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# 2자유도 평면 병진 병렬형 기구의 동역학 해석

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# Dynamics Analysis of a 2-DOF Planar Translational Parallel Manipulator

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Abstract

In this paper, the dynamics of a novel 2-DOF planar Translational Parallel Manipulator (TPM) is analyzed. The suggested TPM is made up of two PPa (Prismatic-planar Parallelogram) legs. Since all the linear actuators are mounted on the base, the proposed TPM can be applied for high speed positioning applications. The Lagrangian equations of the first type is employed to derive the inverse dynamic equations. It is shown that the analytical inverse dynamics equations match very well with ADAMS simulations. These analytical inverse dynamics equations will be used for the real-time computed torque control in the further work.

**Key Words :** Translational parallel manipulator(병진 병렬형 기구), parallelogram mechanism(평행사변형 기구), dynamics analysis (동역학 해석), Lagrangian method(라그랑지안 방법)

## 1. Introduction

In many sectors such as electronics, packing, food, pharmacy, and other light industries, Cartesian-type or SCARA-type serial-kinematic manipulators with 3-DOF or 4-DOF are mainly used. However, serial-kinematic structures suffer from large moving inertia and small ratio of payload to weight. In order to overcome the above shortcomings, parallel manipulators have been investigated. Since heavy actuators locate near or at the fixed base and payload is distributed to several serial chains, parallel manipulators can generate high speed, high stiffness and high accuracy[1,2]. Gogu[3] introduced many different kinds of parallel manipulators.

Recently, many researchers have focused on the Delta-type TPMs[4] for high-speed applications in place of SCARA-type serial manipulators. However, in the applications requiring small operating range along the z-axis, it may be more economic to employ a 2-DOF planar parallel manipulator with small 1-DOF or 2-DOF actuators in series instead of using 3-DOF or 4-DOF spatial parallel manipulators[4~8]. One of the simplest methods to construct 2-DOF planar TPMs is to use 2-PP structure[9] where P denotes a prismatic joint. However, the structure has the disadvantage that large moment may be applied to linear actuators. In order to

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reduce the difficulty, TPMs using parallelogram mechanisms [10~12] have been recently introduced. Liu and Wang designed the 2-PPa TPM with the parallel arrangement of linear actuators[11] and presented 2-DOF to 6-DOF parallel manipulators using parallelograms[12]. Other several 2-DOF mechanisms[13~15] are presented.

In this paper, a novel planar 2-DOF TPM with parallelogram mechanism is proposed. This TPM departs from the existing 2-PPa TPM[11] in the linear actuator arrangement. This perpendicular arrangement of the linear actuators can provide better kinematic and dynamic performance than the previous parallel arrangement. First, the position, velocity, and acceleration relations are analyzed. Then, the inverse dynamics is derived by using the Lagrangian equation of the first type due to the complex kinematics. Finally, the derived inverse dynamic equations are compared with ADAMS simulations.

## 2. Kinematic Analysis

As shown in Fig. 1, the proposed 2-DOF TPM consists of two PPa (Prismatic-planar Parallelogram) legs connecting the moving platform to the fixed base. Kim[16] presented the optimal design method of the 2-PPa TPM. The two prismatic joints are actuated and arranged perpendicularly. Each parallelogram allows the moving platform to move along the perpendicular direction of the corresponding linear actuator. This manipulator also becomes a 2-DOF over-constrained TPM due to the overlapped rotational constraints. In Fig. 1, the circle and square denote active and passive joints, respectively.

In Fig.1, the point $P$ at the center of the moving platform can be expressed with respect to the reference frame $OXY$ as

$$\boldsymbol{p} = -a\boldsymbol{e}_i + d_i\boldsymbol{e}_i + l_a\boldsymbol{s}_i + b\boldsymbol{e}_i \quad \text{for } i=1,2 \tag{1}$$

where, $\boldsymbol{p}=[p_1, p_2]^T$, and $\boldsymbol{e}_i$ and $\boldsymbol{s}_i$ are the unit directional vectors of the prismatic joint and parallelogram links, respectively. Rewriting Eq. (1) gives

$$l_a\boldsymbol{s}_i = \boldsymbol{m}_i - d_i\boldsymbol{e}_i \quad \text{for } i=1,2 \tag{2}$$

where, $\boldsymbol{m}_i \equiv \boldsymbol{p} + a\boldsymbol{e}_i - b\boldsymbol{e}_i$.

