order $a_{m+1} = t_0$ and $a_0 = c$, so that $a_i a_m = a_{i-1} t_0$, $a_1 t = c$, $a_1 a_j = c a_{j+1} = 0$. Then S is a commutative semigroup with zero element 0 such that

(1) $a_i \in S^{n-i-1} \setminus S^{n-i}$ ($\forall 1 \leq i \leq m$). In particular, $S^{n-1} \neq \{0\}$ and $S^n = \{0\}$.

(2) $\Gamma(S) = (G_n \cup T) + \{c\}$ and $\Gamma(S)_c^* = G_n$, where $|V(G_n)| = n - 3$ and G_n is the subgraph induced on $\{a_1, a_2, \dots, a_m\}$. In particular, $\Gamma(S)$ is a refinement of $K_{1, n-3}$ with center c together with finitely many or infinitely many end vertices adjacent to the center c .

Proof. First, the multiplication defined in S is clearly well defined. Without loss of generality, assume $1 \leq i \leq j \leq m$. Then $a_j a_i = a_{j-(j-i)} a_{i+(j-i)} = a_i a_j$ and it follows from the construction that the law of commutativity holds in S .

Second, we need to verify the associativity of the binary operation defined on S . Without loss of generality, it is only necessary to test the case $|T| = 2, m = 3$. Then $S = \{0, c, a_1, a_2, a_3, t_1, t_2\}$, and we have the following multiplication table: The set $\{a_2, t_1, t_2\}$ is a generating set of S under the multiplication table

TABLE 1

\cdot	0	c	a_1	a_2	a_3	t_1	t_2
0	0	0	0	0	0	0	0
c	0	0	0	0	0	0	0
a_1	0	0	0	0	0	c	c
a_2	0	0	0	0	c	a_1	a_1
a_3	0	0	0	c	a_1	a_2	a_2
t_1	0	0	c	a_1	a_2	a_3	a_3
t_2	0	0	c	a_1	a_2	a_3	a_3

above, since $0 = a_2^2, c = a_2 t_1^2, a_1 = a_2 t_1, a_3 = t_1^2$. By Light's associativity test (see [8]), it is enough to verify that $x(uy) = (xu)y$ for all $x, y \in S \setminus \{0, c\}$, and u is any element in the generating set of S . Observe the following two multiplication tables: where the header row of Table 2 is the values of $a_2 a_1, a_2^2, a_2 a_3, a_2 t_1, a_2 t_2$,

TABLE 2

\cdot	0	0	c	a_1	a_1
a_1	0	0	0	0	0
a_2	0	0	0	0	0
a_3	0	0	0	0	0
t_1	0	0	0	c	c
t_2	0	0	0	c	c

where the header column of Table 3 is the values of $a_1 a_2, a_2^2, a_3 a_2, t_1 a_2, t_2 a_2$. We can see that the entries in the various cells in Table 2 agrees with the entries in the corresponding cells of Table 3. This shows that $x(a_2 y) = (x a_2) y$ for all $x, y \in S \setminus \{0, c\}$. Thus $x(a_2 y) = (x a_2) y$ holds for all x, y in S . Similarly, we have $x(t_1 y) = (x t_1) y$ and $x(t_2 y) = (x t_2) y$ for all x, y of S . By Light's test, the law of associativity holds for S . Thus S is a commutative semigroup with zero element 0.

TABLE 3

\cdot	a_1	a_2	a_3	t_1	t_2
0	0	0	0	0	0
0	0	0	0	0	0
c	0	0	0	0	0
a_1	0	0	0	c	c
a_1	0	0	0	c	c

Third, since $T^2 = \{a_m\}$, $a_1T = \{c\}$ and $a_iT = \{a_{i-1}\}$ for all $2 \leq i \leq m$, we obtain by induction $T^p = \{a_{m-p+2}\} = \{a_{n-p-1}\}$ for all $2 \leq p < n - 1$. In particular for $p = n - 1$ we have $T^{n-1} = \{c\}$. Hence $S^p \neq \{0\}$ since $T^p \subseteq S^p$ for all $p < n$, and $S^{n-1} = \{0, c\}$. Thus $S^n = \{0\}$, $S^{n-1} = \{0, c\}$. Obviously, each vertex in T is an end vertex adjacent to the center vertex c , and $\Gamma(S) \setminus T$ is a refinement of the star graph $K_{1,m}$.

