GENERAL NONLINEAR RANDOM SET-VALUED VARIATIONAL INCLUSION PROBLEMS WITH RANDOM FUZZY MAPPINGS IN BANACH SPACES

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Abstract. This paper is dedicated to study a new class of general nonlinear random A-maximal m-relaxed η-accretive (so called (A, η)-accretive [49]) equations with random relaxed cocoercive mappings and random fuzzy mappings in q-uniformly smooth Banach spaces. By utilizing the resolvent operator technique for A-maximal m-relaxed η-accretive mappings due to Lan et al. and Chang's lemma [13], some new iterative algorithms with mixed errors for finding the approximate solutions of the aforesaid class of nonlinear random equations are constructed. The convergence analysis of the proposed iterative algorithms under some suitable conditions are also studied.

1. Introduction

Variational inequality theory has become a very effective and powerful tool for studying a wide range of problems arising in pure and applied sciences which include the work on differential equations, mechanics, contact problems in elasticity, control problems, general equilibrium problems in economics and transportation, and unilateral, obstacle, moving, and free boundary problems. For the applications, physical formulation, numerical methods and other aspects of variational inequalities, see [1–67] and the references therein. Quasi-variational inequalities are generalized forms of variational inequalities in which the constraint set depend on the solution. These were introduced and studied by Bensoussan et al. [12]. In 1991, Chang and Huang [17, 18] introduced and studied some new classes of complementarity problems and variational inequalities for set-valued mappings with compact values in Hilbert spaces. An useful and important generalization of the variational inequalities is called the

variational inclusions, due to Hassouni and Moudafi [32], which have wide applications in the fields of optimization and control, economics and transportation equilibrium, engineering science.

It is known that accretivity of the underlying operator plays indispensable roles in the theory of variational inequality and its generalizations. In 2001, Huang and Fang [38] were the first to introduce generalized m-accretive mapping and gave the definition of the resolvent operator for generalized m-accretive mappings in Banach spaces. Verma [59–62] introduced and studied new notions of A-monotone and (A, η)-monotone operators and studied some properties of them in Hilbert spaces. In [49], Lan et al. first introduced the concept of (A, η)-accretive mappings, which generalizes the existing η-subdifferential operators, maximal η-monotone operators, H-monotone operators, A-monotone operators, (H, η)-monotone operators, (A, η)-monotone operators in Hilbert spaces, H-accretive mapping, generalized m-accretive mappings and (H, η)-accretive mappings in Banach spaces.

The fuzzy set theory which was introduced by Professor Lotfi Zadeh [66] at the university of California in 1965 has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of the fuzzy set theory can be found in many branches of regional, physical, mathematical and engineering sciences, see, for example, $[9, 30, 54, 67]$. In 1989, Chang and Zu [21] first introduced and studied a class of variational inequalities for fuzzy mappings. Since then several classes of variational inequalities with fuzzy mappings were considered by Chang and Haung [16], Ding [27], Ding and Park [29], Haung [34], Noor [55] and Park and Jeong [56, 57] in Hilbert spaces. Note that most of results in this direction for variational inclusions (inequalities) has been done in the setting of Hilbert spaces.

Variational inequalities with fuzzy mapping have been useful in the study of equilibrium and optimal control problem, see, for example, [15]. Recently, Huang and Lan [40], considered nonlinear equations with fuzzy mapping in fuzzy normed spaces and subsequently Lan and Verma [53] considered fuzzy variational inclusion problems in Banach spaces.

On the other hand, the random variational inequality and random quasi-variational inequality problems, random variational inclusion problems and random quasi-complementarity problems have been introduced and studied by Chang [14], Chang and Huang [19, 20], Chang and Zhu [22], Cho et al. [23], Ganguly and Wadhawa [31], Huang and Cho [37], Khan et al. [44] and Lan [47], etc. The concept of random fuzzy mapping was first introduced by Haung [36]. Subsequently, the random variational inclusion problem for random fuzzy mappings is studied by Ahmad and Bazan [4]. Cho and Lan [25] considered and studied a class of generalized nonlinear random (A, η)-accretive equations with random relaxed cocoercive mappings in Banach spaces and by introducing some random iterative algorithms, they proved the convergence of iterative sequences generated by the proposed algorithms.

Recently, Lan et al. [50] introduced and studied a class of general nonlinear random set-valued operator equations involving generalized m-accretive mappings in Banach spaces. They also established the existence theorems of the solution and convergence theorems of the generalized random iterative procedures with errors for these nonlinear random set-valued operator equations in q-uniformly smooth Banach spaces.

Very recently, Uea and Kumam [58] introduced and studied a class of general nonlinear random (H, η)-accretive equations with random fuzzy mappings in Banach spaces and by using the resolvent mapping technique for the (H, η)-accretive mappings proved the existence and convergence theorems of the generalized random iterative algorithms for these nonlinear random equations with random fuzzy mappings in q-uniformly smooth Banach spaces.

In this paper, a new class of general nonlinear random A-maximal m-relaxed η-accretive (so called (A, η)-accretive [49]) equations with random relaxed cocoercive mappings and random fuzzy mappings in Banach spaces is introduced and studied. By using the resolvent operator technique for A-maximal m-relaxed η-accretive mappings due to Lan et al. and Chang's lemma [13], some new iterative algorithms with mixed errors for finding the approximate solutions of the mentioned class of nonlinear random equations are constructed. The existence of random solutions and the convergence of random iterative sequences generated by the suggested iterative algorithms in q-uniformly smooth Banach spaces are also proved. The results presented in this paper improve and extend the corresponding results of $[14, 19, 23, 25, 28, 31, 32, 35\text{--}37, 39, 41, 43, 45, 50, 58]$ and many other recent works.

2. Preliminaries and basic facts

Throughout this paper, let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measure space and X be a separable real Banach space endowed with dual space X^*, norm $\| \cdot \|$ and dual pair $\langle \cdot, \cdot \rangle$ between X and X^*. We denote by $\mathcal{B}(X)$, $CB(X)$ and $\hat{H}(\cdot, \cdot)$ the class of Borel σ-fileds in X, the family of all nonempty closed bounded subsets of X and the Hausdorff metric

$$\hat{H}(A, B) = \max\{\sup_{x \in A} \inf_{y \in B} d(x, y), \sup_{y \in B} \inf_{x \in A} d(x, y)\}$$

on $CB(X)$, respectively. The *generalized duality mapping* $J_q : X \to 2^{X^*}$ is defined by

$$J_q(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in X,$$

where $q > 1$ is a constant. In particular, J_2 is usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2} J_2(x)$ for all $x \neq 0$ and J_q is single-valued if X^* is strictly convex. In the sequel, we always assume that X is a real Banach space such that J_q is single-valued. If X is a Hilbert space, then J_2 becomes the identity mapping on X.

The *modulus of smoothness* of X is the function $\rho_X : [0, \infty) \to [0, \infty)$ defined by

$$\rho_X(t) = \sup\{\frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : \|x\| \le 1, \|y\| \le t\}.$$

A Banach space X is called *uniformly smooth* if

$$\lim_{t \to 0} \frac{\rho_X(t)}{t} = 0.$$

Further, a Banach space X is called *q-uniformly smooth* if there exists a constant $c > 0$ such that

$$\rho_X(t) \le ct^q, \quad q > 1.$$

It is well-known that Hilbert spaces, L_p (or l_p) spaces, $1 < p < \infty$, and the Sobolev spaces $W^{m,p}$, $1 < p < \infty$, are all q-uniformly smooth.

Concerned with the characteristic inequalities in q-uniformly smooth Banach spaces, Xu [63] proved the following result.

Lemma 2.1. *Let X be a real uniformly smooth Banach space. Then X is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in X$,*

$$\|x + y\|^q \le \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

Definition 2.2. A mapping $x : \Omega \to X$ is said to be *measurable* if, for any $B \in \mathcal{B}(X)$, $\{t \in \Omega : x(t) \in B\} \in \mathcal{A}$.

Definition 2.3. A mapping $T : \Omega \times X \to X$ is called a *random* mapping if, for any $x \in X$, $T(\cdot, x) : \Omega \to X$ is measurable. A random mapping T is said to be *continuous* if, for any $t \in \Omega$, the mapping $T(t, \cdot) : X \to X$ is continuous.

Similarly, we can define a random mapping $a : \Omega \times X \times X \to X$. We shall write $T_t(x) = T(t, x(t))$ and $a_t(x, y) = a(t, x(t), y(t))$ for all $t \in \Omega$ and $x(t), y(t) \in X$.

It is well-known that a measurable mapping is necessarily a random mapping.

Definition 2.4. A set-valued mapping $V : \Omega \to 2^X$ is said to be *measurable* if, for any $B \in \mathcal{B}(X)$, $V^{-1}(B) = \{t \in \Omega : V(t) \cap B \ne \emptyset\} \in \mathcal{A}$.

Definition 2.5. A mapping $u : \Omega \to X$ is called a *measurable selection* of a set-valued measurable mapping $V : \Omega \to 2^X$ if, u is measurable and for any $t \in \Omega$, $u(t) \in V(t)$.

Definition 2.6. A set-valued mapping $V : \Omega \times X \to 2^X$ is called a *random* set-valued mapping if, for any $x \in X$, $V(\cdot, x)$ is measurable. A random set-valued mapping $V : \Omega \times X \to 2^X$ is said to be $\hat{H}$-*continuous* if, for any $t \in \Omega$, $V(t, \cdot)$ is continuous in the Hausdorff metric on $CB(X)$.

