Commun. Korean Math. Soc. **28** (2013), No. 2, pp. 371–381
http://dx.doi.org/10.4134/CKMS.2013.28.2.371

ON θ-$(\mathcal{G}, \mathcal{H})$-CONTINUOUS FUNCTIONS IN GRILL TOPOLOGICAL SPACES

Ahmad Al-Omari

Abstract. In this paper, we investigate some properties of θ-$(\mathcal{G}, \mathcal{H})$-continuous functions in a grill topological spaces. Moreover, the relationships with other related functions are investigated.

1. Introduction and preliminaries

In 1968, Veličko [10] introduced the class of θ-open sets. A set A is said to be θ-open [10] if every point of A has an open neighborhood whose closure is contained in A. The θ-interior [10] of a subset A of X is the union of all θ-open subsets of A and is denoted by $Int_\theta(A)$. Naturally, the complement of a θ-open set is said to be θ-closed. Equivalently $Cl_\theta(A) = \{x \in X : Cl(U) \cap A \neq \phi$ for every $U \in \tau(x)\}$ and a set A is θ-closed if and only if $A = Cl_\theta(A)$. Note that all θ-open sets form a topology on X which coarser than τ and is denoted by τ_θ and that a space (X, τ) is regular if and only if $\tau = \tau_\theta$. Note also that the θ-closure of a given set need not be a θ-closed set.

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A in (X, τ), respectively. The power set of X will be denoted by $\mathcal{P}(X)$. A subcollection $\mathcal{G}$ of $\mathcal{P}(X)$ is called a grill [3] on X if $\mathcal{G}$ satisfies the following conditions:

(1) $A \in \mathcal{G}$ and $A \subseteq B$ implies that $B \in \mathcal{G}$,
(2) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

For any point x of a topological space (X, τ), $\tau(x)$ denotes the collection of all open neighborhoods of x.

Definition 1.1 ([9])**.** Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. A mapping $\Phi : \mathcal{P}(X) \to \mathcal{P}(X)$ is defined as follows: $\Phi(A) = \Phi_{\mathcal{G}}(A, \tau) = \{x \in X : A \cap U \in \mathcal{G}$ for all $U \in \tau(x)\}$ for each $A \in \mathcal{P}(X)$. The mapping Φ is called the operator associated with the grill $\mathcal{G}$ and the topology τ.

Received April 20, 2012; Revised January, 2013.
2010 *Mathematics Subject Classification.* Primary 54A05, 54C10.
Key words and phrases. grill topological space, θ-continuous, weakly $\mathcal{H}$-continuous, strongly θ-continuous, θ-$(\mathcal{G}, \mathcal{H})$-continuous.

Proposition 1.2 ([9]). *Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. Then for all $A, B \subseteq X$ the following properties hold*:

1. $A \subseteq B$ *implies that* $\Phi(A) \subseteq \Phi(B)$.
2. $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$.
3. $\Phi(\Phi(A)) \subseteq \Phi(A) = Cl(\Phi(A)) \subseteq Cl(A)$.
4. *If* $U \in \tau$, *then* $U \cap \Phi(A) \subseteq \Phi(U \cap A)$.

Let $\mathcal{G}$ be a grill on a space X. Then we define a map $\Psi : \mathcal{P}(X) \to \mathcal{P}(X)$ by $\Psi(A) = A \cup \Phi(A)$ for all $A \in \mathcal{P}(X)$. The map Ψ is a Kuratowski closure axiom. Corresponding to a grill $\mathcal{G}$ on a topological space (X, τ), there exists a unique topology $\tau_{\mathcal{G}}$ on X given by $\tau_{\mathcal{G}} = \{U \subseteq X : \Psi(X - U) = X - U\}$, where for any $A \subseteq X$, $\Psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}\text{-}Cl(A)$ is $\tau_{\mathcal{G}}$-closure of A and the $\tau_{\mathcal{G}}$-interior of A is denoted by $\tau_{\mathcal{G}}\text{-}Int(A)$. For any grill $\mathcal{G}$ on a topological space (X, τ), $\tau \subseteq \tau_{\mathcal{G}}$. If (X, τ) is a topological space with a grill $\mathcal{G}$ on X, then we call it a grill topological space and denote it by $(X, \tau, \mathcal{G})$.

Definition 1.3 ([2]). Let $(X, \tau, \mathcal{G})$ be a grill topological space and $A \subseteq X$. A point x of X is called a θ-$\mathcal{G}$-cluster point of A if $\Psi(U) \cap A \neq \phi$ for every open set $U \in \tau(x)$. The set of all θ-$\mathcal{G}$-cluster points of A is called the θ-$\mathcal{G}$-closure of A and is denoted by $\tau_{\mathcal{G}}\text{-}Cl_\theta(A)$.

A subset A is said to be θ-$\mathcal{G}$-closed if $\tau_{\mathcal{G}}\text{-}Cl_\theta(A) = A$. The complement of a θ-$\mathcal{G}$-closed set is called a θ-$\mathcal{G}$-open set.

Definition 1.4 ([2]). A point $x \in (X, \tau, \mathcal{G})$ is said to be a θ-$\mathcal{G}$-interior point of A if there exists an open set U containing x such that $U \subseteq \Psi(U) \subseteq A$. The set of all θ-$\mathcal{G}$-interior points of A is said to be the θ-$\mathcal{G}$-interior of A and denoted by $\tau_{\mathcal{G}}\text{-}Int_\theta(A)$.

Definition 1.5. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be θ-continuous [4] (resp. strongly θ-continuous [7], weakly continuous [6]) if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(Cl(U)) \subseteq Cl(V)$ (resp. $f(Cl(U)) \subseteq V$, $f(U) \subseteq Cl(V)$).

Definition 1.6. A function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is said to be θ-$(\mathcal{G}, \mathcal{H})$-continu-ous (resp. weakly $\mathcal{H}$-continuous [1]) if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists an open set U in X containing x such that $f(\Psi(U)) \subseteq \Psi(V)$ (resp. $f(U) \subseteq \Psi(V)$).

