Empirical Correlations for Breakup Length of Liquid Jet in Uniform Cross Flow-A Review

Soo-Young No

Key Words: Liquid jet, Uniform cross flow, Empirical correlation, Column fracture distance, Column fracture height

Abstract

The empirical correlations for the prediction of breakup length of liquid jet in uniform cross flow are reviewed and classified in this study. The breakup length of liquid jets in cross flow was normally discussed in terms of the distances from the nozzle exit to the column breakup location in the *x* and *y* directions, called as column fracture distance and column fracture height, respectively. The empirical correlations for the prediction of column fracture distance can be classified as constant form, momentum flux ratio form, Weber number form and other parameter form, respectively. In addition, the empirical correlations for the prediction of column fracture height can be grouped as momentum flux ratio form, Weber number form and other parameter form, respectively. It can be summarized that the breakup length of liquid jet in a cross flow is a basically function of the liquid to air momentum flux ratio. However, Weber number, liquid-to-air viscosity ratio and density ratio, Reynolds number or Ohnesorge number were incorporated in the empirical correlations depending on the investigators. It is clear that there exist the remarkable discrepancies of predicted values by the existing correlations even though many correlations have the same functional form. The possible reasons for discrepancies can be summarized as the different experimental conditions including jet operating condition and nozzle geometry, measurement and image processing techniques introduced in the experiment, difficulties in defining the breakup location etc. The evaluation of the existing empirical correlations for the prediction of breakup length of liquid jet in a uniform cross flow is required.

1. Introduction

The injection of liquid into a high-speed cross flow in combustion systems is found in numerous practical applications such as diesel engine(1) with plain-orifice atomizer, gasoline engine(2) with pressure-swirl atomizer, gas turbine with plain-orifice atomizer or airblast atomizer(3), ramjet, scramjets with plain-orifice atomizer(4). The cross flow situation in agricultural field will be the application of chemicals to crops by flat-fan nozzles mounted on boom sprayers(5,6). Injection of liquid friction modifiers on to the rail surface with air-blast atomizers mounted to the external undercarriage of trains is another application of cross flow(7).

Numerous studies have been conducted to characterize the liquid jet atomization process in an air cross flow. A comprehensive review of the behavior of liquid jets in high-speed cross flow was undertaken in 1992 by Schetz(8). Recently, Karagozan had reviewed the researches related to liquid jet atomization processes in cross flow extensively because of their widespread application in engineering systems(4).

As one of atomization characteristics, breakup length of spray is of prime importance in diesel engines and air-breathing propulsion systems. It is known that the understanding of the trajectory and breakup of liquid jet in a cross flow is critical to improve the efficiency and performance of liquid-fueled ramjet and scram jet combustors(9), lean pre-vaporized and premixed

---

(2012년 12월 28일 접수 ~2013년 2월 1일 심사완료, 2013년 3월 13일 게재확정)

Dept. of Biosystems Engineering, Chungbuk National University, Cheongju, Korea
E-mail : sooyoung@chungbuk.ac.kr
TEL : (043)261-2583 FAX : (043)271-4413

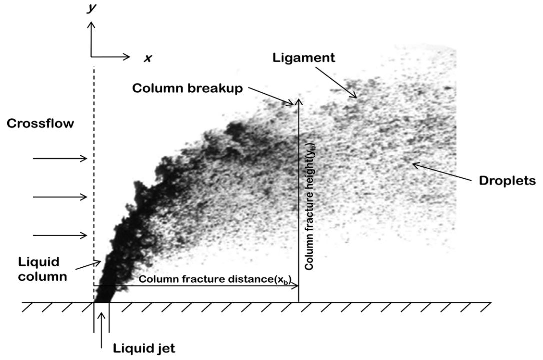

Fig. 1 Typical example of breakup length in cross flow

(LPP) gas turbine technology(10). There are several parameters such as column and surface breakup, penetration height, jet width, droplet size, droplet velocity for the liquid jet in cross flow as shown in Fig. 1. A lot of empirical correlations suggested by many researchers to predict the penetration height of liquid jet in uniform cross flow can be found in the recent review papers(11,12). In all of these empirical correlations, the jet-to-cross flow(liquid-to-air) momentum flux ratio is the only parameter that determines penetration height at a location downstream of the injector. The empirical correlations to predict the breakup length of liquid jet in cross flow has been the subject of considerable research. However, a detailed survey of correlations related to breakup length of liquid jet in uniform cross flow is not available in the literature.