Definition 2.7. Let X be a q-uniformly smooth Banach space, $T : \Omega \times X \to X$ and $\eta : \Omega \times X \times X \to X$ be random single-valued mappings. Then

(a) T is said to be *accretive* if

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t)) \rangle \geq 0, \quad \forall x(t), y(t) \in X, t \in \Omega;$$

(b) T is called *strictly accretive* if T is accretive and

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t)) \rangle = 0,$$

if and only if $x(t) = y(t)$ for all $t \in \Omega$;

(c) T is said to be *r-strongly accretive* if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t)) \rangle \geq r(t)\|x(t) - y(t)\|^q, \quad \forall x(t), y(t) \in X, t \in \Omega;$$

(d) T is said to be *(θ, κ)-relaxed cocoercive* if there exist measurable functions $\theta, \kappa : \Omega \to (0, \infty)$ such that

$$\langle T_t(x) - T_t(y), J_q(x(t) - y(t)) \rangle \geq -\theta(t)\|T_t(x) - T_t(y)\|^q + \kappa(t)\|x(t) - y(t)\|^q,$$
$$\forall x(t), y(t) \in X, t \in \Omega;$$

(e) T is called *ϱ-Lipschitz continuous* if there exists a measurable function $\varrho : \Omega \to (0, \infty)$ such that

$$\|T_t(x) - T_t(y)\| \leq \varrho(t)\|x(t) - y(t)\|, \quad \forall x(t), y(t) \in X, t \in \Omega;$$

(f) η is said to be *τ-Lipschitz continuous* if there exists a measurable function $\tau : \Omega \to (0, \infty)$ such that

$$\|\eta_t(x, y)\| \leq \tau(t)\|x(t) - y(t)\|, \quad \forall x(t), y(t) \in X, t \in \Omega;$$

(g) η is said to be *μ-Lipschitz continuous in the second argument* if there exists a measurable function $\mu : \Omega \to (0, \infty)$ such that

$$\|\eta_t(x, u) - \eta_t(y, u)\| \leq \mu(t)\|x(t) - y(t)\|, \quad \forall x(t), y(t), u(t) \in X, t \in \Omega.$$

In a similar way to part (g), we can define the Lipschitz continuity of the mapping η in the third argument.

Definition 2.8. Let X be a q-uniformly smooth Banach space, $\eta : \Omega \times X \times X \to X$ and $H, A : \Omega \times X \to X$ be three random single-valued mappings. Then a set-valued mapping $M : \Omega \times X \to 2^X$ is said to be:

(a) *accretive* if

$$\langle u(t) - v(t), J_q(x(t) - y(t)) \rangle \geq 0,$$
$$\forall x(t), y(t) \in X, u(t) \in M_t(x), v(t) \in M_t(y), t \in \Omega;$$

(b) *η-accretive* if

$$\langle u(t) - v(t), J_q(\eta_t(x, y)) \rangle \geq 0,$$
$$\forall x(t), y(t) \in X, u(t) \in M_t(x), v(t) \in M_t(y), t \in \Omega;$$

(c) *strictly η-accretive* if M is η-accretive and the equality holds if and only if $x(t) = y(t)$, $\forall t \in \Omega$;

(d) *r-strongly η-accretive* if there exists a measurable function $r : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), J_q(\eta_t(x, y)) \rangle \geq r(t) \|x(t) - y(t)\|^q,$$
$$\forall x(t), y(t) \in X, u(t) \in M_t(x), v(t) \in M_t(y), t \in \Omega;$$

(e) *α-relaxed η-accretive* if there exists a measurable function $\alpha : \Omega \to (0, \infty)$ such that

$$\langle u(t) - v(t), J_q(\eta_t(x, y)) \rangle \geq -\alpha(t) \|x(t) - y(t)\|^q,$$
$$\forall x(t), y(t) \in X, u(t) \in M_t(x), v(t) \in M_t(y), t \in \Omega;$$

(f) *m-accretive* if M is accretive and $(I_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and for any measurable function $\rho : \Omega \to (0, \infty)$, where I denotes the identity mapping on X, $I_t(x) = x(t)$, for all $x(t) \in X$, $t \in \Omega$;

(g) *generalized m-accretive* if M is η-accretive and $(I_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$;

(h) *H-accretive* if M is accretive and $(H_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$, where $H_t(\cdot) = H(t, \cdot)$ for all $t \in \Omega$;

(i) *(H, η)-accretive* if M is η-accretive and $(H_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$;

(j) *A-maximal m-relaxed η-accretive* if M is m-relaxed η-accretive and $(A_t + \rho(t)M_t)(X) = X$ for all $t \in \Omega$ and any measurable function $\rho : \Omega \to (0, \infty)$, where $A_t(\cdot) = A(t, \cdot)$ for all $t \in \Omega$;

(k) *β-Ĥ-Lipschitz continuous* if there exists a measurable function $\beta : \Omega \to (0, +\infty)$ such that

$$\hat{H}(M_t(x), M_t(y)) \leq \beta(t) \|x(t) - y(t)\|, \quad \forall x(t), y(t) \in X, t \in \Omega.$$

Remark 2.9. (1) If $X = \mathcal{H}$ is a Hilbert space, then parts (a)-(i) of the above mentioned definition reduce to the definitions of monotone operators, η-monotone operators, strictly η-monotone operators, strongly η-monotone operators, relaxed η-monotone operators, maximal monotone operators, maximal η-monotone operators, H-monotone operators and (H, η)-monotone operators, respectively.

(2) For appropriate and suitable choices of m, A, η and X, it is easy to see that part (j) of the above mentioned definition includes a number of definitions of monotone operators and accretive mappings (see [49]).

Proposition 2.10 ([49])**.** *Let $A : \Omega \times X \to X$ be an r-strongly η-accretive mapping and $M : \Omega \times X \to 2^X$ be an A-maximal m-relaxed η-accretive mapping. Then the operator $(A_t + \rho(t)M_t)^{-1}$ is single-valued for any measurable function $\rho : \Omega \to (0, +\infty)$ and $t \in \Omega$.*

Definition 2.11. Let $A : \Omega \times X \to X$ be a strictly η-accretive mapping and $M : \Omega \times X \to 2^X$ be an A-maximal m-relaxed η-accretive mapping. Then,

for any measurable function $\rho : \Omega \to (0, +\infty)$, the resolvent operator $J_{\rho(t),A_t}^{\eta_t,M_t} : X \to X$ is defined by:

$$J_{\rho(t),A_t}^{\eta_t,M_t}(u(t)) = (A_t + \rho(t)M_t)^{-1}(u(t)), \quad \forall t \in \Omega, u(t) \in X.$$

Proposition 2.12 ([49]). *Let X be a q-uniformly smooth Banach space and $\eta : \Omega \times X \times X \to X$ be τ-Lipschitz continuous, $A : \Omega \times X \to X$ be an r-strongly η-accretive mapping and $M : \Omega \times X \to X$ be an A-maximal m-relaxed η-accretive mapping. Then the resolvent operator $J_{\rho(t),A_t}^{\eta_t,M_t} : X \to X$ is $\frac{\tau^{q-1}(t)}{r(t)-\rho(t)m(t)}$-Lipschitz continuous, i.e.,*

$$\|J_{\rho(t),A_t}^{\eta_t,M_t}(x(t)) - J_{\rho(t),A_t}^{\eta_t,M_t}(y(t))\| \leq \frac{\tau^{q-1}(t)}{r(t) - \rho(t)m(t)}\|x(t) - y(t)\|,$$
$$\forall x(t), y(t) \in X, t \in \Omega,$$

where $\rho(t) \in (0, \frac{r(t)}{m(t)})$ is a real-valued random variable for all $t \in \Omega$.

3. A new random variational inclusion problem

In what follows, we denote the collection of all fuzzy sets on X by $\mathfrak{F}(X) = \{A \mid A : X \to [0,1]\}$. For any set K, a mapping $\mathcal{S}$ from K into $\mathfrak{F}(X)$ is called a *fuzzy mapping*. If $\mathcal{S} : K \to \mathfrak{F}(X)$ is a fuzzy mapping, then $\mathcal{S}(x)$, for any $x \in K$, is a fuzzy set on $\mathfrak{F}(X)$ (in the sequel, we denote $\mathcal{S}(x)$ by $\mathcal{S}_x$) and $\mathcal{S}_x(y)$, for any $y \in X$, is the degree of membership of y in $\mathcal{S}_x$. For any $A \in \mathfrak{F}(X)$ and $\alpha \in [0,1]$, the set

$$(A)_\alpha = \{x \in X : A(x) \geq \alpha\}$$

is called a α-*cut* set of A.

Definition 3.1. A fuzzy mapping $\mathcal{S} : \Omega \to \mathfrak{F}(X)$ is called *measurable* if, for any $\alpha \in (0,1]$, $(\mathcal{S}(\cdot))_\alpha : \Omega \to 2^X$ is a measurable set-valued mapping.

Definition 3.2. A fuzzy mapping $\mathcal{S} : \Omega \times X \to \mathfrak{F}(X)$ is called a *random fuzzy mapping* if, for any $x \in X$, $\mathcal{S}(\cdot, x) : \Omega \to \mathfrak{F}(X)$ is a measurable fuzzy mapping.

Obviously, the random fuzzy mapping includes set-valued mapping, random set-valued mapping and fuzzy mapping as special cases.

