

# Combined Economic and Emission Dispatch with Valve-point loading of Thermal Generators using Modified NSGA-II

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**Abstract** – This paper discusses the application of evolutionary multi-objective optimization algorithms namely Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and Modified NSGA-II (MNSGA-II) for solving the Combined Economic Emission Dispatch (CEED) problem with valve-point loading. The valve-point loading introduce ripples in the input-output characteristics of generating units and make the CEED problem as a non-smooth optimization problem. IEEE 57-bus and IEEE 118-bus systems are taken to validate its effectiveness of NSGA-II and MNSGA-II. To compare the Pareto-front obtained using NSGA-II and MNSGA-II, reference Pareto-front is generated using multiple runs of Real Coded Genetic Algorithm (RCGA) with weighted sum of objectives. Furthermore, three different performance metrics such as convergence, diversity and Inverted Generational Distance (IGD) are calculated for evaluating the closeness of obtained Pareto-fronts. Numerical results reveal that MNSGA-II algorithm performs better than NSGA-II algorithm to solve the CEED problem effectively.

**Keywords:** Multi-objective optimization, Non-dominated sorting genetic algorithm-II (NSGA-II), Modified NSGA-II (MNSGA-II), Real coded genetic algorithm (RCGA), Pareto optimal solutions, Valve-point loading

## 1. Introduction

The main objective of Economic Dispatch (ED) problem is to find the optimal combination of power generation that minimizes the total fuel cost while satisfying the systems constraints. Various conventional methods like Lambda-iteration, Base point participation factor, Gradient method and Newton method are used to solve ED problem. In all these methods, the fuel cost function is chosen to be of quadratic form [1]. In reality, the input-output characteristics of generating units are non-linear due to valve-point loading effect. To achieve more accurate dispatch of generation, valve-point loading effect is included in the fuel cost function of the thermal generators. Conventional methods have failed to obtain global optimal solution. Hence, stochastic methods such as Genetic Algorithm (GA) [2], Evolutionary Programming (EP) [3], Improved EP [4], Particle Swarm Optimization (PSO) [5], Differential Evolution (DE) [6] and Harmony Search Algorithm (HSA) [7] have been used to solve the ED problem with valve-point loading effect by adding the rectified sinusoidal contribution to the conventional quadratic cost function.

With the increasing awareness of environmental protection in recent years, Economic Emission Dispatch (EED) is proposed as an alternative to achieve simultaneously the minimization of fuel costs and pollutant emissions [8]. However, minimizing the emission and cost are usually two conflicting objectives. Thus, it is not possible to minimize both of them simultaneously and some form of conflicting resolution must be adopted to arrive at a solution [9].

Several EED strategies have appeared in the literature over the years. El-kieb *et al* have applied a Lagrange Relaxation based algorithm to environmental constraints of ED problem [10]. The economic and environmental objectives simultaneously combine them linearly to form a single objective function. By varying the weight, the trade-off between fuel cost and environmental cost was determined by Ramanathan [11]. Yokoyama *et al* have applied  $\epsilon$ -constrained algorithm to treat the optimal dispatch problems with multiple performance indices and to grasp trade-off relations between selected indices [12]. Farag *et al* have proposed a Linear Programming based optimization method, in which the emission function is treated as a constraint [13]. Nanda *et al* introduced the Goal Programming technique and the Gauss-Seidel method for the EED problem [14]. However, these classical methods are highly sensitive and frequently converge at local optimum solution and computational time increases with the increase of the dimensionality of the problem.

Later, the use of heuristic optimization approaches such as GA [15], EP [16] is proposed to solve the multi-