The breakup length of liquid jet is closely related to breakup process or breakup mechanism. However, in this study, focus is concentrated only on the empirical correlation for the prediction of breakup length of liquid jet in cross flow. Discussion will be limited to empirical correlations available in the literature after 1990 for the prediction of breakup length of liquid jet in gaseous cross flow. In addition, the studies related to non-uniform cross flows such as swirling cross flow(13-15) and cross flow containing a shear layer(16) and controlled liquid jet such as exciting liquid jet(17) will not be included. In this article, therefore, empirical correlations for the breakup length of liquid jet in uniform cross flow will be reviewed and discussed.

2. Empirical Correlations for Breakup Length

In the studies of liquid jet in cross flow, breakup length(18-20) is also referred to as jet streamwise and transverse penetration(21), column fracture location(22), column breakup point(10), jet breakdown point (position)(23,24), transverse and streamwise penetration before break-up(25), transverse and x-penetration break-up length(21) etc.

The breakup length of liquid jets in cross flow was normally discussed in terms of the distances from the nozzle exit to the column breakup location in the x and y directions, called as streamwise breakup length and transverse breakup length, or column fracture distance and column fracture height, respectively(22). In this study, the terms “column fracture distance (x_b)” and “column fracture height (y_b)” will be selected as shown in Fig. 1.

3. Column Fracture Distance

Around twelve empirical correlations for the prediction of column fracture distance can be found in the literature. These correlations can be roughly grouped as constant form, momentum flux ratio form, Weber number form and other parameter form, respectively.

3.1 Constant form

Inamura *et al.*(18) had measured the break-up length of liquid jets in cross flow by contact needle probes. They found that the column fracture location in the y direction (y_b) moves downstream as momentum flux ratio q increases, but one in the x direction (x_b) is nearly constant regardless of q as

$$x_b/d = 3\sim3.5. \quad (1)$$

According to Wu *et al.*(22), the streamwise distance from the nozzle exit to the column fracture point normalized by jet nozzle diameter x_b/d was not sensitive to liquid to air momentum flux ratio and liquid properties as follows.

$$x_b/d=8.06 \tag{2}$$

Recently, the breakup location was defined by Tambe *et al.*(20) as the mean location of the formation of the first ligament. They found that the streamwise breakup length is independent of q and has the following constant with some spread in the data.

$$x_b/d=14.97 \tag{3}$$

The breakup length was defined as the distance from the center of the orifice exit to the point of the liquid column fracture. Ahn *et al.*(26) found that for noncavitating flows, the liquid jet has an approximate breakup length of

$$x_b/d=8.02\pm1.43 \tag{4}$$

It is found that this result is very close to Eq. (2), regardless of injection pressure differential or liquid to air momentum flux ratio. However, the liquid jet has a shorter breakup length as injection pressure difference increases in cavitation flows.

Recently, Wang *et al.*(27) suggested the following constant form for column fracture distance in the study of characterization of trajectory, break point and break point dynamics of a liquid jet in a crossflow.

$$x_b/d=6.9 \tag{5}$$

However, they did not report the evaluation of break location, even though the comparison of previous correlations for penetration and breakup time was included.

To summarize, five empirical correlations of constant form for liquid column fracture distance is not sensitive to q and liquid jet properties and can be expressed as

$$x_b/d=C_x \tag{6}$$

where C_x is an empirical constant associated with the cross stream penetration of the liquid column(28). However, it should be noted that C_x of Wu *et al.*(22) is more than double compared with that of Inamura *et al.*(18), and C_x of Tambe *et al.*(20) is slightly less than double compared with that of Wu *et al.*(22). Later, for this axial distance to the column fracture, investigation of Sallam *et al.*(28) yield the surprisingly same result as 8.0 with that of Wu *et al.*(22).

