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An Iterative Approach to the Estimation of CO_2 Abatement Costs

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ABSTRACT : This study proposes an iterative approach to the estimation of the marginal abatement costs of undesirable outputs by computing the slope of the efficient production possibilities frontier on the basis of the efficient projection points generated by the directional output distance function approach due to Fare et al. (2005) based on duality theory. In case of the latter methodology, the estimated marginal abatement costs differ significantly depending on the choice of the directional output vector. In addition, depending on the curvature of the underlying PPF the efficient projection points may be located at a significant distance away from their actually observed counterparts. While it would be more logical to estimate marginal abatement costs as a PPF slope at a point corresponding to the actually observed emissions level, the methodology based on duality theory is likely to produce unstable results due to the problems associated with applying the theorem of implicit function differentiation. Since our methodology is not based on duality theory, our results are immune to both of these problems. We apply our methodology to a sample of Western European countries for the period of 1995-2011 to illustrate our approach.

Keywords : CO_2 emissions, Marginal abatement cost, Distance function

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방향성 벡터 일반화를 통한 이산화탄소의 한계저감비용 연구

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요 약 : 기존 연구에서는 이산화탄소의 한계저감비용을 추정할 경우 쌍대성 이론에 근거하여 임의로 설정된 하나의 방향성 벡터(directional vector) 설정하였으나 본 연구에서는 이러한 한계를 극복하고자 다양한 형태의 방향성 벡터를 사용하여 이산화탄소의 한계저감비용을 추정하였다. 기존의 방법론에서는 임의로 설정된 방향성 벡터가 한계저감비용 추정에 결정적인 역할을 하여 선택된 방향성 벡터에 따라 한계저감 비용 추정치가 상당한 차이가 있음을 알 수 있다. 그리고 45°의 방향성 벡터를 설정하는 경우에는 실제 이산화탄소 배출량 수준과는 다른 배출량 수준에서의 한계저감비용을 추정하게 되지만 본 연구에서 제안한 방법론에 의하여 추정된 한계저감비용은 실제 이산화탄소 배출량 수준에서 한계저감비용을 추정하여 보다 더 현실을 정확하게 반영하는 추정치이다. 새로운 방법론을 서유럽 국가에 적용하여 추정한 이산화탄소의 한계저감비용은 기존 방법론을 사용하는 경우에 비하여 적은 것으로 추정되었다.

주제어 : 이산화탄소 배출, 한계저감비용, 거리함수

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I. Introduction

Recent concerns about global warming prompted the international community to design plans aimed at reducing the level of greenhouse gas emissions. The Kyoto Protocol adopted in 1997 stipulated the individual CO_2 reduction pledges by the countries listed in Annex I. The EU members mentioned in Annex I have reduced their CO_2 emission levels on average by 5.2% between 1990 and 2012. A political agreement was concluded in 2009 in Copenhagen by the conference of parties to the United Nations Framework Convention on Climate Change. More than 120 countries and the European Union have acceded to the accord by pledging to reduce their greenhouse gas emission levels by 2020 (Dellink et al., 2010). In 2011 the European Commission published a roadmap for moving to a low-carbon economy in 2050 that suggests the twenty-seven EU members reduce their CO_2 emission levels by 40% relative to 1990 by the year of 2030. However, such reduction is likely to come at substantial costs. It is the purpose of this study to examine such costs, also referred to as the marginal abatement costs, for the European Community members, in a robust fashion.

Similarly to the seminal contribution of Shephard (1970), the directional distance function approach by Fare et al. (2005) to the computation of CO_2 marginal abatement costs is based on duality theory and computes these costs as a ratio of the two outputs' shadow prices since this ratio can be shown to be equal to the slope of the PPF at the projection of the observed outputs' combination on it along some projection vector. However, depending on the choice of the projection vector the estimates of CO_2 marginal abatement costs can differ quite substantially, part of the reason being that different projection vectors project the observed combinations of GDP and CO_2 emission levels to different parts of the PPF. Secondly, the estimation of marginal abatement costs as a ratio of the two shadow prices is in fact equivalent to applying the implicit function theorem that is likely to produce unstable estimates in case the projected combination of GDP and CO_2 is located on the rather steep or flat part of the PPF.

