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The Design of an Optimal Demand Response Controller Under Real Time Electricity Pricing

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Abstract – The use of a demand response controller is necessary for electric devices to effectively respond to time varying price signals and to achieve the benefits of cost reduction. This paper describes a new formulation with the form of constrained optimization for designing an optimal demand response controller. It is demonstrated that constrained optimization is a better approach for the demand response controller, in terms of the ambiguity of device operation and the practicality of implementation of the optimal control law. This paper also proposes a design scheme to construct a demand response controller that is useful when a system controller is already adapted or optimized for the system. The design separates the demand response function from the original system control function while leaving the system control law unchanged. The proposed formulation is simulated and compared to the system with simple dynamics. The effects of the constraints, the system characteristics and the electricity price are examined further.

Keywords: Demand response, Real time pricing, Optimal control, Dynamic programming

1. Introduction

The traditional power system is configured and operated to meet all power demands [1]. This means that the capacity of power generation should continuously increase as industries continue to develop and as more electrical/ electronic equipment comes into use. Various methods, such as economic dispatch and unit commitment, have been developed to save generation costs while satisfying the demand for electric power [2-3]. However, the unilateral obligations imposed on power generation have limitations and cannot guarantee the reliability and quality of the power system. If the recent introduction of a market system and deregulation of the power industry are both considered, the limitations of the traditional system continue to grow worse. It has been asserted that these limitations were a major cause of California's accident in 2000 [4-5].

Significant attention has been paid to the role of demand side participation, and many theoretical and empirical studies have been carried out. Improved communication capabilities, supported by smart grid technology, will enable demand response (DR) to be a more integral and beneficial part of the future power system. The benefits of DR are generally recognized as follows [6-8]. First, customers can reduce their electricity cost by shifting the load away from the peak price times. Second, load shifting induces the reduction of peak demand, by which the reliability is improved, and the generation efficiency increases. Finally, converting the inelastic demand for electricity into an elastic demand can prevent a generation company from achieving a monopoly or an oligopoly from exercising market power.

Demand response programs are classified into incentive based programs (IBP) and price based programs (PBP) [1]. In particular for real time pricing (RTP), the capacity of the response from human customers is very limited due to the frequent changes (e.g., 5 minutes) in electricity pricing. Thus, for RTP, instead of the customer responding directly to electricity prices, the response should be governed by the devices or appliances themselves [9]. Therefore, the implementation of such smart devices boils down to designing of the DR controllers. Applications of DR controllers can be found in several published investigations. According to [10] and [11], the cost benefit of DR can be obtained with the use of DR control algorithms for various appliances, such as electrical vehicle charging systems, dish washers and HVAC systems. In [12], the need for DR control functions that change the set point with respect to the electricity price for HVAC systems is described.

This paper is organized as follows. The first section is the introduction, given here. In section II, the dynamic programming problem of a proposed constrained optimization for a DR controller is formulated and solved. The solution interpretation is examined in terms of its desirable properties, and it is also proposed that the constrained optimization solution is a reasonable choice for the implementation of a DR controller. Section III describes a design scheme developed to construct a DR controller by attaching the DR function to the original system controller, with the system control law remaining

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unchanged due to considerations of the electricity price. The additive property of the linear system is actively utilized in this process. Section IV is the simulation section in which the proposed formulation is simulated and compared using the system with simple dynamics, which is similar to that of an electric heater. The effects of the constraints, the system characteristics and the electricity price are examined further. The concluding section summarizes the content.

2. Formulation of the Demand Response Controller

2.1 The constrained optimization problem

Dynamic programming (DP) [13] is useful in the formulation of DR controllers that can be defined by the discrete state, the state transition, the state transition cost and admissible control decisions. There are also works in which DP is used to solve load control problems [8, 14, 15]. The common drawbacks of DP for real time control applications include the curse of dimensionality and the computation time. However, that is not the case for the DR controller formulation proposed here, as explained later.

In constructing of the DP formulation, we consider a discrete-time linear system with the dynamics equation

$$x_{k+1} = A_k x_k + B_k u_k + \omega_k, \quad k = 0,1,\ldots,N-1 \tag{1}$$

and the cost function

$$E\left[g_N\left(x_N\right) + \sum_{k=0}^{N-1} g_k\left(x_k, u_k, \omega_k\right) \right] \tag{2}$$

where

A_k	system parameter
B_k	input parameter
x_k	state of the system
u_k	control
ω_k	random disturbance
$g_N(x_N)$	terminal cost
$g_k(x_k, u_k, \omega_k)$	transition cost

N and E denote the number of times the control is applied and the expected value of the random variable, respectively. The system parameter A_k determines the natural response and stability of the system, and the input parameter B_k is related to the system gain. The transition cost of the existing formulation of the composite objective optimization [16] has the form of

$$g_k\left(x_k, u_k, \omega_k\right) = \alpha\left(x_{k+1} - x_{k+1}^{ref}\right)^2 + \rho_k u_k \tag{3}$$

where x_{k+1}^{ref} is the target or the reference value of the system, ρ_k is the electricity price at time k, and α is the econometric scaling factor for converting abstract discomfort into monetary cost. The serious defect in the existing formulation is the fact that the effect of the scaling factor α may confuse the user regarding the operation of the device. As α becomes smaller, the term regarding the electricity price dominates the transition cost and the system cannot keep the state at the reference value. Thus, if α is not tuned appropriately, users find it hard to decide if the under-performance is due to DR to time varying electricity price or due to failure of the device. Additionally, it is very difficult to determine the degree of under-performance caused by α. The confusion associated with the effect of α grows worse when the electricity price does not change because the system will continue to operate in an under-performing state even in cases in which DR choices are not necessary. This makes the formulation of the composite objective optimization far from an ideal approach in terms of the ambiguity and makes the device manufacturers less likely to provide users with options to adjust the intent of participation in DR.

