Finding Impossible Differentials for Rijndael-like and 3D-like Structures

Ting Cui, Chen-hui Jin
Information Science and Technology Institute
Zhengzhou, 450004 - China
[e-mail: cuiting_1209@yahoo.com.cn]
*Corresponding author: Ting Cui

Received October 14, 2012; revised February 13, 2013; revised March 7, 2013;
accepted March 19, 2013; published March 29, 2013

Abstract

Impossible Differential Cryptanalysis (IDC) uses impossible differentials to discard wrong subkeys for the first or the last several rounds of block ciphers. Thus, the security of a block cipher against IDC can be evaluated by impossible differentials. This paper studies impossible differentials for Rijndael-like and 3D-like ciphers, we introduce methods to find 4-round impossible differentials of Rijndael-like ciphers and 6-round impossible differentials of 3D-like ciphers. Using our methods, various new impossible differentials of Rijndael and 3D could be searched out.

Keywords: Block cipher, impossible differential, Rijndael structure, 3D structure.

This research was supported by National Natural Science Foundation of China(Grant No. 61272488，61272041，61202491.) We express our thanks to Dr. Guo-qiang Liu who helped us to improve the quality of this manuscript. We are also grateful to the anonymous referees for their valuable comments.
http://dx.doi.org/10.3837/tiis.2013.03.006

1. Introduction

Impossible differential cryptanalysis(IDC) was first proposed by Knudsen [14] and Biham [1] to attack DEAL and Skipjack. It is known as one of the most powerful attacks on block ciphers. This cryptanalysis has attracted wide attention and many good results are achieved [1,2,3,7,9,10,11,13,17].

Compared with traditional differential cryptanalysis, IDC considers the differential characteristics with probability 0, when a pair of plaintexts satisfies the input difference of the characteristics, the difference of ciphertexts decrypted by the right subkey never satisfy the output difference of characteristics. By this way we can discard wrong subkeys and recover the right subkey. Impossible differential attack is composed of two steps: finding the longest characteristics and recovering the subkeys. Retrieving the characteristics often use the idea of "miss-in-the-middle", namely to find two differential characteristics with probability 1 from encryption and decryption, and connect them together when there are some inconsistencies in the middle. As is suggested by [4], the key step of IDC is to retrieve the longest impossible differentials. In [4,5], two methods were provided to find impossible differentials of various block ciphers, but both of them have their limitations and some important inconsistencies are ignored [6].

This paper focuses on finding new impossible differentials for two block ciphers: the Rijndael-like block ciphers and the 3D-like ciphers. The cipher Rijndael [7] was submitted to the AES (Advanced Encryption Standard) and was later selected as the AES. Since its selection, Rijndael has received a great deal of attentions, both in block cipher design and cryptanalysis. In [9], 4-round impossible differential of AES128 is detected for the first time, and this ID distinguisher was used in most later IDC results(e.g. in [2] and [17]). And in [10], some new impossible differentials of AES are searched out. In CANS 2008, the new iterated block cipher 3D [8] was designed inspired by AES. The novel design of 3D cipher also attracts some research interests: in 2010, Tang et al proposed a 6-round impossible differential of 3D cipher and attack 9-round 3D cipher [10], then later in ISPEC 2011, Jorge launched a 10-round impossible differential attack by using new 6-round distinguisher of 3D cipher [15], and Takuma et al presented 11- and 13-round attacks on 3D with the truncated differential cryptanalysis in [16], now is approved to be the best attack on 3D.

Although impossible differential cryptanalysis does not give the best attack on these two ciphers[18,16], impossible differential properies still need to be sufficiently considered. Up to now, the longest impossible differential for AES-128 is still 4-round[9,10], while the longest impossible differential for 3D cipher is 6-round [11,15]. In this paper, we will present new methods to find impossible differentials of these two structures. By applying our results, various new impossible differentials of these two block ciphers can be searched out.

Our paper is organized as follows. In Section 2, we introduce some basic notions. In Section 3 and 4, we find various impossible differentials of these two structures. In Section 5, we draw conclusions.

2. Preliminaries

We will introduce some basic notations and definitions through this paper.

$\oplus$ the bitwise XOR;

$+$ the addition over real number space;

$\circ$ the compound operation of two functions;
$\#\{\bullet\}$ the number of elements in a set;
Δx the XOR difference of x and x';
$w(X)$ the number of nonzero components of vector X.

Definition 1 [12]**.** Let $f:\{0,1\}^n \to \{0,1\}^n$ and $\alpha, \beta \in \{0,1\}^n$, the differential probability of f is defined by

$$p_f(\alpha \to \beta) = \frac{1}{2^n} \#\{x \in \{0,1\}^n : f(x) \oplus f(x \oplus \alpha) = \beta\}.$$

It is widely known that if f is a bijection with $p_f(\alpha \to \beta) > 0$, then $\alpha \neq 0$ iff $\beta \neq 0$.

Definition 2 [7]**.** (*branch number*) Let $f(x) = M \times x$, where $M = (m_{ij})_{n \times n}$ is a matrix over $GF(2^m)$, and x is a $n \times 1$ vector over $GF(2^m)$. Then the branch number of f is defined by

$$Br(f) = \min\{w(x) + w(Mx) : x \in GF(2^m)^n \setminus \{0\}\}.$$

Definition 3 [12]**.** (*differential active S-box*) A differential active S-box is defined as an S-box whose input difference is non-zero.