Finally, we claim that $a_i \in S^{n-i-1} \setminus S^{n-i}$ holds for each i with $1 \leq i \leq m$. In fact, if this is not the case, then assume that j is the least number such that $a_j \in S^{n-j}$. Then for all $i < j$ we have $a_i \in S^{n-i-1} \setminus S^{n-i}$. Clearly, $j \geq 2$ and hence such i exists. By calculation, we have $S^{n-i-1} = S^{n-i} \cup \{a_i\}$ for all $1 \leq i \leq m$. Let $i = j - 1$, then $S^{n-j} = S^{n-j+1} \cup \{a_{j-1}\}$. Thus $a_j \in S^{n-j+1}$ since $a_j \in S^{n-j}$ and $a_j \neq a_{j-1}$. Let $i = j - 2$, then $S^{n-j+1} = S^{n-j+2} \cup \{a_{j-2}\}$. Thus $a_j \in S^{n-j+2}$ since $a_j \in S^{n-j+1}$ and $a_j \neq a_{j-2}$. Continuing this process until we get $a_j \in S^{n-1}$. Note that $S^{n-1} = \{0, c\}$, a contradiction. The contradiction shows that $a_i \in S^{n-i-1} \setminus S^{n-i}$ holds for all $1 \leq i \leq m$. \square

From the proof above, it is easy to prove that the graph G constructed in Theorem 2.1 has a unique nilpotent semigroup S such that $S^n = \{0\}$, $S^{n-1} \neq \{0\}$ and $\Gamma(S) = (G_n \cup T) + \{c\}$.

Remark 2.2. For $n = 5$ and $n = 6$, we draw the corresponding graphs in the following Figure 1 and Figure 2, respectively. It can be verified that for the graph G in Figure 1 (respectively, in Figure 2), if $S^k = \{0\}$ for some k and $\Gamma(S) = G$, then $S^5 = \{0\}$ ($S^6 = \{0\}$ respectively). Also there exists a unique nilpotent semigroup S such that $S^5 = \{0\}$, $S^4 \neq \{0\}$ (respectively, $S^6 = \{0\}$, $S^5 \neq \{0\}$) and $\Gamma(S) = G$.

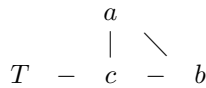


FIGURE 1

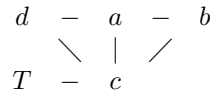


FIGURE 2

3. Counting semigroups

In the following, we will consider the number of non-isomorphic corresponding semigroups of the graph G in Figure 1 and Figure 2. First, we give some known structural results which we have obtained in [9].

Lemma 3.1. *For a semigroup S , the following statements are equivalent:*

- (1) S is a semigroup such that $S^4 = \{0\}$, $S^3 \neq \{0\}$ and $\Gamma(S) = (K_{1,1} \cup T) + \{c\}$, where $|T| \geq 1$ and $\Gamma(S)_c^* = K_{1,1}$.
- (2) $S = \{0, c, a, b\} \cup T$ is a disjoint union of two subsets, $cS = \{0\} = 0S$, $ab = 0 = a^2$, $\{a, b\}T = \{c\}$ and exactly one of the following conditions holds:
 - (i) $b^2 = 0$ and $T^2 = \{a, b\}$, where $|T| \geq 2$. In this case, S is a disjoint union of S^2 and T , where $S^2 = \{0, c, a, b\}$.
 - (ii) $b^2 \in \{0, c\}$ and $T^2 = \{a\}$, where $|T| \geq 1$. In this case, S is a disjoint union of S^2 , $\{b\}$ and T , where $S^2 = \{0, c, a\}$.

Lemma 3.2. *For any semigroup S , the following statements are equivalent:*

- (1) S is a semigroup such that $S^5 = \{0\}$, $S^4 \neq \{0\}$ and $\Gamma(S) = (K_{1,2} \cup T) + \{c\}$, where $|T| \geq 1$ and $\Gamma(S)_c^* = K_{1,2}$.
- (2) $S = \{0, c, a, b, d\} \cup T$ is a disjoint union of two subsets, $cS = \{0\} = 0S$, $ab = ad = 0$, $bd = b^2 = c$, $a^2 = 0$, $aT = \{c\}$, $\{b, d\}T = \{a\}$ and exactly one of the following conditions holds:
 - (i) $d^2 = c$, and $T^2 = \{b, d\}$. This is the case only when $|T| \geq 2$. In this case, S is a disjoint union of S^2 and T , where $S^2 = \{0, c, a, b, d\}$ and $S^3 = \{0, c, a\}$.
 - (ii) $d^2 \in \{0, c\}$ and $T^2 = \{b\}$. In this case, S is a disjoint union of S^2 , $\{d\}$ and T , where $S^2 = \{0, c, a, b\}$ and $S^3 = \{0, c, a\}$.