By the above definitions, we have the following diagram and none of these implications is reversible

$$\begin{array}{ccccc}
\text{strongly } \theta\text{-continuous} & \longrightarrow & \text{continuous} & \longrightarrow & \theta\text{-continuous} \\
\downarrow & & \downarrow & & \downarrow \\
\theta\text{-}(\mathcal{G}, \mathcal{H})\text{-continuous} & \longrightarrow & \text{weakly } \mathcal{H}\text{-continuous} & \longrightarrow & \text{weakly continuous}
\end{array}$$

Remark 1.7. (1) In Example 5.1 of [1], it is shown that not every weakly continuous function is weakly $\mathcal{H}$-continuous.

(2) The following strict implications are well-known:

$$\begin{array}{ccccc} \text{strongly } \theta\text{-continuous} & \longrightarrow & \text{continuous} & \longrightarrow & \theta\text{-continuous} \\ & & & & \downarrow \\ & & & & \text{weakly continuous} \end{array}$$

Example 1.8. Let $X = \{1,2,3,4\}$, $\tau = \{X, \phi, \{1,2,3\}, \{3\}, \{3,4\}\}$ with $\mathcal{G} = \{X, \{2,3,4\}, \{1,3,4\}, \{3,4\}\}$ and $Y = \{a,b,c,d\}$, $\sigma = \{Y, \phi, \{a,b\}, \{b\}, \{d\}, \{b,d\}, \{a,b,d\}, \{b,c,d\}\}$ with $\mathcal{H} = \{Y, \{b\}, \{d\}\}$. We define a function $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ as $f = \{(1,a),(2,b),(3,c),(4,d)\}$. Then f is weakly $\mathcal{H}$-continuous but not θ-$(\mathcal{G},\mathcal{H})$-continuous. First we show that f is weakly $\mathcal{H}$-continuous.

(1) Let $1 \in X$ and $V = \{a,b\}$, $V = \{a,b,d\}$ or $V = Y \in \sigma$ such that $f(1) = a \in V$. Then there exists an open set $U = \{1,2,3\} \subseteq X$ such that $1 \in U$ and $f(U) \subseteq \Psi(V)$ since $\Psi(\{a,b\}) = \{a,b,c\}$ and $\Psi(\{a,b,d\}) = \Psi(Y) = Y$.

(2) Let $2 \in X$ and $V = \{b\}$, $V = \{a,b\}$, $V = \{b,d\}$, $V = \{a,b,d\}$, $V = \{b,c,d\}$ or $V = Y \in \sigma$ such that $f(2) = b \in V$. Then there exists an open set $U = \{1,2,3\} \subseteq X$ such that $2 \in U$ and $f(U) \subseteq \Psi(V)$ since $\Psi(\{b\}) = \Psi(\{a,b\}) = \{a,b,c\}$ and $\Psi(\{b,d\}) = \Psi(\{a,b,d\}) = \Psi(\{b,c,d\}) = \Psi(Y) = Y$.

(3) Let $3 \in X$ and $V = \{b,c,d\}$ or $V = Y \in \sigma$ such that $f(3) = c \in V$. Then there exists an open set $U = \{3\} \subseteq X$ such that $3 \in U$ and $f(U) \subseteq \Psi(V)$ since $\Psi(\{b,c,d\}) = \Psi(Y) = Y$.

(4) Let $4 \in X$ and $V = \{d\}$, $V = \{b,d\}$, $V = \{a,b,d\}$, $V = \{b,c,d\}$ or $V = Y \in \sigma$ such that $f(4) = d \in V$. Then there exists an open set $U = \{3,4\} \subseteq X$ such that $4 \in U$ and $f(U) \subseteq \Psi(V)$ since $\Psi(\{d\}) = \{c,d\}$ and $\Psi(\{b,d\}) = \Psi(\{a,b,d\}) = \Psi(\{b,c,d\}) = \Psi(Y) = Y$.

Then by (1), (2), (3) and (4) f is weakly $\mathcal{H}$-continuous.

Now we show that $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is not θ-$(\mathcal{G},\mathcal{H})$-continuous. Let $1 \in X$ and $V = \{a,b\} \in \sigma$ such that $f(1) = a \in V \in \sigma$. But, for every open set $U \subseteq X$ such that $1 \in U$, where $U = \{1,2,3\}$ or $U = X$, $\Psi(U) = X$. Then $f(\Psi(U)) = Y \nsubseteq \Psi(V) = \{a,b,c\}$. Therefore, $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is not θ-$(\mathcal{G},\mathcal{H})$-continuous.

Example 1.9. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{b,c\}\}$ with $\mathcal{G} = \{X, \{b,c\}\}$ and $Y = \{b,c\}$, $\sigma = \{Y, \phi, \{c\}\}$ with $\mathcal{H} = \{Y, \{c\}\}$. We define a function $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ as $f = \{(a,b),(b,c),(c,b)\}$. Then f is θ-$(\mathcal{G},\mathcal{H})$-continuous but not continuous.

(1) Let $a \in X$ and $V = Y \in \sigma$ such that $f(a) = b \in V$. Then there exists an open set $U = X \in \tau$ containing a such that $f(\Psi(U)) \subseteq \Psi(V) = Y$.

(2) Let $b \in X$ and $V = \{c\}$ or $V = Y$ such that $f(b) = c \in V$. Then there exists an open set $U = \{b, c\}$ or $U = X$ containing b such that $f(\Psi(U)) \subseteq \Psi(V) = Y$.
(3) Let $c \in X$ and $V = Y$ such that $f(c) = b \in V$. Then there exists an open set $U = \{b, c\}$ or $U = X$ containing c such that $f(\Psi(U)) \subseteq \Psi(V) = Y$.