To deal with the defects in the existing formulation, we propose a new formulation of constrained optimization, in which the term of state departure from the reference value is extracted from the transition cost into the constraint as follows:

$$\begin{gathered} J_N(x_N) = g_N(x_N) = \begin{cases} 0 & \text{if } x_N^{\min} \le x_N \le x_N^{\max} \\ \infty & \text{otherwise} \end{cases} \\ g_k\left(x_k, u_k, \omega_k\right) = \rho_k u_k \\ s.t \;\; x_k^{\min} \le E\left\{x_k\right\} \le x_k^{\max} \end{gathered} \tag{4}$$

where $x_k^{\min}$ and $x_k^{\max}$ are the admissible minimum and maximum values at time k. The function $J_k\left(x_k\right)$ is a cost-to-go function, which means the total sum of the transition costs from the current state x_k at time k to the terminal state x_N at time N for the given control decision. The function $J_k\left(x_k\right)$ is given by

$$J_k(x_k) = E\left[g_N\left(x_N\right) + \sum_{i=k}^{N-1} g_i\left(x_i, u_i, \omega_i\right) \right] \tag{5}$$

From the principle of optimality of dynamic programming [13], the optimal cost-to-function $J_k^*\left(x_k\right)$ can be represented as

$$J_k^*\left(x_k\right) = \min_{u_k} E\left[g_k(x_k, u_k, \omega_k) + J_{k+1}^*\left(x_{k+1}\right) \right] \tag{6}$$

It is assumed that the initial value x_0 is given, such that $x_0^{\min} \le x_0 \le x_0^{\max}$ is satisfied. Then, the ambiguous scaling factor α is removed from the transition cost and the degree of under-performance due to DR becomes predictable to some extent.

2.2 The optimal solution

At first, we find the optimal control u_{N-1}^* and the optimal cost-to-go function $J_{N-1}^*(x_{N-1})$. Using (6), $J_{N-1}^*(x_{N-1})$ at time $k=N-1$ is of the form

$$\begin{aligned} J_{N-1}^*(x_{N-1}) &= \min_{u_{N-1}} E\left[\rho_{N-1}u_{N-1} + J_N^*(x_N)\right] \\ &= \min_{u_{N-1}} E\left[\rho_{N-1}u_{N-1} + J_N\left(A_{N-1}x_{N-1} + B_{N-1}u_{N-1} + \omega_{N-1}\right)\right] \end{aligned} \quad (7)$$

If we assume $E\{\omega_{N-1}\}=0$ and use the fact $J_N^*(x_N) = J_N(x_N)$ at final stage N, u_{N-1}^* is determined as the control that forces the next state x_N to become the midpoint between $x_N^{\min}$ and $x_N^{\max}$ regardless of disturbance. Let us denote the midpoint as x_N^{mid} or

$$x_N^{\text{mid}} = \frac{x_N^{\min} + x_N^{\max}}{2} \quad (8)$$

then $J_N(x_N)$ becomes zero from (4) and u_{N-1}^* is determined as

$$u_{N-1}^* = -\frac{A_{N-1}x_{N-1} - x_N^{\text{mid}}}{B_{N-1}} \quad (9)$$

From (7) and (9), $J_{N-1}^*(x_{N-1})$ becomes

$$J_{N-1}^*(x_{N-1}) = -\rho_{N-1} \cdot \left(\frac{A_{N-1}x_{N-1} - x_N^{\text{mid}}}{B_{N-1}}\right) \quad (10)$$

As the next step in the backward DP procedure, $J_{N-2}^*(x_{N-2})$ can be similarly represented as

$$\begin{aligned} &J_{N-2}^*(x_{N-2}) \\ &= \min_{u_{N-2}} E\left[\rho_{N-2}u_{N-2} + J_{N-1}^*(x_{N-1})\right] \\ &= \min_{u_{N-1}} E\left[\rho_{N-2}u_{N-2} + J_{N-1}^*\left(A_{N-2}x_{N-2} + B_{N-2}u_{N-2} + \omega_{N-2}\right)\right] \end{aligned} \quad (11)$$

From (10), Eq. (11) can be arranged as a linear function of u_{N-2}, given as

$$J_{N-2}^*(x_{N-2}) = \min_{u_{N-1}} E\begin{bmatrix} \left\{\rho_{N-2} - \dfrac{A_{N-2}B_{N-2}}{B_{N-1}} \cdot \rho_{N-1}\right\} u_{N-2} \\ -\rho_{N-1}\left\{\dfrac{A_{N-1}\left(A_{N-2}x_{N-2} + \omega_{N-2}\right) - x_N^{\text{mid}}}{B_{N-1}}\right\} \end{bmatrix} \quad (12)$$