Let $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l, \mathcal{Q}, \mathcal{G} : \Omega \times X \to \mathfrak{F}(X)$ be random fuzzy mappings satisfying the following condition (∗): There exist mappings $a_1, a_2, \ldots, a_l, d, e : X \to [0,1]$ such that

(∗) $\quad \begin{aligned} &(\mathcal{S}_{i,t,x(t)})_{a_i(x(t))} \in CB(X) \quad \text{for each } i = 1, 2, \ldots, l, \\ &(\mathcal{Q}_{t,x(t)})_{d(x(t))} \in CB(X),\ (\mathcal{G}_{t,x(t)})_{e(x(t))} \in CB(X), \quad \forall(t, x(t)) \in \Omega \times X. \end{aligned}$

By using the random fuzzy mapping $\mathcal{S}_1$ satisfying (∗) with corresponding function $a_1 : X \to [0,1]$, we can define a random set-valued mapping S_1 as follows:

$$S_1 : \Omega \times X \to CB(X), \quad (t, x(t)) \mapsto (\mathcal{S}_{1,t,x(t)})_{a_1(x(t))}, \quad \forall(t, x(t)) \in \Omega \times X,$$

where $\mathcal{S}_{1,t,x(t)} = \mathcal{S}_1(t, x(t))$. In the sequel, S_i $(i = 1, 2, \ldots, l)$, Q and G are called the random set-valued mappings induced by the random fuzzy mappings $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$ and $\mathcal{G}$, respectively.

Suppose that $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l, \mathcal{Q}, \mathcal{G} : \Omega \times X \to \mathfrak{F}(X)$ are random fuzzy mappings, $A, p, f : \Omega \times X \to X$, $\eta : \Omega \times X \times X \to X$ and $N : \Omega \times \underbrace{X \times X \times \cdots \times X}_{l} \to X$ are random single-valued mappings. Further, let $a_1, a_2, \ldots, a_l, d, e : X \to [0, 1]$ be any mappings and $M : \Omega \times X \times X \to 2^X$ be a random set-valued mapping such that, for each fixed $t \in \Omega$ and $z(t) \in X$, $M(t, \cdot, z(t)) : X \to 2^X$ be an A-maximal m-relaxed η-accretive mapping with $\mathrm{Im}(p) \cap \mathrm{dom}\, M(t, \cdot, z(t)) \neq \emptyset$. Now, we consider the following problem:

For any element $h : \Omega \to X$ and any measurable function $\lambda : \Omega \to (0, +\infty)$, find measurable mappings $x, u_1, u_2, \ldots, u_l, \vartheta, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$ and for each $i = 1, 2, \ldots, l$, $\mathcal{S}_{i,t,x(t)}(u_i(t)) \geq a_i(x(t))$, $\mathcal{Q}_{t,x(t)}(\vartheta(t)) \geq d(x(t))$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$(1) \quad h(t) \in f_t(x) + N_t(u_1, u_2, \ldots, u_l) + \lambda(t) M_t(p_t(x) - \vartheta, w), \quad \forall t \in \Omega.$$

The problem (1) is called *the general nonlinear random A-maximal m-relaxed η-accretive equation with random relaxed cocoercive mappings and random fuzzy mappings* in Banach spaces.

Remark 3.3. For appropriate and suitable choices of X, A, η, λ, p, f, M, N, $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$, $\mathcal{G}$ and h, one can obtain many known classes of random variational inequalities, random quasi-variational inequalities, random complementarity and random quasi-complementarity problems as special cases of the problem (1).

Some special cases of the problem (1) are as follows:

Case I: If $l = 3$, $N : \Omega \times X \times X \times X \to X$ is a random single-valued mapping, $\mathcal{S}_1 = \mathcal{S}$, $\mathcal{S}_2 = \mathcal{T}$, $\mathcal{S}_3 = \mathcal{P}$, $a_1 = a$, $a_2 = b$, $a_3 = c$, $u_1 = \nu$, $u_2 = u$, $u_3 = v$, and A, p, f, η, M, Q, G are the same as in the problem (1), then for any element $h : \Omega \to X$ and any measurable function $\lambda : \Omega \to (0, +\infty)$, the problem (1) collapses to the problem of finding measurable mappings $x, \nu, u, v, \vartheta, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $\mathcal{S}_{t,x(t)}(\nu(t)) \geq a(x(t))$, $\mathcal{T}_{t,x(t)}(u(t)) \geq b(x(t))$, $\mathcal{P}_{t,x(t)}(v(t)) \geq c(x(t))$, $\mathcal{Q}_{t,x(t)}(\vartheta(t)) \geq d(x(t))$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$(2) \quad h(t) \in f_t(x) + N_t(\nu, u, v) + \lambda(t) M_t(p_t(x) - \vartheta, w), \quad \forall t \in \Omega,$$

which appears to be a new problem.

Case II: Let $A, p, f, \eta, M, N, \mathcal{T}, \mathcal{P}, \mathcal{G}$ be the same as in the problem (2) and $S, Q : \Omega \times X \to CB(X)$ be two random set-valued mappings. Now, by using S and Q, we define two random fuzzy mappings $\mathcal{S}, \mathcal{Q} : \Omega \times X \to 2^X$ as follows:

$$\mathcal{S}_{t,x(t)} = \chi_{S_t(x)}, \qquad \mathcal{Q}_{t,x(t)} = \chi_{Q_t(x)}, \qquad \forall (t, x(t)) \in \Omega \times X,$$

where $\chi_{S_t(x)}$ and $\chi_{Q_t(x)}$ are the characteristic functions of the sets $S_t(x)$ and $Q_t(x)$, respectively. It is easy to see that $\mathcal{S}$ and $\mathcal{Q}$ are random fuzzy mappings satisfying the condition $(*)$ with constant functions $a(x(t)) = 1$ and $d(x(t)) = 1$, for all $t \in \Omega$, $x(t) \in X$, respectively. Also,

$$(\mathcal{S}_{t,x(t)})_{a(x(t))} = (\chi_{S_t(x)})_1 = \{y \in X : \chi_{S_t(x)}(y) = 1\} = S_t(x),$$
$$(\mathcal{Q}_{t,x(t)})_{d(x(t))} = (\chi_{Q_t(x)})_1 = \{y \in X : \chi_{Q_t(x)}(y) = 1\} = Q_t(x).$$

Then the problem (2) is equivalent to the following:

Find measurable mappings $x, \nu, u, v, \vartheta, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $\nu(t) \in S_t(x)$, $\mathcal{T}_{t,x(t)}(u(t)) \geq b(x(t))$, $\mathcal{P}_{t,x(t)}(v(t)) \geq c(x(t))$, $\vartheta(t) \in Q_t(x)$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$(3) \qquad h(t) \in f_t(x) + N_t(\nu, u, v) + \lambda(t)M_t(p_t(x) - \vartheta, w), \quad \forall t \in \Omega,$$

which appears to be a new problem.

Case III: If A, p, f, η, M, N, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$ are the same as in the problem (2) and $S, Q : \Omega \times X \to X$ are two random single-valued mappings, then the problem (3) is equivalent to determining measurable mappings $x, u, v, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $\mathcal{T}_{t,x(t)}(u(t)) \geq b(x(t))$, $\mathcal{P}_{t,x(t)}(v(t)) \geq c(x(t))$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$(4) \qquad h(t) \in f_t(x) + N_t(S_t(x), u, v) + \lambda(t)M_t(p_t(x) - Q_t(x), w), \quad \forall t \in \Omega,$$

which appears to be a new problem.

Case IV: If A, p, f, η, M, N, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, S are the same as in the problem (4) and $Q \equiv 0$, then the problem (4) reduces to the following problem:

Find measurable mappings $x, u, v, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $\mathcal{T}_{t,x(t)}(u(t)) \geq b(x(t))$, $\mathcal{P}_{t,x(t)}(v(t)) \geq c(x(t))$, $\mathcal{G}_{t,x(t)}(w(t)) \geq e(x(t))$ and

$$(5) \qquad h(t) \in f_t(x) + N_t(S_t(x), u, v) + \lambda(t)M_t(p_t(x), w), \quad \forall t \in \Omega,$$

which appears to be a new problem. The problem (5) is introduced and studied by Uea and Kumam [58], when $f \equiv 0$, $h(t) = 0$ and $\lambda(t) = 1$, for all $t \in \Omega$.

Case V: Let A, p, f, η, M, N be the same as in the problem (1) and $S_1, S_2, \ldots, S_l, Q, G : \Omega \times X \to CB(X)$ be random set-valued mappings. Like in Case II, by using S_1, S_2, ..., S_l, Q and G, we can define random fuzzy mappings $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l, \mathcal{Q}$ and $\mathcal{G}$ as follows:

$$\mathcal{S}_{i,t,x(t)} = \chi_{S_{i,t}(x)}, \quad (i = 1, 2 \ldots, l),$$
$$\mathcal{Q}_{t,x(t)} = \chi_{Q_t(x)}, \quad \mathcal{G}_{t,x(t)} = \chi_{G_t(x)}, \qquad \forall (t, x(t)) \in \Omega \times X,$$

where $\chi_{S_{i,t}(x)}$ $(i = 1, 2, \ldots, l)$, $\chi_{Q_t(x)}$ and $\chi_{G_t(x)}$ are the characteristic functions of the sets $S_{i,t}(x)$ $(i = 1, 2, \ldots, l)$, $Q_t(x)$ and $G_t(x)$, respectively. It is easy to see that $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$ and $\mathcal{G}$ are random fuzzy mappings satisfying the

condition ($*$) with constant functions $a_i(x(t)) = 1$ ($i = 1, 2, \ldots, l$), $d(x(t)) = 1$ and $e(x(t)) = 1$, for all $t \in \Omega$, $x(t) \in X$, respectively. Also,

$$(\mathcal{S}_{i,t,x(t)})_{a_i(x(t))} = (\chi_{S_{i,t}(x)})_1 = \{y \in X : \chi_{S_{i,t}(x)}(y) = 1\} = S_{i,t}(x),$$
$$(\mathcal{Q}_{t,x(t)})_{d(x(t))} = (\chi_{Q_t(x)})_1 = \{y \in X : \chi_{Q_t(x)}(y) = 1\} = Q_t(x),$$
$$(\mathcal{G}_{t,x(t)})_{e(x(t))} = (\chi_{G_t(x)})_1 = \{y \in X : \chi_{G_t(x)}(y) = 1\} = G_t(x),$$
$$i = 1, 2, \ldots, l.$$

Then the problem (1) is equivalent to the following:

Find measurable mappings $x, u_1, u_2, \ldots, u_l, \vartheta, w : \Omega \to X$ such that for each $t \in \Omega$, $x(t) \in X$, $u_i(t) \in S_{i,t}(x)$ for each $i = 1, 2, \ldots, l$, $\vartheta(t) \in Q_t(x)$, $w(t) \in G_t(x)$ and

$$(6) \quad h(t) \in f_t(x) + N_t(u_1, u_2, \ldots, u_l) + \lambda(t)M_t(p_t(x) - \vartheta, w), \quad \forall t \in \Omega.$$

The problem (6) is called the *general nonlinear random A-maximal m-relaxed η-accretive equation with random relaxed cocoercive mappings* in Banach spaces.