By (1), (2) and (3), f is θ-$(\mathcal{G}, \mathcal{H})$-continuous. On the other hand, let $b \in X$ and $V = \{c\} \in \sigma$ such that $f(b) = c \in V \in \sigma$. But, for every open set $U \subseteq X$ such that $b \in U$, where $U = \{b, c\}$ or $U = X$. Then $f(U) = Y \nsubseteq V = \{c\}$. Therefore, $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is not continuous.

2. Characterizations of θ-$(\mathcal{G}, \mathcal{H})$-continuous functions

In this section, we obtain several characterizations of θ-$(\mathcal{G}, \mathcal{H})$-continuous functions in grill topological spaces.

Theorem 2.1. *For a function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$, the following properties are equivalent*:

(1) *f is θ-$(\mathcal{G}, \mathcal{H})$-continuous*;
(2) *$\tau_{\mathcal{G}}$-$Cl_\theta(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\mathcal{H}}$-$Cl_\theta(B))$ for every subset B of Y*;
(3) *$f(\tau_{\mathcal{G}}$-$Cl_\theta(A)) \subseteq \sigma_{\mathcal{H}}$-$Cl_\theta(f(A))$ for every subset A of X.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Suppose that $x \notin f^{-1}(\sigma_{\mathcal{H}}$-$Cl_\theta(B))$. Then $f(x) \notin \sigma_{\mathcal{H}}$-$Cl_\theta(B)$ and there exists an open set V containing $f(x)$ such that $\Psi(V) \cap B = \phi$. Since f is θ-$(\mathcal{G}, \mathcal{H})$-continuous, there exists an open set U containing x such that $f(\Psi(U)) \subseteq \Psi(V)$. Therefore, we have $f(\Psi(U)) \cap B = \phi$ and $\Psi(U) \cap f^{-1}(B) = \phi$. This shows that $x \notin \tau_{\mathcal{G}}$-$Cl_\theta(f^{-1}(B))$. Thus, we obtain $\tau_{\mathcal{G}}$-$Cl_\theta(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\mathcal{H}}$-$Cl_\theta(B))$.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be an open set of Y containing $f(x)$. Then we have $\Psi(V) \cap (Y - \Psi(V)) = \phi$ and $f(x) \notin \sigma_{\mathcal{H}}$-$Cl_\theta(Y - \Psi(V))$. Therefore, $x \notin f^{-1}(\sigma_{\mathcal{H}}$-$Cl_\theta(Y - \Psi(V)))$ and by (2) we have $x \notin \tau_{\mathcal{G}}$-$Cl_\theta(f^{-1}(Y - \Psi(V)))$. There exists an open set U containing x such that $\Psi(U) \cap f^{-1}(Y - \Psi(V)) = \phi$ and hence $f(\Psi(U)) \subseteq \Psi(V)$. Therefore, f is θ-$(\mathcal{G}, \mathcal{H})$-continuous.

(2) $\Rightarrow$ (3): Let A be any subset of X. Then we have

$$\tau_{\mathcal{G}}\text{-}Cl_\theta(A) \subseteq \tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(f(A))) \subseteq f^{-1}(\sigma_{\mathcal{H}}\text{-}Cl_\theta(f(A)))$$

and hence $f(\tau_{\mathcal{G}}$-$Cl_\theta(A)) \subseteq \sigma_{\mathcal{H}}$-$Cl_\theta(f(A))$.

(3) $\Rightarrow$ (2): Let B be a subset of Y. We have

$$f(\tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(B))) \subseteq \sigma_{\mathcal{H}}\text{-}Cl_\theta(f(f^{-1}(B))) \subseteq \sigma_{\mathcal{H}}\text{-}Cl_\theta(B)$$

and hence $\tau_{\mathcal{G}}$-$Cl_\theta(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\mathcal{H}}$-$Cl_\theta(B))$. □

Definition 2.2 ([1])**.** A grill topological space $(X, \tau, \mathcal{G})$ is called an $A\mathcal{G}$-space if $Cl(A) \subseteq \Phi(A)$ for every open set $A \subseteq X$.

Theorem 2.3 ([1])**.** *For a grill topological space $(Y, \sigma, \mathcal{G})$, the following properties are equivalent*:

(1) $(Y,\sigma,\mathcal{G})$ *is an* $A\mathcal{G}$*-space*;
(2) $\sigma \setminus \{\phi\} \subseteq \mathcal{G}$;
(3) $\Phi(V) = Cl(V) = \Psi(V)$ *for every* $V \in \sigma$.

Theorem 2.4. *For a function* $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$*, the following implications*: (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) *hold. Moreover, the implication* (4) $\Rightarrow$ (1) *holds if* $(Y,\sigma,\mathcal{H})$ *is an* $A\mathcal{H}$*-space.*

(1) f *is* θ-$(\mathcal{G},\mathcal{H})$*-continuous*;
(2) $f^{-1}(V) \subseteq \tau_{\mathcal{G}}\text{-}Int_\theta(f^{-1}(\Psi(V)))$ *for every open set* V *of* Y;
(3) $\tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$ *for every open set* V *of* Y;
(4) *For each* $x \in X$ *and each open set* V *of* Y *containing* $f(x)$*, there exists an open set* U *of* X *containing* x *such that* $f(\Psi(U)) \subseteq Cl(V)$.

Proof. (1) $\Rightarrow$ (2): Suppose that V is any open set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$ and there exists an open set U containing x such that $f(\Psi(U)) \subseteq \Psi(V)$. Therefore, $x \in U \subseteq \Psi(U) \subseteq f^{-1}(\Psi(V))$. This shows that $x \in \tau_{\mathcal{G}}\text{-}Int_\theta(f^{-1}(\Psi(V)))$. Therefore, we obtain $f^{-1}(V) \subseteq \tau_{\mathcal{G}}\text{-}Int_\theta(f^{-1}(\Psi(V)))$.