The length of the linear actuators $d_i$ can be obtained from Eq. (2) by

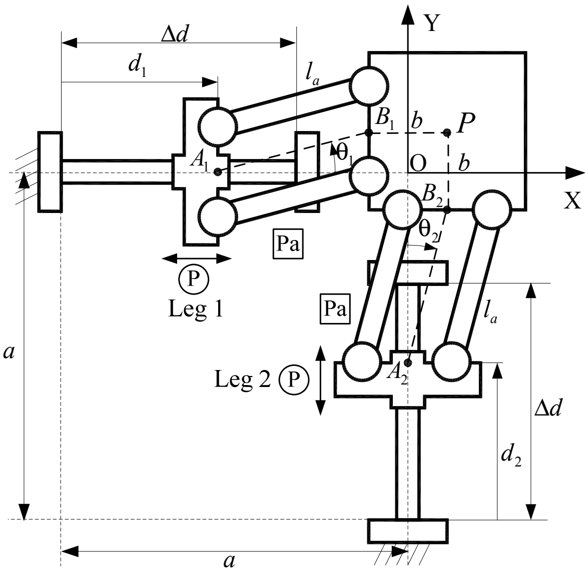


**Fig. 1 2-PPa TPM with linear actuation**

$$d_i = \boldsymbol{m}_i^T\boldsymbol{e}_i \pm \sqrt{(\boldsymbol{m}_i^T\boldsymbol{e}_i)^2 - \boldsymbol{m}_i^T\boldsymbol{m}_i + l_a^2} \quad \text{for } i=1,2 \tag{3}$$

Selecting only the negative square root, two linear actuator lengths are given by

$$d_i = (p_i + a - b) - \sqrt{l_a^2 - p_j^2} \quad \text{for } i \neq j \tag{4}$$

The linear velocity of the moving platform is obtained by taking derivatives of Eq. (1) with respect to time.

$$\dot{\boldsymbol{p}} = \dot{d}_i\boldsymbol{e}_i + l_a(\boldsymbol{\omega}_i \times \boldsymbol{s}_i) \quad \text{for } i=1,2 \tag{5}$$

where, $\omega_i$ denotes the angular velocity vector of links $l_a$ in the $i^{\text{th}}$ parallelogram. In order to eliminate unknown $\omega_i$, taking dot-multiply at the both sides of Eq. (5) yields,

$$\boldsymbol{s}_i^T\dot{\boldsymbol{p}} = (\boldsymbol{s}_i^T\boldsymbol{e}_i)\dot{d}_i \tag{6}$$

Writing Eq. (6) for $i=1,2$, the velocity relation can be determined by

$$J_x\dot{\boldsymbol{p}} = J_q\dot{\boldsymbol{d}} \tag{7}$$

where, $\dot{\boldsymbol{p}}=[\dot{p}_1, \dot{p}_2]^T$ and $\dot{\boldsymbol{d}}=[\dot{d}_1, \dot{d}_2]^T$. The Jacobian submatrices in the term of $\theta_i$ are

$$J_x = \begin{bmatrix} \cos\theta_1 \sin\theta_1 \\ \sin\theta_2 \cos\theta_2 \end{bmatrix},\ J_q = \begin{bmatrix} \cos\theta_1 & 0 \\ 0 & \cos\theta_2 \end{bmatrix} \tag{8}$$

Rewriting Eq. (7) gives,

$$\dot{d} = J\dot{p} \tag{9}$$

where, $J$ is the Jacobian matrix.

$$J = J_q^{-1} J_x = \begin{bmatrix} 1 & \tan\theta_1 \\ \tan\theta_2 & 1 \end{bmatrix} \tag{10}$$

An inverse kinematic singularity(1) occurs when the determinant of $J_q$ goes to zero,

$$\cos\theta_1 \cos\theta_2 = 0. \tag{11}$$

A direct kinematic singularity(1) occurs when the determinant of $J_x$ is equal to zero,

$$\cos(\theta_1 + \theta_2) = 0. \tag{12}$$

Using the principle of virtual works, the statics relation between force at the moving platform and actuator force can be expressed as

$$f = J^T \tau \tag{13}$$

where, $f = [f_1, f_2]^T$ is the applied force vector at the moving platform and $\tau = [\tau_1, \tau_2]^T$ is the actuator force vector.

