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단 보

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Extension for Downward Continuation of the Method of "Upward Continuation of Potential Field on Spherical Patch Area"

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"구면부분지역에서 퍼텐셜마당의 상향연속"의 하향연속 확장적용

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Abstract: We formerly reported a new method for the upward continuation of potential field on spherical patch area including Earth's curvature, which has been neglected in most studies on rectangular area with flat Earth assumption. This new method is applicable to downward continuation as well by only assigning corresponding value for the ratio of two radii; r_2/r_1, *i.e.*, target radius r_2 versus datum radius r_1. In addition, the inherent problem of this method due to spherical surface geometry is described, and its one possible remedy is given.

Keywords: Potential field, Upward/down continuation, Spherical curvature

요 약: 우리는 구면일부지역에서의 퍼텐셜마당에 대하여 구면의 곡률을 고려하는 새로운 상향연속방법을 이전에 보고하였다. 그런데, 이 방법은 실제로 r_2/r_1의 비율(즉, 목표면의 반경 r_2 대 자료면의 반경 r_1) 만을 달리함으로써 하향연속에도 그대로 사용될 수 있다. 한편, 이 방법에 내재된 구면의 기하학적 성질에 기인하는 문제와 그에 대한 대책을 기술하였다.

주요어: 퍼텐셜마당, 상향/하향연속, 구면곡률

Introduction

Although flat Earth has been generally assumed in most continuation of potential field on a localized area, it is desirable to consider the curvature of the Earth, if the dimension of survey area exceeds a few hundred kilometers. In our former report, we presented the new algorithm derived for upward continuation of gravity or magnetic field on a spherical patch area (Na *et al*., 2012). We hereby state that downward continuation can be attained by exactly the same scheme, as explained below in the following section.

Unlike Cartesian coordinate system, in spherical coordinates, Laplace's equation cannot be expressed with complete separation of variables. This defect is unavoidable due to the nature of spherical coordinate system itself, since the Laplacian operator is expressed as follows: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$, *i.e.*, two scale factors r and $r\sin\theta$ exist for the variables θ and ϕ. However, this problem can be made tolerable, when different latitude ranges are separately handled, as shown below.

Spherical Patch Transform: Overview, Problem and Its Remedy

Suppose we know a set of values of potential field $U(\vec{r})$ on a spherical patch area, which is regularly gridded in ranges of latitude and longitude as follows. Denote $U(\vec{r}) = U(r, \theta_1 + k\Delta\theta, \phi_1 + l\Delta\phi)$ as $U(r,k,l)$ and its two dimensional discrete Fourier transform at the same radius (height)

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as $\tilde{U}(r,m,n)$. Then forward and inverse relations between $U(r, k, l)$ and $\tilde{U}(r,m,n)$ can be expressed as follows. More description for definition of terms and indexes are given in the former report (Na *et al*., 2012).

$$\tilde{U}(r,m,n) = \frac{1}{K}\sum_{k=0}^{K-1}\frac{1}{L}\sum_{l=0}^{L-1} U(r,k,l)\exp\left(-2\pi i\frac{km}{K}\right)\exp\left(-2\pi i\frac{ln}{L}\right) \tag{1}$$

$$U(r,k,l) = \sum_{m=0}^{K-1}\sum_{n=0}^{L-1} \tilde{U}(r,m,n)\exp\left(+2\pi i\frac{km}{K}\right)\exp\left(+2\pi i\frac{ln}{L}\right) \tag{2}$$

Then the discrete version of Laplace equation in terms of $\tilde{U}(r,m,n)$ is found as follows.

$$\sum_{m=0}^{K-1}\sum_{n=0}^{L-1}\left(\begin{array}{c} r^2\dfrac{\partial^2\tilde{U}}{\partial r^2} + 2r\dfrac{\partial\tilde{U}}{\partial r} \\ + (\Delta\theta)^{-2}\left(-4\pi^2\dfrac{m^2}{K^2}\right)\tilde{U} + (\Delta\theta)^{-1}\left(2\pi i\dfrac{m}{K}\right)\cot\theta\tilde{U} \\ + (\Delta\phi)^{-2}\left(-4\pi^2\dfrac{n^2}{L^2}\right)\sin^{-2}\theta\tilde{U} \end{array}\right) \times \exp\left(2\pi i\frac{km}{K}\right)\exp\left(2\pi i\frac{ln}{L}\right) = 0 \tag{3}$$

and its solution is attained as follows.