(2) $\Rightarrow$ (1): Let $x \in X$ and $V \in \sigma$ containing $f(x)$. Then, by (2) $f^{-1}(V) \subseteq \tau_{\mathcal{G}}\text{-}Int_\theta(f^{-1}(\Psi(V)))$. Since $x \in f^{-1}(V)$, there exists an open set U containing x such that $\Psi(U) \subseteq f^{-1}(\Psi(V))$. Therefore, $f(\Psi(U)) \subseteq \Psi(V)$ and hence f is θ-$(\mathcal{G},\mathcal{H})$-continuous.

(2) $\Rightarrow$ (3): Suppose that V is any open set of Y and $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \phi$; hence $\Psi(W) \cap V \subseteq Cl(W) \cap V = \phi$. Therefore, we have $f^{-1}(\Psi(W)) \cap f^{-1}(V) = \phi$. Since $x \in f^{-1}(W)$, by (2) $x \in \tau_{\mathcal{G}}\text{-}Int_\theta(f^{-1}(\Psi(W)))$. There exists an open set U containing x such that $\Psi(U) \subseteq f^{-1}(\Psi(W))$. Thus we have $\Psi(U) \cap f^{-1}(V) = \phi$ and hence $x \notin \tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(V))$. This shows that $\tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.

(3) $\Rightarrow$ (4): Suppose that $x \in X$ and V is any open set of Y containing $f(x)$. Then $V \cap (Y - Cl(V)) = \phi$ and $f(x) \notin Cl(Y - Cl(V))$. Therefore $x \notin f^{-1}(Cl(Y - Cl(V)))$ and by (3) $x \notin \tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(Y - Cl(V)))$. There exists an open set U containing x such that $\Psi(U) \cap f^{-1}(Y - Cl(V)) = \phi$. Therefore, we obtain $f(\Psi(U)) \subseteq Cl(V)$.

(4) $\Rightarrow$ (3): Let V be any open set of Y. Suppose that $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \phi$. By (4), there exists an open set U containing x such that $f(\Psi(U)) \subseteq Cl(W)$. Since $V \in \sigma$, $Cl(W) \cap V = \phi$ and $f(\Psi(U)) \cap V \subseteq Cl(W) \cap V = \phi$. Therefore, $\Psi(U) \cap f^{-1}(V) = \phi$ and hence $x \notin \tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(V))$. This shows that $\tau_{\mathcal{G}}\text{-}Cl_\theta(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.

(4) $\Rightarrow$ (1): Since $(Y,\sigma,\mathcal{H})$ is an $A\mathcal{H}$-space, $Cl(V) \subseteq \Psi(V)$ for every open set V of Y and hence f is θ-$(\mathcal{G},\mathcal{H})$-continuous. □

Proposition 2.5. *A function* $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ *from an* $A\mathcal{G}$*-space to an* $A\mathcal{H}$*-space is* θ-$(\mathcal{G},\mathcal{H})$*-continuous if and only if it is* θ*-continuous.*

Proof. This follows from Theorem 2.3. □

3. Some properties of θ-$(\mathcal{G},\mathcal{H})$-continuous functions

A subset A of a grill topological space $(X,\tau,\mathcal{G})$ is said to be pre-$\mathcal{G}$-open [5] if $A \subseteq Int(\Psi(A))$. A function $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is said to be pre-$\mathcal{G}$-continuous [5] if the inverse image of every open set of Y is pre-$\mathcal{G}$-open in X.

Theorem 3.1. *If $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is a pre-$\mathcal{G}$-continuous function and $\Psi(f^{-1}(U)) \subseteq f^{-1}(\Psi(U))$ for every open set U in Y, then f is θ-$(\mathcal{G},\mathcal{H})$-continuous.*

Proof. Let $x \in X$ and U be an open set in Y containing $f(x)$. By hypothesis, $\Psi(f^{-1}(U)) \subseteq f^{-1}(\Psi(U))$. Since f is pre-$\mathcal{G}$-continuous, $f^{-1}(U)$ is pre-$\mathcal{G}$-open in X and so $f^{-1}(U) \subseteq Int(\Psi(f^{-1}(U)))$. Since $x \in f^{-1}(U) \subseteq Int(\Psi(f^{-1}(U)))$, there exists an open set V containing x such that $x \in V \subseteq \Psi(V) \subseteq \Psi(f^{-1}(U)) \subseteq f^{-1}(\Psi(U))$ and so $f(\Psi(V)) \subseteq \Psi(U)$ which implies that f is θ-$(\mathcal{G},\mathcal{H})$-continuous. □

Lemma 3.2 ([1])**.** (1) *A function $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is weakly $\mathcal{H}$-continuous if and only if for each open set V in Y, $f^{-1}(V) \subseteq Int(f^{-1}(\Psi(V)))$.*

(2) *If a grill space $(Y,\sigma,\mathcal{H})$ is an $A\mathcal{H}$-space and a function $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is weakly $\mathcal{H}$-continuous, then $Cl(f^{-1}(V)) \subseteq f^{-1}(\Psi(V))$ for every open set V of Y.*

Theorem 3.3. *Let $(Y,\sigma,\mathcal{H})$ be an $A\mathcal{H}$-space. For a function $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$, the following properties are equivalent*:

(1) *f is weakly $\mathcal{H}$-continuous*;
(2) *$Cl(f^{-1}(V)) \subseteq f^{-1}(\Psi(V))$ for every open set V of Y*;
(3) *f is weakly continuous.*

Proof. (1) $\Rightarrow$ (2): The proof follows from Lemma 3.2(2).

(2) $\Rightarrow$ (3): Let V be any open set of Y. Since $(Y,\sigma,\mathcal{H})$ is an $A\mathcal{H}$-space, by (2) we have $Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$. It follows from Theorem 7 of [8] that f is weakly continuous.