3.2 Momentum flux ratio form

Birouk *et al.*(21) found that at lower liquid viscosity, the column fracture distance tends to be constant and independent of q as pointed out in the above section. However, they concluded at higher liquid viscosities, μ_l=0.033 to 0.058 Pa·s, that the column fracture distance vary with jet and cross flow velocities, in turn, momentum flux ratio q. According to this finding, the same research group had suggested the following two different correlations according to the liquid viscosity in the subsequent works(25,29).

$$x_b/d=0.0037q+14.1 \quad \text{for } \mu_l<0.029\ \text{Pa·s} \tag{7}$$

$$x_b/d=542.64q^{0.87}\,\mathrm{Oh}^5 \quad \text{for } \mu_l>0.029\ \text{Pa·s} \tag{8}$$

It should be noted that they introduced the dimensionless number, i.e. Ohnesorge number, to consider the effect of liquid viscosity on column fracture distance.

An empirical correlation to predict the axial distance to column fracture point for the angled injection instead of normal injection of liquid was proposed by Costa *et al.*(19) after testing the existing correlation of Weber number form which will be discussed in the next section.

$$x_b/d=8.05q^{0.5}\cot\text{an}(\theta) \tag{9}$$

It should be pointed out that the axial distance to the column fracture distance x_b is not independent of q as reported by Wu *et al.*(22) for normal injection of the liquid.

3.3 Weber number form

In the study on breakup regimes of angled liquid injection into subsonic crossflow, Fuller *et al.*(30) defined a breakup regime parameter T_b, and also suggested empirical correlation to predict column fracture distance as follows.

$$x_b/d=9.3+2.6\,\frac{V_l\sin\theta}{U_a-V_l\cos\theta}\rho_r^{0.5} \quad T_b<1 \tag{10}$$

$$x_b/d = 9.3/T_b^2 + 1.7 We_l^{1/3} \cos\theta \quad T_b > 1 \tag{11}$$

where $T_b = \frac{3}{2}\frac{V_l}{U_a - V_l \cos\theta}\rho_r^{0.5} We_l^{-1/3}$, $T_b<1$ indicates a dominance of aerodynamic forces, whereas $T_b>1$ means a dominance of nonaerodynamic, i.e. liquid forces. It should be pointed out that for $T_b<1$ and $\theta=$ 90 deg, Eq. (10) yields a constant of $x_b/d=9.30$. This correlation is incident with the constant form with $C_x=9.30$ discussed in the previous section. These correlations were introduced to predict the experimental data by Costa *et al.*(19), but the results revealed that they do not satisfactorily estimate their data.

Bellofiore *et al.*(24) proposed the empirical correlation for the column fracture distance in terms of Weber number as follows.

$$x_b/d = 3.794\, We^{0.366} \tag{12}$$

It is of interest to note that column fracture distance depends only on the aerodynamic Weber number. In this study, the point where the liquid column is assumed to lose coherence is evaluated as jet breakdown points in the range of q=12~81, Re_g= 5000~26000 and We=10~350. It should be pointed out that data for nozzle orifice diameter of 0.5 mm was only introduced in this study.

In the subsequent publication by the same research group(10), the following correlation for the column breakup distance at room temperature, nozzle orifice of 0.3 and 0.5 mm and ambient pressure of 1 and 2 MPa was suggested.

$$x_b/d = 3.687 q^{-0.068}\, \mathrm{We}^{0.42} \tag{13}$$

It is of interest to note that liquid to air momentum flux ratio is included in the above expression although, of course, it was not considered the former correlations such as Ragucci *et al.*(10,31).

With taking into account of varying the air viscosity according to the increase of air temperature, the following correlation was proposed in their continued study(32). The experimental conditions such as nozzle orifice diameter of 0.3 and 0.5 mm, ambient temperature of 300 to 600 K, air pressure of 1 and 2 MPa were introduced in this study.

$$x_b/d = 4.17 q^{-0.095}\, \mathrm{We}^{0.382}(\mu/\mu_{a,300K})^{0.046} \tag{14}$$

where μ_a is the air viscosity at between 300 K and 600K.