$$\tilde{U}(r,m,n) = Cr^{\lambda} \tag{4}$$

where the exponent $\lambda(m,n)$ is given as

$$\lambda(m,n) = \frac{-1-\sqrt{1-4A}}{2}, \tag{5a}$$

with $A(m,n)$ defined as

$$A(m,n) = -\left(\frac{2\pi m}{K\Delta\theta}\right)^2 + \frac{2\pi i m}{K\Delta\theta}\cot(\theta_1 + k\Delta\theta) - \left(\frac{2\pi n}{L\Delta\phi\sin(\theta_1 + k\Delta\theta)}\right)^2. \tag{5b}$$

Care should be taken in assigning the values for m and n in real evaluations of Eqs. (4) ~ (5). Nominally m and n increase from 0 to K–1 and L–1 as shown in the relations of discrete Fourier transform pairs as Eqs. (1) ~ (2). However, due to the intrinsic structure of discrete Fourier transform, m and n in Eq. (5) should read from 0 to K/2 and L/2, which correspond to two Nyquist wavenumbers, and then from –K/2+1 and –L/2+1 to –1 (see, for example, Press *et al*., 1996).

The continuation of gravity potential U can be done by the

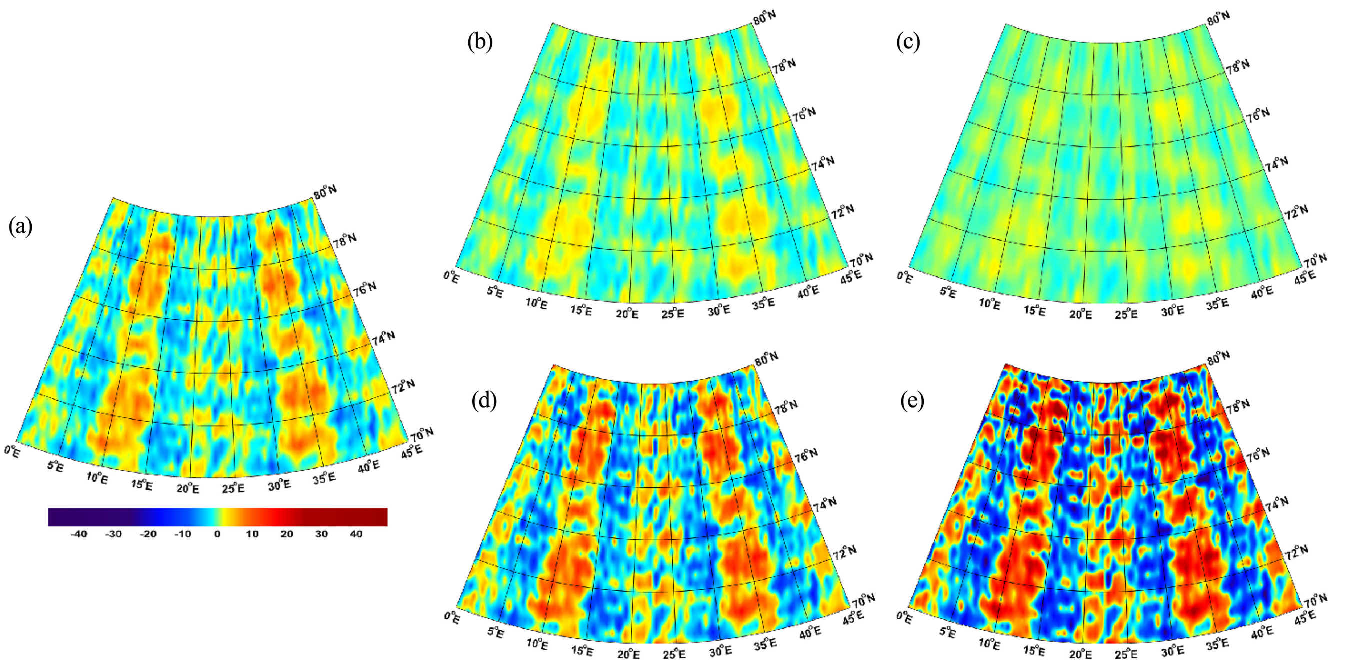

Fig. 1. Illustrations of gravity fields calculated by the algorithm of this study at different heights; (a) a synthesized gravity field at radius of r_1, (b) and (c) - two gravity fields acquired by upward continuation using the spherical patch transform of this study with radii $r_2 = 1.002r_1$ and $1.004r_1$, (d) and (e) - two gravity fields acquired by downward continuation using the spherical patch transform of this study with radii $r_2 = 0.999r_1$ and $0.998r_1$. For example, if r_1 is specified as 6400 km, each target radii correspond to 6412.8, 6425.6, 6393.6, 6387.2 km (b-e), under mass free assumption in the space between r_1 and r_2. The latitude range is between 70° and 80° N, and the longitude range is 45°. Four sectors divided by three latitude parallels were separately taken for upward/downward continuations. [arbitrary unit].

following procedure.

$$U(r_1,k,l) \Rightarrow \tilde{U}(r_1,m,n) \Rightarrow \tilde{U}(r_2,m,n) \Rightarrow U(r_2,k,l)$$

The three successive steps are; 1) acquire Fourier transform $\tilde{U}(r_1,m,n)$ of the given potential $U(r_1,k,l)$, 2) perform the upward/downward continuation in the wave number domain as $\tilde{U}(r_2,m,n) = \tilde{U}(r_1,m,n)\left(\frac{r_2}{r_1}\right)^{\lambda}$, and 3) do the inverse Fourier transform to acquire $U(r_2,k,l)$.

The continuation of gravity field g is attained similarly.

$$g(r_1,k,l) \Rightarrow \tilde{g}(r_1,m,n) \Rightarrow \tilde{g}(r_2,m,n) \Rightarrow g(r_2,k,l)$$

