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NON-EXISTENCE OF TOTALLY GEODESIC SCREEN DISTRIBUTIONS ON LIGHTLIKE HYPERSURFACES OF INDEFINITE KENMOTSU MANIFOLDS

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Abstract. We study lightlike hypersurfaces of indefinite Kenmotsu manifolds. The purpose of this paper is to prove that there do not exist totally geodesic screen distributions on semi-symmetric lightlike hypersurfaces of indefinite Kenmotsu manifolds with flat transversal connection.

1. Introduction

In the classical theory of spacetime, while the rest spaces of timelike curves are spacelike subspaces of the tangent spaces, the rest spaces of null curves are lightlike subspaces of the tangent spaces [14]. Null curves can represent the histories of photons, the effect of the Riemannian curvature tensor will be to distort or focus small bundles of light rays. To investigate this, Hawking and Ellis introduced the notion of so-called screen spaces in Section 4.2 of their book [7]. Since for any semi-Riemannian manifold there is a natural existence of lightlike subspaces, in [3] Duggal-Bejancu published their work on the general theory of degenerate (lightlike) submanifolds to fill a gap in the study of submanifolds. Since then there has been very active study on lightlike geometry of submanifolds (see up-to date results in two books [5, 6]). The geometry of lightlike submanifolds is used in mathematical physics, in particular, in general relativity since lightlike submanifolds can be models of different types of horizons (event horizons, Cauchy's horizons, Kruskal's horizons). Now we have lightlike version of a large variety of Riemannian submanifolds. The lightlike version of totally geodesic submanifolds comes from the classical geometry of non-degenerate submanifolds for which there are only one type [2]. But the totally geodesic properties of lightlike submanifolds M with a screen distribution $S(TM)$ are two types: (1) M is totally geodesic and (2) $S(TM)$ is totally geodesic in M. The lightlike submanifolds with above two kind totally geodesic properties studied by Duggal-Bejancu [3] and Duggal-Jin

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[4]. The notion of totally geodesic screen distributions is more interest that of totally geodesic submanifolds. Some papers on totally geodesic (umbilical) screen distributions recently studied by Jin [8, 9]. On the contrary the rich existing of lightlike submanifolds with totally geodesic screen distributions, the objective of this paper is the study for non-existence of totally geodesic screen distributions on semi-symmetric lightlike hypersurfaces of indefinite Kenmotsu manifolds with flat transversal connection.

2. Lightlike hypersurfaces

An odd dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$ is said to be an *indefinite Kenmotsu manifold* [12, 13] if there exist an almost contact metric structure $(J, \zeta, \theta, \bar{g})$, where J is a $(1,1)$-type tensor field, ζ is a vector field which called the characteristic vector field and θ is a 1-form such that

$$J^2X = -X + \theta(X)\zeta, \quad J\zeta = 0, \quad \theta \circ J = 0, \quad \theta(\zeta) = 1, \\ \theta(X) = \bar{g}(\zeta, X), \quad \bar{g}(JX, JY) = \bar{g}(X, Y) - \theta(X)\theta(Y), \tag{2.1}$$

$$\bar{\nabla}_X \zeta = -X + \theta(X)\zeta, \tag{2.2}$$

$$(\bar{\nabla}_X J)Y = -\bar{g}(JX, Y)\zeta + \theta(Y)JX, \tag{2.3}$$

for any vector fields X, Y on $\bar{M}$, where $\bar{\nabla}$ is the Levi-Civita connection of $\bar{M}$.

A hypersurface (M, g) of $\bar{M}$ is called a *lightlike hypersurface* if the normal bundle $TM^\perp$ of M is a vector subbundle of the tangent bundle TM of M, of rank 1, and coincides with the radical distribution $Rad(TM)$. Then there exists a non-degenerate complementary vector bundle $S(TM)$ of $Rad(TM)$ in TM, which is called a *screen distribution* on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \tag{2.4}$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(\bar{M})$ the algebra of smooth functions on $\bar{M}$ and by $\Gamma(E)$ the $F(\bar{M})$ module of smooth sections of a vector bundle E over $\bar{M}$. It is well-known [3] that, for any null section ξ of $Rad(TM)$, there exists a unique null section N of a unique vector bundle $tr(TM)$ of rank 1 in the orthogonal complement $S(TM)^\perp$ of $S(TM)$ in $\bar{M}$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \ \forall\, X \in \Gamma(S(TM)).$$

In this case, the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM). \tag{2.5}$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen $S(TM)$ respectively.