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objective constrained optimization problem. Prabakar *et al* have applied modified price penalty factor method to Combined Economic Emission Dispatch (CEED) problem and converted to single objective problem [17]. Recently, the Multi-Objective Evolutionary Algorithms (MOEAs) are used to eliminate many difficulties in the classical methods [18]. Because, population of solutions is used in their search and multiple Pareto-optimal solutions can be found in one single simulation run. Some of the popular MOEAs are Non Dominated Sorting Genetic Algorithm (NSGA), Niche Pareto Genetic Algorithm, Strength Pareto Evolutionary Algorithm (SPEA), Non Dominated Sorting Genetic Algorithm-II (NSGA-II), Pareto Archived Evolution Strategy etc. [19]. Abido has applied NSGA approaches for solving the multi-objective CEED problem. In addition, a fuzzy based mechanism is employed to extract the best compromise solution [20]. NSGA suffers from computational complexity, non-elitist approach and the need to specify a sharing parameter. An improved version of NSGA known as NSGA-II, which resolved CEED problems and uses elitism to create a diverse Pareto-optimal front, has been subsequently presented [21, 22]. Although in NSGA-II the crowding distance operator will ensure diversity along the non-dominated front, lateral diversity will be lost. To overcome this, crowding distance operator is replaced by dynamic crowding distance (DCD) and controlled elitism (CE) is incorporated to NSGA-II [23, 24] and named as modified NSGA-II (MNSGA-II) has been used to solve the CEED problem. Jeyadevi *et al* have applied MNSGA-II algorithm to solve multi-objective optimal reactive power dispatch problem by minimizing real power loss and maximizing the system voltage stability [25]. Most recently, the basic HSA is updated using fast non-dominated sorting and diversity with DCD strategy and named as Multi-Objective HSA has been used to solve CEED problem [26]. Wu *et al* have proposed multi-objective DE (MODE) algorithm with elitist archive and crowding entropy based diversity measure to solve the environmental/economic dispatch problem [27]. Youlin Lu et al have proposed an Enhanced Multi-Objective Differential Evolution (E-MODE) algorithm for handling the complicated constraints and improve the convergence performance of EED problem [28].

Very few works are reported for solving CEED problem with valve-point loading effect. Basu analyzed the interactive fuzzy satisfying based Simulated Annealing technique for CEED problem with non-smooth fuel cost and emission level functions. The major advantage of this method is obtaining a compromising solution in the presence of conflicting objectives [29]. MODE algorithm has been applied for solving EED problems with valve-point loading and only extreme points obtained are compared with Partial DE, NSGA-II and SPEA-2 for the three different test systems. However, the performance measures of the different MOEAs with respect to reference Pareto-front are not considered in this paper [30]. Also, the transmission

line losses are calculated through  $B_{mn}$  coefficients [29, 30].

In this paper, NSGA-II and MNSGA-II algorithms are used to solve CEED problem with valve-point effect. RCGA with weighted sum approach is used to generate reference Pareto-front and compare the performance of NSGA-II and MNSGA-II algorithms. Three different performance metrics convergence, diversity and IGD were used for evaluating the closeness to the reference Pareto optimal front. The rest of this paper is organized as follows: Section 2 describes the CEED problem formulation. Implementation of NSGA-II and MNSGA-II for the CEED problem is explained in section 3. Section 4 describes various performance measures. The simulation results of various test cases are presented in section 5 and section 6 concludes.

## 2. Problem Formulation

The multi-objective CEED problem with its constraints is formulated as a non-linear constrained problem as follows.

$$\text{Minimize } [F(P_g), E(P_g)] \quad (1)$$

subject to power balance and generation capacity constraints [24]. Where,  $F(P_g)$ : Total fuel cost (\$/hr), and  $E(P_g)$ : Total emission cost (lb/hr)

### 2.1 Objective functions

Minimization of fuel cost with valve-point loading effect: Large steam turbine generators will have a number of steam admission valves that are opened in sequence to obtain ever-increasing output of the unit and the input-output characteristics are not always smooth. These “valve-points” are illustrated in Fig. 1. Ignoring the valve-point loading effects, some inaccuracy would result in the generation dispatch.