We find it reasonable to estimate the efficient points on the PPF using a multitude of the projection vectors, rather than just one such vector, since the PPF estimation based on a larger number of projection vectors is also based on a larger set of the efficient projection points, and the problems with implicit function differentiation are avoided. Once the PPF estimates are in place, it is straight-forward then to employ the textbook interpretation of the PPF slope as an opportunity cost of one output in terms of the other, i.e. of CO_2 reduction in terms of the foregone GDP. This study decides on the right set of the projection vectors on the basis of which to estimate the PPF by applying an iteration procedure. Starting with a small number of the initial projection vectors (say, three), at each consecutive iteration new projection vectors are added to the set employed at the previous iteration. The iteration process stops when the norm of the difference between the two PPF's computed on the basis of the two consecutive sets of projection vectors becomes negligibly small.

To illustrate the proposed methodology, this study employs an unbalanced panel of fifteen EU countries for the period between 1995 and 2011 to compute marginal abatement costs of reducing CO_2 emissions in terms of the foregone GDP. All of these countries are party to the Kyoto Protocol and are likely to share a similar PPF due to their close geographical and economic proximity.

This paper is organized as follows. Section II describes the theoretical framework for assessing the marginal abatement costs on the basis of distance function estimation. Section III describes the iteration procedure of estimating the marginal abatement costs in the framework of directional output distance functions. Section IV presents the dataset and discusses our empirical results. Section V concludes.

II. Costs of Reducing Pollution and Directional Output Distance Functions

1. Modeling a production technology with desirable and undesirable outputs

Shephard's (1970) output distance function provides a convenient tool of modeling a multi-output technology in that this function inherits the properties of the underlying production set and is dual to the revenue function. The latter property is important since, as shown by Fare et al. (1993, 2005), it allows one to derive revenue-deflated shadow prices of all outputs. Assuming that the shadow price of one of the outputs is equal to its market price, one can compute absolute, i.e. undeflated, shadow prices of all other outputs. Since the ratio of the absolute shadow prices can be interpreted as an opportunity cost of one output in terms of the other, the estimation of such ratios is a natural choice when dealing with the task of inferring the marginal abatement costs of pollutants such as e.g. CO_2 in terms of the amount of GDP that has to be foregone to reduce the amount of pollution.

To formalize the idea, let $P(x)=\{(b,y): x \text{ can produce } (b,y)\}$ be the set of all output vectors (b,y) that can be produced using input vector x. Let b denote the undesirable output, and y be the desirable one. For instance, b could refer to the amount of CO_2 emissions, while y is GDP. All inputs are assumed to be strongly disposable, i.e. if $x' \geq x$ then $P(x) \subseteq P(x')$: increasing the amount of any one or some inputs by any amount will not render the production of previously feasibly output vectors infeasible. The production set $P(x)$ is also assumed to be compact to preclude the possibility of producing infinite amounts of output with finite amounts of inputs. The idea of a costly reduction of an undesirable output is partially captured by the assumption of weak disposability of desirable and undesirable outputs, namely: $(b,y) \in P(x)$ implies $(\theta b, \theta y) \in P(x)$ for any real $\theta \in [0,1]$. In other words, reducing the undesirable output is always feasible if accompanied by a proportional reduction of the desirable output. This assumption, however, is also made when modeling the production technology with desirable outputs only. In order to completely preclude the possibility of a costless reduction of an undesirable output, the strong disposability property is only assumed for the desirable output, but not the undesirable one. That is, if $(b,y) \in P(x)$ then $(b, y-\Delta y) \in P(x), \Delta y \geq 0$: it is possible to costlessly reduce the amount of good output y while keeping the levels of inputs and undesirable output b intact. Strong disposability, however, is not assumed for the undesirable output, thus necessitating the existence of opportunity costs associated with the reduction in b. Finally, the assumption of null jointness of the desirable and undesirable outputs says that no production of the desirable y is possible unless some amount of the undesirable b is produced as well: $(y,0) \in P(x)$ implies y=0.

2. Directional output distance functions and the shadow prices of outputs

The output distance function approach to the measurement of the shadow prices of undesirable outputs pioneered by Fare et al. (1993, 2005) and adopted in many studies (e.g. Coggins and Swinton 1996, Lee 2011, Maradan and Vassiliev 2005) is based on the original idea by Shephard (1970) of measuring the shadow price ratio of the two outputs at an efficient projection point along a vector (1,1) of the observed combination of outputs on the efficient production frontier.