(3) $\Rightarrow$ (1): Let f be weakly continuous. By Theorem 1 of [6], $f^{-1}(V) \subseteq Int(f^{-1}(Cl\ (V)))$ for every open set V of Y. Since $(Y,\sigma,\mathcal{H})$ is an $A\mathcal{H}$-space, $Cl(V) = \Psi(V)$ and we have $f^{-1}(V) \subseteq Int(f^{-1}(\Psi(V)))$. Therefore, by Lemma 3.2(1) f is weakly $\mathcal{H}$-continuous. □

The following corollary follows from Lemma 3.2 and Theorems 3.1 and 3.3.

Corollary 3.4. *Let $f:(X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ be pre-$\mathcal{G}$-continuous and $(Y,\sigma,\mathcal{H})$ is an $A\mathcal{H}$-space. Then the following properties are equivalent*:

(1) *f is θ-$(\mathcal{G},\mathcal{H})$-continuous*;
(2) *$\Psi(f^{-1}(V)) \subseteq f^{-1}(\Psi(V))$ for every open set V in Y*;
(3) *$Cl(f^{-1}(V)) \subseteq f^{-1}(\Psi(V))$ for every open set V in Y*;
(4) *f is weakly $\mathcal{H}$-continuous.*

Definition 3.5. A grill topological space $(X,\tau,\mathcal{G})$ is said to be θ-$\mathcal{G}$-T_2 (resp. $\mathcal{G}$-Urysohn) if for each distinct points $x,y \in X$, there exist two θ-$\mathcal{G}$-open (resp. open) sets $U,V \in X$ containing x and y, respectively, such that $U \cap V = \phi$ (resp. $\Psi(U) \cap \Psi(V) = \phi$).

Theorem 3.6. *If $f,g : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ are θ-$(\mathcal{G},\mathcal{H})$-continuous functions and $(Y,\sigma,\mathcal{H})$ is $\mathcal{H}$-Urysohn, then $A = \{x \in X : f(x) = g(x)\}$ is a θ-$\mathcal{G}$-closed set of $(X,\tau,\mathcal{G})$.*

Proof. We prove that $X - A$ is a θ-$\mathcal{G}$-open set. Let $x \in X - A$. Then $f(x) \neq g(x)$. Since Y is $\mathcal{H}$-Urysohn, there exist open sets V_1 and V_2 containing $f(x)$ and $g(x)$, respectively, such that $\Psi(V_1) \cap \Psi(V_2) = \phi$. Since f and g are θ-$(\mathcal{G},\mathcal{H})$-continuous, there exists an open set U containing x such that $f(\Psi(U)) \subseteq \Psi(V_1)$ and $g(\Psi(U)) \subseteq \Psi(V_2)$. Hence we obtain that $\Psi(U) \subseteq f^{-1}(\Psi(V_1))$ and $\Psi(U) \subseteq g^{-1}(\Psi(V_2))$. From here we have $\Psi(U) \subseteq f^{-1}(\Psi(V_1)) \cap g^{-1}(\Psi(V_2))$. Moreover $f^{-1}(\Psi(V_1)) \cap g^{-1}(\Psi(V_2)) \subseteq X - A$. This shows that $X - A$ is θ-$\mathcal{G}$-open. □

Definition 3.7. A grill topological space $(X,\tau,\mathcal{G})$ is said to be $\mathcal{G}$-regular if for each closed set F and each point $x \in X - F$, there exist an open set V and a $\tau_{\mathcal{G}}$-open set $U \in \tau_{\mathcal{G}}$ such that $x \in V$, $F \subseteq U$ and $U \cap V = \phi$.

Example 3.8. Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}\}$ and $\mathcal{G} = \{X\}$. Then $(X,\tau,\mathcal{G})$ is an $\mathcal{G}$-regular space which is not regular.

Lemma 3.9. *A grill topological space $(X,\tau,\mathcal{G})$ is $\mathcal{G}$-regular if and only if for each open set U containing x there exists an open set V such that $x \in V \subseteq \Psi(V) \subseteq U$.*

Proposition 3.10. *Let $(X,\tau,\mathcal{G})$ be an $\mathcal{G}$-regular space. Then $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is θ-$(\mathcal{G},\mathcal{H})$-continuous if and only if it is weakly $\mathcal{H}$-continuous.*

Proof. Every θ-$(\mathcal{G},\mathcal{H})$-continuous function is weakly $\mathcal{H}$-continuous. Suppose that f is weakly $\mathcal{H}$-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then, there exists an open set U containing x such that $f(U) \subseteq \Psi(V)$. Since X is $\mathcal{G}$-regular, by Lemma 3.9 there exists an open set W such that $x \in W \subseteq \Psi(W) \subseteq U$. Therefore, we obtain $f(\Psi(W)) \subseteq \Psi(V)$. This shows that f is θ-$(\mathcal{G},\mathcal{H})$-continuous. □

Definition 3.11 ([2])**.** A grill topological space $(X,\tau,\mathcal{G})$ is said to be $\mathcal{G}$-extremally disconnected if the $\tau_{\mathcal{G}}$-closure of every open subset of X is open.

Theorem 3.12. *Let a grill topological space $(Y,\sigma,\mathcal{H})$ be an $A\mathcal{H}$-space and $\mathcal{H}$-extre-mally disconnected. Then $f : (X,\tau,\mathcal{G}) \to (Y,\sigma,\mathcal{H})$ is θ-$(\mathcal{G},\mathcal{H})$-continuous if and only if it is weakly $\mathcal{H}$-continuous.*

Proof. It is clear that every θ-$(\mathcal{G},\mathcal{H})$-continuous function is weakly $\mathcal{H}$-continuous. Conversely, suppose that f is weakly $\mathcal{H}$-continuous. Let $x \in X$ and V be an open set of Y containing $f(x)$. Then by Lemma 3.2(1), $x \in f^{-1}(V) \subseteq$