Let P be the projection morphism of TM on $S(TM)$. Then the Gauss and Weingartan formulas of TM and $S(TM)$ are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N, \tag{2.6}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N; \tag{2.7}$$

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi, \tag{2.8}$$

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \tag{2.9}$$

for all $X,\, Y \in \Gamma(TM)$, respectively, where ∇ and ∇^* are the liner connections on TM and $S(TM)$, respectively, B and C are the local second fundamental forms on TM and $S(TM)$, respectively, A_N and A^*_ξ are the shape operators on TM and $S(TM)$, respectively and τ is a 1-form on TM.

Since $\bar{\nabla}$ is torsion-free, ∇ is also torsion-free and B is symmetric on TM. From the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y,\, \xi)$ for all $X,\, Y \in \Gamma(TM)$, we show that B is independent of the choice of a screen distribution $S(TM)$ and satisfies

$$B(X, \xi) = 0, \quad \forall\, X \in \Gamma(TM). \tag{2.10}$$

The above two local second fundamental forms B and C of M and $S(TM)$ are related to their shape operators by

$$B(X,Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{2.11}$$

$$C(X, PY) = g(A_N X, PY), \quad \bar{g}(A_N X, N) = 0. \tag{2.12}$$

From (2.11), we show that A^*_ξ is $S(TM)$-valued self-adjoint such that $A^*_\xi \xi = 0$.

$S(TM)$ is called *totally geodesic* [3] in M if $C = 0$, or equivalently $A_N = 0$. The induced connection ∇ of M is not a metric one and satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\,\eta(Z) + B(X, Z)\,\eta(Y), \tag{2.13}$$

for any $X,\, Y,\, Z \in \Gamma(TM)$, where η is a 1-form such that

$$\eta(X) = \bar{g}(X, N), \quad \forall\, X \in \Gamma(TM). \tag{2.14}$$

But ∇^* is a metric connection. From (2.6), (2.9) and (2.10), we show that

$$\bar{\nabla}_X \xi = -A^*_\xi X - \tau(X)\xi, \ \forall\, X \in \Gamma(TM). \tag{2.15}$$

Denote by $\bar{R}$, R and R^* the curvature tensors of the connections $\bar{\nabla}$, ∇ and ∇^* respectively. Using (2.6)$\sim$(2.9), for any $X,\, Y,\, Z \in \Gamma(TM)$, we obtain the Gauss-Codazzi equation for M and $S(TM)$:

$$\begin{aligned} \bar{R}(X,Y)Z = {} & R(X,Y)Z + B(X,Z)A_N Y - B(Y,Z)A_N X \\ & + \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ & + \tau(X)B(Y,Z) - \tau(Y)B(X,Z)\}N, \end{aligned} \tag{2.16}$$

$$\begin{aligned} \bar{R}(X,Y)N = {} & -\nabla_X(A_N Y) + \nabla_Y(A_N X) + A_N[X,Y] \\ & + \tau(X)A_N Y - \tau(Y)A_N X \\ & + \{B(Y, A_N X) - B(X, A_N Y) + 2d\tau(X,Y)\}N, \end{aligned} \tag{2.17}$$

$$R(X,Y)PZ = R^*(X,Y)PZ + C(X,PZ)A^*_\xi Y - C(Y,PZ)A^*_\xi X + \{(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) + C(X,PZ)\tau(Y) - C(Y,PZ)\tau(X)\}\xi, \quad (2.18)$$

$$R(X,Y)\xi = -\nabla^*_X(A^*_\xi Y) + \nabla^*_Y(A^*_\xi X) + A^*_\xi[X,Y] - \tau(X)A^*_\xi Y + \tau(Y)A^*_\xi X + \{C(Y,A^*_\xi X) - C(X,A^*_\xi Y) - 2d\tau(X,Y)\}\xi. \quad (2.19)$$

For any $X \in \Gamma(TM)$, let $\nabla^\perp_X N = \pi(\bar{\nabla}_X N)$, where π is the projection morphism of $T\bar{M}$ on $tr(TM)$. Then $\nabla^\perp$ is a linear connection on the transversal vector bundle $tr(TM)$ of M. We say that $\nabla^\perp$ is the *transversal connection* of M. We define the curvature tensor $R^\perp$ on $tr(TM)$ by

$$R^\perp(X,Y)N = \nabla^\perp_X\nabla^\perp_Y N - \nabla^\perp_Y\nabla^\perp_X N - \nabla^\perp_{[X,Y]}N, \quad \forall\, X,\, Y \in \Gamma(TM). \quad (2.20)$$