The acceleration relation can be derived by taking derivatives of Eq. (7) with respect to time.

$$\ddot{d} = J_q^{-1}(\dot{J}_x \dot{p} + J_x \ddot{p} - \dot{J}_q \dot{d}) \tag{14}$$

where, the accelerations of the moving platform and linear actuator are $\ddot{p} = [\ddot{p}_1, \ddot{p}_2]^T$, $\ddot{d} = [\ddot{d}_1, \ddot{d}_2]^T$, and time derivatives of $J_x$ and $J_q$ are

$$\dot{J}_x = \begin{bmatrix} -\sin\theta_1 \dot{\theta}_1 & \cos\theta_1 \dot{\theta}_1 \\ \cos\theta_2 \dot{\theta}_2 & -\sin\theta_2 \dot{\theta}_2 \end{bmatrix},\ \dot{J}_q = -\begin{bmatrix} \sin\theta_1 \dot{\theta}_1 & 0 \\ 0 & \sin\theta_2 \dot{\theta}_2 \end{bmatrix} \tag{15}$$

## 3. Dynamics Analysis

Since the inertia, centrifugal and Coriolis terms of the equation of motion become larger as higher speed and acceleration are required, the feedback linearization by compensating the dynamic terms (or computed-torque control) is essential. For real-time computed torque control especially in high speed applications, the derivation of the closed form solution of inverse dynamics is necessary.

Theoretically, the dynamics analysis can be accomplished by using just two generalized coordinates, i.e., $d_1$ and $d_2$ since this is a 2-DOF manipulator. However, this would lead to a cumbersome expression for the Lagrangian function, due to the complex kinematics of the manipulator. Instead, Lagrange's equation of the first type will be employed by introducing two redundant coordinates, $p_1$ and $p_2$. In this paper, the generalized coordinates are defined by

$$\boldsymbol{q} = [p_1, p_2, d_1, d_2] \tag{16}$$

The Lagrange equations of the first type can be written as(1)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{i=1}^{k} \lambda_i \frac{\partial \Gamma_i}{\partial q_j} \quad \text{for } j = 1,2 \ldots n \tag{17}$$

where, $n$ is the number of generalized coordinates, $k$ is the number of constraint functions, $n-k$ is the number of actuated joint variables, $\Gamma_i$ denotes the $i^{th}$ constraint function, and $\lambda_i$ is the Lagragian multiplier.

The first set of equations related to constraints can be written in the form

$$\sum_{i=1}^{2} \lambda_i \frac{\partial \Gamma_i}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} - \hat{Q}_j \quad \text{for } j = 1,2 \tag{18}$$

where, $\hat{Q}_j$ denotes the generalized force contributed by an externally applied force. Once the Lagrangian multipliers are found from Eq. (18), the second set of equations related to actuation forces can be written as

$$Q_j = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} - \sum_{i=1}^{2} \lambda_i \frac{\partial \Gamma_i}{\partial q_j} \quad \text{for } j = 3,4 \tag{19}$$

where, $Q_j$ is the actuator force.

The total kinetic energy of a 2-PPa TPM consists of the sum of the kinetic energy of the moving platform ($K_p$), the parallelogram ($K_{bi}$), and the sliders ($K_{ai}$)

$$K = K_p + \sum_{i=1}^{2} \left( K_{ai} + K_{bi} \right) \quad \text{for } i = 1,2 \tag{20}$$

Since the moving platform and slider part have only translational motion, the kinetic energies can be obtained by

$$K_p = \frac{1}{2} m_p \left( \dot{p}_1^2 + \dot{p}_2^2 \right), K_{ai} = \frac{1}{2} m_a \dot{d}_i^2 \quad \text{for } i = 1,2 \tag{21}$$

