
Original Paper

Flow Analyses Inside Jet Pumps Used for Oil Wells

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Abstract

Jet pump is one type of artificial lifts and is used when depth and deviation of producing wells increases and pressure depletion occurs. In the present study, numerical analysis has been carried out to analyze the flow behavior and find the performance of the jet pump. Reynolds-averaged Navier Stokes equations were solved and k- ϵ turbulence model was used for simulations. Water and light oil as primary fluids were used to pump water, light oil and heavy oil. The ratios of area and length to diameter of the mixing tube were considered as design parameters. The pump efficiency was considered to maximize for the downhole conditions. It was found that the increase in viscosity and density of the secondary fluid reduced efficiency of the system. Water as primary fluid produced better efficiency than the light oil. It was also found that the longer throat length increased efficiency upto 40% if light oil was used as primary fluid and secondary fluid viscosity was 350 cSt.

Keywords: Artificial lift, well pumping, jet pump, hydraulic lift, primary fluid, secondary fluid.

1. Introduction

If a self flowing well ceases to flow or is not able to deliver the required quantity of oil to the surface, additional energy is supplemented either by a mechanical means or by injecting compressed gas. There are many types of artificial lifts (AL) (Fig. 1) and these supplement energy to lift the fluid from the wells. Several criteria are used for selecting any AL (Fig. 2). The jet pump has no moving parts and is tolerant towards various power or primary fluids and can handle corrosive and abrasive well fluids. The pump is suitable for deep wells, directional wells, crooked wells, subsea production wells, wells with high viscosity fluid, high paraffin, high sand content, and particularly for wells with relatively high gas oil ratio.

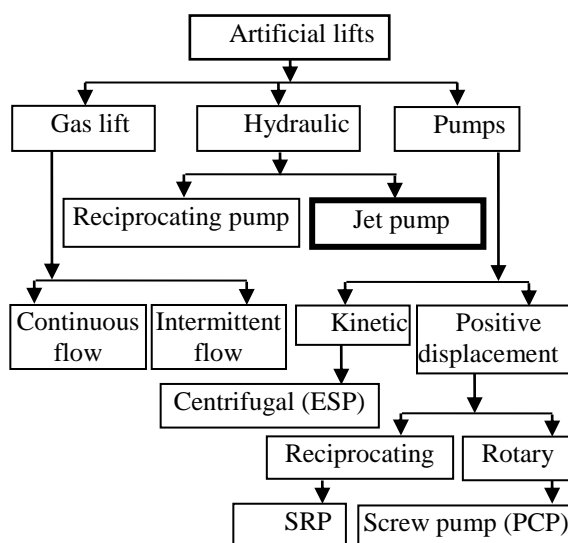


Fig. 1 Artificial lift systems

A typical jet pump (Fig. 3) consists of a throat or mixing chamber section which separates two counter facing cones on either side. A high energy primary fluid is injected at high pressure to the driving nozzle and it becomes a low pressure stream after passing through the nozzle. At this point, the suction of secondary fluid which is basically a reservoir fluid takes place. The both the fluids enter into the mixing chamber and mixes up. The mixed fluid enters the diffuser where the high kinetic energy mixture is converted into potential head which lifts the mixture to the surface.

The different conditions of the well and the type of hydrocarbons present decide the type of primary fluid to be used. There are mainly two types of primary fluids: water and light oil. The usage of water as primary fluid is preferred as it is environmentally less hazardous and relatively safe. The jet pumps driven by liquid hydrocarbons were found to be suitable for lifting heavy oils without any operational problems [1].

Recent use of jet pump is very low and the main reason for this is the low efficiency and also the lack of knowledge about the advantages of the pump. Corteville et al. [2] identified two main reasons for the reduced usage of jet pumps i.e., the energy efficiency of the jet pump is usually in the range of 26-33% which is very low and the sizing methods during the design are considered inaccurate, incomplete and somewhat mysterious. The major losses in the pump arise primarily due to the improper mixing of primary and secondary fluids and the formation of recirculation region near the mixing tube entrance [3]. The pump efficiency reduces with the presence of gas either in the primary or secondary fluid [4].

Several researches were carried out on liquid-liquid flow jet pump and density and viscosity of the primary and secondary fluids were assumed same. But in oil and gas application, the secondary fluid differs from the primary fluid in density and viscosity. Hesham et al. [4] conducted analytical and experimental studies for the jet pump in which the primary and secondary fluids differ in density and viscosity. They performed experiments and established relationships among mass ratio, head ratio and the efficiency for the jet pump. It was found that a longer throat was required in order to get a good mixing for pumping a highly viscous crude oil. The efficiency and life of jet pumps is also influenced by the cavitation occurring in the pumps [5].