Case VI: If $X = \mathcal{H}$ is a Hilbert space, $T : \Omega \times \mathcal{H} \to \mathcal{H}$ is a random single-valued operator, $h(t) = 0$ and $\lambda(t) = 1$, for all $t \in \Omega$, $f \equiv 0$, $p \equiv I$, $N_t(x, y, z) = N_t(x, y)$, for all $t \in \Omega$ and $x(t), y(t), z(t) \in X$, $M(t, x(t), s(t)) = M(t, x(t))$ for all $t \in \Omega$ and $x(t), s(t) \in \mathcal{H}$ and $M(t, \cdot) : \mathcal{H} \to 2^{\mathcal{H}}$ is a A-maximal m-relaxed η-monotone (so-called (A, η)-monotone) operator for all $t \in \Omega$, then the problem (5) reduces to the following generalized random set-valued operator equation involving A-maximal m-relaxed η-monotone operator in Hilbert spaces:

Find measurable mapping $x : \Omega \to \mathcal{H}$ such that

$$(7) \quad x \in J_{\rho(t),A_t}^{\eta_t, M_t}(A_t(x) - \rho(t)N_t(S_t(x), T_t(x))), \quad \forall t \in \Omega,$$

where $\rho(t) > 0$ is a real-valued random variable and $J_{\rho(t),A_t}^{\eta_t, M_t} = (A_t + \rho(t)M_t)^{-1}$.

The problem (7) was introduced and studied by Lan [45].

Case VII: If $T : \Omega \times X \to X$ is a random single-valued mapping, $N_t(x, y, z) = x(t) + y(t)$ for all $t \in \Omega$ and $x(t), y(t), z(t) \in X$, $f \equiv 0$, $p \equiv I$, $M(t, x(t), s(t)) = M(t, x(t))$ for all $t \in \Omega$ and $x(t), s(t) \in X$ and $M(t, \cdot) : X \to 2^X$ is a A-maximal m-relaxed η-monotone operator for all $t \in \Omega$, then the problem (5) changes into finding a measurable mapping $x : \Omega \to X$ such that

$$(8) \quad 0 \in S_t(x) + T_t(x) + \lambda(t)M_t(x), \quad \forall t \in \Omega.$$

The problem (8) was introduced and studied by Cho and Lan [25].

Some other special cases of the problems (1)–(6) can be found in [1–3, 23, 25, 28, 32, 35, 42, 46, 50, 58] and the references therein.

4. Random iterative algorithms

In this section, we develop and analyze a new class of iterative methods and construct some new random iterative algorithms with mixed errors for solving the problems (1)–(6). For this end, we need the following lemmas.

Lemma 4.1 ([13]). *Let $M : \Omega \times X \to CB(X)$ be a $\hat{H}$-continuous random set-valued mapping. Then, for any measurable mapping $x : \Omega \to X$, the set-valued mapping $M(\cdot, x(\cdot)) : \Omega \to CB(X)$ is measurable.*

Lemma 4.2 ([13]). *Let $M, V : \Omega \to CB(X)$ be two measurable set-valued mappings, $\epsilon > 0$ be a constant and $x : \Omega \to X$ be a measurable selection of M. Then there exists a measurable selection $y : \Omega \to X$ of V such that, for any $t \in \Omega$,*

$$\|x(t) - y(t)\| \le (1 + \epsilon)\hat{H}(M(t), V(t)).$$

The following lemma offers a good approach for solving the problem (1).

Lemma 4.3. *The set of measurable mappings $x, u_1, u_2, \ldots, u_l, \vartheta, w : \Omega \to X$ is a random solution of the problem (1) if and only if, for each $t \in \Omega$, $u_i(t) \in S_{i,t}(x)$ for each $i = 1, 2, \ldots, l$, $\vartheta(t) \in Q_t(x)$, $w(t) \in G_t(x)$ and*

$$\begin{aligned} p_t(x) = {}& \vartheta(t) + J^{\eta_t, M_t(\cdot, w)}_{\rho(t)\lambda(t), A_t}[A_t(p_t(x) - \vartheta) - \rho(t)(f_t(x) \\ & + N_t(u_1, u_2, \ldots, u_l) - h(t))], \end{aligned}$$

where $J^{\eta_t, M_t(\cdot, w)}_{\rho(t)\lambda(t), A_t} = (A_t + \rho(t)\lambda(t)M_t(\cdot, w))^{-1}$ and $\rho : \Omega \to (0, \infty)$ is a measurable function.

Proof. The fact follows directly from the definition of $J^{\eta_t, M_t(\cdot, w)}_{\rho(t)\lambda(t), A_t}$. $\qquad\square$

Now, by using Chang's lemma [13] and based on Lemma 4.3, we can construct the new following iterative algorithm for solving the problem (1).

Algorithm 4.4. *Let A, p, f, η, M, N, $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$, $\mathcal{G}$, h, λ be the same as in the problem (1) and let S_i $(i = 1, 2, \ldots, l)$, Q, G be $\hat{H}$-continuous random set-valued mappings induced by $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$ and $\mathcal{G}$, respectively. Assume that $\alpha : \Omega \to (0, 1]$ is a measurable step size function. For any measurable mapping $x_0 : \Omega \to X$, the set-valued mappings $S_i(\cdot, x_0(\cdot)), Q(\cdot, x_0(\cdot)), G(\cdot, x_0(\cdot)) : \Omega \to CB(X)$ $(i = 1, 2, \ldots, l)$, are measurable by Lemma 4.1. Hence there exist measurable selections $u_{0,i} : \Omega \to X$ of $S_i(\cdot, x_0(\cdot))$ $(i = 1, 2, \ldots, l)$, $\vartheta_0 : \Omega \to X$ of $Q(\cdot, x_0(\cdot))$ and $w_0 : \Omega \to X$ of $G(\cdot, x_0(\cdot))$ by Himmelberg [33]. For each $t \in \Omega$, set*

$$\begin{aligned} x_1(t) = {}& (1 - \alpha(t))x_0(t) + \alpha(t)\{x_0(t) - p_t(x_0) + \vartheta_0(t) + J^{\eta_t, M_t(\cdot, w_0)}_{\rho(t)\lambda(t), A_t}[A_t(p_t(x_0) \\ & - \vartheta_0) - \rho(t)(f_t(x_0) + N_t(u_{0,1}, u_{0,2}, \ldots, u_{0,l}) - h(t))]\} \\ & + \alpha(t)e_0(t) + r_0(t), \end{aligned}$$

where $\rho(t)$ is the same as in Lemma 4.3 and $e_0, r_0 : \Omega \to X$ are measurable functions. It is easy to know that $x_1 : \Omega \to X$ is measurable. Since $u_{0,i}(t) \in S_{i,t}(x_0) \in CB(X)$ for each $i = 1, 2, \ldots, l$, $\vartheta_0(t) \in Q_t(x_0) \in CB(X)$ and $w_0(t) \in G_t(x_0) \in CB(X)$, by Lemma 4.2, there exist measurable selections $u_{1,i}, w_1, \vartheta_1 :$

$\Omega \to X$ $(i = 1, 2, \ldots, l)$ *of set-valued measurable mappings* $S_i(\cdot, x_1(\cdot))$ $(i = 1, 2, \ldots, l)$, $Q(\cdot, x_1(\cdot))$ *and* $G(\cdot, x_1(\cdot))$, *respectively, such that, for all* $t \in \Omega$,

$$\|u_{0,i}(t) - u_{1,i}(t)\| \leq (1 + \frac{1}{1})\hat{H}(S_{i,t}(x_0), S_{i,t}(x_1)),$$

$$\|\vartheta_0(t) - \vartheta_1(t)\| \leq (1 + \frac{1}{1})\hat{H}(Q_t(x_0), Q_t(x_1)),$$

$$\|w_0(t) - w_1(t)\| \leq (1 + \frac{1}{1})\hat{H}(G_t(x_0), G_t(x_1)).$$

Letting

$$\begin{aligned} x_2(t) &= (1 - \alpha(t))x_1(t) + \alpha(t)\{x_1(t) - p_t(x_1) + \vartheta_1(t) + J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w_1)}[A_t(p_t(x_1) \\ &\quad - \vartheta_1) - \rho(t)(f_t(x_1) + N_t(u_{1,1}, u_{1,2}, \ldots, u_{1,l}) - h(t))]\} \\ &\quad + \alpha(t)e_1(t) + r_1(t), \quad \forall t \in \Omega, \end{aligned}$$