The kinetic energy of the $i^{\text{th}}$ parallelogram is given by

$$K_{bi} = \frac{1}{6} m_b l_a^2 \dot{\theta}_i^2 + \frac{1}{2} m_b (\dot{d}_i^2 - l_a \dot{d}_i \dot{\theta}_i \sin\theta_i) \quad \text{for } i = 1,2 \tag{22}$$

where, $m_p$ is the mass of the moving platform, $m_a$ the mass of slider part, and $m_b$ the mass of two connecting links of each parallelogram. In order to express $K_{bi}$ as a function with only the generalized coordinates, the relations of $\theta_i$ and $\dot{\theta}_i$ to the generalized coordinates should be derived. From Eq. (1), the relation between $\theta_i$ and the generalized coordinates can be obtained as

$$\tan\theta_i = \frac{p_j}{p_i - d_i + a - b} \quad \text{for } i \neq j \tag{23}$$

Since $\omega_1 = \dot{\theta}_1 \hat{\boldsymbol{k}}$ and $\omega_2 = -\dot{\theta}_2 \hat{\boldsymbol{k}}$, the relation between $\dot{\theta}_i$ and the generalized coordinates can be obtained from Eq. (5) by

$$\dot{\theta}_i = \frac{1}{l_a \cos\theta_i} \dot{p}_j \quad \text{for } i \neq j \tag{24}$$

Since the potential $U$ is zero, the Lagrangian function ($L = K - U$) can be reduced to

$$\begin{aligned} L = K = & \frac{1}{2} m_p (\dot{p}_1^2 + \dot{p}_2^2) \\ & + \sum_{i=1}^{2} \left\{ \frac{1}{2} (m_a + m_b) \dot{d}_i^2 + \frac{1}{6} m_b l_a^2 \dot{\theta}_i^2 - \frac{1}{2} m_b l_a \sin\theta_i \dot{d}_i \dot{\theta}_i \right\} \end{aligned} \tag{25}$$

Using Eqs. (23) and (24), the Largrangian function in Eq. (25) can be expressed with only the generalized coordinates by

$$\begin{aligned} L = K = & \frac{1}{2} m_p (\dot{p}_1^2 + \dot{p}_2^2) \\ & + \sum_{i=1}^{2} \left\{ \begin{aligned} & \frac{1}{2} (m_a + m_b) \dot{d}_i^2 \\ & + \frac{1}{6} m_b (1 + \frac{p_j^2}{(p_i - d_i + a - b)^2}) \dot{p}_j^2 \\ & - \frac{1}{2} m_b \frac{p_j}{p_i - d_i + a - b} \dot{d}_i \dot{p}_j \end{aligned} \right\} \end{aligned} \tag{26}$$

where, $j = 2$ for $i = 1$, and $j = 1$ for $i = 2$.

The constraint functions are obtained from the fact that the distance between joints $A_i$ and $B_i$ is always equal to the length of the parallelogram links. From Eq. (2), two constraint functions are given by

$$\Gamma_i = (p_i - d_i + a - b)^2 + p_j^2 - l_a^2 = 0 \quad \text{for } i \neq j \tag{27}$$

Taking the partial derivatives of the Lagrangian and constraint functions with respect to the four generalized coordinates, substituting all the derivatives into Eq. (18), and solving the resulting equation for Lagrangian multiplier $\lambda_1$ and $\lambda_2$ yields

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial \Gamma_1}{\partial p_1} & \frac{\partial \Gamma_2}{\partial p_1} \\ \frac{\partial \Gamma_1}{\partial p_2} & \frac{\partial \Gamma_2}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{p}_1} \right) - \frac{\partial L}{\partial p_1} - f_x \\ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{p}_2} \right) - \frac{\partial L}{\partial p_2} - f_y \end{bmatrix} \tag{28}$$

where, $f_x$ and $f_y$ denote the externally applied forces along the x- and y-axes. Assuming there is no external force, the Lagrangian multipliers are given by