Using the assumption $E\{\omega_{N-2}\}=0$, u_{N-2}^* is determined depending on the sign of the coefficient term, given as

$$u_{N-2}^* = \begin{cases} u_{N-2}^{\max} & \text{if } \rho_{N-2} < \dfrac{A_{N-1}B_{N-2}}{B_{N-1}} \cdot \rho_{N-1} \\ u_{N-2}^{\min} & \text{if } \rho_{N-2} \geq \dfrac{A_{N-1}B_{N-2}}{B_{N-1}} \cdot \rho_{N-1} \end{cases} \quad (13)$$

where $u_{N-2}^{\min}$ and $u_{N-2}^{\max}$ are the minimum and maximum controls, which satisfy the constraint of the next state x_{N-1}. The backward DP procedure given here can be applied to determine the optimal control u_k^* and the optimal cost-to-go function $J_k^*(x_k)$ at time k. To carry the algorithm forward, $J_k^*(x_k)$ is derived:

$$\begin{aligned} J_k^*(x_k) &= \min_{u_k} E\left[\rho_k u_k + J_{k+1}^*(x_{k+1})\right] \\ &= \min_{u_k} E\left[\rho_k u_k + \rho_{k+1}u_{k+1}^* + J_{k+2}^*(x_{k+2})\right] \end{aligned} \quad (14)$$

Let us denote the result of the application of u_{k+1}^* as x_{k+2}^*. Then, using the following relationship

$$x_{k+2}^* = A_{k+1}x_{k+1} + B_{k+1}u_{k+1}^* + \omega_{k+1} \quad (15)$$

and the conditions $E\{\omega_k\}=0$ and $E\{\omega_{k+1}\}=0$, Eq. (14) can be written as

$$\begin{aligned} &J_k^*(x_k) \\ &= \min_{u_k} E\left[\rho_k u_k + \rho_{k+1}u_{k+1}^* + J_{k+2}^*\left(x_{k+2}^*\right)\right] \\ &= \min_{u_k} E\begin{bmatrix} \rho_k u_k + \rho_{k+1}\left\{\dfrac{x_{k+2}^* - A_{k+1}x_{k+1} - \omega_{k+1}}{B_{k+1}}\right\} \\ +J_{k+2}^*\left(x_{k+2}^*\right) \end{bmatrix} \\ &= \min_{u_k} E\begin{bmatrix} \rho_k u_k \\ +\rho_{k+1}\left\{\dfrac{x_{k+2}^* - A_{k+1}\left[A_k x_k + B_k u_k + \omega_k\right] - \omega_{k+1}}{B_{k+1}}\right\} \\ +J_{k+2}^*\left(x_{k+2}^*\right) \end{bmatrix} \\ &= \min_{u_k} E\begin{bmatrix} \left\{\rho_k - \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1}\right\} u_k \\ +\rho_{k+1}\left\{\dfrac{x_{k+2}^* - A_{k+1}A_k x_k}{B_{k+1}}\right\} + J_{k+2}^*\left(x_{k+2}^*\right) \end{bmatrix} \end{aligned} \quad (16)$$

It should be noted that $J_k^*(x_k)$ is linear to the control u_k. Thus, the optimal control u_k^* can be determined as

$$u_k^* = \begin{cases} u_k^{\max} & \text{if } \rho_k < \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \\ u_k^{\min} & \text{if } \rho_k \geq \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \end{cases} \tag{17}$$

where $u_k^{\min}$ and $u_k^{\max}$ are the minimum and maximum controls, which satisfy the constraint of the next state x_{k+1}.

2.3 Remarks on the optimal solution

A desirable characteristic of the optimal solution in (17) is that it depends only on the system dynamics and the electricity price at the current time index k and the next time index k+1. Thus, there is no curse of dimensionality problem, and the optimal control law (17) holds when N approaches infinity, so that (17) can be used as a general optimal control law.

However, the form of the solution to the constrained optimization approach has a beneficial property in terms of its ambiguity. In this case, users can anticipate the result of DR because the state of the system lies within the range of the constraints. That is, the interpretation of the constraints is clear to the users. Thus, it is easier for the device manufacturer to adopt the options to adjust the intent of participation in the DR by setting the admissible range of the system state. Additionally, it is possible to separate the DR function from the original system control function due to the absence of the state variable from the objective function. This separation property enables the solution to be easily applied to real-life devices while leaving the existing system control function unchanged. This result will be formulated and explained in the next section.