2.1. Brief Description of Rijndael-like Structure

The Rijndael-like structure operates on n^2-word state, which is represented as a $n \times n$ state of words(a $n \times n$ matrix), and the state for a n^2 data-block, $(x_{0,0}, \cdots, x_{n-1,0}, x_{0,1}, \cdots, x_{n-1,n-1})$, is denoted by the word matrix $X = (x_{i,j})_{n \times n}$. Each round of Rijndael-like structure is composed of four operations:

$$Round_{Rijndael}(X) = ARK \circ MC \circ SR \circ SB(X),$$

where

-*SubBytes* (SB): applying the bijective S-box s on each word, i.e.

$$SB:(x_{i,j})_{n \times n} \to (y_{i,j})_{n \times n}, y_{i,j} = s(x_{i,j});$$

-*ShiftRows* (SR): cyclically shifting each row, i.e.

$$SR:\left(x_{i,j}\right)_{n \times n} \to \left(r_{i,j}\right)_{n \times n}, r_{i,j} = x_{i,(j+t_i) \bmod n}, \text{ where } \{t_i : 0 \le i \le n-1\} = Z_n;$$

-*MixColumns* (MC): multiplication of each column by a constant $n \times n$ matrix, i.e.

$$MC:(x_{i,j})_{n \times n} \to M_{n \times n} \times (x_{i,j})_{n \times n};$$

-*AddRoundKey* (ARK): XORing the state and a n^2-word subkey, i.e.

$$ARK(X) = X \oplus K_i,$$

where K_i is the round key.

Like all other works on Rijndael cipher, we assume the last MC operation is omitted.

2.2 Brief Description of 3D-like Structure

The 3D-like structure also has an SPN structure, message block is represented as a 3-dimensional cube($n \times n \times n$ state of words, see **Fig. 1**), and in this paper, we represented the cube as a matrix

$$X=\begin{pmatrix} x_{0,0,0} & \cdots & x_{0,0,n-1} & \cdots & x_{i,0,0} & \cdots & x_{i,0,n-1} & \cdots & x_{n-1,0,0} & \cdots & x_{n-1,0,n-1} \\ \vdots & \ddots & \vdots & & \vdots & \ddots & \vdots & & \vdots & \ddots & \vdots \\ x_{0,n-1,0} & \cdots & x_{0,n-1,n-1} & \cdots & x_{i,n-1,0} & \cdots & x_{i,n-1,n-1} & \cdots & x_{n-1,n-1,0} & \cdots & x_{n-1,n-1,n-1} \end{pmatrix}$$

For any fixed $0 \le k \le n-1$, the matrix $(x_{k,i,j})_{n\times n}$ is said to be the k-th *vertical slice* of the cube(see **Fig. 1**).

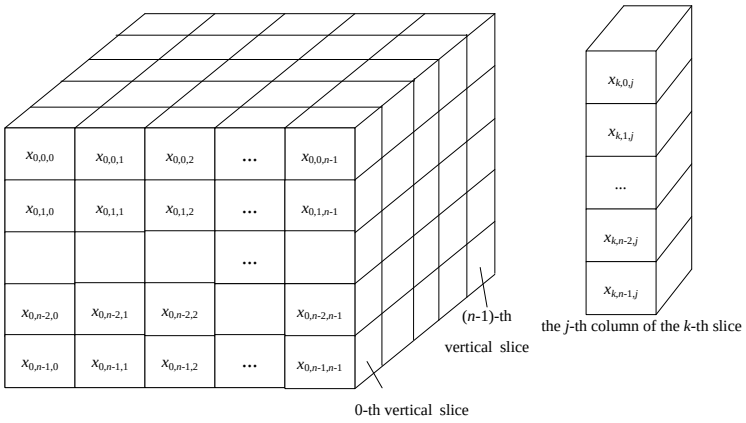


Fig. 1. State cube of 3D-like structure

The i-th round of 3D-like structure is composed of four operations:

$$\tau_i(X)=\pi\circ\theta_{i \bmod 2+1}\circ\gamma\circ\kappa_i(X),$$

where

- κ_i : XORing the state and a n^3-word i-th round subkey, i.e.

$$\kappa_i(X)=X\oplus K_i$$

where K_i is the i-th round subkey;

- γ : applying the bijective S-box s on each word, i.e.

$$\gamma(X)=(s(x_{0,0,0}),\cdots,s(x_{n-1,n-1,n-1}));$$

- θ_1,θ_2 : cyclically shifting operations, where θ_1 operates within each vertical slice and θ_2 operates between different vertical slices, i.e.

$\theta_1:\left(x_{k,i,j}\right)_{n\times n\times n}\to\left(y_{k,i,j}\right)_{n\times n\times n}$, $y_{k,i,j}=x_{k,i,(j+t_i)\bmod n}$, where $\{t_i:0\le i\le n-1\}=Z_n$;

and

$\theta_2:\left(x_{k,i,j}\right)_{n\times n\times n}\to\left(z_{k,i,j}\right)_{n\times n\times n}$, $z_{k,i,j}=x_{(k+c_i)\bmod n,i,j}$, where $\{c_i:0\le i\le n-1\}=Z_n$;

For briefness, we call θ_1 *SWS*(*Shift within Slice*) and θ_2 *SBS*(*Shift between Slices*) for short ;

- π : multiplication of each column of the state cube by a constant $n\times n$ matrix, i.e.

$$\pi(X)=M_{n\times n}\times X\ .$$

Likewise, we omit the last π operation.

3. Retrieving Impossible Differentials for Rijndael-like Cipher

In this section, we will provide some 4-round impossible differentials for Rijndael-like

structure by using the inconsistency of the **MixColumn layer** in the 2nd round. The transformation we considered is

$$T(X) = (ARK_4 \circ SR_4 \circ SB_4) \circ (ARK_3 \circ MC_3 \circ SR_3 \circ SB_3)$$
$$\circ (ARK_2 \circ MC_2 \circ SR_2 \circ SB_2) \circ (ARK_1 \circ MC_1 \circ SR_1 \circ SB_1)(X)$$

Definition 4. (*collection set of SR*). Let $SR: \left(x_{i,j}\right)_{n\times n} \to \left(r_{i,j}\right)_{n\times n}$ with $r_{i,j} = x_{i,(j+t_i) \bmod n}$, then the j-th($0 \le j \le n-1$) collection set of SR is defined by

$$\Omega_j = \left\{\left(i, (j+t_i) \bmod n\right) : 0 \le i \le n-1\right\}.$$

For the input state matrix $X_{n\times n} = \left(x_{i,j}\right)$, if $(i, j) \in \Omega_l$, then the SR operation will move $x_{i,j}$ to the l-th column.