To model the effects of non-smooth fuel cost functions, a recurring rectified sinusoidal contribution is added to the

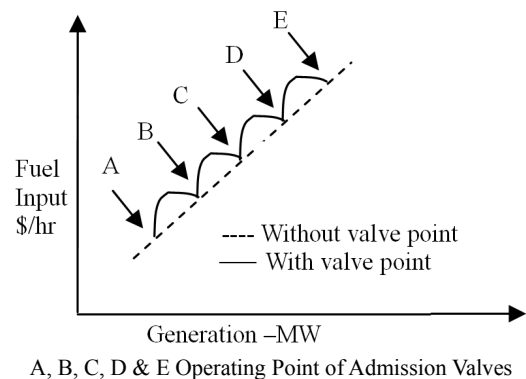


Fig. 1. Incremental fuel cost curve for 5-valve steam turbine unit.

second order polynomial functions to represent the input-output equation (2) as follows. The total fuel cost in terms of real power output can be expressed as [2],

$$F(P_g) = \sum_{i=1}^N a_i + b_i P_{gi} + c_i P_{gi}^2 + \left| d_i \sin \left\{ e_i \left( P_{gi}^{\min} - P_{gi} \right) \right\} \right| \$ / hr. \quad (2)$$

Where,  $F(P_g)$ : Total fuel cost (\$/hr),  $N$ : Number of generators,  $a_i, b_i, c_i, d_i, e_i$ : Fuel cost coefficients of generator  $i$ ,  $P_{gi}$ : Power generated by generator  $i$  and  $P_{gi}^{\min}$ : Minimum power generation limit.

Minimization of pollutant emission: The total emission of atmospheric pollutants such as Sulphur Oxides (SO<sub>x</sub>) and Nitrogen Oxides (NO<sub>x</sub>) from a fossil-fired thermal generating unit can be approximately modelled as a direct sum of a quadratic function and an exponential term of the active power output of the generating units and is expressed in the following form [29].

$$E(P_g) = \sum_{i=1}^N \alpha_i + \beta_i P_{gi} + \gamma_i P_{gi}^2 + \eta_i \exp(\delta_i P_{gi}) \text{ lb} / hr. \quad (3)$$

Where,  $E(P_g)$ : Total emission cost and  $\alpha_i, \beta_i, \gamma_i, \eta_i, \delta_i$ : Emission coefficients of generator  $i$ .

## 2.2 Problem constraints

Generation capacity constraint: For stable operation, real power output of each generator is restricted by lower and upper limits as follows [26]:

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, \quad i = 1, \dots, N \quad (4)$$

Where,  $P_{gi}^{\min}$ : Minimum power generated and  $P_{gi}^{\max}$ : Maximum power generated.

Power balance constraint: The total power generated must supply the total load demand and the transmission losses [22].

$$\sum_{i=1}^N P_{gi} - P_d - P_{loss} = 0 \quad (5)$$

Where,  $P_d$ : Total load demand and  $P_{loss}$ : Power loss in the transmission network.

The real power loss  $P_{loss}$  can be calculated from Newton-Raphson load flow solution, which gives all bus voltage magnitudes and angles; it can be described as follows:

$$P_{loss} = \sum_{k=1}^{N_L} g_k \left[ V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j) \right] \quad (6)$$

Where  $i$  and  $j$  are the total number of buses,  $k$  is the  $k$ th network branch that connects bus  $i$  to bus  $j$ ,  $N_L$  is the

number of transmission lines,  $V_i$  and  $V_j$  are the voltage magnitudes at bus  $i$  and  $j$ ,  $g_k$  is the transfer conductance between bus  $i$  and  $j$ ,  $\theta_i$  and  $\theta_j$  are the voltage angles at bus  $i$  and  $j$  respectively [27].

## 3. Implementation of NSGA-II and MNSGA-II

The NSGA-II, MNSGA-II algorithms, Dynamic Crowding Distance (DCD), Controlled Elitism (CE) and MNSGA-II algorithm computational flow are described.

### 3.1 NSGA-II

NSGA-II is a fast and elitist MOEA and implements elitism for multi-objective search, using an elitism-preserving approach. Elitism enhances the convergence properties towards the true Pareto-optimal set. A parameter-less diversity preserving mechanism is adopted. Diversity and spread of solutions are guaranteed without the use of sharing parameters. When two solutions belong to the same Pareto-optimal front, the one located in a lesser crowded region of the front is preferred. Crowded comparison operator is used for good spread of solutions in the obtained non-dominated solutions [21].

### 3.2 MNSGA-II

Although the crowded comparison operator ensures diversity along the non-dominated front in NSGA-II, the uniform diversity and lateral diversity are lost and hence leads to slowing down the search. These drawbacks are overcome by introducing a new diversity strategy called dynamic crowding distance (DCD) and controlled elitism (CE) into the NSGA-II algorithm for solving the CEED problem. Thus, the search algorithm needs diversity along the Pareto front and lateral to the Pareto front for better convergence [24, 25].