3. Separation of the Demand Response Function

3.1 Separation of the demand response function

Let us consider again the discrete-time linear system with the dynamics equation

$$x_{k+1} = A_k x_k + B_k u_k + \omega_k, \quad k = 0,1,\ldots,N-1 \tag{18}$$

and the optimal control u_k^* such that the application of the control locates the expectation of the next state $E\{x_{k+1}\}$ within the range $x_{k+1}^{\min} \leq E\{x_{k+1}\} \leq x_{k+1}^{\max}$ determined by the constraint. Let us define the state of the system without DR as x_k^s and the state with DR as x_k^{dr} such that

$$x_k = x_k^s + x_k^{dr} \tag{19}$$

We define the lower and upper margins of state departure from the reference value as x_k^{low} and x_k^{upp}, respectively, such that

$$x_k^{low} = x_k^{\min} - x_k^{ref}, \quad x_k^{upp} = x_k^{\max} - x_k^{ref} \tag{20}$$

Then the constraint of the state can be represented as

$$x_{k+1}^{ref} + x_{k+1}^{low} \leq E\{x_{k+1}\} \leq x_{k+1}^{ref} + x_{k+1}^{upp} \tag{21}$$

If we assume that there is a perfect control of system u_k^{s*} such that the expectation of the next state becomes reference value x_{k+1}^{ref} which is set in the system without respect to electricity price, then the optimal control u_k^* can be divided into two components as

$$u_k^* = u_k^{s*} + u_k^{dr*} \tag{22}$$

where u_k^{dr*} is defined as the optimal control for DR. Then, from the additive property of the linear system, dynamics equation with optimal control can be represented as

$$x_{k+1}^s + x_{k+1}^{dr} = A_k\left\{x_k^s + x_k^{dr}\right\} + B_k\left\{u_k^{s*} + u_k^{dr*}\right\} + \omega_k \tag{23}$$

From the assumption that u_k^{s*} is the perfect control for x_{k+1}^{ref}, u_k^{s*} becomes the optimal solution of

$$x_{k+1}^s = A_k x_k^s + B_k u_k^s + \omega_k \tag{24}$$

with the objective function of

$$E\left[\sum_{k=0}^{N-1}\left(x_k^s - x_k^{ref}\right)^2\right] \tag{25}$$

Then, from (18), u_k^{dr*} becomes the optimal solution of

$$x_{k+1}^{dr} = A_k x_k^{dr} + B_k u_k^{dr} \tag{26}$$

with the objective function of

$$\sum_{k=0}^{N-1} \rho_k u_k \tag{27}$$

subject to the constraint of possible state transition

$$x_k^{low} \leq x_k^{dr} \leq x_k^{upp} \tag{28}$$

Then, solving the constrained optimization problem in (18) and (21), the problem is separated into two optimization problems: one is the normal optimal control problem with the quadratic cost given in (24)-(25), and the other is the state-displaced constrained optimization problem defined in (26)-(28). The optimal control law u_k^{dr*} of the state-displaced constrained optimization problem is represented similarly as (17), given by:

$$u_k^{dr*} = \begin{cases} u_k^{upp} & \text{if } \rho_k < \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \\ u_k^{low} & \text{if } \rho_k \geq \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \end{cases} \tag{29}$$

where u_k^{low} is the control which makes the expectation value of the next state x_{k+1}^{dr} into x_{k+1}^{low} and u_k^{upp} is the control which makes x_{k+1}^{dr} into x_{k+1}^{upp}. The concept of the separation of DR function from the system control function is illustrated in Fig. 2 as an expansion of the general schematic diagram of the DR controller depicted in Fig. 1. From Fig. 2, the DR function can be considered to follow a completely different control law, which is applied to a different system in the proposed method of separation.

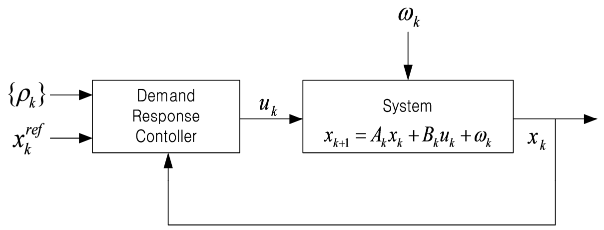

Fig. 1. The DR controller for the entire system

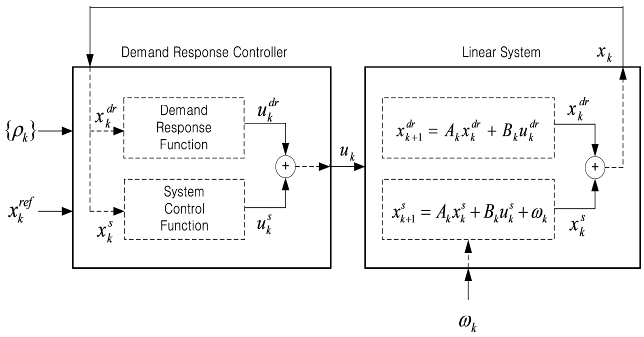

Fig. 2. Separation of the DR function from the system control function, as applied to conceptually different systems

As a further step, if the system controller is the fixed component which is adapted or optimized to its own functional objective, the design and implementation of the DR controller is adding the DR function to the existing system controller. The effect of the additional control associated with the DR function can be observed as a disturbance from the view of the existing system controller and is reflected in the next control action. It is guaranteed that the next control action will attempt to carry the state of the system to the reference value. Then, the DR function becomes equal to determining the control that is making the transition from the reference state to the state of either $x_{k+1}^{\min}$ or $x_{k+1}^{\max}$, and can be implemented as a simple feed-forward controller with the control law of

$$u_k^{dr*} = \begin{cases} u_k^{ref-to-\max} & \text{if } \rho_k < \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \\ u_k^{ref-to-\min} & \text{if } \rho_k \geq \dfrac{A_{k+1}B_k}{B_{k+1}} \cdot \rho_{k+1} \end{cases} \tag{30}$$

where $u_k^{ref-to-\min}$ and $u_k^{ref-to-\max}$ are the controls required to change the reference state into the states $x_k^{\min}$ and $x_k^{\max}$, respectively. The controls $u_k^{ref-to-\min}$ and $u_k^{ref-to-\max}$ may be negative values, even though the real control u_k to the system cannot be negative. Fig. 3 presents this design scheme for the DR controller. With this separation design method, the DR function can be implemented irrespective of which control method in the field of control theory is used and whether the control method is optimal or not, as long as the system is controlled within a moderate range around the reference x_k^{ref} by the existing system controller. This will cause the given DR controller to be more easily accepted by manufacturers.