then $x_2 : \Omega \to X$ *is measurable. By induction, we can define sequences* $\{x_n(t)\}$, $\{u_{n,i}(t)\}$ $(i = 1, 2, \ldots, l)$, $\{\vartheta_n(t)\}$ *and* $\{w_n(t)\}$ *for solving the problem (1) inductively satisfying*

$$(9) \quad \begin{cases} x_{n+1}(t) = (1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + \vartheta_n(t) \\ \qquad + J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_n) - \vartheta_n) - \rho(t)(f_t(x_n) \\ \qquad + N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l}) - h(t))]\} \\ \qquad + \alpha(t)e_n(t) + r_n(t), \quad \forall t \in \Omega, \\ u_{n,i}(t) \in S_{i,t}(x_n), \vartheta_n(t) \in Q_t(x_n), w_n(t) \in G_t(x_n), \\ \|u_{n,i}(t) - u_{n+1,i}(t)\| \leq (1 + \frac{1}{1+n})\hat{H}(S_{i,t}(x_n), S_{i,t}(x_{n+1})), \\ \|\vartheta_n(t) - \vartheta_{n+1}(t)\| \leq (1 + \frac{1}{1+n})\hat{H}(Q_t(x_n), Q_t(x_{n+1})), \\ \|w_n(t) - w_{n+1}(t)\| \leq (1 + \frac{1}{1+n})\hat{H}(G_t(x_n), G_t(x_{n+1})), \\ i = 1, 2, \ldots, l, \end{cases}$$

where for all $n \geq 0$ *and* $t \in \Omega$, $e_n(t), r_n(t) \in X$ *are real-valued random errors to take into account a possible inexact computation of the random resolvent operator point satisfying the following conditions:*

$$(10) \quad \begin{cases} \lim_{n \to \infty} \|e_n(t)\| = \lim_{n \to \infty} \|r_n(t)\| = 0, \quad \forall t \in \Omega; \\ \sum_{n=0}^{\infty} \|e_n(t) - e_{n-1}(t)\| < \infty, \quad \forall t \in \Omega; \\ \sum_{n=0}^{\infty} \|r_n(t) - r_{n-1}(t)\| < \infty, \quad \forall t \in \Omega. \end{cases}$$

Algorithm 4.5. *Let* A, p, f, η, M, N, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, h, λ *be the same as in the problem (2) and let* S, T, P, Q, G *be* $\hat{H}$-*continuous random set-valued mappings induced by* $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$ *and* $\mathcal{G}$, *respectively. Suppose further that* $\alpha : \Omega \to (0, 1]$ *is a measurable step size function. In a similar way to Algorithm*

4.4, for any measurable mapping $x_0 : \Omega \to X$, we can define sequences $\{x_n(t)\}$, $\{\nu_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ for solving the problem (2) in the following way:

$$
\begin{cases}
x_{n+1}(t) = (1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + \vartheta_n(t) \\
\qquad + J_{\rho(t)\lambda(t),A_t}^{\eta_t,M_t(\cdot,w_n)}[A_t(p_t(x_n) - \vartheta_n) - \rho(t)(f_t(x_n) + N_t(\nu_n, u_n, v_n) \\
\qquad - h(t))]\} + \alpha(t)e_n(t) + r_n(t), \quad \forall t \in \Omega, \\
\nu_n(t) \in S_t(x_n), \ \|\nu_n(t) - \nu_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(S_t(x_n), S_t(x_{n+1})), \\
u_n(t) \in T_t(x_n), \ \|u_n(t) - u_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(T_t(x_n), T_t(x_{n+1})), \\
v_n(t) \in P_t(x_n), \ \|v_n(t) - v_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(P_t(x_n), P_t(x_{n+1})), \\
\vartheta_n(t) \in Q_t(x_n), \ \|\vartheta_n(t) - \vartheta_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(Q_t(x_n), Q_t(x_{n+1})), \\
w_n(t) \in G_t(x_n), \ \|w_n(t) - w_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(G_t(x_n), G_t(x_{n+1})),
\end{cases}
$$

where for all $n \ge 0$, $e_n(t), r_n(t) \in X$ are the same as in Algorithm 4.4 satisfying the conditions (10).

Algorithm 4.6. *Let $A, p, f, \eta, M, N, \mathcal{T}, \mathcal{P}, \mathcal{G}, S, Q, h, \lambda$ be the same as in the problem (4) and $T, P, G : \Omega \times X \to CB(X)$ be $\hat{H}$-continuous random set-valued mappings induced by $\mathcal{T}, \mathcal{P}, \mathcal{G}$, respectively. Further, let $\alpha : \Omega \to (0, 1]$ be a measurable step size function. In similar to Algorithm 4.4, for any measurable mapping $x_0 : \Omega \to X$, we can define sequences $\{x_n(t)\}, \{u_n(t)\}, \{v_n(t)\}$ and $\{w_n(t)\}$ for solving the problem (4) as follows:*

$$
\begin{cases}
x_{n+1}(t) = (1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + Q_t(x_n) \\
\qquad + J_{\rho(t)\lambda(t),A_t}^{\eta_t,M_t(\cdot,w_n)}[A_t(p_t(x_n) - Q_t(x_n)) - \rho(t)(f_t(x_n) \\
\qquad + N_t(S_t(x_n), u_n, v_n) - h(t))]\} + \alpha(t)e_n(t) + r_n(t), \quad \forall t \in \Omega, \\
u_n(t) \in T_t(x_n), \ \|u_n(t) - u_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(T_t(x_n), T_t(x_{n+1})), \\
v_n(t) \in P_t(x_n), \ \|v_n(t) - v_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(P_t(x_n), P_t(x_{n+1})), \\
w_n(t) \in G_t(x_n), \ \|w_n(t) - w_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(G_t(x_n), G_t(x_{n+1})),
\end{cases}
$$

where for all $n \ge 0$, $e_n(t), r_n(t) \in X$ are the same as in Algorithm 4.4 satisfying the conditions (10).

Algorithm 4.7. *Let $A, p, f, \eta, M, N, \mathcal{T}, \mathcal{P}, \mathcal{G}, S, Q, h, \lambda$ be the same as in the problem (5) and $T, P, G : \Omega \times X \to CB(X)$ be $\hat{H}$-continuous random set-valued mappings induced by $\mathcal{T}, \mathcal{P}, \mathcal{G}$, respectively. Further, let $\alpha : \Omega \to (0, 1]$ be a measurable step size function. Like in Algorithm 4.4, for any measurable mapping $x_0 : \Omega \to X$, we can define sequences $\{x_n(t)\}, \{u_n(t)\}, \{v_n(t)\}$ and $\{w_n(t)\}$ for solving the problem (5) as follows:*

$$
\begin{cases}
x_{n+1}(t) = (1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + J_{\rho(t)\lambda(t),A_t}^{\eta_t,M_t(\cdot,w_n)}[A_t(p_t(x_n)) \\
\qquad - \rho(t)(f_t(x_n) + N_t(S_t(x_n), u_n, v_n) - h(t))]\} \\
\qquad + \alpha(t)e_n(t) + r_n(t), \quad \forall t \in \Omega, \\
u_n(t) \in T_t(x_n), \ \|u_n(t) - u_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(T_t(x_n), T_t(x_{n+1})), \\
v_n(t) \in P_t(x_n), \ \|v_n(t) - v_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(P_t(x_n), P_t(x_{n+1})), \\
w_n(t) \in G_t(x_n), \ \|w_n(t) - w_{n+1}(t)\| \le (1 + \frac{1}{1+n})\hat{H}(G_t(x_n), G_t(x_{n+1})),
\end{cases}
$$

where for all $n \geq 0$, $e_n(t), r_n(t) \in X$ are the same as in Algorithm 4.4 satisfying the conditions (10).

Remark 4.8. (1) If $h(t) = 0$, $\lambda(t) = 1$, $e_n(t) = 0$, $r_n(t) = 0$, for all $n \geq 0$ and $t \in \Omega$, $f \equiv 0$ and $M : \Omega \times X \times X \to 2^X$ is a (H, η)-accretive mapping, then Algorithm 4.7 reduces to Algorithm 3.2 in [58]. Also, for a suitable and appropriate choice of the involved mappings and constants in Algorithm 4.7, one can obtain Algorithms 3.3 and 3.4 in [58].

(2) If $h(t) = 0$, $\lambda(t) = 1$, $r_n(t) = 0$, for all $n \geq 0$ and $t \in \Omega$, $f \equiv 0$ and $M : \Omega \times X \times X \to 2^X$ is a generalized m-accretive mapping, then Algorithm 4.7 reduces to Algorithm 2.4 in [50].

(3) If $p \equiv I$, $f \equiv 0$, $h(t) = 0$, $\lambda(t) = 1$, $r_n(t) = 0$, for all $n \geq 0$ and $t \in \Omega$, $G : \Omega \times X \to X$ be a random single-valued mapping, $N_t(x, , y, z) = f_t(z) + g_t(x, y)$, for all $t \in \Omega$ and $x(t), y(t), z(t) \in X$, where $f : \Omega \times X \to X$ and $g : \Omega \times X \times X \to X$ are random single-valued mappings and $M : \Omega \times X \times X \to 2^X$ is a generalized m-accretive mapping, then Algorithm 4.7 reduces to Algorithm 2.5 in [50].

(4) If $f \equiv 0$, $N_t(S_t(x), u, v) = S_t(x) + u(t)$, $M(t, x(t), y(t)) = M(t, x(t))$, for all $t \in \Omega$ and $x(t), y(t) \in X$, $\alpha(t) = 1$, $r_n(t) = 0$, for all $n \geq 0$ and $t \in \Omega$, then Algorithm 4.7 reduces to Algorithm 3.1 in [25].