The main difference between two studies by Ragucci *et al.*(10,32) in their experimental condition is the ambient temperature and air viscosity. It should be noted that the above two correlations are a significant improvement on the previous correlations proposed by Bellofiore *et al.*(24) and Ragucci *et al.*(31), which will be discussed in the next section. As pointed out by the authors, it is clear that the influence of air viscosity variation on the column fracture distance is nearly negligible in the above correlation.

It is clear that column fracture distance in the momentum flux ratio form and Weber number form depends on the operating condition, widely different from what has been reported by the earlier studies(18,22,27,30). It can be concluded that this difference is due to the introduction of experimental data at different air temperature and pressures.

3.4. Other parameter form

In the study of dynamics and coherence breakdown of kerosene and water jets in cross flow, the experimental results of Ragucci *et al.*(31) indicated that the column breakup distance depends on liquid-to-air velocity ratio and surface tension ratio as follows.

$$x_b/d = 25.6(\sigma_w/\sigma)^{0.5}(V_l/V_a)^{0.5} \tag{15}$$

However, it is important to note that this correlation was not introduced in their continued works.

To summarize, it should be noted that column fracture distance increases with the increase of q at higher liquid viscosities, even though at lower liquid viscosities, the column fracture distance tends to be constant as same as the references(18,22,30). However, Ohnesorge number, Weber number, air viscosity, density ratio, liquid-to-air velocity ratio, or surface tension ratio were incorporated in the empirical correlations depending on the researchers.

4. Column Fracture Height

Around ten empirical correlations for the prediction of column fracture height can be found in the literature. These correlations can be roughly grouped as momentum flux ratio form, Weber number form and other parameter form.

4.1 Momentum flux ratio form

Inamura *et al.*(18) found in their experiment that the column fracture height is dependent of liquid to air momentum flux ratio q. They, however, didn't suggest the empirical correlation for column fracture point in the y direction. Wu *et al.*(22) developed the correlations of cross fracture locations from the shadowgraph images analysis. For column fracture height, it should be noted that two correlations, i.e. one is theoretical and another empirical, were proposed by them as follows.

$$y_b/d = 3.44\,q^{0.5}\ \text{(theoretical correlation)} \quad (16)$$

$$y_b/d = 3.07\,q^{0.53}\ \text{(empirical correlation)} \quad (17)$$

For noncavitation flows, it was found by Ahn *et al.*(26) that Eq. (16) correlates very well the experimental results. However, breakup length shows a smaller value in cavitation flows, similar to the column fracture distance, i.e. x- directional breakup length. For hydraulic flip flows, the breakup length expressed by the effective jet diameter and effective liquid to air momentum flux ratio also reveals a slightly smaller than the value of Wu *et al.*(22).

Birouk *et al.*(21) found that the correlation proposed by Wu *et al.*(22) can be applicable only to liquids with low viscosity and suggested an empirical extension to their correlation as follows.

$$y_b/d = 3.13(We_j/We_{cf})^{0.53} \quad (18)$$

Due to the ratio of the jet to cross flow Weber number is equal to the momentum flux ratio q, it is clear that the above correlation is exactly coincident with the empirical correlation suggested by Wu *et al.*(22) with slightly different coefficient of proportionality.

In their subsequent studies(25,29), they found that at liquid viscosities higher than 0.019 Pa·s, the jet penetration before break-up appears to depend on the liquid viscosity in addition to the jet/cross-air flow momentum ratio. Accordingly they proposed the new empirical correlations capable of predicting the column fracture height as follows.

$$y_b/d = 3.13q^{0.53} \quad \text{for } \mu_l < 0.019\ \text{Pa·s} \quad (19)$$

$$y_b/d = 8.60q^{0.87}\,Oh^2 \quad \text{for } \mu_l > 0.019\ \text{Pa·s} \quad (20)$$

Eq. (19) for liquid viscosities up to 0.019 Pa·s shows a good agreement with the correlation suggested by Wu *et al.*(22). It should be pointed out that Wu *et al.*(22) used liquids with viscosity much lower than the one used in this study. This means that liquids with viscosities up to 0.019 Pa·s do not affect the column fracture height. However, for liquids viscosities higher than 0.019 Pa·s, the column fracture height depends on the liquid viscosity as well as liquid-to-air momentum flux ratio.

