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# PID auto-tuning controller design via fuzzy logic

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**Abstract** PID auto-tuning controller was designed via fuzzy logic. Typical values such as error and error derivative feedback were changed as heuristic expressions, and they determine PID gain through fuzzy logic and defuzzification process. Fuzzy procedure and PID controller design were considered separately, and they are combined and analyzed. Obtained auto-tuning PID controller by Fuzzy Logic showed the ability for less than 3rd order plant control.

• **Key Words** : Auto-tuning PID, fuzzy logic, heuristic design, 2<sup>nd</sup> order system

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## 1 Introduction

Importance of PID controller has been emphasized particularly on the industrial fields. By its strong and easy handling feature PID control has been widely used and preferred from engineers and operators[1]. Generally, PID gains are decided by operations of output error and error derivative. By doing such determination of gain it needs manual operation such as trial and error.

However, we have difficulty in tuning PID gains manually. Comparing with its robust and easy maneuvering operator is needed whenever references are changed. For such requirements, auto-tuning schemes are essential for automation. There are lots of researches related with auto-tuning technique [2-3].

In this literature, we focus on PID auto-tuning via heuristic approach. Actually, PID controller has much strong points to control, it has robustness and rather easy for tuning. However, if it is not provided auto-tuning algorithm, it shows poor performance for reference varying problem. Hence, to make automation auto-tuning structure is strongly needed. Well-known fuzzy theory, feedback data are fuzzified, and control input variables to plant are

determined by fuzzy logic, that is, IF THEN rule. Feedback values are considered as feedback error and error derivative to construct PID controller make easily. Each roles of proportional, integral, and derivative gain are also used for design of fuzzy logic. This control structure provides mixed algorithms of analytic approach and heuristic knowledge.

In the following chapter, auto-tuning schemes are illustrated. PID structure and fuzzy logic are illustrated briefly. In Chapter 3, we have combined auto-tuning structure with combination of PID and fuzzy logic structure. In Chapter 4, discussion for simulation, specially, to tracking problem was carried out with computer simulation. Finally, we derived conclusions in Chapter 5.

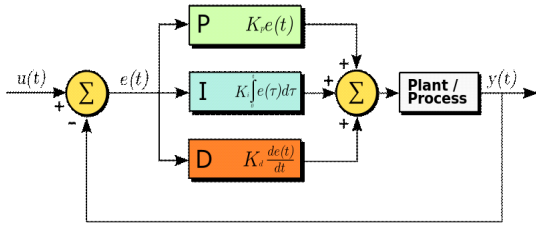
## 2. PID control

PID control is a typical feedback control mechanism, and usually deals within the SISO system. These controllers used so widely and are found in large numbers in all industries. As *Karl Johar* mentioned PID come in many different forms and are manufactured by the hundred thousand

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yearly [4]. The simple diagram of a general PID controller is shown below.



[Fig. 1] PID control structure

For PID tuning, inputs are  $e$  (the control error  $e = r - y$ ) and derivative of error  $de$  (change rate of error).

When dealing with continuously control system as shown above,  $u(t)$  is the control variable and  $e$  is the difference between set point  $r$  and measured value  $y$  (output of the system). The basic algorithm for a typical PID has the following form:

$$u(t) = K \left[ e(t) + \frac{1}{T_i} \int e(s) ds + T_d \frac{de(t)}{dt} \right] \quad (1)$$

and can be shown in transfer function form:

$$G(s) = \frac{U(s)}{E(s)} = k_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (2)$$

Where  $k_p$  is proportional gain,  $T_i$  is integral time, and  $T_d$  is derivative time.

The equations above show that the control variable is the addition effect of three terms: the P-term (means proportional to the error), the I-term (represents integral of the error), and the D-term (is the derivative of the error). Parameters  $K$ ,  $T_i$  and  $T_d$  are for proportional gain, integral time and derivative time respectively. Actually, the input of the whole PID control system is the error and its error derivative. Thus it is easy to find out the main

functions of the controller are provide feedback; due to the integral action it can eliminate steady state error; based on derivative action it can anticipate the change trend and response in advance.