### 3.3 Dynamic crowding distance (DCD)

NSGA-II uses crowding distance (CD) measure in population maintenance, to remove excess individuals in the non-dominated set (NDS) when the number of non-dominated solutions exceeds population size. The individuals having lower value of CD are preferred over individuals with higher value of CD in removal process. Individual's CD can be calculated as follows:

$$CD_i = \frac{1}{N_{obj}} \sum_{g=1}^{N_{obj}} \left| f_{i+1}^g - f_{i-1}^g \right| \quad (7)$$

Where  $N_{obj}$  is the number of objectives,  $f_{i+1}^g$  is the  $g^{th}$  objective of the  $i+1^{th}$  individual and  $f_{i-1}^g$  is the  $g^{th}$  objective of the  $i-1^{th}$  individual after sorting the

population according to CD value. The major drawback of CD is lack of uniform diversity in the obtained non-dominated solutions as illustrated in [24, 25]. If normal CD is applied, some of the individuals helps to maintain uniform spread are removed.

To overcome this problem, dynamic crowding distance (DCD) method is suggested in [19, 23]. The individuals CD are calculated only once during the process of population maintenance but the individuals DCD are varying dynamically during the process of population maintenance. In the DCD approach, one individual with lowest DCD value every time is removed and recalculates DCD for the remaining individuals. The individuals DCD are calculated as follows:

$$DCD_i = \frac{CD_i}{\log\left(\frac{1}{Var_i}\right)} \quad (8)$$

Where  $CD_i$  is calculated by eqn. (10),  $Var_i$  is based on

$$Var_i = \frac{1}{N_{obj}} \sum_g^{N_{obj}} (|f_{i+1}^g - f_{i-1}^g| - CD_i)^2 \quad (9)$$

$Var_i$  is the variance of CDs of individuals which are neighbours of the  $i^{th}$  individual.  $Var_i$  can give information about the difference variations of CD in different objectives. Therefore, if DCD is used in population maintenance, individuals in the NDS will have more chance to maintain.

### 3.4 Controlled elitism (CE)

A controlled elitism is incorporated in NSGA-II algorithm which will control the extent of exploitation rather than controlling the extent of exploration. In this approach, algorithm restricts the number of individuals in the current best non-dominated front adaptively and maintains a predefined distribution of number of individuals in each front. A geometric distribution is employed for this purpose,

$$N_j = N \frac{1-r}{1-r^K} r^{j-1} \quad (10)$$

Where K is the number of nondominated front,  $N_j$  is the maximum number of allowed individuals in the  $j^{th}$  front and r is the reduction rate. Since  $r < 1$ , the maximum allowable number of individual in the first front is the highest. Thereafter, each front is allowed to have an exponentially reducing number of solutions. It is clear that the new population obtained under the controlled NSGA-II procedure will generally be more diverse than that obtained by using the usual NSGA-II approach [19, 24].

### 3.5 MNSGA-II computational flow

- Step 1:** Generate a random parent population of size N within control variable limits.
- Step 2:** The population is sorted based on non-domination. Each population is assigned a rank equal to its non-domination level. Calculate the crowding distance (CD) of populations in each non-domination level and sort populations in descending order of its CD.
- Step 3:** Tournament selection: Select two individuals at random and then compare their front number and its crowding distance. Select the better one and copy it to the mating pool.
- Step 4:** Create offspring population of size N by Simulated Binary Crossover (SBX) and polynomial mutation. The crossover probability of  $P_c = 0.85$  and a mutation probability of  $P_m = 1/n$  (where n is the number of decision variables) are used.
- Step 5:** Combine the parent population and offspring population. The size of combined population is 2N.
- Step 6:** Perform non-dominated sorting to combined population and identify different fronts.
- Step 7:** Applying Controlled Elitism (CE) concept, restricts the number of individuals in the current best non-dominated front adaptively and maintains a predefined distribution of number of individuals in each front.
- Step 8:** If the size of non-dominated set M is greater than the population size N, then remove M-N individuals from non-dominated set by using DCD based strategy, elsewhere, go to step 4. The new population obtained under the MNSGA-II will, in general be more diverse than that obtained by using NSGA-II approach.
- Step 9:** Stopping rule: The process can be stopped after a fixed number of iterations. If the criterion is not satisfied then the procedure is repeated from step 3 after creating the new population [25].