Example 1. For AES-128, the *SR* layer is defined as

$$\begin{pmatrix} x_{0,0} & x_{0,1} & x_{0,2} & x_{0,3} \\ x_{1,0} & x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,0} & x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,0} & x_{3,1} & x_{3,2} & x_{3,3} \end{pmatrix} \xrightarrow{SR} \begin{pmatrix} x_{0,0} & x_{0,1} & x_{0,2} & x_{0,3} \\ x_{1,1} & x_{1,2} & x_{1,3} & x_{1,0} \\ x_{2,2} & x_{2,3} & x_{2,0} & x_{2,1} \\ x_{3,3} & x_{3,0} & x_{3,1} & x_{3,2} \end{pmatrix},$$

where $t_0 = 0, t_1 = 1, t_2 = 2, t_3 = 3$, thus

$$\Omega_0 = \left\{(0,0),(1,1),(2,2),(3,3)\right\}, \Omega_1 = \left\{(0,1),(1,2),(2,3),(3,0)\right\},$$
$$\Omega_2 = \left\{(0,2),(1,3),(2,0),(3,1)\right\}, \Omega_3 = \left\{(0,3),(1,0),(2,1),(3,2)\right\}.$$

The properties below are trivial.

Property 1. For a given input difference state matrix:

1. $ARK, SB, ARK^{-1}, SB^{-1}$ change neither the number nor the coordinates of differential active S-boxes;

2. SR, SR^{-1} do not change the number of differential active S-boxes;

3. MC, MC^{-1} only influence current column.

Theorem 1. Let $M_{n\times n}$ be the matrix representation of the MixColumn transformation with branch number $d+1$. Let the collection sets of SR and SR^{-1} be $\Omega_0, \cdots, \Omega_{n-1}$ and $\Phi_0, \cdots, \Phi_{n-1}$, respectively. If $\Delta x_{p_1,i_1}, \cdots, \Delta x_{p_s,i_s}, \Delta y_{q_1,j_1}, \cdots, \Delta y_{q_r,j_r}$ are nonzero, and for some $z_1 + z_2 \le d$ hold

$$\{(p_1,i_1), \cdots, (p_s,i_s)\} \subseteq \bigcup_{u=1}^{z_1} \Omega_{k_u}, \{(q_1,j_1), \cdots, (q_r,j_r)\} \subseteq \bigcup_{v=1}^{z_2} \Phi_{h_v},$$

then

$$(0, \cdots, 0, \Delta x_{p_1,i_1}, 0, \cdots, 0, \Delta x_{p_s,i_s}, 0, \cdots, 0) \to (0, \cdots, 0, \Delta y_{q_1,j_1}, 0, \cdots, 0, \Delta y_{q_r,j_r}, 0, \cdots, 0)$$

is a 4-round impossible differential of Rijndael-like cipher.

Proof. Assume the input difference is $\Delta X = (0, \cdots, 0, \Delta x_{p_1,i_1}, 0, \cdots, 0, \Delta x_{p_s,i_s}, 0, \cdots, 0)$, we

will bound the number of active S-boxes in

$$(SR_2 \circ SB_2) \circ (ARK_1 \circ MC_1 \circ SR_1 \circ SB_1)(\Delta X).$$

Since $\{(p_1, i_1), \cdots, (p_s, i_s)\} \subseteq \bigcup_{u=1}^{z_1} \Omega_{k_u}$, then the active S-boxes in $(MC_1 \circ SR_1 \circ SB_1)(\Delta X)$ only appear in the $k_1, \cdots, k_{z_1} - th$ columns. Thus there are at most nz_1 active S-boxes in

$$(SR_2 \circ SB_2) \circ (ARK_1 \circ MC_1 \circ SR_1 \circ SB_1)(\Delta X).$$

From the decrypt direction, assume the output difference is $\Delta Y = (0, \cdots, 0, \Delta y_{q_1, j_1}, 0, \cdots, 0, \Delta y_{q_r, j_r}, 0, \cdots, 0)$, and we will discuss the number of active S-boxes in

$$ARK_2^{-1} \circ (SB_3^{-1} \circ SR_3^{-1} \circ MC_3^{-1} \circ ARK_3^{-1}) \circ (SB_4^{-1} \circ SR_4^{-1} \circ ARK_4^{-1})(\Delta Y).$$

Since $\{(q_1, j_1), \cdots, (q_r, j_r)\} \subseteq \bigcup_{v=1}^{z_2} \Phi_{h_v}$, then the active S-boxes of

$$(MC_3^{-1} \circ ARK_3^{-1}) \circ (SB_4^{-1} \circ SR_4^{-1} \circ ARK_4^{-1})(\Delta Y)$$

only appear in the $h_1, \cdots, h_{z_2}$-th columns. Hence there are at most nz_2 active S-boxes in

$$ARK_2^{-1} \circ (SB_3^{-1} \circ SR_3^{-1} \circ MC_3^{-1} \circ ARK_3^{-1}) \circ (SB_4^{-1} \circ SR_4^{-1} \circ ARK_4^{-1})(\Delta Y).$$

Taking MC_2 into consideration: in the input state matrix of MC_2, we affirm that there is at least one column α satisfies

$$w(\alpha) + w(MC_2(\alpha)) \le d$$

(otherwise the total number of active S-boxes in the state matrices before and after MC_2 will be at least $nd \ge nz_1 + nz_2$, this leads contradiction). On the other hand, we notice that $Br(MC_2) = d + 1$, this indicates $w(\alpha) + w(MC_2(\alpha)) \ge d + 1$. Thus $\Delta X \to \Delta Y$ is a 4-round impossible differential of Rijndael-like cipher.