## 2.1 The utility of three terms

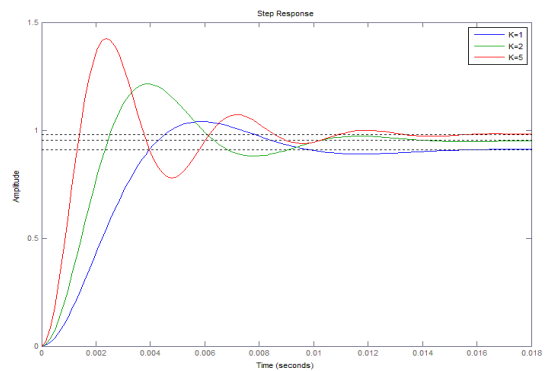
### 2.1.1 Proportional action

The proportional term can make an output proportional to the current error value. The proportional gain constant, known as  $K_p$ , is introduced here.

In the case of pure proportional control, the control law of equation (1) reduces to

$$u(t) = K e(t) \quad (3)$$

This control action works like an amplifier and is simply making a proportional to the control error. This is the simplest form of feedback. The proportional gain should be high to ensure that process output approach to set point  $r$  faster; the change on the output could be large due to the error as well. However, if the proportional gain is too large, the system could tend to be unstable and insensitive to load disturbance. In fact, the proportional term should contribute the most of the output change. As to the error derivative, the proportional gain does have effect on the damping frequency of the system. It could cause overshoot and oscillation.



[Fig. 2] control effect of P

Figure above shows the effect of different values of proportional control. With an increasing value of  $p$ , the response time decrease, meanwhile the overshoot increase largely which makes the system unstable. Proportional control is a tradeoff between response time and system stability.

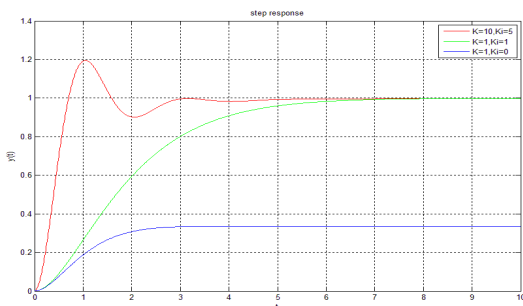
### 2.1.2 Integral action

The main function of the integral action is to make sure that in finite steady state, the system output approaches and eventually becomes the same as the set point which is called zero steady-state error. With proportional control, error is enlarged and it is necessary to have an error in order to have a non-zero control signal. With integral action, any small positive error can always give arise control signal, and a negative error will lead to a reducing control signal no matter how small the error is.

The control formula of an integral action is expressed as (4):

$$I = K_i \int_0^t e dt \tag{4}$$

Integral action can also be motivated as a device that automatically adjusts the reset of a proportional controller. It is clear that the integral term is related to both the input error and the duration of the error. The function of this term is to accelerate the movement of the process to the set point and regulate the steady-state error. Meanwhile, the accumulated errors could be integrated with past, the value of overshoot could be increased.



[Fig. 3] control effect of I

From figure 3, the parameter of I increase from 0 to 1, and the steady state error decrease from around 0.7 to 0. However keep increasing the value of I to 5, a serious overshoot appears as introduced before.

### 2.1.3 Derivative action

The purpose of the derivative action is to improve the closed-loop stability. It can anticipate the process output by extrapolating the error by tangent to the error curve; it also determines the rate of change of the controller output. The trend of the error derivative could be modified with the change of the gain. Derivative is also used to reduce the overshoot produced by integral term and improve stability at the same time. Plus to the error derivative, it slows the transient response of the controller, which could offer a smooth trend comparing to the original ones.

The control rule for derivative action is shown below:

$$D = K_d \frac{de(t)}{dt} \tag{5}$$

It is important to choose proper parameters for a PID controller since these parameters have different influence on both dynamic and static systems.

## 2.2 Characteristics of PID

PID controller is so popular because of its varieties advantages. Introduced by Yongli Huang, this type of controllers has simple control structure; is easy of design and also inexpensive in cost. The PID formulas are simple and can be easily adapted to different controlled part [10]. Due to the simple structure, PID controllers also very easy to realize into practice which means that it can be used into varieties situations; it is a primer controller which can fulfill the most of demands. However as W. D. Chang stated, this controller is sensitive to the unknown parameters and external random

disturbance. It is thus difficult to get the ideal dynamic performance with fixed PID parameters [9]. Also the too simple algorithm structure makes PID controller suitable for the minimum phase of SISO system; when it comes to open loop unstable systems, it usually takes several PID controllers working together to get a better control effect which may make the control system complicated.