(5) If all the conditions in (4) hold, $p \equiv I$, $T : \Omega \times X \to X$ is a random single-valued mapping, and $h(t) = 0$ for all $t \in \Omega$, then Algorithm 4.7 collapses to Algorithm 3.2 in [25].

(6) If $h(t) = 0$, $\lambda(t) = 1$, $r_n(t) = 0$, for all $n \geq 0$ and $t \in \Omega$, $f \equiv 0$ and $M : \Omega \times X \to 2^X$ is a generalized m-accretive mapping, then Algorithm 4.7 reduces to Algorithm 2.6 in [50].

Remark 4.9. In brief, for a suitable and appropriate choice of the mappings A, p, f, η, M, N, $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, S_i $(i = 1, 2, \ldots, l)$, S, T, P, Q, G, α, h, λ, the sequences $\{e_n\}$, $\{r_n\}$ and the space X, Algorithms 4.4–4.7 include many known algorithms which due to classes of variational inequalities and variational inclusions (see, for example, $[14, 19, 23, 25, 28, 31, 32, 35$–$37, 39, 41, 43, 50, 58]$).

5. Main results

In this section, we prove the existence of solutions for the problems (1)–(6) and the convergence of iterative sequences generated by Algorithms 4.4–4.7 in q-uniformly smooth Banach spaces.

Definition 5.1. Let X be a q-uniformly smooth Banach space and let $N : \Omega \times \underbrace{X \times X \times \cdots \times X}_{l} \to X$ be a random single-valued mapping. N is said to be $(\epsilon_1, \epsilon_2, \ldots, \epsilon_l)$-mixed Lipschitz continuous if there exist measurable functions

$\epsilon_i : \Omega \to (0, +\infty)$ $(i = 1, 2, \ldots, l)$ such that

$$\|N_t(x_1, x_2, \ldots, x_l) - N_t(y_1, y_2, \ldots, y_l)\| \leq \sum_{i=1}^{l} \epsilon_i(t) \|x_i(t) - y_i(t)\|,$$

$$\forall x_i(t), y_i(t) \in X, t \in \Omega.$$

Theorem 5.2. *Let X be a q-uniformly smooth Banach space, A, p, f, η, M, N, $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$, $\mathcal{G}$, h, λ be the same as in the problem (1) and $S_i, Q, G : \Omega \times X \to CB(X)$ $(i = 1, 2, \ldots, l)$, be random set-valued mappings induced by $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{Q}$, $\mathcal{G}$, respectively. Suppose further that*

(a) *p is (γ, ϖ)-relaxed cocoercive and π-Lipschitz continuous;*
(b) *A is r-strongly η-accretive and σ-Lipschitz continuous;*
(c) *η and f are τ-Lipschitz continuous and ε-Lipschitz continuous, respectively;*
(d) *for each $i = 1, 2, \ldots, l$, S_i is ξ_i-$\hat{H}$-Lipschitz continuous, and Q and G are ϱ-$\hat{H}$-Lipschitz continuous and ι-$\hat{H}$-Lipschitz continuous, respectively;*
(e) *N is $(\epsilon_1, \epsilon_2, \ldots, \epsilon_l)$-mixed Lipschitz continuous;*
(f) *there exist measurable functions $\mu : \Omega \to (0, +\infty)$ and $\rho : \Omega \to (0, +\infty)$ with $\rho(t) \in (0, \frac{r(t)}{\lambda(t)m(t)})$, for all $t \in \Omega$, such that*

(11)
$$\|J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, x)}(z(t)) - J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, y)}(z(t))\| \leq \mu(t)\|x(t) - y(t)\|,$$
$$\forall t \in \Omega, x(t), y(t), z(t) \in X$$

and

(12)
$$\begin{cases} \varphi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)} < 1, \\ \sigma(t)(\pi(t) + \varrho(t)) + \rho(t)(\varepsilon(t) + \sum_{i=1}^{l} \epsilon_i(t)\xi_i(t)) \\ < \tau^{1-q}(t)(1 - \varphi(t))(r(t) - \rho(t)\lambda(t)m(t)), \end{cases}$$

where c_q is the same as in Lemma 2.1.

Then there exists a set of measurable mappings $x^*, u_i^*, v^*, \vartheta^*, w^* : \Omega \to X$ $(i = 1, 2, \ldots, l)$, which is a random solution of the problem (1) and for each $t \in \Omega$, $x_n(t) \to x^*(t)$, $u_{n,i}(t) \to u_i^*(t)$ for each $i = 1, 2, \ldots, l$, $v_n(t) \to v^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$, $w_n(t) \to w^*(t)$ as $n \to \infty$, where $\{x_n(t)\}$, $\{u_{n,i}(t)\}$ $(i = 1, 2, \ldots, l)$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are the iterative sequences generated by Algorithm 4.4.

Proof. It follows from (9), Proposition 2.12 and (11) that

(13)
$$\|x_{n+1}(t) - x_n(t)\|$$
$$\leq \|(1 - \alpha(t))x_n(t) + \alpha(t)\{x_n(t) - p_t(x_n) + \vartheta_n(t)$$
$$+ J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_n) - \vartheta_n) - \rho(t)(f_t(x_n)$$
$$+ N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l}) - h(t))]\}$$
$$+ \alpha(t)e_n(t) + r_n(t) - (1 - \alpha(t))x_{n-1}(t)$$

$$
\begin{aligned}
&- \alpha(t)\{x_{n-1}(t) - p_t(x_{n-1}) + \vartheta_{n-1}(t) \\
&\quad + J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_{n-1})}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(f_t(x_{n-1}) \\
&\quad + N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}) - h(t))]\} \\
&- \alpha(t)e_{n-1}(t) - r_{n-1}(t)\| \\
\leq\; &(1 - \alpha(t))\|x_n(t) - x_{n-1}(t)\| \\
&+ \alpha(t)\Big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| \\
&+ \|\vartheta_n(t) - \vartheta_{n-1}(t)\| \\
&+ \|J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_n) - \vartheta_n) \\
&- \rho(t)(f_t(x_n) + N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l}) - h(t))] \\
&- J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_{n-1})}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(f_t(x_{n-1}) \\
&+ N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}) - h(t))]\|\Big) \\
&+ \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\| \\
\leq\; &(1 - \alpha(t))\|x_n(t) - x_{n-1}(t)\| \\
&+ \alpha(t)\Big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| \\
&+ \|\vartheta_n(t) - \vartheta_{n-1}(t)\| + \|J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_n) - \vartheta_n) \\
&- \rho(t)(f_t(x_n) + N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l}) - h(t))] \\
&- J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(f_t(x_{n-1}) \\
&+ N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}) - h(t))]\| \\
&+ \|J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_n)}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(f_t(x_{n-1}) \\
&+ N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}) - h(t))] \\
&- J_{\rho(t)\lambda(t),A_t}^{\eta_t, M_t(\cdot, w_{n-1})}[A_t(p_t(x_{n-1}) - \vartheta_{n-1}) - \rho(t)(f_t(x_{n-1}) \\
&+ N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}) - h(t))]\|\Big) \\
&+ \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\| \\
\leq\; &(1 - \alpha(t))\|x_n(t) - x_{n-1}(t)\| \\
&+ \alpha(t)\Big(\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\| \\
&+ \|\vartheta_n(t) - \vartheta_{n-1}(t)\| + \mu(t)\|w_n(t) - w_{n-1}(t)\| \\
&+ \frac{\tau^{q-1}(t)}{r(t) - \rho(t)\lambda(t)m(t)}(\|A_t(p_t(x_n) - \vartheta_n) \\
&- A_t(p_t(x_{n-1}) - \vartheta_{n-1})\| + \rho(t)\|f_t(x_n) - f_t(x_{n-1})\| \\
&+ \rho(t)\|N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l})
\end{aligned}
$$

$$- N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l}\|)\Big)$$
$$+ \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\|.$$

By Lemma 2.1, there exists a constant $c_q > 0$ such that we have

$$\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\|^q$$
$$\leq \|x_n(t) - x_{n-1}(t)\|^q - q\langle p_t(x_n) - p_t(x_{n-1}), J_q(x_n(t) - x_{n-1}(t))\rangle$$
$$+ c_q\|p_t(x_n) - p_t(x_{n-1})\|^q.$$

Since p is (γ, ϖ)-relaxed cocoercive and π-Lipschitz continuous, it follows that

(14)
$$\|x_n(t) - x_{n-1}(t) - (p_t(x_n) - p_t(x_{n-1}))\|^q$$
$$\leq \|x_n(t) - x_{n-1}(t)\|^q + (q\gamma(t) + c_q)\|p_t(x_n) - p_t(x_{n-1})\|^q$$
$$- q\varpi(t)\|x_n(t) - x_{n-1}(t)\|^q$$
$$= (1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t))\|x_n(t) - x_{n-1}(t)\|^q.$$

From (9), ϱ-$\hat{H}$-Lipschitz continuity of Q and ι-$\hat{H}$-Lipschitz continuity of G, deduce that

(15)
$$\|\vartheta_n(t) - \vartheta_{n-1}(t)\| \leq (1 + \frac{1}{n})\hat{H}(Q_t(x_n), Q_t(x_{n-1}))$$
$$\leq \varrho(t)(1 + \frac{1}{n})\|x_n(t) - x_{n-1}(t)\|$$

and

(16)
$$\|w_n(t) - w_{n-1}(t)\| \leq (1 + \frac{1}{n})\hat{H}(G_t(x_n), G_t(x_{n-1}))$$
$$\leq \iota(t)(1 + \frac{1}{n})\|x_n(t) - x_{n-1}(t)\|.$$

By using σ-Lipschitz continuity of A, π-Lipschitz continuity of p and (15), we obtain