□

In Rijndael cipher (see Appendix A), the branch number of the Mixcolumn transformation reaches 5 [7]. By applying Theorem 1, we find various impossible differentials. For briefness, we denote the input word matrix of Rijndael by $X = \begin{bmatrix} x_0 & x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_9 & x_{13} \\ x_2 & x_6 & x_{10} & x_{14} \\ x_3 & x_7 & x_{11} & x_{15} \end{bmatrix}$ and the output word matrix of 4-round Rijndael encryption by $Y = \begin{bmatrix} y_0 & y_4 & y_8 & y_{12} \\ y_1 & y_5 & y_9 & y_{13} \\ y_2 & y_6 & y_{10} & y_{14} \\ y_3 & y_7 & y_{11} & y_{15} \end{bmatrix}$.

Corollary 1. Let $\Delta X = (x_0, \cdots, x_{15}), \Delta Y = (y_0, \cdots, y_{15})$ be input difference and output difference of 4-round Rijndael, respectively. If $\{i : \Delta x_i \ne 0\} \subseteq I, I \in Input_k$ and $\{j : \Delta y_j \ne 0\} \subseteq O, O \in Output_k$ hold for some $1 \le k \le 3$, then $\Delta X \to \Delta Y$ is a 4-round

impossible differential of Rijndael. Where the index set $Input_k$ and $Output_k$ are listed in **Table 1.**

Table 1. Index set $Input_k$ and $Output_k$

k	$Input_k$	$Output_k$
1	{0,5,10,15}, {3,4,9,14}, {2,7,8,13}, {1,6,11,12}.	{0,7,10,13}, {1,4,11,14}, {2,5,8,15}, {3,6,9,12}.
2	{0,3,4,5,9,10,14,15}, {0,2,5,7,8,10,13,15}, {0,1,5,6,10,11,12,15}, {2,3,4,7,8,9,13,14}, {1,3,4,6,9,11,12,14}, {1,2,6,7,8,11,12,13}.	{0,1,4,7,10,11,13,14}, {0,2,5,7,8,10,13,15}, {0,3,6,7,9,10,12,13}, {1,2,4,5,8,11,14,15}, {1,3,4,6,9,11,12,14}, {2,3,5,6,8,9,12,15}.
3	{1,2,3,4,6,7,8,9,11,12,13,14}, {0,1,2,5,6,7,8,10,11,12,13,15}, {0,1,3,4,5,6,9,10,11,12,14,15}, {0,2,3,4,5,7,8,9,10,13,14,15}.	{0,1,2,4,5,7,8,10,11,13,14,15}, {0,1,3,4,6,7,9,10,11,12,13,14}, {0,2,3,5,6,7,8,9,10,12,13,15}, {1,2,3,4,5,6,8,9,11,12,14,15}.

Example 2. We choose input difference whose 0,3,5,9,10,14,15-th words are nonzero, and output difference whose 0,1,4,7,10,11,13,14-th words are nonzero, thus we can construct impossible differential

$$(\Delta_0,0,0,\Delta_3,0,\Delta_5,0,0,0,\Delta_9,\Delta_{10},0,0,0,\Delta_{14},\Delta_{15})$$
$$\not\to(\delta_0,\delta_1,0,0,\delta_4,0,0,\delta_7,0,0,\delta_{10},\delta_{11},0,\delta_{13},\delta_{14},0)$$

via Corollary 1. We depict such impossible differential of Rijndael in **Fig. 2**, where we ignore the KeyAddition operation since it does not affect the differential state.

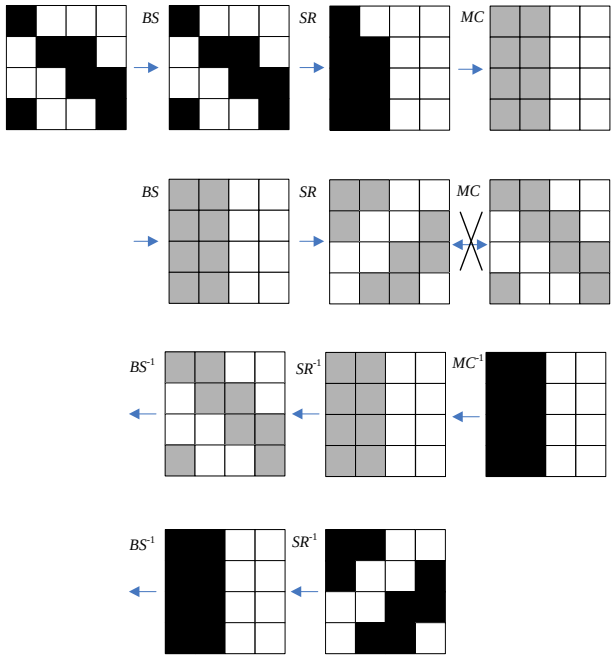


Fig. 2. A 4-round impossible differential of Rijndael
■denotes a nonzero difference, □ denotes a zero difference, ▩denotes a difference affected

By observation, the impossible differentials proposed by [9] and [10](see **Fig.3**) are special cases of ours.

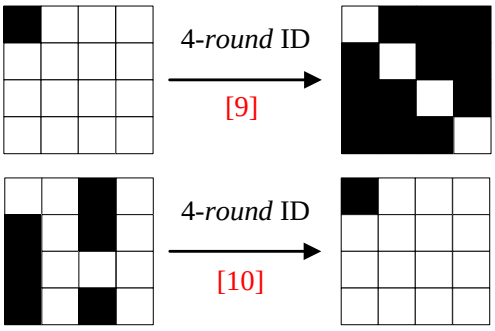


Fig. 3. Typical 4-round impossible differential of Rijndael in [9] and [10]

4. Retrieving Impossible Differentials for 3D-like Cipher

In this section, we will provide some 6-round impossible differentials for 3D-like structure by using the inconsistency of the π **layer** in the 3rd round function. The cipher which we study excludes the last π operation, i.e.