## 4. Performance Metrics

To evaluate the performances of multi-objective optimization algorithms some measures of performances are essential. The existing performance metrics can be classified into three classes: metrics for convergence ( $\gamma$ ), metrics for diversity ( $\Delta$ ) and metrics for both convergence and diversity. These metrics are helpful for evaluating closeness of the obtained Pareto-front with the reference Pareto-front and evaluating diversity among non-dominated solutions [19].

### 4.1 Convergence metric or Distance metric ( $\gamma$ )

$\gamma$  evaluates average distance between non-dominated

solutions found and the actual Pareto-optimal front, as follows:

$$\gamma = \frac{\sum_{i=1}^N d_i}{N} \quad (11)$$

Where,  $d_i$  is the distance between non-dominated solutions found and the actual Pareto-optimal front and  $N$  is the number of solutions in the front. The smaller the value of this metric, the better convergence toward the Pareto-optimal front [21, 24].

#### 4.2 Spread metric or diversity metric ( $\Delta$ )

$\Delta$  measures the extent of spread achieved among the obtained solutions. We calculate the Euclidean distance  $d_i$  between consecutive solutions in the obtained non-dominated set of solutions and then we calculate the average  $\bar{d}$  of these distances. Thereafter, from the obtained set of non-dominated solutions, we first calculate the extreme solutions and then we use the following metric to calculate the nonuniformity in the distribution:

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}} \quad (12)$$

Where,  $d_f$  and  $d_l$  are the Euclidean distances between the extreme solutions and the boundary solutions of the obtained non-dominated set. The parameter  $\bar{d}$  is the average of all distances  $d_i$ ,  $i = 1, 2, \dots, (N-1)$ , assuming that there are  $N$  solutions on the best non-dominated front and  $(N-1)$  consecutive distances. According to this metric, if an algorithm finds a smaller  $\Delta$  value is able to find a better diverse set of non-dominated solutions [21].

#### 4.3 Inverted generational distance (IGD)

IGD is designed for both convergence and diversity. IGD is calculated as shown below

$$IGD = \frac{\sum_{v \in P^*} d(v, P)}{|P^*|} \quad (13)$$

Where,  $P^*$  is a set of uniformly distributed points in true Pareto-front,  $P$  is the non-dominated solutions obtained by MOEAs,  $d(v, P)$  is the minimum Euclidean distance between  $v$  and the point in  $P$ . A value of IGD equal zero indicates that  $P$  should be close to  $P^*$  [19].

### 5. Results and Discussion

The NSGA-II and MSGA-II algorithms are coded in MATLAB version 7.11 on a PC with Pentium-IV Intel (R)

Core(TM) i3-2310M CPU operating at 2.10 GHz speed with 4 GB RAM.

#### 5.1 Test system description

The NSGA-II and MNSGA-II algorithms are applied to IEEE 57-bus and IEEE 118-bus systems. The control parameters used for NSGA-II and MNSGA-II simulations are shown in Table 1. The fuel cost coefficients, emission coefficients, the lower power limits and the upper power limits are taken from [4, 17, 22] and [31]. Valve-point loading coefficients for IEEE 57-bus and IEEE 118-bus systems are appropriately assumed. In general, the population size of six times the number of decision variables is considered. The bus data and the line data are taken from [32]. Power flow calculations are made using MATPOWER software [32]. Control elitism rate ( $r$ ) of MNSGA-II is assumed as 0.55.