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 2(p_1 - d_1 + a - b) & 2p_1 \\ 2p_2 & 2(p_2 - d_2 + a - b) \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{29}$$

where,

$$\begin{aligned} v_i = & (m_p + \frac{1}{3} m_b) \ddot{p}_i - \frac{m_b p_j \dot{d}_i \dot{p}_j}{2(p_i - d_i + a - b)^2} + \frac{m_b p_j^2 \dot{p}_j^2}{3(p_i - d_i + a - b)^3} \\ & + \frac{m_b p_i (-\dot{d}_j^2 + \dot{d}_j \dot{p}_j - (p_j - d_j + a - b) \ddot{d}_j)}{2(p_j - d_j + a - b)^2} + \frac{m_b p_i p_i^2}{3(p_j - d_j + a - b)^2} \\ & + \frac{m_b p_i^2 (2 \dot{p}_i (\dot{d}_j - \dot{p}_j) + (p_j - d_j + a - b) \ddot{p}_i)}{3(p_j - d_j + a - b)^3}. \end{aligned}$$

Substituting Eq. (28) into Eq. (19), the actuator force can be determined by

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}_1}\right)-\frac{\partial L}{\partial d_1}\\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}_2}\right)-\frac{\partial L}{\partial d_2}\end{bmatrix}-\begin{bmatrix}\frac{\partial \Gamma_1}{\partial d_1} & \frac{\partial \Gamma_2}{\partial d_1}\\ \frac{\partial \Gamma_1}{\partial d_2} & \frac{\partial \Gamma_2}{\partial d_2}\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\end{bmatrix} \quad (30)$$

Equation (30) yields

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}w_1\\ w_2\end{bmatrix}+2\begin{bmatrix}\lambda_1(p_1-d_1+a-b)\\ \lambda_2(p_2-d_2+a-b)\end{bmatrix} \quad (31)$$

where,

$$w_i=(m_a+m_b)\ddot{d}_i-\frac{m_b\dot{p}_j^2}{2(p_i-d_i+a-b)}-\frac{m_b p_j^2\dot{p}_j^2}{3(p_i-d_i+a-b)^3}+\frac{m_b p_j(\dot{p}_i\dot{p}_j-(p_i-d_i+a-b)\ddot{p}_j)}{2(p_i-d_i+a-b)^2}.$$

When the trajectories of the moving platform ($p_i$, $\dot{p}_i$, $\ddot{p}_i$) are given, the procedure to calculate actuator forces ($\tau_i$) can be summarized as follows.

(a) $d_i$: inverse kinematics (Eq. (4))

(b) $\dot{d}_i$: velocity relation (Eq. (9))

(c) $\ddot{d}_i$: acceleration relation (Eq. (14))

(d) Lagrangian multiplier ($\lambda_i$): Eq. (29)

(e) Actuator forces ($\tau_i$): Eq. (31)

**Table 1 Kinematic parameters of the prototype 2-PPa TPM**

| Parameters | Values |
|---|---|
| Workspace | 260×260 mm2 |
| Stroke ($\Delta d$) | 300 mm |
| Total size ($a$) | 482 mm |
| Link length($l_a$) | 260 mm |
| Moving platform($b$) | 92 mm |

**Table 2 Mass properties of the prototype 2-PPa TPM**

| Parameters | Values |
|---|---|
| slider ($m_a$) | 0.95 kg |
| moving platform ($m_p$) | 2.21 kg |
| parallelogram ($m_b$) | 2×0.42 kg |

## 4. Dynamic Simulations

The prototype 2-PPa manipulator and the ADAMS modeling are shown in Fig. 2 and Fig. 3, respectively. Table 1 and Table 2 present the kinematic parameters and mass properties of the prototype TPM. The actuator forces are calculated with the inverse dynamics and the results are compared with ADAMS simulations. Fig. 4 and Fig. 5 show the actuator forces for the line and circle trajectories with cubic polynomials, respectively. It is noted that two results match very well and the difference is less than 9N (about 5% max. error).