(17)
$$\|A_t(p_t(x_n) - \vartheta_n) - A_t(p_t(x_{n-1}) - \vartheta_{n-1})\|$$
$$\leq \sigma(t)(\|p_t(x_n) - p_t(x_{n-1})\| + \|\vartheta_n(t) - \vartheta_{n-1}(t)\|)$$
$$\leq \sigma(t)(\pi(t) + \varrho(t)(1 + \frac{1}{n}))\|x_n(t) - x_{n-1}(t)\|.$$

From ε-Lipschitz continuity of f, it follows that

(18)
$$\|f_t(x_n) - f_t(x_{n-1})\| \leq \varepsilon(t)\|x_n(t) - x_{n-1}(t)\|.$$

Since N is $(\epsilon_1, \epsilon_2, \ldots, \epsilon_l)$-mixed Lipschitz continuous, and for each $i = 1, 2, \ldots,$ l, S_i is ξ_i-$\hat{H}$-Lipschitz continuous, by (9), we get

$$\|N_t(u_{n,1}, u_{n,2}, \ldots, u_{n,l}) - N_t(u_{n-1,1}, u_{n-1,2}, \ldots, u_{n-1,l})\|$$

(19)
$$\leq \sum_{i=1}^{l} \epsilon_i(t)\|u_{n,i}(t) - u_{n-1,i}(t)\|$$

$$\leq \sum_{i=1}^{l} \epsilon_i(t)(1 + \frac{1}{n})\hat{H}(S_{i,t}(x_{n-1}), S_{i,t}(x_n))$$

$$\leq \sum_{i=1}^{l} \epsilon_i(t)\xi_i(t)(1 + \frac{1}{n})\|x_n(t) - x_{n-1}(t)\|.$$

Combining (14)–(19) with (13), we obtain

(20)
$$\|x_{n+1}(t) - x_n(t)\| \leq (1 - \alpha(t) + \alpha(t)\psi(t,n))\|x_n(t) - x_{n-1}(t)\| + \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\|,$$

where

$$\psi(t,n) = (\varrho(t) + \mu(t)\iota(t))(1 + \frac{1}{n}) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)}$$
$$+ \frac{\tau^{q-1}(t)\Gamma(t,n)}{r(t) - \rho(t)\lambda(t)m(t)},$$

$$\Gamma(t,n) = \sigma(t)(\pi(t) + \varrho(t)(1 + \frac{1}{n})) + \rho(t)(\varepsilon(t) + \sum_{i=1}^{l} \epsilon_i(t)\xi_i(t)(1 + \frac{1}{n})).$$

Letting $\theta(t,n) = 1 - \alpha(t) + \alpha(t)\psi(t,n)$, we know that $\theta(t,n) \to \theta(t) = 1 - \alpha(t) + \alpha(t)\psi(t)$, as $n \to \infty$, where

$$\psi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)}$$
$$+ \frac{\tau^{q-1}(t)\Gamma(t)}{r(t) - \rho(t)\lambda(t)m(t)},$$

$$\Gamma(t) = \sigma(t)(\pi(t) + \varrho(t)) + \rho(t)(\varepsilon(t) + \sum_{i=1}^{l} \epsilon_i(t)\xi_i(t)).$$

In view of the condition (12), $\psi(t) \in (0,1)$ for all $t \in \Omega$ and so $0 < \theta(t) < 1$ for all $t \in \Omega$. Hence there exist $n_0 \in \mathbb{N}$ and a measurable function $\hat{\theta} : \Omega \to (0,\infty)$ (Take $\hat{\theta}(t) = \frac{\theta(t)+1}{2} \in (\theta(t),1)$ for each $t \in \Omega$) such that $\theta(t,n) \leq \hat{\theta}(t)$ for all $n \geq n_0$ and $t \in \Omega$. Accordingly, for all $n > n_0$, by (20), deduce that, for all $t \in \Omega$,

(21) $\|x_{n+1}(t) - x_n(t)\|$

$$\leq \hat{\theta}(t)\|x_n(t) - x_{n-1}(t)\| + \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\|$$

$$\leq \hat{\theta}(t)[\hat{\theta}(t)\|x_{n-1}(t) - x_{n-2}(t)\| + \alpha(t)\|e_{n-1}(t) - e_{n-2}(t)\|$$

$$+ \|r_{n-1}(t) - r_{n-2}(t)\|] + \alpha(t)\|e_n(t) - e_{n-1}(t)\| + \|r_n(t) - r_{n-1}(t)\|$$

$$= \hat{\theta}^2(t)\|x_{n-1}(t) - x_{n-2}(t)\| + \alpha(t)[\hat{\theta}(t)\|e_{n-1}(t) - e_{n-2}(t)\|$$

$$+ \|e_n(t) - e_{n-1}(t)\|] + \hat{\theta}(t)\|r_{n-1}(t) - r_{n-2}(t)\| + \|r_n(t) - r_{n-1}(t)\|$$

$$\leq$$

$$\vdots$$

$$\leq \hat{\theta}^{n-n_0}(t)\|x_{n_0+1}(t) - x_{n_0}(t)\|$$

$$+ \sum_{i=1}^{n-n_0} \alpha(t)\hat{\theta}^{i-1}(t)\|e_{n-(i-1)}(t) - e_{n-i}(t)\|$$

$$+ \sum_{i=1}^{n-n_0} \hat{\theta}^{i-1}(t)\|r_{n-(i-1)}(t) - r_{n-i}(t)\|.$$

By using the inequality (21), it follows that, for any $m \geq n > n_0$,

$$\|x_m(t) - x_n(t)\| \leq \sum_{j=n}^{m-1} \|x_{j+1}(t) - x_j(t)\|$$

(22)
$$\leq \sum_{j=n}^{m-1} \hat{\theta}^{j-n_0}(t)\|x_{n_0+1}(t) - x_{n_0}(t)\|$$

$$+ \sum_{j=n}^{m-1} \sum_{i=1}^{j-n_0} \alpha(t)\hat{\theta}^{i-1}(t)\|e_{n-(i-1)}(t) - e_{n-i}(t)\|$$

$$+ \sum_{j=n}^{m-1} \sum_{i=1}^{j-n_0} \hat{\theta}^{i-1}(t)\|r_{n-(i-1)}(t) - r_{n-i}(t)\|.$$

Since $\hat{\theta}(t) < 1$ for all $t \in \Omega$, it follows from (10) and (22) that $\|x_m(t) - x_n(t)\| \to 0$ as $n \to \infty$ and so $\{x_n(t)\}$ is a Cauchy sequence in X. In view of completeness of X, there exists $x^*(t) \in X$ such that $x_n(t) \to x^*(t)$ for all $t \in \Omega$. By using (9), ξ_i-$\hat{H}$-Lipschitz continuity of S_i $(i = 1, 2, \ldots, l)$, ϱ-$\hat{H}$-Lipschitz continuity of Q and ι-$\hat{H}$-Lipschitz continuity of G, we have

(23)
$$\begin{cases} \|u_{n,i}(t) - u_{n+1,i}(t)\| \leq (1 + \frac{1}{1+n})\xi_i(t)\|x_n(t) - x_{n+1}(t)\|, \\ \|\vartheta_n(t) - \vartheta_{n+1}(t)\| \leq (1 + \frac{1}{1+n})\varrho(t)\|x_n(t) - x_{n+1}(t)\|, \\ \|w_n(t) - w_{n+1}(t)\| \leq (1 + \frac{1}{1+n})\iota(t)\|x_n(t) - x_{n+1}(t)\|, \\ i = 1, 2, \ldots, l. \end{cases}$$

It follows from (23) that $\{u_{n,i}(t)\}$ $(i = 1, 2, \ldots, l)$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are also Cauchy sequences in X. Accordingly, there exist $u_i^*(t), \vartheta^*(t), w^*(t) \in X$ $(i = 1, 2, \ldots, l)$, such that for all $t \in \Omega$ and for each $i = 1, 2, \ldots, l$, $u_{n,i}(t) \to u_i^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$ and $w_n(t) \to w^*(t)$ as $n \to \infty$. Since $\{x_n(t)\}$, $\{u_{n,i}(t)\}$ $(i = 1, 2, \ldots, l)$, $\{\vartheta_n(t)\}$ and $\{w_n(t)\}$ are sequences of measurable mappings,

we know that $x^*, u_i^*, \vartheta, w : \Omega \to X$ $(i = 1, 2, \ldots, l)$, are measurable. Further, for each $t \in \Omega$ and $i = 1, 2, \ldots, l$, we have

$$
\begin{aligned}
d(u_i^*(t), S_{i,t}(x^*)) &= \inf\{\|u_i^*(t) - z\| : z \in S_{i,t}(x^*)\} \\
&\leq \|u_i^*(t) - u_{n,i}(t)\| + d(u_{n,i}(t), S_{i,t}(x^*)) \\
&\leq \|u_i^*(t) - u_{n,i}(t)\| + \hat{H}(S_{i,t}(x_n), S_{i,t}(x^*)) \\
&\leq \|u_i^*(t) - u_{n,i}(t)\| + \xi_i(t)\|x_n(t) - x^*(t)\|.
\end{aligned}
$$

The right side of the above inequality tends to zero as $n \to \infty$. Hence, for each $i = 1, 2, \ldots, l$, $u_i^*(t) \in S_{i,t}(x^*)$.