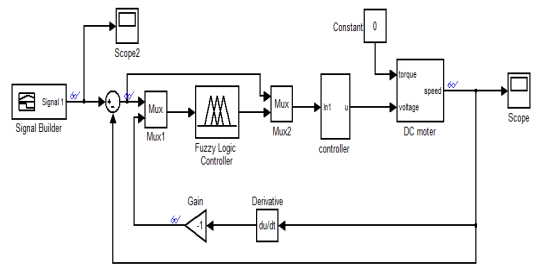
### 3. Fuzzy logic

There have been developed many ways for auto tuning PID such as genetic algorithm (GA). Here fuzzy theory was involved. Fuzzy logic is actually probabilistic logic. Instead of real numbers, fuzzy set has values that range from 0 to 1 and the number indicates membership of the set. Wikipedia introduced that fuzzy set theory defines fuzzy operators on fuzzy sets, and it usually uses "if-then" rules. A proper example has been made of fuzzy logic control. Assume a simple temperature regulator that uses a fan might look like this:

- IF temperature IS very cold THEN stop fan
- IF temperature IS cold THEN turn down fan
- IF temperature IS normal THEN maintain level
- IF temperature IS hot THEN speed up fan.

As the temperature could be at two conditions at the same time to different degree, the membership of the four conditions should have overlap area with neighborhood.

Claimed by Charles P. Coleman, fuzzy logic has been successfully applied into controller design in the last twenty years [1]. In this project, fuzzy rules are used to regulate parameters of the three terms; fuzzy rules and membership functions would be displayed later in design and simulation section.



[Fig. 4] PID Simulink construction

### 3.1 Design and simulation

Fuzzy PID auto tuning is to find out the fuzzy relationship between the three parameters and the two inputs ( $e$  and  $de$ ), changing the three parameters based on the fuzzy control theory to fulfill different inputs, thus the plant can perform good both in dynamic and static state. The model construction is based on the Simulink function of the software Matlab, which is quite useful and efficient in the model designing and simulation. According to instruction, the whole PID auto-tuning controller is designed like that:

Three main blocks make the system: fuzzy logic controller, PID controller and the plant (which is a DC motor in this example). Fuzzy logic controller is responsible for finding a rough range for the three parameters used in the PID controller. Inputs are error  $e$  and the derivative of error  $de$ , while the outputs are three gains. To find out the control appearance, the three gain terms are constructed separately as shown below.

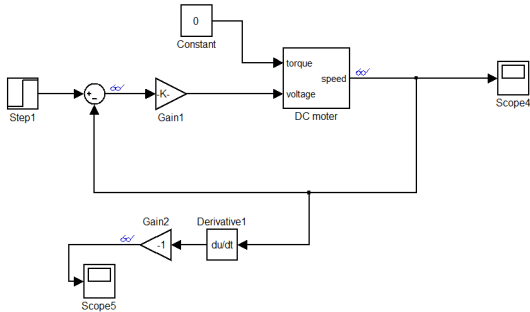
#### 3.1.1 Proportional P

Proportional gain  $K_p$  is to make the system response faster, improve the accuracy of the adjustment; but a large  $K_p$  may cause overshoot. For pure p gain, the output is proportional to the error, and the transfer function of the controller is

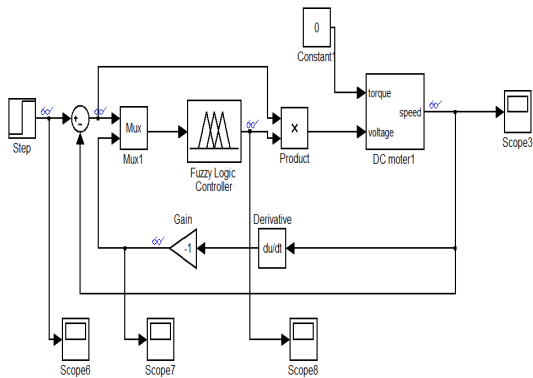
$$G = \frac{U(s)}{e(s)} = K_p \quad (6)$$