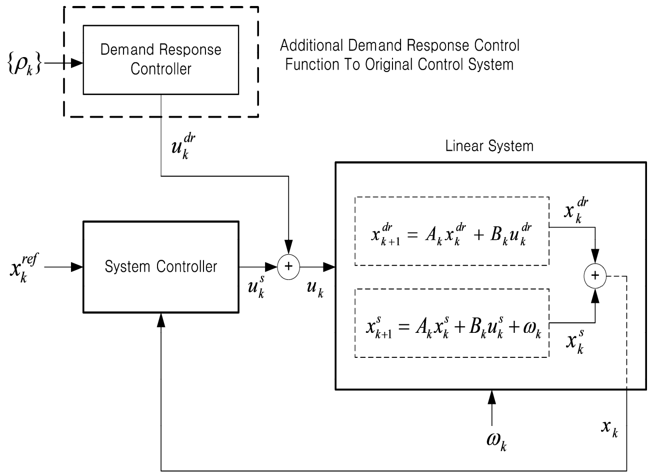

Fig. 3. Design scheme of DR controller with system control function is given and unknown

3.2 Selection of the lower and upper margins of departure

It is desirable that the DR controllers do not harm the expectations of the device users. That is, the benefit of the DR should not be obtained by sacrificing the convenience associated with using the devices. If this is not the case, even though the users are willing to participate in the DR program initially, the good intentions of the users are significantly undermined over time.

The degree of under-performance is represented as the lower margin x_k^{low} and the upper margin x_k^{upp} in the proposed design scheme of the DR controller. Therefore, care should be taken when the values of the lower and upper margins are selected, so that discomfort from the DR does not occur. For example, if the device is an electric heater, we can set $x_k^{low} = 0$ and $x_k^{upp} = 1$. Then, the room temperature will not be less than the reference value set by the user due to DR, and instead, the room will tend to become warmer than the reference temperature. There may be some sort of misunderstanding that DR inevitably causes inconvenience. Of course, the benefit of cost reduction can be gained, if users endure the discomfort associated with low temperatures in the winter. However, this consequence is a saving and not a true reflection of DR. The DR should become the intelligent response to the electricity price, so that it is possible to reduce the associated cost, even when the electricity consumption increases. This situation is examined in the simulation section with concrete numbers.

4. The Simulation Results

4.1 The conditions

To begin with, we consider the discrete-time linear system with the dynamics equation

$$x_{k+1} = 0.7x_k + 0.2u_k, \quad k = 0,1,\ldots,N-1$$

This may be regarded as an electric heater, where x_k is the room temperature in units of °C and u_k is the electricity consumption in units of kWh. The simulation time N is 60 and k is in units of 5 minutes. It is assumed that control u_k is greater than or equal to zero and there is no limit on the maximum value of u_k. It is also assumed that the initial value x_0 is 20°C and the reference value x_k^{ref} is also 20°C for all k. The disturbance ω_k is ignored when we check the effect of the parameters, such as the DR margins, the system dynamics and the electricity price, on the DR benefits of cost reduction because the disturbance does not affect the optimal control law. However, the simulation is also performed in light of the disturbance to verify that the existence of the disturbance does not affect the DR benefit and the optimal control law. A set of 60 electricity prices is generated using the statistical data of PJM hourly real time prices from 2008. The statistical analysis indicated that the behavior of the market price may resemble a log-normal distribution [17].

These are the basic simulation conditions. We will change the conditions, such as the DR margins, the system dynamics and the electricity price, in the specific simulation cases and examine the effect of the changes.

4.2 The quantitative verification of optimality

Before we examine the effects of the various cases, let us first verify the optimal control law in (17) for the constrained optimization formulation with the system under simulation. For verification, the branch condition in (17) is changed by multiplication with a constant. The constant is in the range of [0.5, 1.5], and the electricity price is fixed with one set of values generated by the log-normal distribution. Fig. 4 presents the simulation results. The optimal condition is displayed with a dotted line.

It can be seen that cost reduction takes the form of a quadratic function and is lowest in proximity of the optimal condition, as expected. Below the optimal condition, the average temperature and the electricity consumption decrease; additionally, they increase above the optimal condition. The case of 'above the optimal condition' is easily understandable because it is the usual pattern of increased electricity consumption followed by an increase in the room temperature and cost. However, in the case of 'below the optimal condition', the cost increased even though the electricity consumption and the temperature decrease. This shows the necessity of an intelligent DR method, one of which is proposed in this paper.