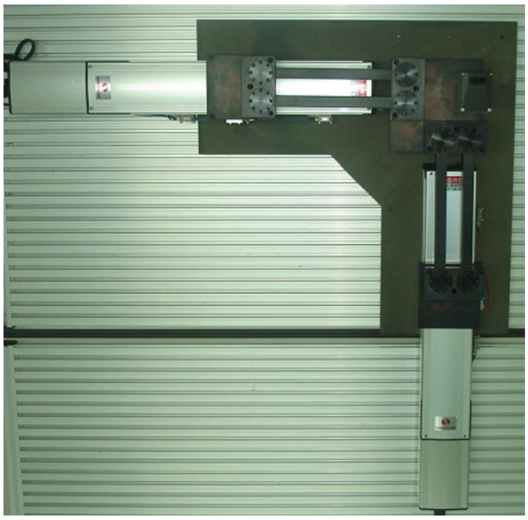

**Fig. 2 Prototype 2-PPa TPM with linear actuation**

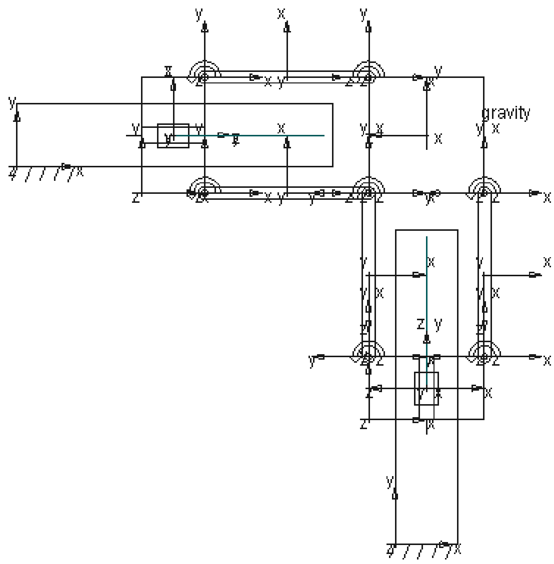


**Fig. 3 ADAMS modeling of 2-PPa TPM**

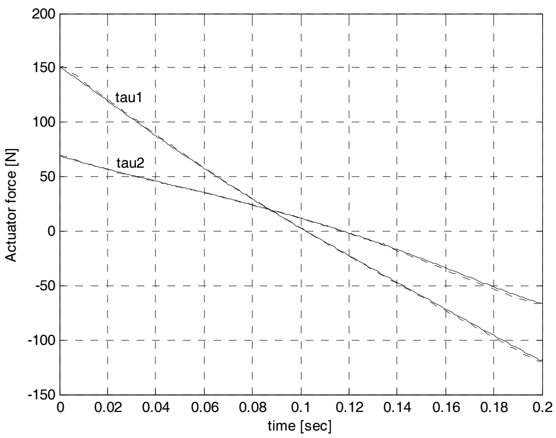


**Fig. 4 Line trajectory from (-100, -50) to (+100, +50) mm (solid: dynamics equation, dotted: ADAMS simulation)**

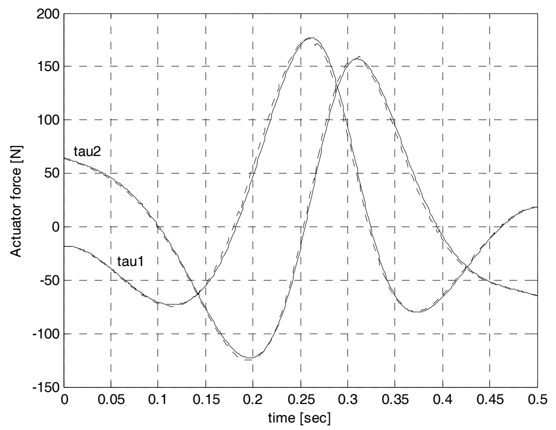


**Fig. 5 Circle trajectory with origin (0, 0) and radius 100 mm (solid: dynamics equation, dotted: ADAMS simulation)**

## 5. Conclusions

A novel 2-DOF planar TPM using parallelogram is presented for the high-speed positioning applications. For real-time computed torque control, the derivation of the closed form solution of inverse dynamics is essential. The position, velocity, singularity, and acceleration analyses are performed. Due to complex kinematics, the inverse dynamics is derived by using Lagrangian equation of the first type including two redundant coordinates. The procedure to calculate the inverse dynamics for given trajectories of the moving platform is summarized in the Section 3. The numerical simulations with ADAMS are performed to verify the accuracy of the analytical inverse dynamics. It is shown that numerical and analytical results match very well for line and circle trajectories. The derived inverse dynamic equations will be used for the real-time computed torque control.

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