**Table 1.** Parameters setting of NSGA-II and MNSGA-II

Parameters	IEEE 57-bus	IEEE 118-bus
Population size	50	100
Number of iteration	200	300
Crossover probability, $P_c$	0.85	0.85
Mutation probability, $P_m$	1/n (n=7)	1/n (n=19)
Crossover index, $\eta_c$	5	5
Mutation index, $\eta_m$	15	15

#### 5.2. Generation of reference Pareto-optimal front

To compare the performance of NSGA-II and MNSGA-II algorithms, a reference Pareto front obtained by using multiple runs of Real Coded Genetic Algorithm (RCGA) with weighted sum approach is considered. In reference Pareto-front generation, CEED problem is treated as single objective optimization problem by linear combination of objectives as follows:

$$\text{Minimize } C = w f_1 + (1-w) f_2 \quad (14)$$

Where,  $w$  is a weighing factor and the sum of weighting factor must be 1.  $f_1$  is the cost objective and  $f_2$  is the emission objective.

To get 50 non-dominated solutions, the algorithm is applied 50 times with varying weight factors as a uniform random number varying between 0 and 1 in each trial. Different population sizes and iteration numbers are selected depending upon the number of decision variables [24].

#### 5.3. Simulation results

Simulations are performed on IEEE 57-bus and IEEE 118-bus systems with valve-point loading for the demand of 1250.8 MW and 3668 MW respectively. Best Pareto-fronts obtained using NSGA-II and MNSGA-II for IEEE

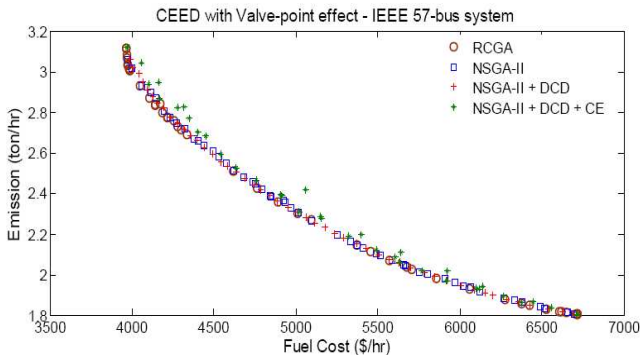


Fig. 2. Reference Pareto-front using RCGA and best obtained Pareto-front of NSGA-II, NSGA-II + DCD and NSGA-II + DCD + CE – IEEE 57-bus system

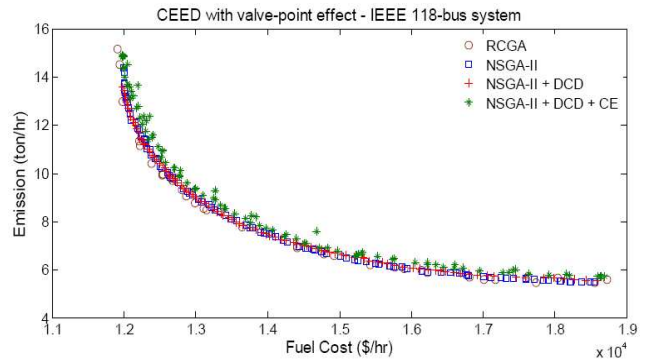


Fig. 3. Reference Pareto-front using RCGA and best obtained Pareto-front of NSGA-II, NSGA-II + DCD and NSGA-II + DCD + CE – IEEE 118-bus system

Table 2. Extreme solutions for cost & emission with valve-point effect using NSGA-II, NSGA-II + DCD and NSGA-II + DCD + CE for IEEE 57-bus system for the demand of 1250.8 MW

Power generation /loss (MW)	NSGA-II		NSGA-II + DCD		NSGA-II + DCD + CE	
	Cost (\$/hr)	Emission (ton/hr)	Cost (\$/hr)	Emission (ton/hr)	Cost (\$/hr)	Emission (ton/hr)
PG <sub>1</sub>	474.6765	289.8145	481.5357	284.5546	484.7586	293.2694
PG <sub>2</sub>	10.0310	99.0938	10.0080	99.7148	10.0110	99.9849
PG <sub>3</sub>	20.2225	138.4096	20.2204	139.3946	20.0000	139.9826
PG <sub>4</sub>	10.0031	99.9033	10.0379	99.8978	10.0054	99.9974
PG <sub>5</sub>	451.3608	283.7511	475.6759	284.6528	471.4728	283.4342
PG <sub>6</sub>	10.0575	99.6018	10.0417	99.4584	10.0006	99.9999
PG <sub>7</sub>	302.3281	263.9579	273.0187	266.5959	274.3374	258.3628
Total Generation (MW)	1278.6795	1274.532	1280.538	1274.269	1280.586	1275.0312
Total Loss (MW)	27.8795	23.732	29.7383	23.4689	29.7858	24.2312
Cost (\$/hr)	<b>3971.6742</b>	6697.938	<b>3968.614</b>	6710.511	<b>3967.273</b>	6717.0476
Emission (ton/hr)	3.0565	<b>1.8075</b>	3.1188	<b>1.8071</b>	3.1169	<b>1.8044</b>