The extent of production efficiency can be measured as a distance between the observable output mix and the PPF measured along the projection vector.

Fare et al. (2005) notice that the direction of the projection vector when both y and b are expanded simultaneously runs counter to the general idea of *reducing* the amount of undesirable output b while simultaneously increasing the desirable y. Second, the distance between the observable output mix and its efficient projection has to be inflated to ensure the correct sign of the estimated abatement costs. To avoid such problems, Fare et al. (2005) introduce the concept of a directional output distance function that explicitly depends on a projection vector $(-g_b, g_y)$ along which the observable output mix is projected on the PPF.

Fare et al. (2005) define a directional output distance function as follows:

$$\vec{D}_0(x, b, y| -g_b, g_y) = \max\{\alpha : (b - \alpha g_b, y + \alpha g_y) \in P(x)\} \tag{1}$$

The definition in (1) reflects the idea of moving towards the PPF by reducing the undesirable output while increasing the desirable one.

3. Empirical estimation of the directional distance functions and marginal abatement costs

One way to parameterize the directional distance function in (1) is by approximating its values in the vicinity of the observable output mixes by a second-order Taylor polynomial. The distance function's parameters are then estimated by minimizing the sum of the distances between the observable output mixes and their projections on the PPF taken along the projection vector $(-g_b, g_y)$ subject to a set of constraints reflecting the properties of the distance function:

$$\begin{cases} Min\sum_1^T \vec{D}_0 - 0 = \\ = Min\sum_1^T \alpha_0 + \sum \alpha_n x_n + \beta_1 y + \gamma_1 b + \frac{1}{2}\sum\sum \alpha_{nn'} x_n x_{n'} \\ \quad + \frac{1}{2}\beta_2 y^2 + \frac{1}{2}\gamma_2 b^2 + \sum \nu_n x_n b + \sum \delta_n x_n y + \mu y b \\ \vec{D}_0 \geq 0 \\ \frac{\partial \vec{D}_0}{\partial y} \leq 0 \\ \frac{\partial \vec{D}_0}{\partial b} \geq 0 \\ \vec{D}_0(x, b - \alpha g_b, y + \alpha g_y | g_b, g_y) = \vec{D}_0(x, b, y | g_b, g_y) - \alpha, \alpha \in \Re \\ \alpha_{nn'} = \alpha_{n'n}, n = 1..K \end{cases} \tag{2}$$

where K is the number of inputs. The first constraint guarantees that the distance function is non-negative for all observable output mixes. The next two constraints ensure the right sign of the directional distance function's partial derivatives, while the second to last constraint in (2) expresses the translation property, which is essentially restating the definition of a distance function as a distance between the observable output mix and its efficient counterpart on the production possibilities frontier. The last constraint ensures the symmetry of coefficients in the quadratic form.

Fare et al. (2005) employ duality between the directional output distance function and the revenue function to argue that the ratio of the shadow price of the undesirable output to that of the desirable one (i.e. the opportunity cost of the former in terms of the latter) is equal to the negative of the ratio of the two partial derivatives of the distance function estimated at the observable output mix:

$$\frac{p_b}{p_y} = -\left.\frac{\partial \vec{D}_0 / \partial b}{\partial \vec{D}_0 / \partial y}\right|_{(b_0, y_0)} > 0 \qquad (3)$$

where p_b and p_y are the shadow prices of undesirable b and desirable y, respectively, and (b_0, y_0) is the observed output mix, and the shadow price of one of the two outputs is equal to its market price.[1)]

It is worthwhile noticing that the right hand side of (3) is the slope of an implicit function $y = f(b)$ defined by the relationship $\vec{D}_0(x, b_0, y_0 | -g_b, g_y)\big|_{(b_0, y_0)} = \alpha^*$, where $\alpha^* : \vec{D}_0(x, b_0, y_0 | -g_b, g_y)\big|_{(b_0, y_0)} = 0$. This follows directly from the theorem on implicit function differentiation (see e.g. Sydsaeter and Hammond 2008, p. 412) and the definition of the distance function. It follows that the ratio of the two shadow

1) This formula has to be slightly modified in case variable entering the distance functions are in logs.

prices in (3) is equal to the slope of the PPF at the projection of the observable (b_0, y_0) on the PPF along projection vector $(-g_b, g_y)$. The latter statement is made possible by the translation property.