with exponent in the second procedure replaced by $\lambda-1$ due to the different radial dependence, *i.e.*, $\tilde{g}(r_2,m,n) = \tilde{g}(r_1,m,n)\left(\frac{r_2}{r_1}\right)^{\lambda-1}$. This is readily conceivable from the relation between the gravity and its potential: $g = \frac{\partial U}{\partial r}$. Because the radial dependence is isolated from angular dependences as shown above, this method (may be called 'Spherical Patch Transform') can be applied either to upward or downward continuation. In Fig. 1, both the upward and downward continuations are exemplified. While shorter wavelength components are comparatively more weakened in the upward continued fields (b-c), they are strengthened in the downward continued fields (d-e). The curvature effect is stronger in higher latitude region.

The discrete version of spherical coordinate Laplacian includes two terms having latitude dependence as $(\Delta\theta)^{-1}\left(2\pi i\frac{m}{K}\right)\cot\theta\tilde{U}$ and $(\Delta\phi)^{-2}\left(-4\pi^2\frac{n^2}{L^2}\right)\sin^{-2}\theta\tilde{U}$, while other terms of $r^2\frac{\partial^2\tilde{U}}{\partial r^2} + 2r\frac{\partial\tilde{U}}{\partial r} + (\Delta\theta)^{-2}\left(-4\pi^2\frac{m^2}{K^2}\right)\tilde{U}$ are free from the θ-dependence. As a result, the parameter $A(m,n)$ includes $\cot(\theta_1 + k\Delta\theta)$ and $\sin(\theta_1 + k\Delta\theta)$, although it is not affected by variation in ϕ. This awkward situation is unavoidable, as long as spherical geometry is to be considered. In evaluation of $A(m,n)$ and $\lambda(m,n)$, one has to take the value of $\theta = \theta_1 + k\Delta\theta$ at the mid-point of the given latitude range. This means overall insufficient calculation by improper treatment of θ-variation. Such problem can be alleviated by treating the original patch area not as a single region but as multiply connected one of each narrow sector separated by latitude parallels. By doing so, more faithful calculation can be attained but with cost of repeating several similar calculations. In fact, upward/downward continuation illustrated in Fig. 1, were acquired by such calculations on four sectors with latitude ranges as 80-77.5, 77.5-75, 75-72.5, and 72.5-70 in degrees. The five sample gravity fields are gridded as $K \times L$ = 32×64. Power spectra of the given dataset and two downward continued ones are given in Fig. 2. It is noted that the gravity field at $r = r_1$ was synthesized in the wavelength domain. Each the three sets of power spectra are consisted of four individual ones for the separate sectors. Obviously short/long wavelength