Similarly, we can verify that for each $t \in \Omega$, $\vartheta^*(t) \in Q_t(x^*)$ and $w^*(t) \in G_t(x^*)$. The condition (11) and $w_n(t) \to w^*(t)$, for all $t \in \Omega$, as $n \to \infty$, imply that for each $t \in \Omega$, $J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w_n)} \to J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w^*)}$ uniformly on X, as $n \to \infty$. Since for each $t \in \Omega$, the mappings $J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w_n)}$, p_t, f_t, N_t and A_t are continuous, it follows from (9) and (10) that for each $t \in \Omega$,

$$
\begin{aligned}
p_t(x^*) = \vartheta^*(t) + J_{\rho(t)\lambda(t), A_t}^{\eta_t, M_t(\cdot, w^*)}[A_t(p_t(x^*) - \vartheta^*) - \rho(t)(f_t(x^*) \\
+ N_t(u_1^*, u_2^*, \ldots, u_l^*) - h(t))].
\end{aligned}
$$

Now, Lemma 4.3 implies that measurable mappings $x^*, u_i^*, \vartheta^*, w^* : \Omega \to X$ $(i = 1, 2, \ldots, l)$ are a random solution of the problem (1). This completes the proof. $\square$

Remark 5.3. If X is a 2-uniformly smooth Banach space and there exists a measurable function $\rho : \Omega \to (0, \infty)$ with $\rho(t) \in (0, \frac{r(t)}{\lambda(t)m(t)})$, for all $t \in \Omega$, such that

$$
\begin{aligned}
&\varphi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt{1 - 2\varpi(t) + (2\gamma(t) + c_2)\pi^2(t)} < 1, \\
&2\varpi(t) - (2\gamma(t) + c_2)\pi^2(t) < 1, \\
&\rho(t) < \frac{r(t)(1 - \varphi(t)) - \tau(t)\sigma(t)(\pi(t) + \varrho(t))}{\tau(t)[\varepsilon(t) + \sum\limits_{i=1}^{l} \epsilon_i(t)\xi_i(t)] + (1 - \varphi(t))\lambda(t)m(t)},
\end{aligned}
$$

then (12) holds. As we know, Hilbert spaces and L_p(or l_p) spaces, $2 \leq p < \infty$, are 2-uniformly smooth.

Theorem 5.4. *Let X be a q-uniformly smooth Banach space, A, p, f, η, M, N, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, h, λ be the same as in the problem (2) and let $S, T, P, Q, G : \Omega \times X \to CB(X)$ be five random set-valued mappings induced by $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, respectively. Suppose that p is (γ, ϖ)-relaxed cocoercive and π-Lipschitz continuous, A is r-strongly η-accretive and σ-Lipschitz continuous, and η and f are τ-Lipschitz continuous and ε-Lipschitz continuous, respectively. Let S, T, P, Q and G be ξ-$\hat{H}$-Lipschitz continuous, ζ-$\hat{H}$-Lipschitz continuous, ς-$\hat{H}$-Lipschitz continuous, ϱ-$\hat{H}$-Lipschitz continuous and ι-$\hat{H}$-Lipschitz continuous, respectively. Assume that N is ϵ-Lipschitz continuous in the second argument,*

δ-*Lipschitz continuous in the third argument and* κ-*Lipschitz continuous in the fourth argument. Let there exist measurable functions* $\mu : \Omega \to (0, +\infty)$ *and* $\rho : \Omega \to (0, +\infty)$ *with* $\rho(t) \in (0, \frac{r(t)}{\lambda(t)m(t)})$, *for all* $t \in \Omega$, *such that* (11) *holds and*

$$\begin{cases} \varphi(t) = \varrho(t) + \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)} < 1, \\ \sigma(t)(\pi(t) + \varrho(t)) + \rho(t)(\epsilon(t)\xi(t) + \delta(t)\zeta(t) + \kappa(t)\varsigma(t)) \\ \quad < \tau^{1-q}(t)(1 - \varphi(t))(r(t) - \rho(t)\lambda(t)m(t)), \end{cases}$$

where c_q *is the same as in Lemma 2.1.*

Then there exists a set of measurable mappings $x^*, \nu^*, u^*, v^*, \vartheta^*, w^* : \Omega \to X$ *which is a random solution of the problem* (2) *and for each* $t \in \Omega$, $x_n(t) \to x^*(t)$, $\nu_n(t) \to \nu^*(t)$, $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$, $w_n(t) \to w^*(t)$, *as* $n \to \infty$, *where* $\{x_n(t)\}$, $\{\nu_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$, $\{\vartheta_n(t)\}$ *and* $\{w_n(t)\}$ *are the iterative sequences generated by Algorithm 4.5.*

Theorem 5.5. *Let* X *be a* q-*uniformly smooth Banach space,* A, p, f, η, M, N, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, S, Q, h, λ *be the same as in the problem* (3) *and* $T, P, G : \Omega \times X \to CB(X)$ *be random set-valued mappings induced by* $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, *respectively. Further, suppose that the conditions* (a)-(f) *in Theorem 5.4 hold. Then there exists a set of measurable mappings* $x^*, \nu^*, u^*, v^*, \vartheta^*, w^* : \Omega \to X$ *which is a random solution of the problem* (3) *and for each* $t \in \Omega$, $x_n(t) \to x^*(t)$, $\nu_n(t) \to \nu^*(t)$, $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$, $w_n(t) \to w^*(t)$, *as* $n \to \infty$, *where* $\{x_n(t)\}$, $\{\nu_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$, $\{\vartheta_n(t)\}$ *and* $\{w_n(t)\}$ *are the iterative sequences generated by Algorithm 4.5.*

Theorem 5.6. *Suppose that* X *is a* q-*uniformly smooth Banach space and* A, p, f, η, M, N, S, T, P, Q, G, h, λ *are the same as in the problem* (6). *Further, assume that the conditions* (a)-(f) *in Theorem 5.4 hold. Then there exists a set of measurable mappings* $x^*, \nu^*, u^*, v^*, \vartheta^*, w^* : \Omega \to X$ *which is a random solution of the problem* (6) *and for each* $t \in \Omega$, $x_n(t) \to x^*(t)$, $\nu_n(t) \to \nu^*(t)$, $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $\vartheta_n(t) \to \vartheta^*(t)$, $w_n(t) \to w^*(t)$, *as* $n \to \infty$, *where* $\{x_n(t)\}$, $\{\nu_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$, $\{\vartheta_n(t)\}$ *and* $\{w_n(t)\}$ *are the iterative sequences generated by Algorithm 4.5.*

Like in the proof of Theorem 5.4, one can verify the convergence of the iterative sequences generated by Algorithms 4.6 and 4.7 and we omit their proofs.

Theorem 5.7. *Assume that* X *is a* q-*uniformly smooth Banach space,* A, p, f, η, M, N, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, S, Q, h, λ *are the same as in the problem* (4) *and let* $T, P, G : \Omega \times X \to CB(X)$ *be random set-valued mappings induced by* $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, *respectively. Further, assume that the conditions* (a)-(f) *in Theorem 5.4 hold. Then there exists a set of measurable mappings* $x^*, u^*, v^*, w^* : \Omega \to X$ *which is a random solution of the problem* (4) *and for each* $t \in \Omega$, $x_n(t) \to x^*(t)$, $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $w_n(t) \to w^*(t)$ *as* $n \to \infty$, *where* $\{x_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$ *and* $\{w_n(t)\}$ *are the iterative sequences generated by Algorithm 4.6.*

Theorem 5.8. *Let X be a q-uniformly smooth Banach space, A, p, f, η, M, N, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, S, Q, h, λ be the same as in the problem* (5) *and let $T, P, G : \Omega \times X \to CB(X)$ be random set-valued mappings induced by $\mathcal{T}$, $\mathcal{P}$, $\mathcal{G}$, respectively. Moreover, assume that the conditions* (a)-(e) *in Theorem 5.4 hold. If there exist measurable functions $\mu : \Omega \to (0, +\infty)$ and $\rho : \Omega \to (0, +\infty)$ with $\rho(t) \in (0, \frac{r(t)}{\lambda(t)m(t)})$, for all $t \in \Omega$, such that* (11) *holds and*

$$
\begin{cases}
\varphi(t) = \mu(t)\iota(t) + \sqrt[q]{1 - q\varpi(t) + (q\gamma(t) + c_q)\pi^q(t)} < 1, \\
\sigma(t)\pi(t) + \rho(t)(\epsilon(t)\xi(t) + \delta(t)\zeta(t) + \kappa(t)\varsigma(t)) \\
\quad < \tau^{1-q}(t)(1 - \varphi(t))(r(t) - \rho(t)\lambda(t)m(t)),
\end{cases}
$$

where c_q is the same as in Lemma 2.1, then there exists a set of measurable mappings $x^*, u^*, v^*, w^* : \Omega \to X$ which is a random solution of the problem (5) *and for each $t \in \Omega$, $x_n(t) \to x^*(t)$, $u_n(t) \to u^*(t)$, $v_n(t) \to v^*(t)$, $w_n(t) \to w^*(t)$, as $n \to \infty$, where $\{x_n(t)\}$, $\{u_n(t)\}$, $\{v_n(t)\}$ and $\{w_n(t)\}$ are the iterative sequences generated by Algorithm 4.7.*

Remark 5.9. Theorems 5.2 and 5.4–5.8 generalize and improve Theorems 3.1 and 3.2 in [25], Theorems 3.1, 3.3 and 3.4 in [50] and Theorems 4.1, 4.3 and 4.4 in [58]. In brief, for an appropriate choice of the mappings A, p, f, η, M, N, $\mathcal{S}_i$ $(i = 1, 2, \ldots, l)$, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{G}$, S_i $(i = 1, 2, \ldots, l)$, S, T, P, Q, G, h, λ, the measurable step size function α, the sequences $\{e_n\}$, $\{r_n\}$ and the space X, Theorems 5.2 and 5.4–5.8 include many known results of generalized variational inclusions as special cases (see [14, 19, 23, 25, 28, 31, 32, 35–37, 39, 41, 43, 45, 50, 58] and the references therein).

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