III. The Iteration Procedure for Estimating the Marginal Abatement Costs

1. Choosing the directional output vector

To estimate the parameters of a directional output distance function by program (2), Fare et al. (2005) suggest employing a projection vector $(-g_b, g_y) = (-1,1)$. However, there appears to be no economic theory that would justify preferring (-1,1) over, say, (-0.5,1) or any other vector moving towards the PPF in the "North-Western" direction. Since for each observable output mix (b_0, y_0) the point $\left(b_0 - \vec{D}_0(x, b_0, y_0 | -g_b, g_y), y_0 + \vec{D}_0(x, b_0, y_0 | -g_b, g_y)\right)$ is by definition of the directional output distance function a projection point of (b_0, y_0) on the PPF, the estimation of program (2) for any valid projection vector $(-g_b, g_y)$ results in a set of projection points on the PPF for each observable output mix (b_0, y_0). Consequently, estimating the distance function parameters (and the associated projections on the PPF) for a variety of vectors renders one with more information about the PPF compared to the estimation based on just one projection vector.

2. The iterations-based approach to the estimation of marginal abatement costs

In this study we suggest estimating the marginal abatement costs of the undesirable output based directly on the well-known interpretation of the slope of the production possibility frontier as an opportunity cost of one output in terms of the other. We suggest the following parametric specification for the PPF: $\hat{y}_{it} = \rho_1 b_{it} + \rho_2 b_{it}^{\ 2}$ where the intercept term is not included due to the property of null-jointness, and the quadratic form was chosen on the basis of the principle of parsimony[2)]. The quadratic specification of the PPF function is clearly not the only possible one, and is a univariate analogue of the more general translog functional form often used in the multivariate case. Marginal abatement costs can be computed at each level of the CO_2 emissions b_{it} as $2\rho_2 b_{it} + \rho_1$.

The efficient level of output $\rho_1 b_{it} + \rho_2 b_{it}^2$ corresponding to the observed level of CO_2 emissions b_{it} is the sum of the projection $\tilde{y}_{it}$ of the observed (inefficient) y_{it} on the PPF along some projection vector $\varphi = (-g_b, g_y)$, and a non-negative value ε_{it}:

$$\rho_1 b_{it} + \rho_2 b_{it}^2 = \tilde{y}_{it} + \varepsilon_{it} = y_{it} + g_y \vec{D}_0 \left(b_{it}, y_{it} | \varphi\right) + \varepsilon_{it} \qquad (4)$$

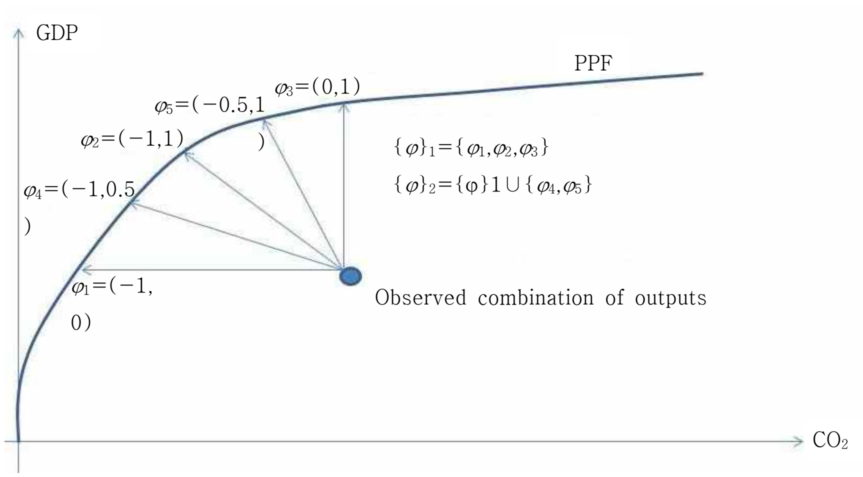
The number of observations to estimate (4) is equal to $S_j \times N \times T$ where S_j is the number of projection vectors in the vector set $\{\varphi\}_j$ used at iteration j, N is the number of countries, and T is the number of years covered by the sample. We estimate (4) by minimizing the sum of squared deviations between $y_{it} + g_y \vec{D}_0 \left(b_{it}, y_{it} | \varphi\right)$ and $\rho_1 b_{it} + \rho_2 b_{it}^2$ constraining ε_{it} to be non-negative and the resulting PPF to be a concave function to satisfy the property of increasing marginal abatement costs with the extent of *reduction* of pollution levels:

2) In case the methodology is extended to accommodate more than one undesirable output, the flexible translog specification can be estimated to represent the PPF.

$$\begin{cases} \underset{\rho_1,\rho_2}{Min} \sum_{i=1}^{N} \sum_{t=1}^{T} \left[\rho_1 b_{it} + \rho_2 b_{it}^2 - y_{it} - g_y \vec{D}_0 \left(b_{it}, y_{it} | \varphi \right) \right]^2 \\ \rho_1 b_{it} + \rho_2 b_{it}^2 - y_{it} - g_y \vec{D}_0 \left(b_{it}, y_{it} | \varphi \right) \geq 0 \\ 2\rho_2 b_{it} + \rho_1 \leq 0 \end{cases} \tag{5}$$

At the first iteration, we choose an initial set of projection vectors $\{\varphi\}_1$ that belong to the space of all vectors pointing from the observed output combinations towards the PPF in the "North-Western" direction, e.g. $\{\varphi\}_1 = \{(-1,0),(-1,1),(0,1)\}$. At the second iteration, we add more vectors to $\{\varphi\}_1$ by dissecting the angles formed by the constituents of $\{\varphi\}_1$ in half, which is illustrated by <Figure 1> where we add φ_4 and φ_5 to $\{\varphi\}_1$ in order to obtain $\{\varphi\}_2$, the set of projection vectors used at the second iteration.

<Figure 1> The Iteration Vectors



Note: $\{\varphi\}_1$ is the set of projection vectors at the first iteration, $\{\varphi\}_2$ is obtained from $\{\varphi\}_1$ by adding two more vectors φ_4 and φ_5 to be used at iteration 2. The vectors in the figure have been scaled to reach the PPF.

At each consecutive iteration *j+1* we increase the number of projection vectors in the set $\{\varphi\}_{j+1}$ in the way illustrated by Figure 1 for the first two iterations. The iteration process stops at some iteration number J if the norm of the difference between two successively estimated PPFs is smaller than a particular $\delta \in \Re$:

$$\left\| f_J(b) - f_{J-1}(b) \right\|_2 \equiv \left(\int_0^{b_{\max}} \left(f_J(b) - f_{J-1}(b) \right)^2 db \right)^{\frac{1}{2}} < \delta \tag{6}$$

It is important to keep in mind that projection vectors (-1,0) and (0,1) are not suitable for the estimation purposes in case the inference is to be made on the basis of duality theory since the implicit functions $y = PPF = f(b)$ and $b = PPF^{-1}(y)$ are not defined in the vicinity of projection points corresponding to projection vectors (-1,0) and (0,1), respectively. On the contrary, computing the efficient projections on the PPF along these vectors does not pose any problem.

3. Parameter constraints to satisfy the translation property

Similarly to Fare et al. (2005) we employ the quadratic specification of the directional distance function:

$$\begin{aligned} D(x, y | -g_b, g_y) = \alpha_0 + \sum_{n=1}^{3} \alpha_n x_n + \beta_1 y + \gamma_1 b + \sum_{n=1}^{3} \sum_{n'=1}^{3} \alpha_{nn'} x_n x_{n'} + \\ + \beta_2 y^2 + \gamma_2 b^2 + \mu y b + \sum_{n=1}^{3} \nu_n x_n b + \sum_{n=1}^{3} \delta_n x_n y \end{aligned} \tag{7}$$

The specification above has to satisfy the translation property that we formulate similarly to Fare et al. (2005):

$$D(b, y|b - \alpha g_b, y + \alpha g_y) - D(b, y| - g_b, g_y) = -\alpha \tag{8}$$

where α is any real number. It can be shown that the translation property for an arbitrary direction vector $(-g_b, g_y)$ is satisfied if the following constraints are imposed on the parameters of the distance function:

$$\begin{cases} \beta_1 g_y - \gamma_1 g_b = -1 \\ \delta_n g_y - \nu_n g_b = 0, n = 1..3 \\ 2\beta_2 g_y = \mu g_b \\ 2\gamma_2 g_b = \mu g_y \\ \beta_2 g_y^2 + \gamma_2 g_b^2 = \mu g_y g_b \end{cases} \tag{9}$$

The derivation of (9) can be found in the Appendix.