Lemma 3.3. *For any semigroup S , the following statements are equivalent:*

- (1) S is a semigroup such that $S^4 = \{0\}$, $S^3 \neq \{0\}$ and $\Gamma(S) = (K_{1,2} \cup T) + \{c\}$, where $|T| \geq 1$ and $\Gamma(S)_c^* = K_{1,2}$.
- (2) $S = \{0, c, a, b, d\} \cup T$ is a disjoint union of two subsets, where $|T| \geq 1$, $cS = \{0\} = 0S$, $ab = ad = 0$, $a^2 = 0$, $bd = c$, $\{a, b\}T = \{c\}$ and exactly one of the following conditions holds:
 - (i) $b^2 = 0$, and either $d^2 \in \{0, c\}$, $dT = \{a\}$, $T^2 = \{b\}$; or $d^2 = a$, $dT = \{b\}$, $T^2 = \{b\}$. In each subcase, $S^2 = \{0, c, a, b\}$.
 - (ii) $\{b^2, d^2\} \subseteq \{0, c\}$, $dT = \{c\}$ and $T^2 = \{a\}$. In this case, $S^2 = \{0, c, a\}$.

Let $X = \{i \mid 1 \leq i \leq n\}$ and $Y = \{0, 1\}$. Let H be the cyclic group of order two which is generated by the permutation $(0, 1)$. Let F be the set of all maps

from $X \times X$ to Y . Define in F two equivalent relations respectively in the following ways:

- (i) $h \sim_2 k$ if and only if there exists a permutation $\sigma \in S_n$ such that $h(i, j) = k(\sigma(i), \sigma(j))$ for all $1 \leq i, j \leq n$.
- (ii) $h \sim_1 k$ if and only if there exists a permutation $\sigma \in S_n$ and a permutation $\tau \in H$ such that $h(i, j) = \tau k(\sigma(i), \sigma(j))$ for all $1 \leq i, j \leq n$.

Let $A(n)$ and $B(n)$ be the number of equivalent classes relative to \sim_1 and \sim_2 , respectively. Recall that in S_n two permutations are conjugate if and only if they have the same type when decomposed into products of mutually disjoint cyclic permutations. Thus Burnside’s Lemma could be used to calculate the $B(n)$. We have a program to compute the $B(n)$ via a computer for all $n \leq 50$. When $4 \nmid n$, we observe that $A(n) = \frac{B(n)}{2}$.

Another way to interpret the numbers $A(n)$ and $B(n)$ is illustrated in the following. Consider the following graphs with the given set X of n vertices. Each vertex is allowed to have at most one loop and every pair of distinct vertices can have at most one edge. Then the largest such graph is the complete graph K_n together with n loops. We denote this graph as K_n° . Then each f in F is just the adjacency matrix of a corresponding graph G_f . Thus $B(n)$ is just the number of non-isomorphic such graphs obtained on the given set of n vertices. For $\tau = (0, 1)$, notice that $\tau(f)$ corresponds to the complement graph of G_f in K_n° . In the following Figure 3, $G = K_3^\circ$ and H is the complement graph of L in K_3° .

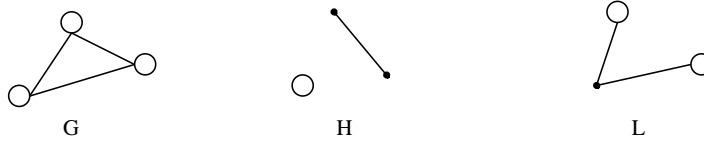


FIGURE 3

This gives another explanation to the number $A(n)$. Let E be the set of all subgraphs G of K_n° such that G has the n vertices. Define in E an equivalent relation in the following way: G_1 and G_2 are equivalent if and only if either $G_1 \cong G_2$ or G_1 is isomorphic to the complement of G_2 in K_n° . Then $A(n)$ is the number of the equivalent classes.

Theorem 3.4. (1) For any finite set T with $|T| \geq 2$, let $G = (K_{1,1} \cup T) + \{c\}$ be a graph with $G_c^* = K_{1,1}$. Then G has $f(n)$ mutually non-isomorphic nilpotent semigroups, where $f(n) = A(n) + 3n + 5$, $n = |T|$.

(2) The graph $K_3 + 1$ (i.e., $(K_{1,1} \cup \{b\}) + \{c\}$) has 9 mutually non-isomorphic nilpotent semigroups.