Table 3. Extreme solutions for cost & emission with valve-point effect using NSGA-II, NSGA-II + DCD and NSGA-II + DCD + CE for IEEE 118-bus system for the demand of 3668 MW

Power generation /loss (MW)	NSGA-II		NSGA-II + DCD		NSGA-II + DCD + CE	
	Cost (\$/hr)	Emission (ton/hr)	Cost (\$/hr)	Emission (ton/hr)	Cost (\$/hr)	Emission (ton/hr)
PG <sub>1</sub>	632.4282	311.7665	635.1131	296.3940	666.3349	292.2280
PG <sub>2</sub>	74.2346	395.6262	79.3146	430.4591	59.2010	422.8407
PG <sub>3</sub>	78.9754	85.1979	89.9439	80.7764	88.9817	88.9017
PG <sub>4</sub>	299.0123	299.8937	296.7635	297.9175	298.7317	298.1514
PG <sub>5</sub>	40.1179	396.7205	40.1746	387.6696	40.8803	397.4317
PG <sub>6</sub>	4.3664	9.7513	1.6219	5.1443	9.1456	9.76915
PG <sub>7</sub>	8.6015	17.7179	16.4082	15.6464	16.1480	17.6033
PG <sub>8</sub>	30.0638	239.3228	31.6779	237.2091	30.7899	237.6172
PG <sub>9</sub>	47.1728	49.8313	48.0858	44.0231	23.7377	36.2583
PG <sub>10</sub>	153.6396	198.7127	153.7741	198.8161	98.8878	197.7378
PG <sub>11</sub>	191.7919	190.3027	194.3582	196.3595	190.1879	182.4525
PG <sub>12</sub>	394.5165	343.1486	393.6483	316.0513	395.6221	377.8778
PG <sub>13</sub>	393.0788	382.3798	393.8913	399.6602	399.7133	399.2092
PG <sub>14</sub>	599.6361	173.1232	546.3082	191.2300	598.7838	161.7066
PG <sub>15</sub>	2.2816	2.9509	2.4881	4.4129	2.6834	2.7229
PG <sub>16</sub>	670.5328	303.0844	678.6703	289.2167	672.3679	294.1894
PG <sub>17</sub>	255.8913	285.1653	268.1255	299.3443	294.2095	294.5967
PG <sub>18</sub>	5.2667	47.3609	6.4409	42.7302	5.1802	43.6933
PG <sub>19</sub>	4.9966	39.5015	4.8769	39.6567	4.0648	20.9354
Total Generation (MW)	3886.6042	3771.559	3881.685	3772.717	3895.652	3775.9231
Total Loss (MW)	218.6042	103.5581	213.6853	104.7174	227.6515	107.9231
Cost (\$/hr)	<b>11994.316</b>	18538.93	<b>11986.88</b>	18589.15	<b>11979.75</b>	18643.362
Emission (ton/hr)	14.3669	<b>5.4869</b>	13.5777	<b>5.5613</b>	14.8153	<b>5.7154</b>

**Table 4.** Statistical results of performance measures – IEEE 57-bus system

Measure	Algorithm	Best	Worst	Mean	Standard deviation
$\gamma$	NSGA-II	30.3899	40.7000	35.7631	2.9537
	NSGA-II + DCD	34.4786	36.6809	35.3886	0.8552
	NSGA-II + DCD + CE	27.8033	38.3118	31.8269	3.7204
$\Delta$	NSGA-II	0.3730	0.5912	0.5166	0.0710
	NSGA-II + DCD	0.1157	0.2214	0.1703	0.0323
	NSGA-II + DCD + CE	0.9471	1.2757	1.1016	0.1126
IGD	NSGA-II	0.0069	0.0092	0.0086	0.0006
	NSGA-II + DCD	0.0080	0.0084	0.0082	0.0001
	NSGA-II + DCD + CE	0.0092	0.0129	0.0103	0.0011