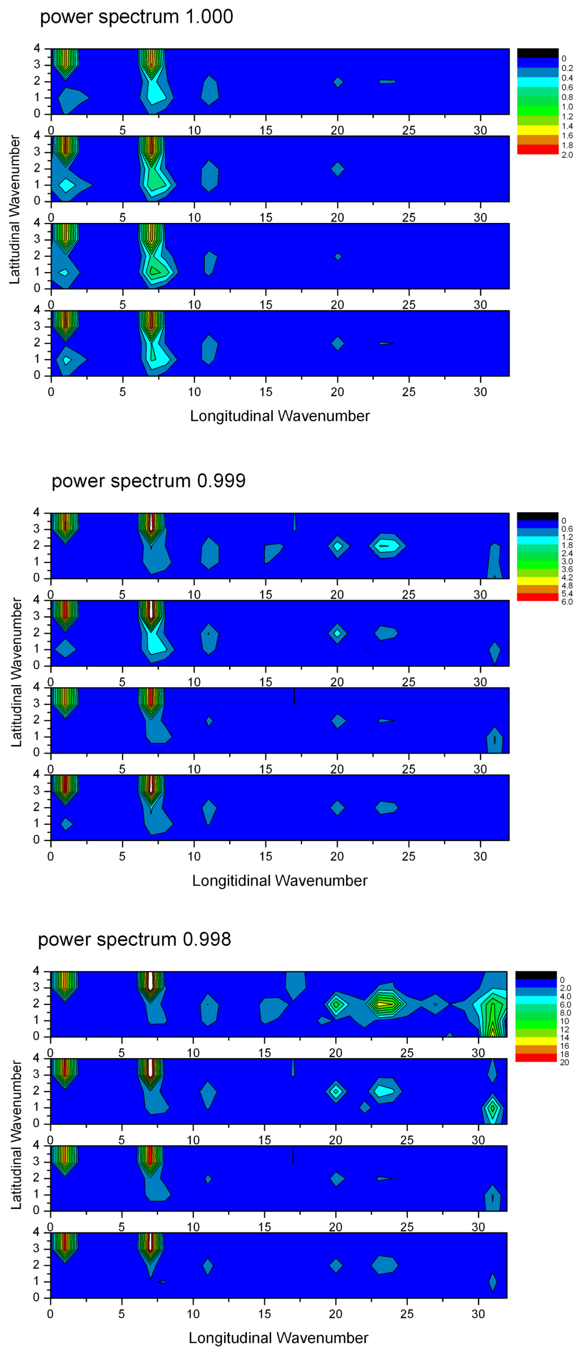

Fig. 2. Power spectra for the three gravity fields in Fig. 1 (a), (d), and (e), which correspond to radii r = r_1, r_2 = 0.999r_1, and r_2 = 0.998r_1. Each set of spectrum is consisted of four separated power spectra for the four sectors. Short wavelength components increase by downward continuation is most prominent in the high latitude sector.

components increase/decrease in their relative compositions by downward continuation. The reverse situation would happen by upward continuation, which may be directly read from Fig. 1. As can be expected, short wavelength components increase/ decrease more abruptly in the high latitude sector (directly shown in Fig. 2), because actual wavelength is relatively shorter for the same longitudinal wave number in that region.

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