From the equal that proportional controller is actually a gain adjustable

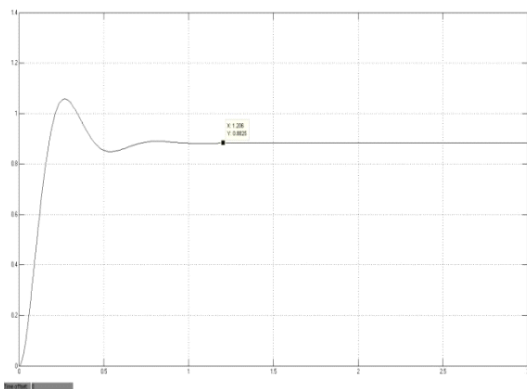
amplifier. Here are the models with constant gain and tuning proportional gain respectively, the initial gain values are both 75.5. The results of error can be observed at the scope 4 and 3 respectively. The output characteristics curves are illustrated in the following graph.



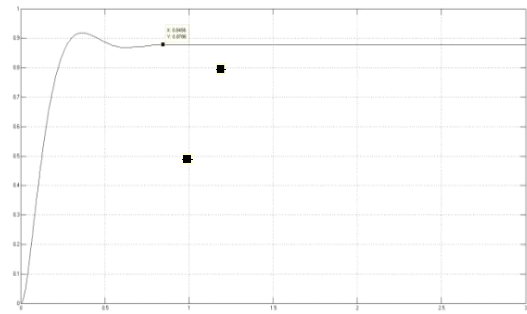
[Fig. 5a] traditional P control



[Fig. 5b] fuzzy P control



[Fig. 6a] output of traditional p control

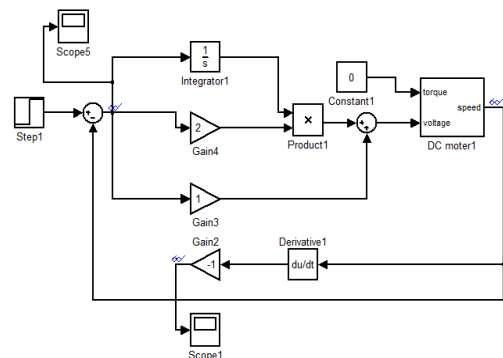


[Fig. 6b] output of fuzzy p control

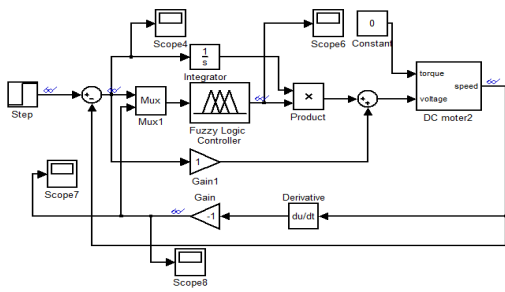
Two points marked on the graphs are highest and steady point respectively. Compared to the constant P case, the one with fuzzy logic control react faster: it reaches at steady state at around 0.84 second while the other one used 1.2 second to do so; also the largest value of the latter one is 0.9 instead of more than 1 in the former model.

### 3.1.2 Integral I

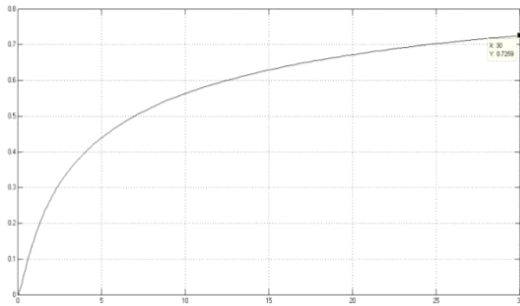
The utility of the integral gain is to reduce the steady state error. Larger the  $K_i$  is, faster the reduction; but too large  $K_i$  may cause overshoot. The simulation procedure is quiet similar with the previous one. The steady state error is observed by scope 1 and scope 8 shown in Figure 7a and 7b.