IV. Data and Empirical Results

1. Data

In this study we treat each country as a producer of two outputs: a desirable good GDP, and an undesirable "bad" pollution, e.g. CO_2 emissions. Inputs are capital, labor and energy use. We use Penn World Table data version 7, by Heston et al. (2011) for data on constant prices GDP, investment flows, and labor input. The GDP and investment flows data are in 2005 constant U.S. dollars, and the capital stocks are constructed by means of the perpetual inventory method. We use the World Bank Development Indicators database for data on the CO_2 emissions and energy use, variable codes being EG.USE.COMM.KT.OE for energy use, and EN.ATM.CO2E.KT for CO_2 emissions. After deleting the missing observations, we end up with a panel of 255 observations on fifteen countries and years between 1995 and 2011. <Table 1> below presents summary statistics.

<Table 1> Summary Statistics, Mean Values

	GDP (bn USD)	CO_2 (kt)	Population (mn people)	Capital (bn USD)	Energy use (kt of oil equivalent)
Austria	283.99	67351	8.15	719.04	31090
Belgium	335.91	112202	10.32	859.99	57907
Denmark	179.32	53535	5.39	412.44	19702
Finland	155.56	60038	5.19	383.65	34302
France	1871.50	388751	62.31	4005.94	260107
Germany	2596.81	830298	82.27	6136.73	341537
Greece	246.47	94811	10.65	604.27	27908
Hungary	148.06	59508	10.13	314.39	26225
Ireland	142.32	41403	3.99	302.41	13848
Italy	1687.91	463280	58.74	4420.39	173087
Netherlands	573.14	176656	16.21	1185.57	76729
Poland	484.51	329449	38.76	824.79	96161
Spain	1132.06	315656	42.69	2879.35	126318
Sweden	283.94	52484	8.99	508.73	50636
UK	1885.89	543156	60.05	3148.59	217655

Source: Heston et al. (2011) Penn World Tables, World Bank Development Indicators (2011); 2005 constant USD where applicable

The summary statistics reveal the Germany, UK and Italy to be major pollutants among the fifteen European countries amounting for 53% of the total CO2 emissions with Germany accounting for more than 22%. The emissions' amount is strongly related to those countries' GDP (the R^2 in a simple OLS regression being 96%).

2. Empirical Results

<Table 2> presents our estimates of the directional distance function parameters for the projection vectors $\varphi_i, = 1..5$ employed at the first two iterations and depicted in <Figure 1>:

<Table 2> Directional Output Distance Function Parameter Estimates

Parameter	φ_3	φ_5	φ_2	φ_4	φ_1
Constant	19.76	-15.24	-18.60	-31.55	-13.50
GDP	-0.09	-0.006	-0.010	-0.89	-0.0003
CO_2	1.84	1.15	1.40	0.74	1.01
K	0.03	-0.03	-0.04	0.29	-0.03
L	-11.70	-1.43	-1.74	15.51	-1.24
E	-0.12	-0.65	-0.80	-1.19	-0.57
K^2	-0.0001	0	0	-0.0002	0
L^2	0.35	0.18	0.23	-0.86	0.14
E^2	0.04	0.06	0.07	0.07	0.05
$CO_2{}^2$	0	0	0	0	0
Y^2	0	0	0	0	0
Y x CO_2	0	0	0	0	0
K x L	0.005	0.008	0.01	0.01	0.007
K x E	0.003	-0.001	-0.001	0.008	-0.001
L x E	-0.38	-0.303	-0.38	-0.63	-0.26
K x Y	0	0	0	-0.00025	0
K x CO_2	0	0	0	-0.0009	0
Y x E	-0.004	0	0	-0.005	0
CO_2 x E	-0.007	0	0	-0.03	0
Y x L	0.01	0.0002	0.0004	0.03	0
CO_2 x L	0.02	0.0001	0.0004	0.21	0

Note: All variables are in logs. Y=GDP, CO2 emissions are measured in terms of equivalent kilotons of oil, K=capital, L=population,E=consumption of energy in terms of equivalent kilotons of oil.

The estimated parameters of the distance function differ a lot depending on the value of the projection vector. The marginal abatement costs vary considerably as well depending on the projection vector, as evidenced by the left-hand part of <Table 3> below.