Tambe *et al.*(20) defined the breakup location as the mean location of the formation of the first ligament. In their study, the transverse breakup lengths (y_b) have been observed to increase with liquid to air momentum flux ratio q, which is due to the accompanying increase in penetration. However, they did not suggest the empirical correlation for the prediction of column fracture height.

Two empirical correlations to predict the breakup lengths for normal and angled liquid injection into subsonic crossflows, i.e. ones of Wu *et al.*(22), Eq. (16) and Fuller *et al.*(30), Eqs. (23) and (24) which belongs to Weber number form in the next section were tested by Costa *et al.*(19) in the study of spray characteristics of angled liquid injection into subsonic cross flows. However, because of unsatisfactory prediction of their data from these two correlations due to mainly the different jet operating condition, they suggested the following correlations.

$$y_b/d = 8.05q^{0.5} \quad (21)$$

Much higher values of cross-fracture locations (y_b/d) for a constant q than those provided by Wu *et al.*

(1997a) were obtained due to the low range of relative air Weber number associated to higher breakup lengths.

As recent one of the momentum flux ratio form, Wang *et al.*(28) suggested the following correlation for the prediction of column fracture height in the range of q=1~54 as follows.

$$y_b/d=2.5q^{0.53} \tag{22}$$

It is clear that in momentum flux ratio form for the liquid with lower viscosity, the column fracture height normalized by the nozzle diameter is a linear function of the square root of momentum flux ratio with the different coefficient of proportionality. However, for the liquid with higher viscosity, the variation of the normalized column fracture height shows a non-linear relationship affected by the liquid viscosity (Eq. 20).

4.2 Weber number form

In the study of effects of injection angle on the breakup processes of turbulent liquid jets in a subsonic cross-flow of air, the analytical/empirical correlations for column fracture height was suggested by Fuller *et al.*(30).

$$y_b/d=2.6\frac{V_l\sin\theta}{U_a-V_l\cos\theta}\rho_r^{0.5} \qquad T_b<1 \tag{23}$$

$$y_b/d=1.7\,We_1^{1/3}\sin\theta \qquad T_b>1 \tag{24}$$

where $T_b=\frac{3}{2}\frac{V_l}{U_a-V_l\cos\theta}\rho_r^{0.5}We_l^{-1/3}$

As pointed out in the previous section, these correlations did not gave the satisfactory prediction of data by Costa *et al.*(19).

In the study of empirical correlation for liquid jet trajectory in high density air cross flow, Ragucci *et al.*(10) proposed the empirical correlation of the Weber number form for column breakup height as follows.

$$y_b/d=4.355q^{0.416}\,We^{0.085} \tag{23}$$

In the subsequent publication by the same research group, the following modified version by taking into account the variation of air viscosity with temperature was suggested(32).

$$y_b/d=3.85q^{0.387}\,We^{0.126}(\mu/\mu_{a,300\mathrm{K}})^{0.202} \tag{24}$$

where μ_a is the air viscosity at between 300 K and 600 K. It is interesting to note that they took into account the effect of the increase of air viscosity instead of liquid viscosity.

4.3 Other parameter form

In the typical operating conditions of premixing duct of LPP gas turbines, Ragucci *et al.*(31) found that the values of column fracture height can be directly correlated to the air flow and initial jet velocities as follows.

$$y_b/d=45.5\,(V_l/V_a) \tag{25}$$

where V_l is the liquid jet velocity and V_a air flow velocity.