Proof. (1) Assume $|T| \geq 2$ and let S be a nilpotent semigroup whose zero-divisor graph is $\Gamma(S) = (K_{1,1} \cup T) + \{c\}$. By Theorem 2.1, $S^5 = \{0\}$ and there

is exactly one semigroup S such that $S^5 = \{0\}$ and $S^4 \neq \{0\}$. From Lemma 3.1 we can see that in case (i), S is completely determined by the tt' for all $t, t' \in T$, which could be freely chosen from the set $\{a, b\}$ such that $T^2 = \{a, b\}$. In case (ii), S is completely determined by b^2 , which is 0 or c . Hence, when $S^4 = \{0\}$ and $S^3 \neq \{0\}$, the number of mutually non-isomorphic nilpotent semigroups is $(A(n) - 1) + 2$, where $n \geq 2$. The remaining case is $S^3 = \{0\}$, $S^2 = \{0, c\}$. By hypothesis, there exist distinct elements $a, b \in S \setminus \{0, c\}$ such that $ab = 0$. Clearly, a, b are symmetric. In this case, S is completely determined by the vector (a^2, b^2) which could be one of the $(0, 0), (c, 0), (c, c)$, and the number of end vertices adjacent to c whose square is c . Thus the number of mutually non-isomorphic nilpotent semigroups is $3(n + 1)$. Therefore, the total number of mutually non-isomorphic nilpotent semigroups is $f(n) = A(n) + 3n + 5$, when $n \geq 2$.

(2) In this case, we can see that $|T| = 1$. Thus case (i) of Lemma 3.1 does not exist. Similar to (1) observation, we can easily find that the total number of mutually non-isomorphic nilpotent semigroups is 9 when $\Gamma(S) = K_3 + 1$. \square

Theorem 3.5. (1) *Suppose $G = (K_{1,2} \cup T) + \{c\}$ is a graph satisfying $G_c^* = K_{1,2}$, where $|T| \geq 2$. Then G has $g(n)$ mutually non-isomorphic nilpotent semigroups, where $g(n) = A(n) + 6n + 14$ and $n = |T|$.*

(2) *The graph $(K_{1,2} \cup \{b\}) + \{c\}$ has 21 mutually non-isomorphic nilpotent semigroups.*

Proof. (1) Assume $|T| \geq 2$ and let S be a nilpotent semigroup such that $\Gamma(S) = (K_{1,2} \cup T) + \{c\}$. By Theorem 2.1, there is exactly one nilpotent semigroup S with $S^6 = \{0\}$ and $S^5 \neq \{0\}$. Then, there are still three remaining cases: (1) $S^5 = \{0\}$ but $S^4 \neq \{0\}$; (2) $S^4 = \{0\}$ but $S^3 \neq \{0\}$; (3) $S^3 = \{0\}$ but $S^2 \neq \{0\}$. When $S^5 = \{0\}$, $S^4 = \{0, c\}$, by Lemma 3.2, we find that in case (i), S is completely determined by an $n \times n$ matrix taken from in T^2 , where $T^2 = \{b, d\}$. In case (ii), we easily know that the number is 2. So in this case, the number of mutually non-isomorphic nilpotent semigroups is $(A(n) - 1) + 2$, where $n \geq 2$. When $S^4 = \{0\}$, $S^3 = \{0, c\}$, from the multiplication formula we have given in Lemma 3.3 we can easily get that in case (i), the number is 3. While in case (ii), the number is also 3. Hence, in this condition, the total number of mutually non-isomorphic nilpotent semigroups is 6. Finally, we will discuss $S^3 = \{0\}$, $S^2 = \{0, c\}$. By assumption, there exist distinct elements $a, b, d \in S \setminus \{0, c\}$ such that $ab = ad = 0$, $bd \neq 0$. Obviously, b, d are symmetric. S is completely determined by a^2, b^2, d^2 and the number of end vertices in T whose square is c . So in this case, the number of mutually non-isomorphic nilpotent semigroups is $2 \times 3 \times (n + 1) = 6(n + 1)$. To conclude, the total number of mutually non-isomorphic nilpotent semigroups is $g(n) = 1 + (A(n) - 1) + 2 + 6 + 6(n + 1) = A(n) + 6n + 14$, when $n \geq 2$.

(2) When $|T| = 1$, we can see that Lemma 3.2(i) does not exist. So the total number of mutually non-isomorphic nilpotent semigroups is 21, follows directly from the argument of (1). \square

In the end of the paper, we list the first six values for $B(n)$, $A(n)$, $f(n)$ and $g(n)$ respectively in the following table:

TABLE 4

n	1	2	3	4	5	6
$B(n)$	0	6	20	90	544	5096
$A(n)$	0	3	10	46	272	2548
$f(n) = A(n) + 3n + 5$ ($n \geq 2$)	9	14	24	63	292	2571
$g(n) = A(n) + 6n + 14$ ($n \geq 2$)	21	29	42	84	316	2598

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