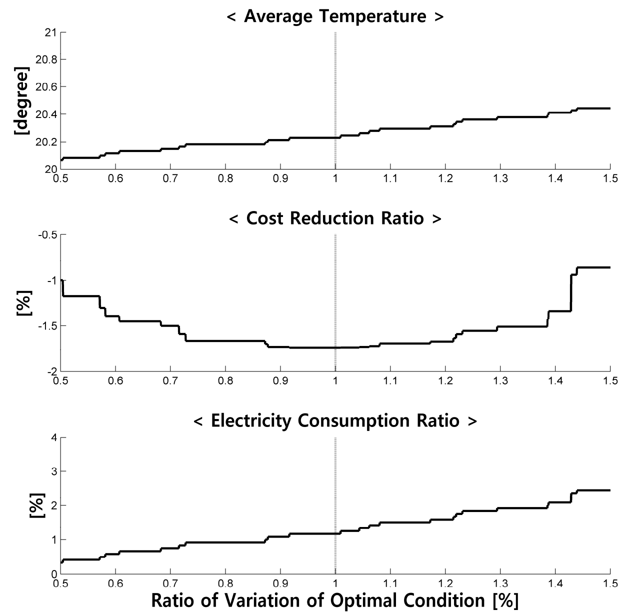

Fig. 4. Quantitative verification of optimal control law of constrained optimization

4.3 Adjusting the upper and lower margins

Three cases of constraints are simulated and compared:

$$\begin{aligned} &\text{Case I} : && x_k^{low} = 0, && x_k^{upp} = 1 && \forall k \\ &\text{Case II} : && x_k^{low} = -1, && x_k^{upp} = 1 && \forall k \\ &\text{Case III} : && x_k^{low} = 0, && x_k^{upp} = 2 && \forall k \end{aligned}$$

Simulations of 100 trials are performed. The electricity consumption increases immediately before the rise in electricity price, as the optimal control law (17) suggests. Fig. 5 represents the temperature and electricity consumption for one trial. The average results of 100 trials are as follows:

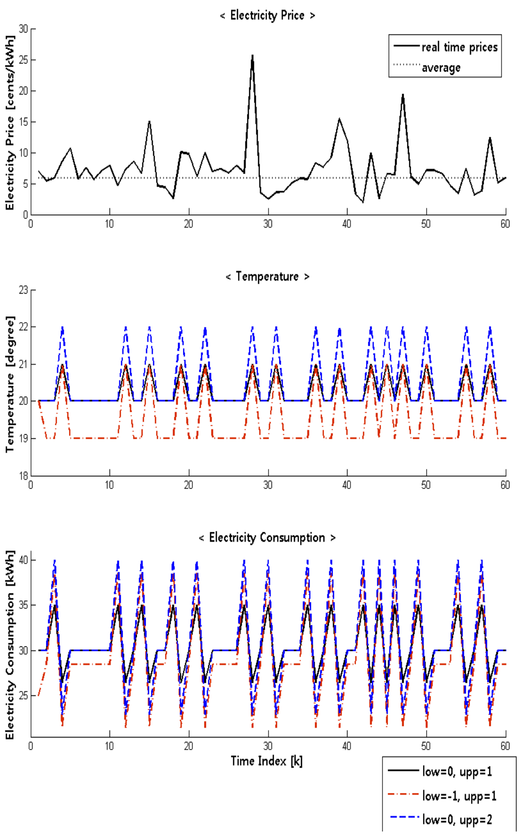

Fig. 5. The simulation results: an individual trial selected from 100 trials for 3 cases of constraints ({upper, lower}={0, 1}, {-1, 1}, {0, 2})

$$\text{< Average of average temperature >}$$
$$\text{Case I : } 20.30°\text{C}, \quad \text{Case II : } 19.62°\text{C}, \quad \text{Case III : } 20.60°\text{C}$$
$$\text{< Average cost reduction ratio >}$$
$$\text{Case I : } 1.82\%, \quad \text{Case II : } 8.84\%, \quad \text{Case III : } 3.63\%$$

$$\text{< Average electricity consumption ratio >}$$
$$\text{Case I : } 1.54\%, \quad \text{Case II : } -2.12\%, \quad \text{Case III : } 3.07\%$$

where the cost reduction ratio is computed as the ratio of the reduced cost to the cost without DR, that is

$$\text{Cost reduction ratio[\%]} = \frac{\text{Cost without DR-Cost with DR}}{\text{Cost without DR}} \times 100$$

and the electricity consumption ratio is computed as the ratio of the increased electricity consumption to the electricity consumption without DR, that is

$$\text{Consumption ratio[\%]} = \frac{\text{Consumption without DR-Consumption with DR}}{\text{Consumption without DR}} \times 100$$

It should be noted that the cost reduction is achieved for Case I and Case III, even though the electricity consumption increases. Accordingly, the average temperature also increases. Although the cost reduction is the best for Case II, the electricity consumption decreases in this case. It is natural that reduced electricity consumption should result in reduced electricity cost, as in Case II. However, this type of saving causes user discomfort and is not true DR. DR can be more meaningful when the cost reduction is achieved without user discomfort. User discomfort can be confirmed by the lowest average temperature for Case II in the simulation results.