Based on liquid jet momentum coherence breakdown concept, empirical correlation for the jet breakdown point was proposed by Bellofiore *et al.*(24) as follows.

$$y_b/d=1.449q^{0.476}\,Re_g^{\,0.135} \tag{26}$$

In this study, the point where the liquid column is assumed to lose coherence is evaluated as jet breakdown points in the range of q=12~81, $\mathrm{Re_g}$= 5000~26000 and We=10~350. It should be noted that column fracture distance in this study depends on the operating condition, widely different from what has been reported by the earlier studies(18,22,27,30). They mentioned that this difference is due to the introduction of experimental data at different air temperature and pressures.

5. Summary

Many empirical correlations have been developed to predict the breakup length of liquid jet in a cross flow. In the most case of liquid jets in cross flow, the definitions of jet breakup are related to the column breakup location where the liquid column breaks into separate ligaments or droplets. As a different defini-

Table 1 Various empirical correlations for liquid column breakup

Investigators	Column fracture height	Column fracture distance
Inamura *et al.*[18]		$x_b/d = 3 \sim 3.5$
Wu *et al.*[22]	$y_b/d = 3.07\ q^{0.53}$	$x_b/d = 8.06$
Fuller *et al.*[30]	$y_b/d = 2.6\frac{v_l \sin\theta}{u_a - v_l \cos\theta}\rho_r^{0.5}\ T_b < 1$ $y_b/d = 1.7 We_l^{1/3} \sin\theta \quad T_b > 1$	$x_b/d = 9.3 + 2.6\frac{v_l \sin\theta}{u_a - v_l \cos\theta}\rho_r^{0.5} \quad T_b < 1$ $x_b/d = 9.3/T_b^2 + 1.7 We_l^{1/3} \cos\theta \quad T_b > 1$
Birouk *et al.*[25]	$y_b/d = 3.13q^{0.53}$	
Birouk *et al.*[29]	$y_b/d = 3.13q^{0.53} \quad \mu_l < 0.019\ P_a \cdot s$ $y_b/d = 8.60q^{0.87}Oh^2 \quad \mu_l > 0.019\ P_a \cdot s$	$y_b/d = 0.0037q + 14.1 \quad \mu_l < 0.029P_a \cdot s$ $y_b/d = 542.64q^{0.87}Oh^5 \quad \mu_l > 0.029P_a \cdot s$
Ragucci *et al.*[31]	$y_b/d = 45.5\frac{V_l}{V_a}$	$x_b/d = 25.6\left(\frac{V_l}{V_a}\right)^{0.5}\left(\frac{\sigma_\omega}{\sigma}\right)^{0.5}$
Tambe *et al.*[20]		$x_b/d = 14.97$
Ahn *et al.*[26]		$x_b/d = 8.02 \pm 1.43$
Costa *et al.*[19]	$y_b/d = 8.05q^{0.5}$	$x_b/d = y_{b/d} \cdot \text{cotan}(\theta)$
Bellofiore *et al.*[23]	$y_b/d = 3.85q^{0.387}We^{0.126}\left(\frac{\mu}{\mu_{air}}\right)^{0.202}$	$x_b/d = 4.17q^{-0.095}We^{0.382}\left(\frac{\mu}{\mu_{air}}\right)^{0.046}$
Ragucci *et al.*[10]	$y_b/d = 4.355q^{0.416}W^{0.085}$	$x_b/d = 3.687q^{-0.068}We^{0420}$
Bellofiore *et al.*[24]	$y_b/d = 1.449q^{0.476}Re_g^{0.135}$	$x_b/d = 3.794We^{0.366}$
Wang *et al.*[28]	$y_b/d = 2.5q^{0.53}$	$x_b/d = 6.90$

$$\rho = \sqrt{\rho_j/\rho_\infty} \quad T_b = \frac{3}{2}\frac{v_l}{u_a - v_l \cos\theta}\rho^{0.5}We_l^{1/3}$$

tion, the jet breakdown point was defined as the point where the liquid column is assumed to lose its momentum coherence. The breakup length can be divided into column fracture height and column fracture distance. Around ten and twelve different correlations have been developed to predict the column fracture height and column fracture distance, respectively. These are summarized in Table 1.