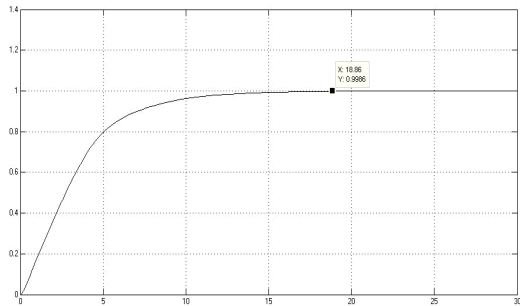
[Fig. 7a] traditional I control



[Fig. 7b] fuzzy I control



[Fig. 8a] output of traditional I control



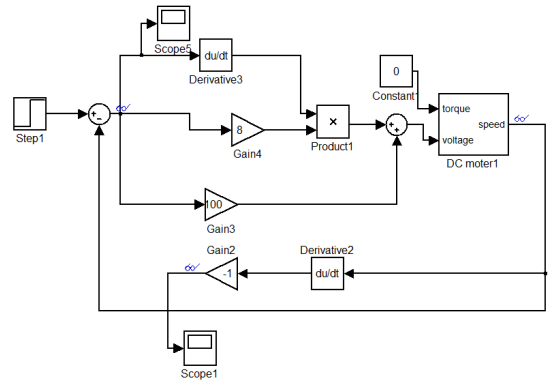
[Fig. 8b] output of fuzzy I control

The results show quiet clear that the auto-tuning model integral gain can reach to steady statemuch quicker than the one without auto-tuning: it reaches zero steady-state at around 18.86 seconds while the traditional one uses far more than 30 seconds and still cannot get to zero steady state.

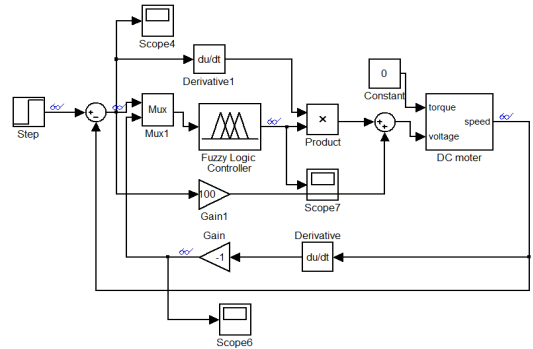
### 3.1.3 Differential D

The function of derivative gain is to control the error varies by predicting the trend of error change

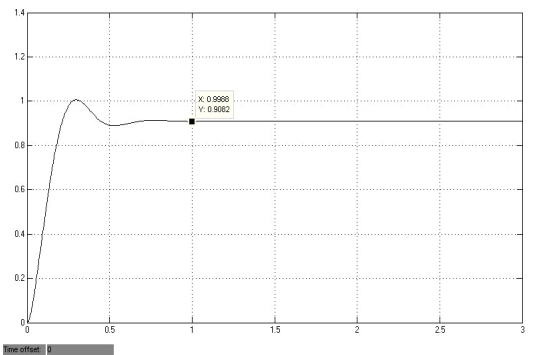
so that proportional and integral gain could adjust in advance to get to set point r faster; but a large Kd may makes the system adjust too early that extend the regulation time. Results can be obtained on scope 1 and 6 respectively.



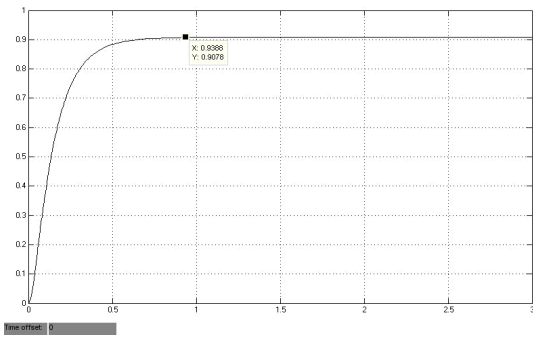
[Fig. 9a] traditional D control



[Fig. 9b] fuzzy D control



[Fig. 10a] output of traditional D control



[Fig. 10b] output of fuzzy D control

An oscillation occurs during the simulation of classical integral gain control while the control of fuzzy involved is more smooth and steady; it minimizes the maximum derivative gain: the classical case reaches almost 1 while the fuzzy one never go more than 0.1. Plus the latter case gets to zero 0.1 second ahead of the former one. Similar trend of the results in Figure 10a and 10b, an obvious oscillation of output appears in the classical control case while a much more smooth curve shown in Figure 9b of the fuzzy integral control one with no overshoot at all.