**Table 5.** Statistical results of performance measures – IEEE 118-bus system

Measure	Algorithm	Best	Worst	Mean	Standard deviation
$\gamma$	NSGA-II	55.6728	66.9460	61.5347	3.2044
	NSGA-II + DCD	58.4553	66.5498	62.3828	2.3237
	NSGA-II + DCD + CE	56.7404	69.5889	62.2870	5.5128
$\Delta$	NSGA-II	0.3384	0.5224	0.4401	0.0561
	NSGA-II + DCD	0.2105	0.3467	0.2574	0.0375
	NSGA-II + DCD + CE	0.6356	0.9133	0.8153	0.0796
IGD	NSGA-II	0.0120	0.0238	0.0210	0.0033
	NSGA-II + DCD	0.0108	0.0237	0.0203	0.0035

57-bus and IEEE 118-bus systems are respectively shown in Fig. 2 and Fig. 3. For validation purposes, the reference Pareto-front generated using RCGA is given as well in the respective figures. Furthermore, the Pareto-fronts generated using NSGA-II, MNSGA-II and multiple runs Pareto-front obtained using RCGA are almost identical.

Extreme solutions of Pareto-front, obtained out of ten trial runs by approaches using NSGA-II and MNSGA-II for IEEE 57-bus and IEEE 118-bus systems are reported in Table 2 and Table 3 respectively. In Table 2, optimum power obtained for the IEEE 57-bus are shown in the first seven rows, the eighth row represents the total generation, the ninth row represents the losses and the remaining two rows represent the total fuel cost and total emission. Similarly the results of other test systems are tabulated. The results show that NSGA-II and MNSGA-II algorithms are the effective tool for handling multi-objective optimization problem where multiple Pareto optimal solutions can be arrived in a single run with a best computational time compared to RCGA method. From the Tables 2 and 3, it can be concluded that, the NSGA-II with DCD and CE is capable of providing better results than the others for the CEED with valve-point effect problem.

The statistical analysis like best, worst, mean and standard deviation results of multi-objective performance metrics are reported for IEEE 57-bus and IEEE 118-bus systems, in Table 4 and Table 5 respectively. It can be seen that, for most of the performance metrics, values obtained by NSGA-II with DCD is smaller than NSGA-II and NSGA-II with DCD and CE, which means that for NSGA-II with DCD is giving better convergence and diversity consistently.

In order to ensure better convergence, a search algorithm may need diversity in both aspects - along the Pareto-optimal front and lateral to the Pareto-optimal front. For increasing number of generations, the number of fronts drops to one in case of NSGA-II with DCD and on the other hand, NSGA-II with DCD and CE approach maintains certain number of fronts. As a result of this, Controlled elitism will be helpful for maintaining lateral diversity in the solutions across various fronts. Since the lateral diversity characteristics of NSGA-II with DCD and CE is better than NSGA-II with DCD, there is a way for getting better extreme solution even though poor convergence.

## 6. Conclusion

In this paper, NSGA-II and MNSGA-II algorithms are applied to solve CEED problem with valve-point loading. The performance of NSGA-II and MNSGA-II algorithms are validated on the standard IEEE 57-bus and IEEE 118-bus systems. RCGA algorithm is employed for generating reference Pareto-front by minimizing weighted sum of objectives. Best-obtained Pareto-front of NSGA-II and MNSGA-II are very close to the reference Pareto-front using RCGA for all the test systems. Pareto-front obtained by MNSGA-II show significant improvement on lateral diversity and uniform distribution of non-dominated solutions compared to NSGA-II. The performance of NSGA-II, NSGA-II with DCD and NSGA-II with DCD and CE are compared with respect to various statistical performance measures such as convergence metric, diversity metric and inverted generational distance metric.

By using the statistical performance measures, it can be concluded that the NSGA-II with DCD is better with respect to most of the multi-objective performance metrics.

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