$$T(X)=(\theta_1\circ\gamma_6\circ\kappa_6)\circ\overset{5}{\underset{i=1}{\circ}}(\pi_i\circ\theta_{i\bmod 2+1}\circ\gamma_i\circ\kappa_i)(X)$$

Definition 5(*collection set of SWS*). Let $\theta_1:(x_{k,i,j})_{n\times n\times n}\to(y_{k,i,j})_{n\times n\times n}$ with $y_{k,i,j}=x_{k,i,(j+t_i)\bmod n}$ be the *SWS* of 3D-like cipher, then the (k,j)-collection set of θ_1 is defined as

$$\Omega^1_{(k,j)}=\{(k,i,(j+t_i)\bmod n):0\le i\le n-1\}.$$

Definition 6(*collection set of SBS*). Let $\theta_2:(x_{k,i,j})_{n\times n\times n}\to(z_{k,i,j})_{n\times n\times n}$ with $z_{k,i,j}=x_{(k+c_i)\bmod n,i,j}$ be the *SBS* of 3D-like cipher，then the (k,j)-collection set of θ_2 is defined as

$$\Omega^2_{(k,j)}=\{((k+c_i)\bmod n,i,j):0\le i\le n-1\}.$$

For the cube $X=\left(x_{k,i,j}\right)_{n\times n\times n}$, if the subscripts of x_{k_1,i_1,j_1} and x_{k_2,i_2,j_2} satisfies $(k_1,i_1,j_1),(k_2,i_2,j_2)\in\Omega^{\bullet}_{(k,j)}$, then transformation $\theta_{\bullet},\square\in\{1,2\}$ will move x_{k_1,i_1,j_1} and x_{k_2,i_2,j_2} to the j-th column of the k-th slice.

Similar to Rijndael-like cipher, we have the following properties for 3D-like ciphers.

Property 2. For a given input difference state cube:

1. $\kappa,\gamma,\kappa^{-1},\gamma^{-1}$ change neither the number nor the coordinates of differential active S-boxes;
2. $\theta_1,\theta_2,\theta_1^{-1},\theta_2^{-1}$ do not change the number of active S-boxes;
3. π,π^{-1} only influence the current column;
4. θ_1,θ_1^{-1} only influence the current slice.

Theorem 2. Let $M_{n\times n}$ be the matrix representation of the π layer with branch number

$d+1$. Let collection sets of θ_1 and θ_1^{-1} be $\Omega^1_{(0,0)},\Omega^1_{(0,1)}\cdots,\Omega^1_{(n-1,n-1)}$ and $\Phi^1_{(0,0)},\Phi^1_{(0,1)},\cdots,\Phi^1_{(n-1,n-1)}$ respectively, collection sets of θ_2 and θ_2^{-1} are $\Omega^2_{(0,0)},\Omega^2_{(0,1)},\cdots,\Omega^2_{(n-1,n-1)}$ and $\Phi^2_{(0,0)},\Phi^2_{(0,1)},\cdots,\Phi^2_{(n-1,n-1)}$, respectively. If $\Delta x_{o_1,p_1,i_1},\cdots,\Delta x_{o_s,p_s,i_s}$, $\Delta y_{g_1,q_1,j_1},\cdots,\Delta y_{g_r,q_r,j_r}$ are nonzero value, then for any $z_1+z_2\le d$, such that

(1) $\{(o_1,p_1,i_1),\cdots,(o_s,p_s,i_s)\}\subseteq\bigcup_{u=1}^{z_1}\left(\bigcup_{i=1}^{C_u}\Omega^2_{(k_u,T(u)_i)}\right)$,

(2) $\{(k_u,w,T(u)_i):1\le u\le z_1,0\le w\le n-1,1\le i\le C_u\}\subseteq\bigcup_{u=1}^{z_1}\left(\bigcup_{i=1}^{c_u}\Omega^1_{(k_u,t(u)_i)}\right)$,

(3) $\{(g_1,q_1,j_1),\cdots,(g_r,q_r,j_r)\}\subseteq\bigcup_{v=1}^{z_2}\left(\bigcup_{j=1}^{C_v}\Phi^1_{(h_v,B(v)_j)}\right)$,

(4) $\{(h_v,w,B(v)_j:1\le v\le z_2,0\le w\le n-1,1\le j\le S_v\}\subseteq\bigcup_{v=1}^{z_2}\left(\bigcup_{j=1}^{c_v}\Phi^2_{(H_v,b(v)_j)}\right)$.

hold synchronously, then

$$(0,\cdots,0,\Delta x_{i_1},0,\cdots,0,\Delta x_{i_s},0,\cdots,0)\to(0,\cdots,0,\Delta y_{j_1},0,\cdots,0,\Delta y_{j_r},0,\cdots,0)$$

is a 6-round impossible differential of 3D-like cipher.

Proof. Let $\Delta X=(0,\cdots,0,\Delta x_{o_1,p_1,i_1},0,\cdots,0,\Delta x_{o_s,p_s,i_s},0,\cdots,0)$ be the input difference of 3D-like cipher. Since

$$\{(o_1,p_1,i_1),\cdots,(o_s,p_s,i_s)\}\subseteq\bigcup_{u=1}^{z_1}\left(\bigcup_{i=1}^{C_u}\Omega^2_{(k_u,T(u)_i)}\right),$$

according to Property 2, the active S-boxes of $\gamma_2\circ\kappa_2\circ(\pi_1\circ\theta_2\circ\gamma_1\circ\kappa_1)(\Delta X)$ only appear in the $T(u)_1,\cdots,T(u)_{C_u}$ -th columns of the k_u -th slice, where $1\le u\le z_1$.