#### 4. PI controller with fuzzy logic

Transfer function for a PI controller is

$$D(s) = \frac{K}{s} \left( s + \frac{1}{T_i} \right) \tag{7}$$

During rising time (e is P),  $\Delta K_p$  should be positive –increase  $K_p$ ; when overshoot (e is N),  $\Delta K_p$  should be negative–decrease  $K_p$ . When e is Z, it depends on de: de is N, take  $\Delta K_p$  negative to avoid large overshoot; when de is Z or P, increase  $K_p$  to decrease e.

[Table 1] rules for  $K_p$

e	de	$\Delta K_p$	$K_p$
N	P/Z/N	N	decrease
Z	N	N	decrease
Z	P/Z	P	increase
p	P/Z/N	p	increase

According to integral separation, when error is around zero, make  $K_i$  positive; otherwise zero.

[Table 2] rules for  $K_i$

e	de	$\Delta K_i$	$K_i$
N	P/Z/N	Z	0
Z	P/Z/N	P	Increase
P	P/Z/N	Z	0

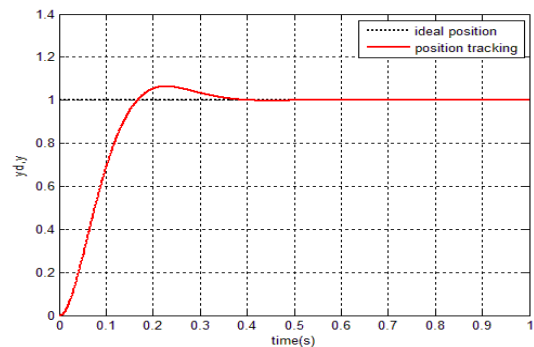
So the rules for PI control can be obtained by combine the rules above:

e	de	$\Delta k_p$	$\Delta k_i$
N	N	N	Z
N	Z	N	Z
N	P	N	Z
Z	N	N	P
Z	Z	P	P
Z	P	P	P
P	N	P	Z
P	Z	P	Z
P	P	P	Z

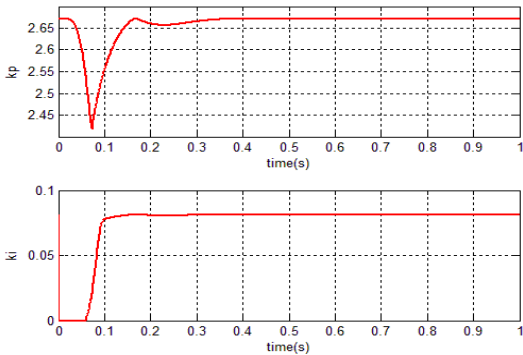
[Fig. 11] control rules for PI

Note: when e is P, it means the error is positive and is response; when N means it is overshoot (e is negative); and Z means value of error is around zero.

Base on the rules above, the tracing curves are shown below.



[Fig. 12] output of PI controller

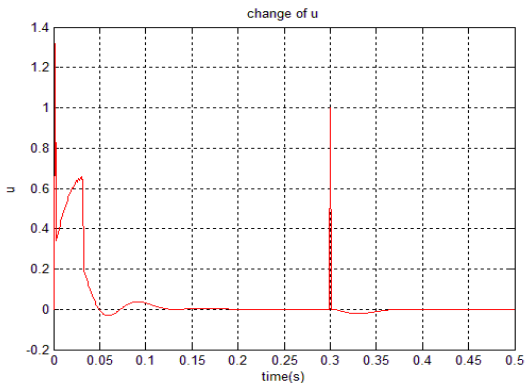


[Fig. 13] trace of change of Kp and Ki for PI controller

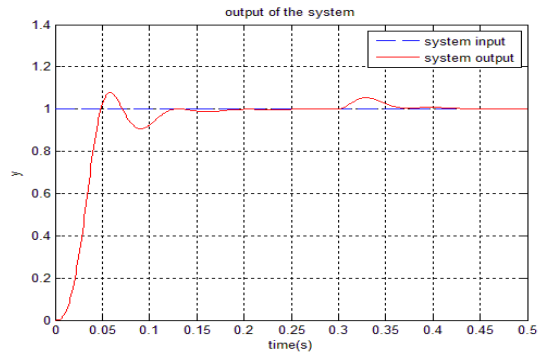
At the first half of 0.1 second, error e is quit large since the signal output starts at 0. So Ki becomes 0; after 0.06second, output increases and becomes closer to the ideal position, which makes error decrease and becomes around 0, Ki increases immedietly. By combing the PI and PD controller, we obtain PID controller:

$$D(s) = \frac{K}{s} \left( (T_D s + 1) \left( s + \frac{1}{T_I} \right) \right) \quad (8)$$