The existing correlations for column fracture height can be classified as three groups such as basic power-law form, Weber number form, and other parameter form. For column fracture distance, the existing correlations can be categorized as four groups such as the constant form, momentum flux ratio form, Weber number form, and other parameter form. It can be summarized that the breakup length of liquid jet in a cross flow is a basically function of the liquid to air momentum flux ratio. However, Weber number, liquid-to-air viscosity ratio and density ratio, Reynolds number or Ohnesorge number were incorporated in the empirical correlations depending on the investigators.

The variation of normalized column fracture height with the various q in four correlations belonged to momentum flux ratio form is shown in Fig. 2. It can be seen that there is no big difference between the correlations suggested by Wu *et al.*(22) and Birouk et al.(23). However, it is clear that there exist the signifi-

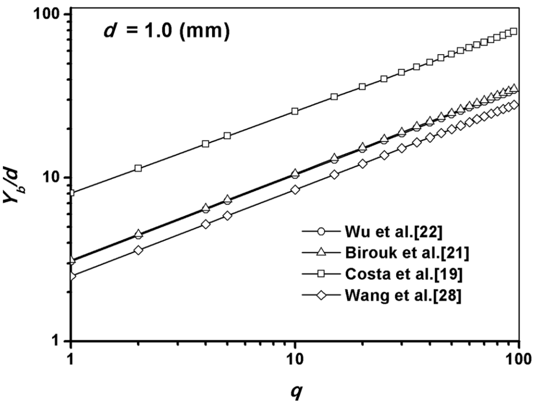

Fig. 2 Comparison of four correlations for column fracture height

cant discrepancies of predicted values by the existing correlations even though many correlations have the same functional form. The possible reasons for discrepancies can be summarized as the different experimental conditions including jet operating condition and nozzle geometry, measurement and image processing techniques, difficulties in defining the breakup location etc. The evaluation of the existing empirical correlations for the prediction of breakup length of liquid jet in a uniform cross flow is required.

Acknowledgement

This work was supported by the research grant of Chungbuk National University in 2012.

Nomenclature

d : discharge orifice diameter (mm)
Oh : Ohnesorge number $[\mu_l/(\rho_l \text{ d } \sigma_l)^{0.5}]$
q : liquid-to-air momentum flux ratio
Re : Reynolds number $(\rho \text{Vd}/\mu)$
T_b : breakup regime parameter
V : velocity (m/s)
We : Weber number $(\rho \text{V}^2 \text{ d}/\sigma)$
x_b : column fracture distance (mm)
y_b : column fracture height (mm)
θ : liquid injection angle
μ : dynamic viscosity (Pa·s)
ρ : density (kg/m^3)
ρ : liquid-to-air density ratio
σ : surface tension (N/m)

Subscript
a : air
cf : cross airflow
g : gas
j : jet
l : liquid
w : water

References

(1) J. M. Desantes, J. Arregle, J. Lopez and J. M. Garcia, *Atomization and Sprays*, 16, 511~530, 2006.
(2) J. M. Nouri and J. H. Whitelaw, *Atomization and Sprays*, 17, 621~640, 2007.
(3) O. M. Elshamy and S. M. Jeng, *18th Annual Conf. on Liquid Atomization and Spray Systems,* ILASS-Americas, Irvine, CA, May, 2005.
(4) A. R. Karagozian, *Progress in Energy and Combustion Science*, 36, 531~553, 2010.
(5) S. Ghosh and J. C. R. Hunt, *Journal of Fluid Mechanics*, 365, 109~136, 1998.
(6) J. C. Phillips and P. C. H. Miller, *Journal of Agricultural Engineering Research*, 72, 161~170, 1999.
(7) L. K. B. Li, S. L. Green, M. H. Davy and D. T. Eadie, *Atomization and Sprays,* 20(8), 697~720, and 721~735, 2010.
(8) J. A. Schetz, *First Symposium (ILASS-Japan) on Atomization*, Yokohama, Japan, 21~22 Dec, 1992, 1~13.
(9) A. H. Lefebvre, *Atomization and Sprays*, Hemisphere, 1989.
(10) R. Ragucci, A. Bellofiore and A. Cavaliere, *Atomization and Sprays*, 17, 47~70, 2007.
(11) S. Y. No, *24th European Conference on Liquid Atomization and Spray Systems*, Estoril, Portugal, 5~7 Sept. 2011.
(12) S. Y. No, *ILASS-Korea Journal*, 16(4), 176~185, 2011.
(13) J. M. Desantes, J. Arregle, J. J. Lopez and J. M. Garcia, *Atomization and Sprays*, 16, 511~530, 2006.
(14) J. M. Desantes, J. Arregle, J. J. Lopez and J. M. Garcia, *Fuel*, 85, 2120~2132, 2006.