The transversal connection $\nabla^\perp$ of M is said to be *flat* [8] if $R^\perp$ vanishes identically. From (2.7), (2.20) and the definition of the connection $\nabla^\perp$, we have

$$\nabla^\perp_X N = \tau(X)N, \;\; R^\perp(X,Y)N = 2d\tau(X,Y)N, \;\; \forall\, X,\, Y \in \Gamma(TM).$$

From this result we have the following result:

Theorem 2.1 ([10]). *Let M be a lightlike hypersurface of a semi-Riemannian manifold $\bar{M}$. Then the transversal connection $\nabla^\perp$ of M is flat if and only if the 1-form τ is closed, i.e., $d\tau = 0$, on any coordinate neighborhood $\mathcal{U} \subset M$.*

We know that $d\tau$ is independent of the choice of the section $\xi \in \Gamma(Rad(TM))$. In fact if we take $\tilde{\xi} = \gamma\xi$ for some function γ, it follows that $\tilde{N} = 1/\gamma$ and $\tau(X) = \tilde{\tau}(X) + X(\mathrm{In}\,\gamma)$. Taking the exterior derivative d on the last equation, we have $d\tau = d\tilde{\tau}$.

3. Non-existence theorem

A lightlike hypersurface M of a semi-Riemannian manifold $\bar{M}$ is called a *semi-symmetric* lightlike hypersurface [6, 12] if

$$R(X,Y)R = 0, \quad \forall\, X,\, Y \in \Gamma(TM).$$

Theorem 3.1. *Let M be a semi-symmetric lightlike hypersurface of an indefinite Kenmotsu manifold $\bar{M}$ with flat transversal connection and* $\dim M > 2$. *Then the screen distribution $S(TM)$ of M is not totally geodesic in M.*

Proof. Assume that $S(TM)$ is totally geodesic in M. If ζ is tangent to M, then it belongs to $S(TM)$ due to Călin [1]. Applying $\bar{\nabla}_X$ to $\bar{g}(\zeta, N) = 0$ and using (2.2) and (2.7), we have $\eta(X) = 0$ for all $X \in \Gamma(TM)$. It is a contradiction as $\eta(\xi) = 1$. Thus ζ is not tangent to M. From this result and the decomposition (2.5) of $T\bar{M}$, the vector field ζ is decomposed by

$$\zeta = W + fN, \quad (3.1)$$

where W is a smooth non-vanishing vector field on M and $f = \theta(\xi)$ is a smooth function. Substituting (3.1) into (2.2) and using (2.6) and (2.7), we obtain

$$\nabla_X W = -X + \theta(X)W, \tag{3.2}$$

$$Xf + f\tau(X) + B(X, W) = f\theta(X) \tag{3.3}$$

for all $X \in \Gamma(TM)$. Substituting (3.3) into $[X, Y]f = X(Yf) - Y(Xf)$ and using (3.2) and (3.3), we have

$$\begin{aligned}(\nabla_X B)(Y, W) - (\nabla_Y B)(X, W) + \tau(X)B(Y, W)\\ - \tau(Y)B(X, W) + 2fd\tau(X, Y)\\ = 2fd\theta(X, Y), \forall X, Y \in \Gamma(TM).\end{aligned} \tag{3.4}$$

Using (2.16), (2.17), (3.1) and the fact $A_{_N} = 0$, the equation (3.4) reduce to

$$\bar{g}(\bar{R}(X, Y)\zeta, \xi) = 2fd\theta(X, Y), \quad \forall X, Y \in \Gamma(TM). \tag{3.5}$$

Substituting (3.2) into $R(X, Y)W = \nabla_X\nabla_Y W - \nabla_Y\nabla_X W - \nabla_{[X,Y]}W$ and using (2.16), (2.17), (3.1) $\sim$ (3.5) and the fact ∇ is torsion-free, we have

$$\bar{R}(X, Y)\zeta = \theta(X)Y - \theta(Y)X + 2d\theta(X, Y)\zeta, \ \forall X, Y \in \Gamma(TM). \tag{3.6}$$

Taking the scalar product with ζ to (3.6) and using $g(\bar{R}(X, Y)\zeta, \zeta) = 0$ and (2.1), we show that $d\theta = 0$ on TM. Substituting (3.1) into (3.6) and using (2.16), (2.17) and (3.4), we show that the curvature tensor R of M is given by

$$R(X, Y)W = \theta(X)Y - \theta(Y)X, \quad \forall X, Y \in \Gamma(TM). \tag{3.7}$$