The benefit of cost reduction increases as the admissible margin of departure is increased. However, the sudden rise in temperature also results in discomfort, and there is a limit to the rate of state change in the real system. Therefore, the margin increase is not always a better decision, and careful selection of the margin is necessary. Even so, the results show that the constrained optimization approach is a better choice for the design of a DR controller than the composite objective optimization approach, in terms of the clarity of the effect of DR and customer convenience.

4.4 Variation in the system dynamics

Two cases of disturbance are simulated and compared:

$$\text{Case I} \;:\; x_{k+1} = 0.7x_k + 0.2u_k$$
$$\text{Case II} \;:\; x_{k+1} = 0.3x_k + 0.2u_k$$

Case II can be interpreted as a more severe weather condition, such as a significantly reduced ambient temperature. Fig. 6 represents the temperature and electricity consumption for one trial. The average results of 100 trials are

$$\text{< Average of average temperature >}$$
$$\text{Case I : } 20.30°\text{C}, \quad \text{Case II : } 20.04°\text{C}$$

$$\text{< Average cost reduction ratio >}$$
$$\text{Case I : } 1.82\%, \quad \text{Case II : } 0.04\%$$

$$\text{< Average electricity consumption ratio >}$$
$$\text{Case I : } 1.50\%, \quad \text{Case II : } 0.22\%$$

In Case II, the effect of DR is quite small. The average temperature is also close to the reference value. This denotes that the DR cannot achieve the desired purpose for some systems, and the only possible method is to decrease the electricity consumption to obtain the desired cost reduction. For example, DR can be effectively applied to HVAC systems that use air as a medium to store or memorize energy. However, we cannot help reduce the electricity consumption to apply DR to the system without memory, such as lighting and television. That is, the lights and the television should be turned off to obtain the benefit of cost reduction.

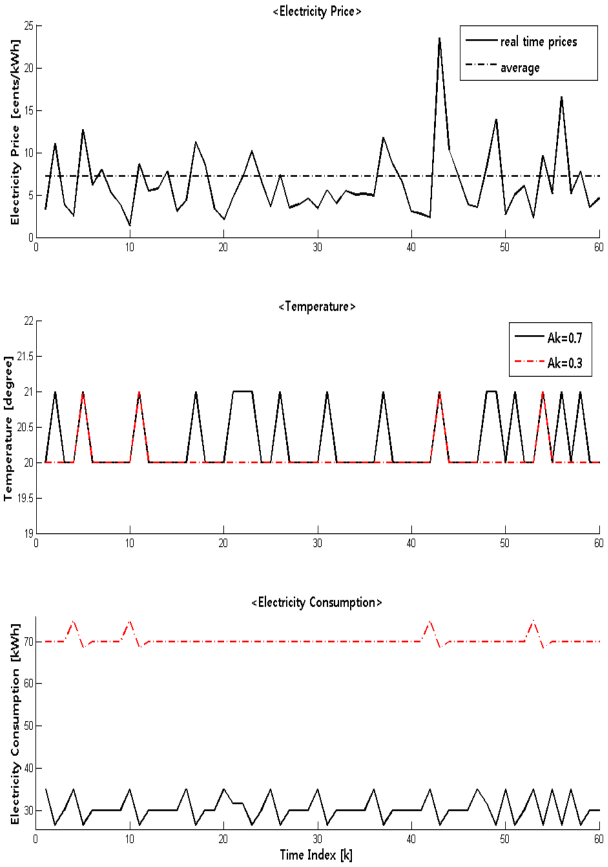

Fig. 6. The simulation results: one representative trial among 100 trials for 2 cases of system dynamics (A_k=0.7 and A_k=0.3)

4.5 Variation in the distribution of the electricity price

Two cases of the probability distribution of electricity price are simulated and compared:

Case I : variance=9
Case II : variance=36

The average is set to 7.1784, as for the PJM data. Fig. 7 represents the temperature and electricity consumption for one trial. Simulations of 100 trials are also performed and the average results of 100 trials are

< Average of average temperature >
Case I : 20.25°C, Case II : 20.36°C

< Average cost reduction ratio >
Case I : 1.15%, Case II : 3.17%

< Average electricity consumption ratio >
Case I : 1.30%, Case II : 1.82%

It is said that variation in the electricity price provides an opportunity for cost reduction [18]. That is, there are more possibilities to reduce electricity cost when the price is significantly more variable. This is verified by the simulation result in which the benefit of cost reduction is greater for Case II with the larger price variance, even with increased electricity consumption and higher average temperatures.

4.6 The existence of disturbances

Three cases of the disturbance are simulated and compared:

Case I : no disturbance
Case II : 5% of set point
Case III : 10% of set point

Random disturbance is generated in accordance with the uniform distribution. The DR margin is set as {upper margin, lower margin}={0, 1}. Simulations of 100 trials are performed. The results are represented in Fig. 8, and the average values are as follows:

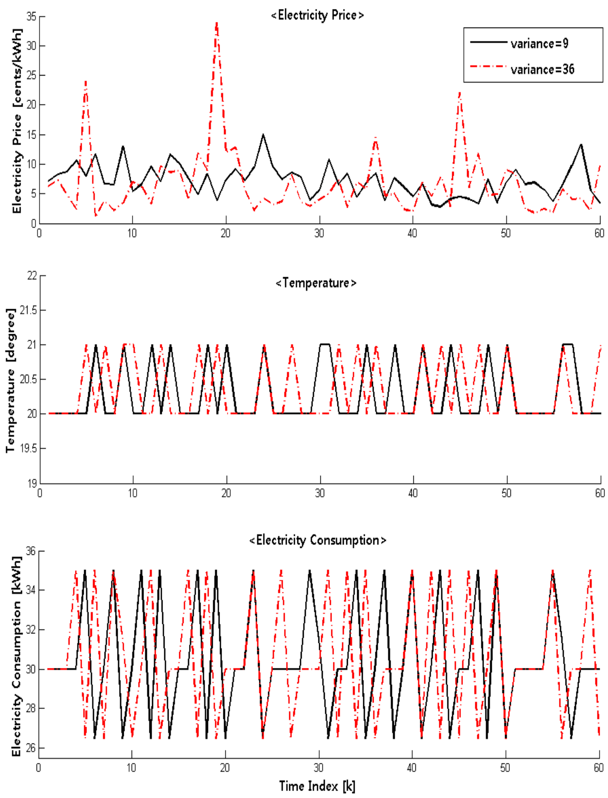

Fig. 7. The simulation results: one representative trial among 100 trials for 2 cases of statistics of electricity price (variance=9 and variance=36)

< Average of average temperature >

Case I : 20.29°C, Case II : 20.29°C, Case III : 20.29°C

< Average cost reduction ratio >

Case I : -1.87%, Case II : −1.87%, Case III : −1.68%

< Average electricity consumption ratio >

Case I : 1.50%, Case II : 1.54%, Case III : 1.55%

It should be noted that the average temperature, the average cost reduction and the average increase in electricity consumption are approximately the same for all cases. That is, the addition of the disturbance and the magnitude of the disturbance do not significantly change the effect of the demand response. This verifies that the DR benefit and the optimal control law do not depend on the existence of a disturbance with the assumption of a zero expectation value for the disturbance.

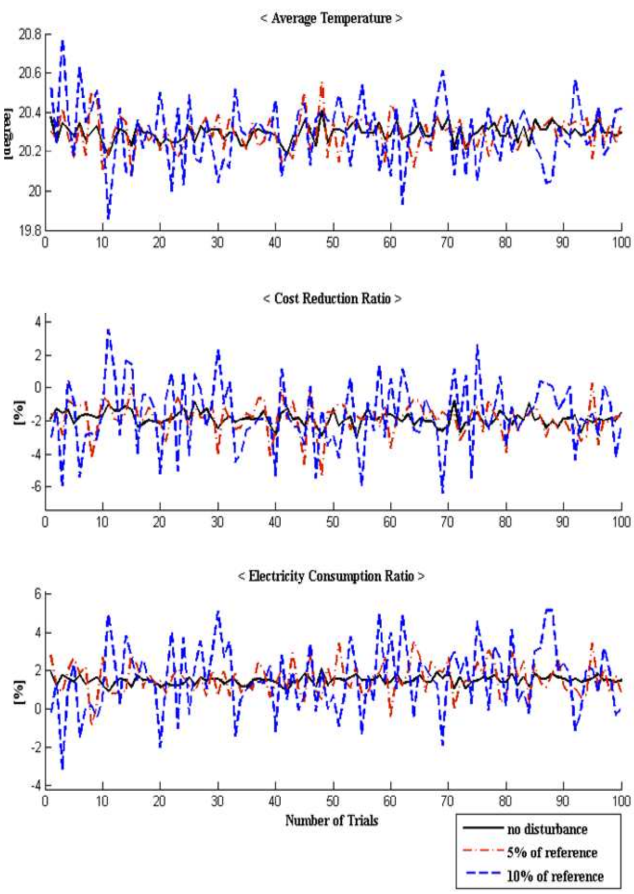

Fig. 8. The simulation results: an average of 100 trials for 3 cases of disturbance (No disturbance, 5% of set point, 10% of set point)

5. Conclusion

One of the generally recognized benefits of DR is the reduction in electricity costs by shifting the load at times of high price. A DR controller is necessary for electric devices to effectively respond to time varying price signals, such as the real time price. Constrained optimization is proposed as the new formulation versus the use of existing composite objective optimization for the design of an optimal DR controller. The proposed constrained optimization is better in terms of the ambiguity of device operation and the practicality of implementation of the optimal control law. This is verified by the simulations with a discrete time system with simple dynamics, which is similar to that of an electric heater. The constrained optimization approach has the additional desirable feature of separation of the DR function from the original system control function. We can construct a DR controller by attaching the DR function to the original system controller using this feature.

In the simulation section, various simulation cases were performed for the proposed method. The optimality condition was checked by quantitative results. The effects of margins, electricity prices, and system characteristics were also examined. It was found that an increase in the margin resulted in greater cost reductions. However, careful selection of the margins is necessary in light of customer convenience. The analysis verified that the benefits of DR may be limited by the type of system. In particular, for a system without memory, such as lighting and television systems, we cannot help reducing electricity consumption to obtain the benefit of cost reduction. Finally, the cost reductions increase when the price variance is greater, verifying many opportunities for severe changes in electricity prices.

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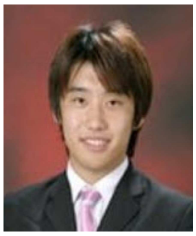

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