Further, since

$$\{(k_u,w,T(u)_i):1\le u\le z_1,0\le w\le n-1,1\le i\le C_u\}\subseteq\bigcup_{u=1}^{z_1}\left(\bigcup_{i=1}^{c_u}\Omega^1_{(k_u,t(u)_i)}\right),$$

then the active S-boxes in $\gamma_3\circ\kappa_3\circ(\pi_2\circ\theta_1\circ\gamma_2\circ\kappa_2)\circ(\pi_1\circ\theta_2\circ\gamma_1\circ\kappa_1)(\Delta X)$ only located in the $t(u)_1,\cdots,t(u)_{c_u}$ -th column of k_u -th slice, where $u=1,\cdots,z_1$. So in the state cube, at most $\sum_{u=1}^{z_1}c_u$ columns have active S-boxes, i.e., there are at most $n\sum_{u=1}^{z_1}c_u\left(\le n^2z_1\right)$ active S-boxes in

$$(\gamma_3\circ\kappa_3)\circ(\pi_2\circ\theta_1\circ\gamma_2\circ\kappa_2)\circ(\pi_1\circ\theta_2\circ\gamma_1\circ\kappa_1)(\Delta X).$$

Note θ_2 does not change the number of active S-boxes, we claim that state cube of $(\theta_2\circ\gamma_3\circ\kappa_3)\circ(\pi_2\circ\theta_1\circ\gamma_2\circ\kappa_2)\circ(\pi_1\circ\theta_2\circ\gamma_1\circ\kappa_1)(\Delta X)$, i.e. the cube before the π_3 layer, has at most n^2z_1 differential active S-boxes.

Let the output difference be $\Delta Y=(0,\cdots,0,\Delta y_{g_1,q_1,j_1},0,\cdots,0,\Delta y_{g_r,q_r,j_r},0,\cdots,0)$, we firstly focus on

$$\pi_3^{-1}\circ(\kappa_4^{-1}\circ\gamma_4^{-1}\circ\theta_1^{-1}\circ\pi_4^{-1})\circ(\kappa_5^{-1}\circ\gamma_5^{-1}\circ\theta_2^{-1}\circ\pi_5^{-1})\circ(\kappa_6^{-1}\circ\gamma_6^{-1}\circ\theta_1^{-1})(\Delta Y).$$

Since $\{(g_1,q_1,j_1),\cdots,(g_r,q_r,j_r)\}\subseteq\bigcup_{v=1}^{z_2}\left(\bigcup_{j=1}^{C_v}\Phi^1_{(h_v,B(v)_j)}\right)$, by Property 2 we know active S-boxes of $\pi_5^{-1}\circ(\kappa_6^{-1}\circ\gamma_6^{-1}\circ\theta_1^{-1})(\Delta Y)$ could only exist in the $B(v)_1,\cdots,B(v)_{C_v}$-th column of the h_v-th slice(for $v=1,\cdots,z_2$). Since

$$\{(h_v,w,B(v)_j:1\le v\le z_2,0\le w\le n-1,1\le j\le S_v\}\subseteq\bigcup_{v=1}^{z_2}\left(\bigcup_{j=1}^{c_v}\Phi^2_{(H_v,b(v)_j)}\right),$$

the active S-boxes of $(\kappa_5^{-1}\circ\gamma_5^{-1}\circ\theta_2^{-1}\circ\pi_5^{-1})\circ(\kappa_6^{-1}\circ\gamma_6^{-1}\circ\theta_1^{-1})(\Delta Y)$ could only appear in the $b(v)_1,\cdots,b(v)_{c_v}$-th columns of H_v-th slice (for $v=1,\cdots,z_2$). So there are at most $\sum_{v=1}^{z_2}c_v$ columns have active S-boxes. This indicates that there are at most $n\sum_{v=1}^{z_2}c_v\le n^2z_2$ active S-boxes after π_3 layer, i.e. in

$$(\kappa_4^{-1}\circ\gamma_4^{-1}\circ\theta_1^{-1}\circ\pi_4^{-1})\circ(\kappa_5^{-1}\circ\gamma_5^{-1}\circ\theta_2^{-1}\circ\pi_5^{-1})\circ(\kappa_6^{-1}\circ\gamma_6^{-1}\circ\theta_1^{-1})(\Delta Y).$$

In the input differential cube of π_3, we affirm there exists one column who has α active S-box, where α satisfies $w(\alpha)+w(\pi_3\alpha)\le d$ (otherwise the total number of active S-boxes in the differential cubes before and after π_3 will be at least $n^2d>n^2z_1+n^2z_2$, this leads a contradiction). Notice that the branch number of π_3 is $d+1$, this indicates $w(\alpha)+w(\pi_3\alpha)>d$. By this mean, $\Delta X\to\Delta Y$ is a 6-round impossible differential of 3D-like cipher.

□

A typical example of impossible differential in 3D block cipher is described in **Fig. 4.**

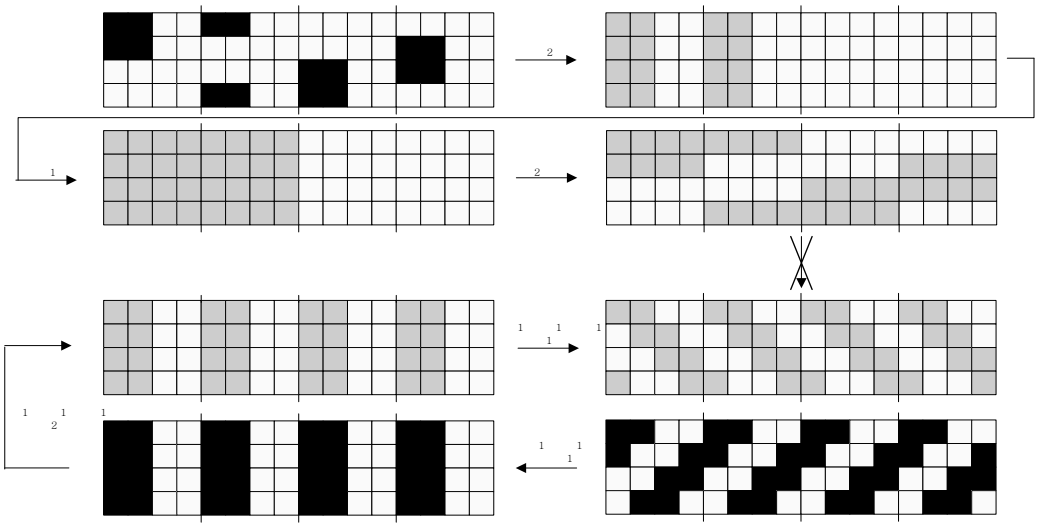


Fig. 4. A 6-round impossible differential of 3D.
■denotes a nonzero difference, □ denotes a zero difference, ▒denotes a difference unsure

By observation, the impossible differential characteristics proposed by [11,15](**see Fig.5**) are special cases of Theorem 2.