<Table 3> Marginal Abatement Costs Measured as Shadow Prices Ratio and as a Slope of the Iterated PPF, thousand USD per ton of CO_2 emissions

	Shadow Prices Ratio				Estimated PPF Slope			
	Median	SD	Min	Max	Median	SD	Min	Max
φ_3	NA				3.91	0.14	3.71	4.50
φ_5	15.84	575.9	0.38	6854	5.87	1.55	0.08	6.78
φ_2	176.82	12600	137.9	196929	4.87	1.19	0.92	5.44
φ_4	237.29	1691	187	13368	4.61	0.97	1.34	4.87
φ_1	NA				4.45	0.86	1.57	4.69

Note: The five vectors referred to in this table are depicted in <Figure 1>.

The right-hand part of <Table 3> reports estimates of the CO_2 marginal abatement costs obtained as a result of estimating the PPF slope based on the set of efficient projections on the PPF of the observed combinations of CO_2 emissions and GDP. While the same projection vectors are used for the estimation, the resulting marginal abatement costs vary considerably less.

In <Table 4> we report the estimates of CO_2 marginal abatement costs obtained as a result of applying the iteration process described in Section III. The iteration process converged after four iterations for $\delta = 0.001$ in (7) with seventeen projection vectors, resulting in the estimated PPF of $\hat{y}_{it} = 7.446 b_{it} - 0.004 b_{it}^2$ where $\hat{y}_{it}$ is the estimated "vertical" projection of the observed GDP level y_{it} onto the PPF, and b_{it} are the observed levels of CO_2 emissions.

<Table 4> CO_2 Reduction in the European Countries

	CO_2 Emissions in 2008, Mton	MAC= PPF Slope, Th USD/ton CO_2	$\frac{GDP}{CO_2}$	Costs of 10% CO_2 Reduction Relative to 2008, % of GDP
Austria	67.73	6.81	4.32	14.29%
Belgium	104.88	6.49	3.4	18.09%
Denmark	46.02	7.00	3.9	16.38%
Finland	56.51	6.89	2.88	20.89%
France	376.99	4.01	5.24	7.34%
Germany	786.66	0.22	3.37	0.61%
Greece	97.81	6.57	2.89	21.85%
Hungary	54.64	6.94	2.99	21.77%
Ireland	43.6	7.06	4.01	17.03%
Italy	445.12	3.29	3.84	8.15%
Netherlands	173.75	5.9	3.59	15.67%
Poland	316.07	4.65	1.75	24.20%
Spain	329.29	4.34	3.7	10.68%
Sweden	49.05	6.99	6.27	10.47%
UK	522.86	2.63	3.93	6.37%

Source: World Development Indicators (2012) and authors' calculations; MAC stands for "marginal abatement costs"

Germany features a conspicuously low level of the CO_2 marginal abatement costs evaluated at $220 per ton of reduced CO_2 emissions. In the fourteen other countries in our sample these costs are estimated to be significantly higher at the average of 5.7 thousand USD per one ton of CO_2. These estimates suggest Germany as a prime candidate for bearing most of the brunt of the CO_2 reduction in Europe, while Denmark, Hungary and Ireland will find it most expensive to reduce CO_2. The opportunity costs of CO_2 reduction measured in terms of the GDP share that has to be sacrificed for a given reduction level appear to be rather significant even for a modest reduction of 10% of the 2008 levels. Thus, such a reduction will reduce Poland's GDP by a quarter, with a similar toll taken on Finland, Hungary and Greece. Germany is likely to forego less than one percent of its GDP in the same case. Of course, our estimates of the GDP percentage opportunity costs are based on the assumption that the marginal abatement costs of CO_2 reduction are constant along the reduction path. However, this assumption seems realistic at least for modest reduction levels since the estimated PPF is rather flat.

Our estimates suggest that, even if the Kyoto goals of reducing the CO_2 emissions by 20% were met by 2008, a further reduction of as little as 10% will entail significant opportunity costs in terms of the foregone GDP. Yet, such further reductions are being planned, as was recently indicated by the Communication of the European Commission made in 2010 that stated the EU’s willingness to undertake an additional reduction of 10% so as to achieve a reduction of 30% relative to the 1990 level by 2020. Given the impressive GDP opportunity costs likely to be associated with such a reduction, the latter appears to be rather improbable.