$Int(f^{-1}(\Psi(V)))$. Let $U = Int(f^{-1}(\Psi(V)))$. Since $(Y, \sigma, \mathcal{H})$ is an $A\mathcal{H}$-space and $\mathcal{H}$-extremally disconnected, by using Lemma 3.2(2) we have

$$\begin{aligned} f(\Psi(U)) &= f(\Psi(Int(f^{-1}(\Psi(V)))) \subseteq f(\Psi(f^{-1}(\Psi(V))) \\ &\subseteq f(f^{-1}(\Psi(\Psi(V))) \subseteq \Psi(V). \end{aligned}$$

Hence f is θ-$(\mathcal{G}, \mathcal{H})$-continuous. □

Corollary 3.13. *Let a grill space $(Y, \sigma, \mathcal{H})$ be an $A\mathcal{H}$-space and $\mathcal{H}$-extremally disconnected. For a function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$, the following properties are equivalent*:

(1) *f is θ-$(\mathcal{G}, \mathcal{H})$-continuous*;
(2) *f is weakly $\mathcal{H}$-continuous*;
(3) *$f^{-1}(V) \subseteq Int(f^{-1}(\Phi(V)))$ for every open set V of Y*;
(4) *$f^{-1}(V) \subseteq Int(f^{-1}(Cl(V)))$ for every open set V of Y*;
(5) *f is weakly continuous.*

Proof. By Theorem 3.12, we have the equivalence of (1) and (2). The equivalences of (2), (3) and (4) follow from Lemma 3.2(1) and Theorem 2.3. The equivalence of (4) and (5) is shown in Theorem 1 of [6]. □

Definition 3.14. A function $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is said to be θ-$(\mathcal{G}, \mathcal{H})$-irresolute if for every θ-$\mathcal{H}$-open set U in Y, $f^{-1}(U)$ is θ-$\mathcal{G}$-open in X.

Theorem 3.15. *Every θ-$(\mathcal{G}, \mathcal{H})$-continuous function is θ-$(\mathcal{G}, \mathcal{H})$-irresolute.*

Proof. Let $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ be a θ-$(\mathcal{G}, \mathcal{H})$-continuous function and U be a θ-$\mathcal{H}$-open set in Y. Let $x \in f^{-1}(U)$. Then, $f(x) \in U$. Since U is θ-$\mathcal{H}$-open, there exists an open set V in Y such that $f(x) \in V \subseteq \Psi(V) \subseteq U$. By θ-$(\mathcal{G}, \mathcal{H})$-continuity of f, there exists an open set W in X containing x such that $f(\Psi(W)) \subseteq \Psi(V) \subseteq U$. Thus $x \in W \subseteq \Psi(W) \subseteq f^{-1}(U)$. Hence $f^{-1}(U)$ is θ-$\mathcal{G}$-open and hence f is θ-$(\mathcal{G}, \mathcal{H})$-irresolute. □

Let (X, τ) be a space with a grill $\mathcal{G}$ on X and $D \subseteq X$. Then $\mathcal{G}_D = \{D \cap A : A \in \mathcal{G}\}$ is obviously a grill on D.

Theorem 3.16. *Let $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ be a function, D be a dense subset in the topological space $(Y, \sigma_{\mathcal{H}})$ and $f(X) \subseteq D$. Then the following properties are equivalent*:

(1) *$f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is θ-$(\mathcal{G}, \mathcal{H})$-continuous*;
(2) *$f : (X, \tau, \mathcal{G}) \to (D, \sigma_D, \mathcal{H}_D)$ is θ-$(\mathcal{G}, \mathcal{H}_D)$-continuous.*

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and W be any open set of D containing $f(x)$, that is $f(x) \in W \in \sigma_D$. Then there exists a $V \in \sigma$ such that $W = D \cap V$. Since $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is θ-$(\mathcal{G}, \mathcal{H})$-continuous and $f(x) \in V \in \sigma$, there exists $U \in \tau$ such that $x \in U$ and $f(\Psi(U)) \subseteq \Psi(V)$. If D is a dense subset in the topological space $(Y, \sigma_{\mathcal{H}})$, then D is a dense subset in the topological space (Y, σ) since $\Psi(D) \subseteq Cl(D)$. Since $\sigma \subseteq \sigma_{\mathcal{H}}$, $V \in \sigma_{\mathcal{H}}$. So, $\Psi(D \cap V) = \Psi(V)$

since D is dense. Thus $f(\Psi(U)) \subseteq \Psi(V) \cap f(X) \subseteq \Psi(D \cap V) \cap D \subseteq \Psi(V) \cap D$. Since $W = D \cap V$, $\Psi_D(W) = \Psi(V) \cap D$ we have $f(\Psi(U)) \subseteq \Psi_D(W)$. Hence we obtain that $f : (X, \tau, \mathcal{G}) \to (D, \sigma_D, \mathcal{H}_D)$ is θ-$(\mathcal{G}, \mathcal{H}_D)$-continuous.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be any open set of Y containing $f(x)$. Since $f(x) \in D \cap V$ and $D \cap V \in \sigma_D$, by (2) there exists $U \in \tau$ containing x such that $f(\Psi(U)) \subseteq \Psi_D(D \cap V) = \Psi(D \cap V) \cap D \subseteq \Psi(V)$. This shows that f is θ-$(\mathcal{G}, \mathcal{H})$-continuous. □

4. Preservation theorems

A subset A of a space X is said to be quasi $\widetilde{H}$-closed relative to X if for every cover $\{V_\alpha : \alpha \in \Lambda\}$ of A by open sets of X, there exists a finite subset Λ_0 of Λ such that $A \subseteq \cup\{\Psi(V_\alpha) : \alpha \in \Lambda_0\}$. A space X is said to be quasi $\widetilde{H}$-closed if X is quasi $\widetilde{H}$-closed relative to X.