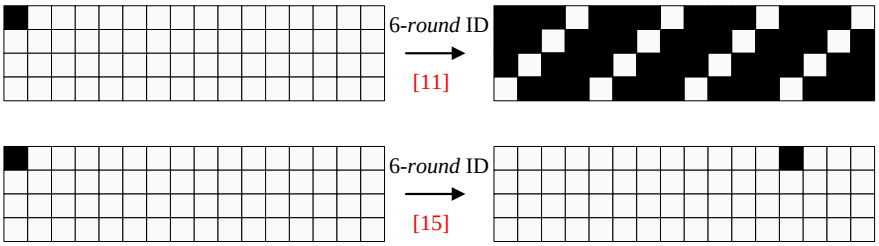


Fig. 5. Typical 6-round impossible differential of 3D in [11] and [15]

5 Conclusions

Since IDC is very powerful in analyzing the security of block ciphers, it is worthwhile for us to evaluate the resistance of block cipher against IDC. The existence of impossible differentials is an evaluation of block cipher against IDC. This paper proposed methods to find impossible differentials of AES and 3D structures and lots of new impossible differentials could be searched out. Our work can be used as a tool to evaluate the vulnerability of new block ciphers employ these two structures against IDC. Although we are not sure that whether these new impossible differentials can improve attacks on AES and 3D, we hope this will be helpful in future.

References

[1] E. Biham, A. Biryukov and A. Shamir, "Cryptanalysis of Skipjack reduced to 31 rounds using impossible differentials," *Journal of Cryptology*, vol.18, no.4, pp.291-311. September 2005. Article (CrossRef Link)

[2] W. Zhang, W. Wu, and D. Feng, "New results on impossible differential cryptanalysis of reduced AES," in *Proc. of ICISC'07*, pp. 239-250, November 29-30, 2007. Article (CrossRef Link)

[3] J. Lu, J. Kim, N. Keller, O. Dunkelman, "Improving the efficiency of impossible differential cryptanalysis of reduced Camellia and MISTY1," in *Proc. of CT-RSA 2008*, pp. 370–386, April 8-11, 2008. Article (CrossRef Link)

[4] J. Kim, S. Hong, J. Sung, S. Lee and J. Lim, "Impossible differential cryptanalysis for block cipher structures," in *Proc. of INDOCRYPT 2003*, pp. 82-96, December 8-10, 2003. Article (CrossRef Link)

[5] Yiyuan Luo, Zhongming Wu, Xuejia Lai, "A unified method for finding impossible differentials of block cipher structures," *Cryptology ePrint Archive*, Report 2009/627. Article (CrossRef Link)

[6] Ruilin Li, Bing Sun, Chao Li, "Impossible differential cryptanalysis of SPN ciphers," *IET Information Security*, vol.5, issue.2, pp.111-120, June, 2011. Article (CrossRef Link)

[7] J. Daemen and V. Rijmen. *The design of Rijndael: AES - the advanced encryption standard*. Springer-verlag, 2002. Article (CrossRef Link)

[8] Nakahara J Jr. "3D: A three-dimensional block cipher" in *Proc. of CANS 2008*, pp. 252-267, December 2-4, 2008. Article (CrossRef Link)

[9] E.Biham,N Keller. "Cryptanalysis of reduced variants of Rijndael," in *Proc. of 3rd AES Conference*, April 13-14, 2000. Article (CrossRef Link)

[10] Hamid Mala, Mohammad Dakhilalian,Vincent Rijmen, et al, "Improved impossible differential cryptanalysis of 7-round AES-128," in *Proc. of INDOCRYPT 2010* , pp. 282–291, December 12-15, 2010. Article (CrossRef Link)

[11] Tang Xue-hai ,Li Chao, Wang Mei-yi, "Impossible differential attack on 3D cipher," *Journal of Electronics & Information Technology*. vol.32, no.10, pp. 2516-2520. October, 2010. (in Chinese)

Article (CrossRef Link)
[12] Kang Ju-sung, Hong Seokhie, Lee Sangjin, et al, "Practical and provable security against differential and linear cryptanalysis for substitution-permutation networks," *ETRI Journal*, vol. 23, no. 4, pp.158-167. April, 2001. Article (CrossRef Link)
[13] Ruilin Li, Bing Sun, Peng Zhang and Chao Li, "New impossible differential cryptanalysis of ARIA," *Cryptology ePrint Archive*, Report 2008/227. Article (CrossRef Link)
[14] L. Knudsen, "DEAL-a 128-bit block cipher," *Technical Report 151*, Department of Informatics, University of Bergen, Bergen, Norway, Feb. 1998. Article (CrossRef Link)
[15] Jorge Nakahara Jr, "New impossible differential and known-key distinguishers for the 3D cipher," in *Proc. of ISPEC 2011*, pp. 208–221, May 30 - June 1, 2011. Article (CrossRef Link)
[16] Takuma Koyama, LeiWang, Yu Sasaki, Kazuo Sakiyama, and Kazuo Ohta, "New truncated differential cryptanalysis on 3D block cipher," in *Proc. of ISPEC 2012*, pp. 109–125, April 9-12, 2012. Article (CrossRef Link)
[17] Jie Chen, Yupu Hu, Yueyu Zhang, "Impossible differential cryptanalysis of advanced encryption standard" *Sci China Ser F-Inf Sci*, vol. 50, no.3, pp. 342-350. June, 2007. Article (CrossRef Link)
[18] Andrey Bogdanov, Dmitry Khovratovich, and Christian Rechberger, "Biclique cryptanalysis of the full AES," in *Proc. of ASIACRYPT 2011*, pp. 344–371, December 4-8, 2011. Article (CrossRef Link)