V. Conclusions

This study offers a new method of estimating the marginal abatement costs of an undesirable output by directly computing the slope of the production possibilities frontier on the basis of the efficient projections estimated by the directional distance function method. We are essentially extending the estimation framework due to Fare et al. (2005) by adding more projection vectors pointing to the PPF in the “North-Western” direction, rather than relying on just one such vector.

Since for the rather flat or horizontal projection vectors estimating the marginal abatement costs on the basis of the duality theory is problematic due to the violated conditions of the theorem on implicit function differentiation, we suggest computing the marginal abatement costs as the magnitude of the slope of the PPF. We suggest estimating the latter by an iterative procedure that uses increasing numbers of the projection vectors so that a larger set of efficient projection points is employed to compute the PPF at each iteration. The iteration process stops once the norm of the difference between the two consecutive PPFs becomes negligibly small.

We use a sample of fifteen developed Western European countries for the period of 1995-2011 to test our approach. We find that the marginal abatement costs estimated according to a set of alternative output projection vectors differ significantly with each other, exacerbating the problem of choosing the 'right' projection vector. In contrast, computing the marginal abatement costs as slopes of the estimated PPF result in much smaller variation between the estimates corresponding to different projection vectors. Evaluating the total opportunity costs of reducing the CO_2 emission levels by 10% relative to their 2008 level, we find that such a reduction will entail significant losses in terms of the GDP, which is consistent with the findings in Fare et al. (2012).

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Appendix

Derivation of the constraints on the distance function to satisfy the translation property.

For the translation property to hold, the distance function has to satisfy

$$D\left(y+\alpha g_y, b-\alpha g_b \middle| g_y, g_b\right)-D\left(y, b \middle| g_y, g_b\right)=-\alpha \tag{A1}$$

Consider the left-hand side of (A1) :

$$\begin{aligned}
& D\left(y+\alpha g_y, x-\alpha g_b \middle| g_y, g_b\right)-D\left(y, x \middle| g_y, g_b\right)= \\
& =\alpha_0+\sum_{n=1}^{3} \alpha_n x_n+\beta_1\left(y+\alpha g_y\right)+\gamma_1\left(b-\alpha g_b\right)+\sum_{n=1}^{3} \sum_{n'=1}^{3} x_n x_{n'}+ \\
& \beta_2\left(y+\alpha g_y\right)^2+\gamma_2\left(b-\alpha g_b\right)^2+\sum_{n=1}^{3} v_n x_n\left(b-\alpha g_b\right)+ \\
& \sum_{n=1}^{3} \delta_n x_n\left(y+\alpha g_y\right)+\mu\left(y+\alpha g_y\right)\left(b-\alpha g_b\right)-\alpha_0-\sum_{n=1}^{3} \alpha_n x_n- \\
& \beta_1 y-\gamma_1 b-\sum_{n=1}^{3} \sum_{n'=1}^{3} x_n x_{n'}-\beta_2 y^2-\gamma_2 b^2-\mu y b-\sum_{n=1}^{3} v_n x_n b-\sum_{n=1}^{3} \delta_n x_n y= \\
& \alpha\left(\beta_1 g_y-\gamma_1 g_b\right)+\alpha \sum_{n=1}^{3} x_n\left(\delta_n g_y-v_n g_b\right)+\alpha y\left(2 \beta_2 g_y-\mu g_b\right)+ \\
& \alpha b\left(\mu g_y-2 \gamma_2 g_b\right)+\alpha^2\left(\beta_2 g_y^2+\gamma_2 g_b^2-\mu g_y g_b\right)
\end{aligned} \tag{A2}$$

The sum on the right-hand side of (A2) must equal $-\alpha$ according to (A1), which implies that all expressions in the round brackets but the first one must equal zero, i.e.

$$\begin{cases} \beta_1 g_y - \gamma_1 g_b = -1 \\ \delta_n g_y - \nu_n g_b = 0, n = 1..3 \\ 2\beta_2 g_y = \mu g_b \\ 2\gamma_2 g_b = \mu g_y \\ \beta_2 g_y^2 + \gamma_2 g_b^2 = \mu g_y g_b \end{cases} \tag{A3}$$

which is exactly (DIR5).

For any direction vector $(g_y, g_b) = (\sin\varphi, |\cos\varphi|)$ the set of constraints (A3) will be different, implying a different set of points on the efficient production frontier generated according to $y^* = y + D(x, y | g_y, g_b)$ and $b^* = b - D(x, y | g_y, g_b)$, where asterisk denotes efficient observations.