Theorem 4.1. *If $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is θ-$(\mathcal{G}, \mathcal{H})$-continuous and K is quasi $\widetilde{H}$-closed relative to X, then $f(K)$ is quasi $\widetilde{H}$-closed relative to Y.*

Proof. Suppose that $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is a θ-$(\mathcal{G}, \mathcal{H})$-continuous function and K is quasi $\widetilde{H}$-closed relative to X. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a cover of $f(K)$ by open sets of Y. For each point $x \in K$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is θ-$(\mathcal{G}, \mathcal{H})$-continuous, there exists an open set U_x containing x such that $f(\Psi(U_x)) \subseteq \Psi(V_{\alpha(x)})$. The family $\{U_x : x \in K\}$ is a cover of K by open sets of X and hence there exists a finite subset K_* of K such that $K \subseteq \cup_{x \in K_*} \Psi(U_x)$. Therefore, we obtain $f(K) \subseteq \cup_{x \in K_*} \Psi(V_{\alpha(x)})$. This shows that $f(K)$ is quasi $\widetilde{H}$-closed relative to Y. □

Definition 4.2. (1) A grill space $(X, \tau, \mathcal{G})$ is said to be θ-$\mathcal{G}$-compact if every cover of X by θ-$\mathcal{G}$-open sets admits a finite subcover.

(2) A subset A of a grill space $(X, \tau, \mathcal{G})$ is said to be θ-$\mathcal{G}$-compact relative to X if every cover of A by θ-$\mathcal{G}$-open sets of X admits a finite subcover.

Proposition 4.3. *Every quasi $\widetilde{H}$-closed space $(X, \tau, \mathcal{G})$ is θ-$\mathcal{G}$-compact.*

Proof. More generally, we show that if A is quasi $\widetilde{H}$-closed relative to a space X, then A is θ-$\mathcal{G}$-compact relative to X. Let $A \subseteq \cup\{V_\alpha : \alpha \in \Lambda\}$, where each V_α is θ-$\mathcal{G}$-open, and A be quasi $\widetilde{H}$-closed relative to X, then for each $x \in A$ there exists an $\alpha(x) \in \Lambda$ with $x \in V_{\alpha(x)}$. Then there exists an open set $U_{\alpha(x)}$ with $x \in U_{\alpha(x)}$ such that $\Psi(U_{\alpha(x)}) \subseteq V_{\alpha(x)}$. Since $\{U_{\alpha(x)} : x \in A\}$ is a cover of A by open sets in X, then there is a finite subset $\{x_1, x_2, \ldots, x_n\} \subseteq A$ such that $A \subseteq \cup\{\Psi(U_{\alpha(x_i)}) : i = 1, 2, \ldots, n\} \subseteq \cup\{V_{\alpha(x_i)} : i = 1, 2, \ldots, n\}$. Hence A is θ-$\mathcal{G}$-compact relative to X. □

Theorem 4.4. *If $f : (X, \tau, \mathcal{G}) \to (Y, \sigma, \mathcal{H})$ is a θ-$(\mathcal{G}, \mathcal{H})$-irresolute surjection and $(X, \tau, \mathcal{G})$ is θ-$\mathcal{G}$-compact, then Y is θ-$\mathcal{H}$-compact.*

Proof. Let $\mathcal{V}$ be a θ-$\mathcal{H}$-open covering of Y. Then, since f is θ-$(\mathcal{G},\mathcal{H})$-irresolute, the collection $\mathcal{U}=\{f^{-1}(U):U\in\mathcal{V}\}$ is a θ-$\mathcal{G}$-open covering of X. Since X is θ-$\mathcal{G}$-compact, there exists a finite subcollection $\{f^{-1}(U_i):i=1,\dots,n\}$ of $\mathcal{U}$ which covers X. Now since f is onto, $\{U_i:i=1,\dots,n\}$ is a finite subcollection of $\mathcal{V}$ which covers Y. Hence Y is a θ-$\mathcal{H}$-compact space. □

Corollary 4.5. *The θ-$(\mathcal{G},\mathcal{H})$-continuous surjective image of a θ-$\mathcal{G}$-compact space is θ-$\mathcal{H}$-compact.*

Definition 4.6. A grill topological space $(X,\tau,\mathcal{G})$ is said to be $\mathcal{G}$-Lindelöf if for every open cover $\{U_\alpha:\alpha\in\Lambda\}$ of X there exists a countable subset $\{\alpha_n:n\in\mathbb{N}\}\subseteq\Lambda$ such that $X=\cup_{n\in\mathbb{N}}\Psi(U_{\alpha_n})$.

Theorem 4.7. *Let $f:(X,\tau,\mathcal{G})\to(Y,\sigma,\mathcal{H})$ be a θ-$(\mathcal{G},\mathcal{H})$-continuous (resp. weakly $\mathcal{H}$-continuous) surjection. If X is $\mathcal{G}$-Lindelöf (resp. Lindelöf), then Y is $\mathcal{H}$-Lindelöf.*

Proof. Suppose that f is θ-$(\mathcal{G},\mathcal{H})$-continuous and X is $\mathcal{G}$-Lindelöf. Let $\{V_\alpha:\alpha\in\Lambda\}$ be an open cover of Y. For each $x\in X$, there exists $\alpha(x)\in\Lambda$ such that $f(x)\in V_{\alpha(x)}$. Since f is θ-$(\mathcal{G},\mathcal{H})$-continuous, there exists an open set $U_{\alpha(x)}$ of X containing x such that $f(\Psi(U_{\alpha(x)}))\subseteq\Psi(V_{\alpha(x)})$. Now $\{U_{\alpha(x)}:x\in X\}$ is an open cover of the $\mathcal{G}$-Lindelöf space X. So there exists a countable subset $\{U_{\alpha(x_n)}:n\in\mathbb{N}\}$ such that $X=\cup_{n\in\mathbb{N}}(\Psi(U_{\alpha(x_n)}))$. Thus $Y=f(\cup_{n\in\mathbb{N}}(\Psi(U_{\alpha(x_n)})))\subseteq\cup_{n\in\mathbb{N}}f(\Psi(U_{\alpha(x_n)}))\subseteq\cup_{n\in\mathbb{N}}\Psi(V_{\alpha(x_n)})$. This shows that Y is $\mathcal{H}$-Lindelöf. In case X is Lindelöf the proof is similar. □

A function $f:(X,\tau,\mathcal{G})\to(Y,\sigma,\mathcal{H})$ is said to be θ-$(\mathcal{G},\mathcal{H})$-closed if for each θ-$\mathcal{G}$-closed set F in X, $f(F)$ is θ-$\mathcal{H}$-closed in Y.