As the transversal connection $\nabla^\perp$ is flat, we have $d\tau = 0$ on TM by Theorem 2.1. Replacing Z by W to (2.16) and using (3.4) and the fact $A_N = 0$, we have

$$\bar{R}(X, Y)W = \theta(X)Y - \theta(Y)X, \quad \forall X, Y \in \Gamma(TM).$$

Taking the scalar product with W to this and using the facts $g(\bar{R}(X, Y)W, W) = 0$, $\theta(X) = g(X, W) + f\eta(X)$ and $f \neq 0$, we have

$$\theta(Y)\eta(X) - \theta(X)\eta(Y) = 0, \quad \forall X, Y \in \Gamma(TM).$$

Replacing Y by ξ to this equation, we have $g(X, W) = 0$ for all $X \in \Gamma(TM)$. This implies $W = e\xi$, where $e = \theta(N) \neq 0$, and ζ is decomposed as

$$\zeta = e\xi + fN. \tag{3.8}$$

Thus ζ belongs to $S(TM)^\perp$. From the fact $\bar{g}(\zeta, \zeta) = 1$ and (3.8), we show that $2ef = 1$. Applying $\bar{\nabla}_X$ to (3.8) and using (2.2), (2.7) and (2.15), we have

$$\begin{aligned}&- eA^*_\xi X + \{X[e] - e\tau(X)\}\xi + \{X[f] + f\tau(X)\}N\\ &= - PX + \{e\theta(X) - \eta(X)\}\xi + f\theta(X)N, \quad \forall X \in \Gamma(TM).\end{aligned}$$

Taking the scalar product with ξ and N to this result by turns, we get

$$X[f] + f\tau(X) = f\theta(X), \ X[e] - e\tau(X) = e\theta(X) - \eta(X), \ eA^*_\xi X = PX, \tag{3.9}$$

respectively. From (2.11) and $(3.9)_3$, we show that

$$eB(X, Y) = g(X, Y), \quad \forall X, Y \in \Gamma(TM). \tag{3.10}$$

It is known that, for any lightlike hypersurface of an indefinite almost contact metric manifold $\bar{M}$, $J(Rad(TM))$ and $J(tr(TM))$ are vector subbundles of $S(TM)$ of rank 1 [10, 11]. Applying J to (3.8) and using $J\zeta = 0$ and $2ef = 1$, we get $J\xi = -2f^2 JN$. Thus $J(Rad(TM)) \cap J(tr(TM)) \neq \{0\}$. This implies

$$J(Rad(TM)) = J(tr(TM)).$$

Using this result, the tangent bundle TM of M is decomposed as follow:

$$TM = Rad(TM) \oplus_{orth} \{J(Rad(TM)) \oplus_{orth} H\}, \tag{3.11}$$

where H is a non-degenerate and almost complex distribution on $S(TM)$ with respect to J. Consider a timelike vector field V and its 1-form v defined by

$$V = -f^{-1}J\xi = e^{-1}JN, \qquad v(X) = -g(X, V), \ \forall\, X \in \Gamma(TM). \tag{3.12}$$

Applying J to $(3.12)_1$ and using $(2.1)_1$ and $2ef = 1$, we get

$$JV = e\xi - fN. \tag{3.13}$$

Denote by Q the projection morphism of TM on H with respect to the decomposition (3.11). Then any vector field X on M is expressed as follow:

$$X = QX + v(X)V + \eta(X)\xi.$$

Applying J to this and using (3.13) and the fact $\theta(X) = f\eta(X)$, we have

$$JX = FX - \theta(X)V + ev(X)\xi - fv(X)N, \tag{3.14}$$

where F is a tensor field of type $(1, 1)$ globally defined on M by

$$FX = JQX, \quad \forall\, X \in \Gamma(TM).$$

Applying $\bar{\nabla}_X$ to $fV = -J\xi$ and $v(Y) = -g(Y, V)$ by turns and using (2.1), (2.3), (2.6), (2.11), (2.15), (3.8) $\sim$ (3.14) and the fact $F\xi = 0$, we get

$$\nabla_X V = FX, \quad (\nabla_X v)Y = -g(FX, Y) + 2v(X)\theta(Y). \tag{3.15}$$

Applying $\bar{\nabla}_X$ to $\theta(Y) = g(Y, \zeta)$ and using (2.2), (2.6) and (3.10), we have

$$(\nabla_X \theta)(Y) = \theta(X)\theta(Y), \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.16}$$