Appendix

A.1 Brief Description of Rijndael Cipher

Rijndael:

Rijndael is an SPN cipher. The length of the block and the length of the key can be specified to be 128, 192 or 256 bits, independently of each other. In this paper we discuss the variant with 128-bit blocks and 128-bit keys. In this variant, the cipher consists of 10 rounds. We represent 128-bit data

$$X = (x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15})$$

in 4×4 matrix as

$$\begin{bmatrix} x_0 & x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_9 & x_{13} \\ x_2 & x_6 & x_{10} & x_{14} \\ x_3 & x_7 & x_{11} & x_{15} \end{bmatrix}.$$

Each round except for the last consists of 4 transformation:
- ***ByteSubstitution*** is applied to each byte separately and is a nonlinear byte-wise substitution to use the S-box.
- ***ShiftRow*** is a cyclic shift of the bytes of each row by 0, 1, 2, or 3, respectively.
- ***MixColumn*** is a linear transformation applied to columns of the matrix. The branch number of this layer is 5.
- ***AddRoundKey*** is a key XOR. Before the first round AddRoundKey is performed using the key as the round key.In the last round the MixColumn is omitted.

A.2 Brief Description of 3D Cipher

3D:

The 3D block cipher operates on 512-bit blocks and uses 512-bit keys, both of which are represented as a $4\times4\times4$ state of bytes (a 3-dimensional cube). The state for a 64-byte data

block $A=(a_0,\cdots,a_{63})$ is denoted

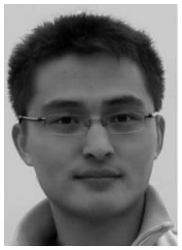
$$A=\left(\begin{array}{cccc|cccc|cccc|cccc} a_0 & a_4 & a_8 & a_{12} & a_{16} & a_{20} & a_{24} & a_{28} & a_{32} & a_{36} & a_{40} & a_{44} & a_{48} & a_{52} & a_{56} & a_{60} \\ a_1 & a_5 & a_9 & a_{13} & a_{17} & a_{21} & a_{25} & a_{29} & a_{33} & a_{37} & a_{41} & a_{45} & a_{49} & a_{53} & a_{57} & a_{61} \\ a_2 & a_6 & a_{10} & a_{14} & a_{18} & a_{22} & a_{26} & a_{30} & a_{34} & a_{38} & a_{42} & a_{46} & a_{50} & a_{54} & a_{58} & a_{62} \\ a_3 & a_7 & a_{11} & a_{15} & a_{19} & a_{23} & a_{27} & a_{31} & a_{35} & a_{39} & a_{43} & a_{47} & a_{51} & a_{55} & a_{59} & a_{63} \end{array}\right)$$

with bytes inserted columnwise. Each square set of 16 bytes is called a slice of the state. Then the i-th round of 3D is calculated by $\tau_i(X)=\pi\circ\theta_{(i \bmod 2)+1}\circ\gamma\circ k_i(X)$, where

- k_i: subkey XOR to the i-th round state;
- γ: nonlinear byte-wise substitution to use the S-box;
- π: linear transformation applied to columns of A with branch number 5;
- θ_1,θ_2: cyclic shifts of the bytes:

$$\theta_1(A)=\left(\begin{array}{cccc|cccc|cccc|cccc} a_0 & a_4 & a_8 & a_{12} & a_{16} & a_{20} & a_{24} & a_{28} & a_{32} & a_{36} & a_{40} & a_{44} & a_{48} & a_{52} & a_{56} & a_{60} \\ a_5 & a_9 & a_{13} & a_1 & a_{21} & a_{25} & a_{29} & a_{17} & a_{37} & a_{41} & a_{45} & a_{33} & a_{53} & a_{57} & a_{61} & a_{49} \\ a_{10} & a_{14} & a_2 & a_6 & a_{26} & a_{30} & a_{18} & a_{22} & a_{42} & a_{46} & a_{34} & a_{38} & a_{58} & a_{62} & a_{50} & a_{54} \\ a_{15} & a_3 & a_7 & a_{11} & a_{31} & a_{19} & a_{23} & a_{27} & a_{47} & a_{35} & a_{39} & a_{43} & a_{63} & a_{51} & a_{55} & a_{59} \end{array}\right)$$

$$\theta_2(A)=\left(\begin{array}{cccc|cccc|cccc|cccc} a_0 & a_4 & a_8 & a_{12} & a_{16} & a_{20} & a_{24} & a_{28} & a_{32} & a_{36} & a_{40} & a_{44} & a_{48} & a_{52} & a_{56} & a_{60} \\ a_{17} & a_{21} & a_{25} & a_{29} & a_{33} & a_{37} & a_{41} & a_{45} & a_{49} & a_{53} & a_{57} & a_{61} & a_1 & a_5 & a_9 & a_{13} \\ a_{34} & a_{38} & a_{42} & a_{46} & a_{50} & a_{54} & a_{58} & a_{62} & a_2 & a_6 & a_{10} & a_{14} & a_{18} & a_{22} & a_{26} & a_{30} \\ a_{51} & a_{55} & a_{59} & a_{63} & a_3 & a_7 & a_{11} & a_{15} & a_{19} & a_{23} & a_{27} & a_{31} & a_{35} & a_{39} & a_{43} & a_{47} \end{array}\right)$$



Ting Cui was born in 1985. He is currently pursuing for the Ph.D degree in Applied Mathematics at the Information Science Technology Institute. His current research interests include block cipher design and cryptanalysis.

Chen-hui Jin was born in 1965. He is currently professor at the Information Science T echnology Institute. His current research interests are cryptography and information s ecurity.