The following characterization of θ-$(\mathcal{G},\mathcal{H})$-closed functions will be used in the sequel.

Theorem 4.8. *A surjective function $f:(X,\tau,\mathcal{G})\to(Y,\sigma,\mathcal{H})$ is θ-$(\mathcal{G},\mathcal{H})$-closed if and only if for each set $B\subseteq Y$ and for each θ-$\mathcal{G}$-open set U containing $f^{-1}(B)$, there exists a θ-$\mathcal{H}$-open set V containing B such that $f^{-1}(V)\subseteq U$.*

Proof. Necessity. Suppose that f is θ-$(\mathcal{G},\mathcal{H})$-closed. Since U is θ-$\mathcal{G}$-open in X, $X-U$ is θ-$\mathcal{G}$-closed and so $f(X-U)$ is θ-$\mathcal{H}$-closed in Y. Now, $V=Y-f(X-U)$ is θ-$\mathcal{H}$-open, $B\subseteq V$ and $f^{-1}(V)=f^{-1}(Y-f(X-U))=X-f^{-1}(f(X-U))\subseteq X-(X-U)=U$.

Sufficiency. Let A be a θ-$\mathcal{G}$-closed set in X. To prove that $f(A)$ is θ-$\mathcal{H}$-closed, we shall show that $Y-f(A)$ is θ-$\mathcal{H}$-open. Let $y\in Y-f(A)$. Then $f^{-1}(y)\cap f^{-1}(f(A))=\phi$ and so $f^{-1}(y)\subseteq X-f^{-1}(f(A))\subseteq X-A$. By hypothesis there exists a θ-$\mathcal{H}$-open set V containing y such that $f^{-1}(V)\subseteq X-A$. So $A\subseteq X-f^{-1}(V)$ and hence $f(A)\subseteq f(X-f^{-1}(V))=Y-V$. Thus $V\subseteq Y-f(A)$ and so the set $Y-f(A)$ being the union of θ-$\mathcal{H}$-open sets is θ-$\mathcal{H}$-open by Theorem 2.6 [2]. □

Theorem 4.9. *Let $f:(X,\tau,\mathcal{G})\to(Y,\sigma,\mathcal{H})$ be a θ-$(\mathcal{G},\mathcal{H})$-closed surjection such that for each $y\in Y$, $f^{-1}(y)$ is θ-$\mathcal{G}$-compact relative to X. If Y is θ-$\mathcal{H}$-compact, then X is θ-$\mathcal{G}$-compact.*

Proof. Let $\mathcal{U}=\{U_\alpha:\alpha\in\Lambda\}$ be a θ-$\mathcal{G}$-open covering of X. Since for each $y\in Y$, $f^{-1}(y)$ is θ-$\mathcal{G}$-compact relative to X, we can choose a finite subset Λ_y of Λ such that $\{U_\beta:\beta\in\Lambda_y\}$ is a covering of $f^{-1}(y)$. Now, by Theorem 4.8, there exists a θ-$\mathcal{H}$-open set V_y containing y such that $f^{-1}(V_y)\subseteq\cup\{U_\beta:\beta\in\Lambda_y\}$. The collection $\mathcal{V}=\{V_y:y\in Y\}$ is a θ-$\mathcal{H}$-open covering of Y. In view of θ-$\mathcal{H}$-compactness of Y there exists a finite subcollection $\{V_{y_1},\dots,V_{y_n}\}$ of $\mathcal{V}$ which covers Y. Then the finite subcollection $\{U_\beta:\beta\in\Lambda_{y_i}, i=1,\dots,n\}$ of $\mathcal{U}$ covers X. Hence X is a θ-$\mathcal{G}$-compact space. □

Acknowledgement. The author wish to thank the referees for useful comments and suggestions.

References

[1] A. Al-Omari and T. Noiri, *Weakly Φ-continuous functions in grill topological spaces*, Hacettepe J. Math. Stat. **41** (2012), no. 6, 785–793.
[2] A. Al-Omari, *On θ-$\mathcal{G}$-open sets and related sets in grill topological spaces*, (submitted).
[3] G. Choqet, *Sur les notions de filter et grill*, Comptes Rendus Acad. Sci. Paris **224** (1947), 171–173.
[4] S. Fomin, *Extensions of topological spaces*, Ann. of Math. **44** (1943), 471–480.
[5] E. Hatir and S. Jafari, *On some new classes of sets and a new decomposition of continuity via grills*, J. Adv. Math. Stud. **3** (2010), no. 1, 33–40.
[6] N. Levine, *A decomposition of continuity in topological spaces*, Amer. Math. Monthly **68** (1961), 44–46.
[7] T. Noiri, *On δ-continuous functions*, J. Korean Math. Soc. **16** (1980), no. 2, 161–166.
[8] D. A. Rose, *Weak continuity and almost continuity*, Internat. J. Math. Math. Sci. **7** (1984), no. 2, 311–318.
[9] B. Roy and M. N. Mukherjee, *On a typical topology induced by a grill*, Soochow J. Math. **33** (2007), no. 4, 771–786.
[10] N. V. Veličko, *H-closed topological spaces*, Amer. Math. Soc. Transl. **78** (1968), 103–118.

Department of Mathematics
Faculty of Science
Al al-Bayt University
P.O. Box 130095
Mafraq 25113, Jordan
E-mail address: omarimutah1@yahoo.com