Applying $\bar{\nabla}_Y$ to (3.14) and using (2.3), (2.6), (2.7), (2.15), (3.8)$\sim$(3.10) and (3.14)$\sim$(3.16), we have

$$(\nabla_X F)Y = v(Y)PX + 2\theta(Y)FX + g(X, Y)V \tag{3.17}$$

for all $X,\, Y \in \Gamma(TM)$. Applying $\bar{\nabla}_Y$ to $(3.15)_1$ and using (3.17), we have

$$\nabla_X \nabla_Y V = v(Y)PX + 2\theta(Y)FX + g(X, Y)V + F(\nabla_X Y).$$

Substituting this and $(3.15)_1$ into $R(X, Y)V = \nabla_X\nabla_Y V - \nabla_Y\nabla_X V - \nabla_{[X,\, Y]}V$ for all $X,\, Y \in \Gamma(TM)$ and using the fact ∇ is torsion-free, we have

$$R(X, Y)V = v(Y)PX - v(X)PY + 2\{\theta(Y)FX - \theta(X)FY\}. \tag{3.18}$$

Applying ∇_Z to (3.7) and using (3.2), (3.7) and (3.16), we have

$$(\nabla_Z R)(X, Y)W = R(X, Y)Z, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM). \tag{3.19}$$

Applying ∇_U to (3.19) and using (3.2) and (3.19), we have

$$\begin{aligned}(3.20)\qquad &(\nabla_U\nabla_Z R)(X,Y)W\\ &= (\nabla_U R)(X,Y)Z + (\nabla_Z R)(X,Y)U - \theta(U)R(X,Y)Z\\ &\quad + R(X,Y)\nabla_U Z,\ \forall\, X,\, Y,\, Z,\, U \in \Gamma(TM).\end{aligned}$$

As M is semi-symmetric, i.e., $R(U,Z)R = 0$, from (3.19) and (3.20), we have

$$\theta(Z)R(X,Y)U = \theta(U)R(X,Y)Z, \quad \forall\, X,\, Y,\, Z,\, U \in \Gamma(TM).$$

Replacing U by W to this and using (3.7) and $W = e\xi$ and $2ef = 1$, we get

$$R(X,Y)Z = 2\theta(Z)\{\theta(X)Y - \theta(Y)X\}, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM).$$

Replacing Z by V to this and using (3.18) and the fact $\theta(V) = 0$, we have

$$v(Y)PX - v(X)PY + 2\{\theta(Y)FX - \theta(X)FY\} = 0, \quad \forall\, X,\, Y \in \Gamma(TM).$$

Replacing Y by V to this and using the facts $\theta(V) = 0$ and $FV = 0$, we have

$$PX = v(X)V, \quad \forall\, X \in \Gamma(TM).$$

It is a contradiction as $\dim M > 2$. Thus $S(TM)$ is not totally geodesic. □

An odd dimensional semi-Riemannian manifold $(\tilde{M}, \tilde{g})$ is called an *indefinite Sasakian manifold* [6, 10] if there exists an almost contact metric structure $(\phi, \mathcal{U}, u, \tilde{g})$, where F is a $(1,1)$-type tensor field, $\mathcal{U}$ is a vector field, u is a 1-form and $\tilde{g}$ is a semi-Riemannian metric on $\tilde{M}$ such that

$$\begin{aligned}\phi^2 X &= -X + u(X)\mathcal{U},\ \phi\mathcal{U} = 0,\ u\circ\phi = 0,\ u(\mathcal{U}) = 1,\\ \tilde{g}(\mathcal{U},\mathcal{U}) &= \epsilon, \quad \tilde{g}(\phi X, \phi Y) = \tilde{g}(X,Y) - \epsilon u(X)u(Y),\\ u(X) &= \epsilon\tilde{g}(X,\mathcal{U}), \quad du(X,Y) = \tilde{g}(\phi X, Y),\ \epsilon = \pm 1,\\ \tilde{\nabla}_X\mathcal{U} &= -\epsilon\phi X,\ (\bar{\nabla}_X\phi)Y = \tilde{g}(X,Y)\mathcal{U} - \epsilon u(Y)X,\end{aligned}$$

for any $X,\, Y \in \Gamma(T\tilde{M})$, where $\tilde{\nabla}$ is the Levi-Civita connection on $\tilde{M}$.

The following result is a very marvellous one in the common sense.

Theorem 3.2. *Let M be a lightlike hypersurface of an indefinite Kenmotsu manifold $\bar{M}$ with flat transversal connection. If $S(TM)$ is totally geodesic in M, then the leaf M^* of $S(TM)$ is an indefinite Sasakian manifold.*

Proof. Take $X,\, Y \in \Gamma(S(TM))$, then $(3.15)_1$ and (3.17) reduce to

$$\nabla_X V = FX, \quad (\nabla_X F)Y = g(X,Y)V + v(Y)X,$$

respectively. By direct calculations, we show that the set (F, V, v, g) is an almost contact metric structure on the leaf M^* of $S(TM)$. Thus M^* is an indefinite Sasakian manifold. □

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