Membership functions are set to two conditions (low and high) for each component instead of three (negative, zero and positive) in previous; thus the rules are quite simple as shown below (low is for 1 and high is 2):



[Fig. 14] input of PID controller



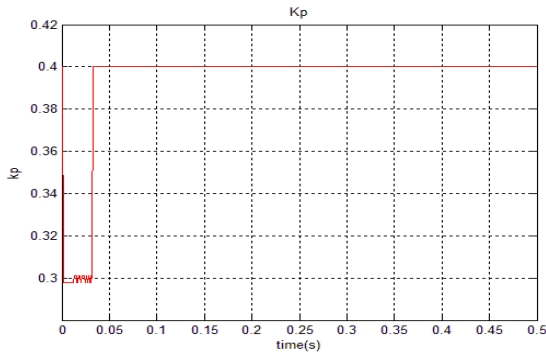
[Fig. 15] output of PID controller



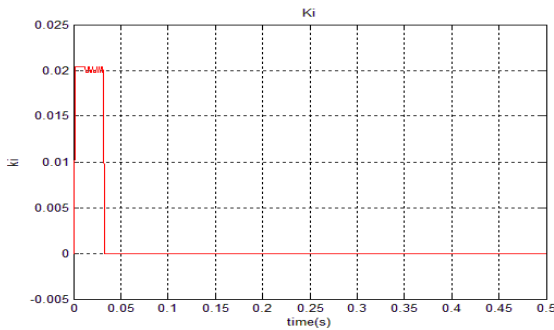
[Fig. 16] error change of PID control

These are the outputs of the combined auto-tuning PID controller. At first since initial output of system is 0 and set point is 1, error is quite large, so input is large in the starting moment. With the increasing of input, error decreasing rapidly and rising time is less than 0.05 second. Then after an overshoot, the system becomes steady after 0.1 second. At around 0.3 second, a disturbance occurs and causes a little oscillation error and input u; the system response fast and after no more than 0.35 second it back to zero steady state error again.

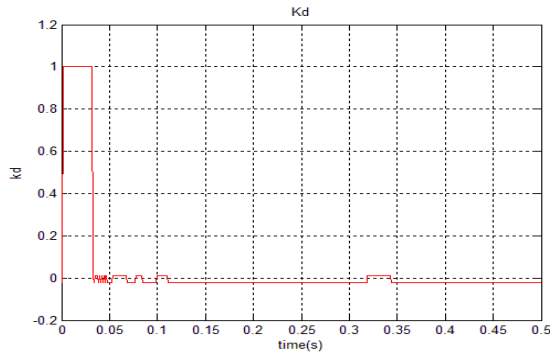




[Fig. 17] change of  $K_p$



[Fig. 18] change of  $K_i$



[Fig. 19] change of  $K_d$

The three figures above show change track of the three parameters of the PID controller. One thing noticeable is near 0.3 second where the unexpected disturbance occurs. Since the interruption makes the system overshoot, derivative makes its effort to reduce the overshoot soon after the oscillation of signal. The result is an optimization of all the

previous cases: response quickly and no overshoot or oscillation.

## 5. Conclusions

Auto-tuning design by heuristic knowledge was provided in this literature. We have determined control input for plant by combining PID control input and fuzzy logic. Fuzzy logic is composed by fuzzification, fuzzy rule, and defuzzification of the error and error derivative. Transforming error values to heuristic meaning, each control components, proportional, integral, and derivative fuzzy rule also considered and verified via simulation.

By combining three fuzzy rules, we provided unitary PID fuzzy rule as unified form. Finally, DC motor simulation was done with auto-tuning and also applied to tracking problem, that is, reference variation. Obtained result can be mainly applied to 2<sup>nd</sup> order system, and we have left as a future research for actual application.

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