(15) S. B. Tambe and S.-M. Jeng, *21st Annual Conf. on Liquid Atomization and Spray Systems,* ILASS-Americas, Orlando, Florida, May 18~21, 2008.

(16) O. M. Elshamy, S. B. Tambe, J. Cai and S.-M. Jeng, *45th AIAA Aerospace Sciences Meeting and Exhibit*, 8-11 Jan. 2007, Reno, Nevada, AIAA 2007~1340.

(17) S. B. Tambe, O. M. Elshamy and S.-M. Jeng, *43rd AIAA /ASME/SAE/ASEE Joint Propulsion Conf. & Exhibit*, 8-11 July 2007, Cincinnati, OH, AIAA 2007~5695.

(18) T. Inamura, N. Nagai, T. Hirai and H. Asano, *Proc. of ICLASS-91,* Gaithersburg, MD, USA, July, 1991, 839~846.

(19) M. Costa, M. J. Melo, J. M. M. Sousa and Y. Levy, *AIAA Journal*, 44(3), 646~653, 2006.

(20) S. B. Tambe, S.-M. Jeng, H. Mongia and G. Hsiao, *43rd AIAA Aerospace Sciences Meeting and Exhibit,* 10-13 Jan. 2005, Reno, Nevada, AIAA paper 2005~731, 2005.

(21) M. Birouk, T. Stabler and B. J. Azzopardi, *Particle and Particle Systems Characterization*, 20, 39~46, 2003.

(22) P.-K.Wu, K. A. Kirkendall and R. P. Fuller, *Journal of Propulsion and Power,* 13(1), 64~73,1997.

(23) A. Bellofiore, R. Ragucci, P. Di Martino and A. Cavaliere, *Proc. of 10th Int'l Conf. on Liquid Atomization and Spray Systems*, Kyoto, Japan, Aug. 27-Sept. 1, 2006, Paper ID F4-01-248.

(24) A. Bellofiore, A. Cavaliere and R. Ragucci, *Combustion Science and Technology*, 179, 319~342, 2007.

(25) M. Birouk, B. J. Azzopardi and T. Stabler, *Particle and Particle Systems Characterization*, 20, 283~289, 2003.

(26) K. Ahn, J. Kim and Y. Yoon, *Atomization and Sprays,* 16, 15~34, 2006.

(27) Q. Wang, U. M. Mondragon, C. T. Brown and V. G. McDonell, *Atomization and Sprays*, 21(3), 203~219, 2011.

(28) K. A. Sallam, C. Aalburg and G. M. Faeth, *AIAA Journal*, 42(12), 2529~2540, 2004.

(29) M. Birouk, B. J. Azzopardi and T. Stablend, *Proc. of 9th Int'l Conf. on Liquid Atomization and Spray Systems*-ICLASS 2003, Sorrento, Italy, July 13-17, 2003, paper ID 1~12.

(30) R. P. Fuller, P.-K. Wu and K. A. Kirkendall, *AIAA Journal*, 38(1), 64~72, 2000.

(31) R. Ragucci, A. Bellofiore and A. Cavaliere, *19th Annual Meeting of Liquid Atomization and Spray Systems*-ILASS-Europe, Notingham, UK, 6~8 Sept. 2004.

(32) R. Ragucci, A. Bellofiore and A. Cavaliere, *Proceedings of the Combustion Institute*, 31, 2231~2238, 2007.