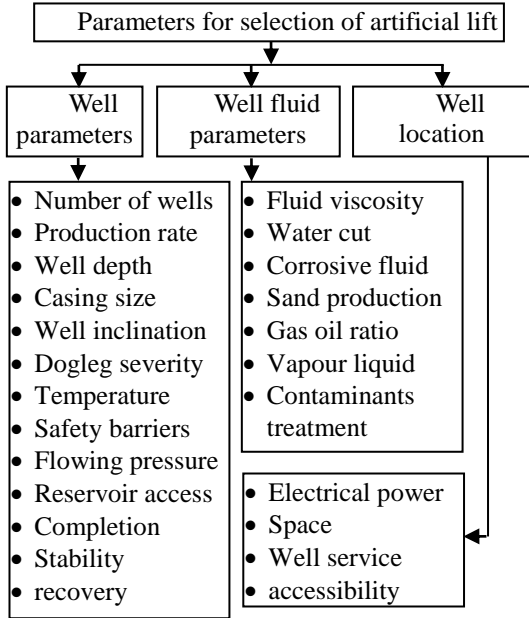


Fig. 2 Parameters to be considered while selecting an AL

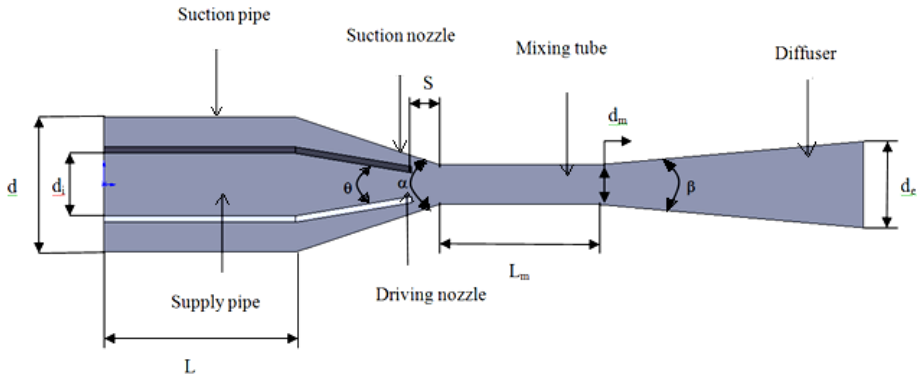


Fig. 3 Components of the jet pump

The geometric and fluid parameters which affect the performance of jet pumps are mixing tube length, nozzle position, diffuser dimensions etc. fluid molecular weight, feed temperature, primary fluid velocity, Reynolds number, pressure ratio, specific heat ratio etc., respectively [3, 6-8]. Hatzlavramidis et al. [5] proposed several methodologies for selecting the optimum size and flow rate of the pump for different production rates and concluded that the pressure loss in the pump was mainly due to the factors like area ratio, pressure effects and viscosity effects. To select a proper pump setting depth, the secondary fluid should have proper viscosity to maintain good flow capacity and also the wellhead pressure should not exceed the allocated surface facility pressure [9].

The performance of a jet pump is highly affected by the viscosity difference of the primary fluid and secondary fluid [4]. The primary fluid may be water or light oil. On the other hand, the secondary fluid may be water, light viscous oil and highly viscous oil. The viscosity of the primary fluid can be controlled at the surface by adding certain additives or by changing the type of the primary fluid. The viscosity of the secondary fluid is the most influential as it varies during the course of production. Hence care must be taken during the sizing of jet pumps [10] and also during the selection of primary fluid.

The viscosity of the primary fluid is influenced by the temperature, pressure, density and the amount of dissolved gases present in the secondary fluid [11]. The results obtained by De Ghetto et al. [8] showed that the average pump performance was reduced by 30% than expected because of high well fluid viscosity. The physical law which governs the viscosity of the secondary fluid when light oil is used as primary fluid is given as [12-13]:

$$\log(\log\mu_p) = \frac{I}{M+1} \log(\log\mu_i) + \frac{M}{M+1} \log(\log\mu_r) \quad (1)$$

The viscosity of the light oil significantly affected the production of reservoir fluid and the effect of viscosity of the reservoir fluid is insignificant [14]. Increased viscosity of primary or secondary fluid also reduces the efficiency factor by increasing the frictional and momentum losses [15]. The loss of performance of a jet pump is more noticeable above 20 cSt and empirical data or pilot testing is used to determine sizing parameters. By blending light oil with the reservoir fluid in the wellbore, the viscosity of the reservoir fluid can be reduced by more than 1600 times [11].

The pressure plays an important role in the design and sizing of the jet pumps. It decides the required horsepower at the surface. The pressure inside the throat must remain above the liquid-vapor pressure in order to prevent cavitation damage [10]. Maximum efficiency occurs when the throat intake pressure is 35% of pump intake pressure [16]. A jet pump will not operate properly if the pressure is even a few hundred Pascals below its design motive pressure [17].

In most of the deep heavy oil reservoirs light oil is used as the primary fluid. This helps in the reduction of the pressure loss because it instantaneously and thoroughly blends with the reservoir fluid in the jet pump throat [8]. If gas is used as a primary fluid, exhaust pressure should be less than half primary gas pressure and less than twice the suction pressure to operate the jet pump properly [18]. The efficiency of a jet pump is a function of the primary pressure drop in the suction chamber [19]. In the presence of gas, the suction pressure is a highly influential parameter and as the intake pressure increases the energy efficiency also increases [2].

Fan et al. [20] was concerned with the design and analysis of supersonic gas flows in jet pumps and a CFD analysis and experimentation was combined in order to improve the jet pump efficiency by optimizing jet pump geometrical and operating parameters. The CFD model is shown to be capable of capturing important flow features and making accurate predictions of performance for most of the operating conditions. Mallela and Chatterjee [3] carried out numerical simulations in order to determine the effects of area ratio, setback distance, mixing tube length, and shape of the driving nozzle on the performance of the jet pump. They observed that the major losses which arise in the jet pumps were primarily due to the improper mixing of primary and secondary fluids because of the formation of recirculation near the mixing tube entrance. Based on these studies, a normalized mixing tube length (Lm/dm) of 7 to 9, area ratio of 0.28, setback ratio of 1.54 to 2.02, and a modified shape of the outer wall of driving nozzle were recommended. Song et al. [21] successfully employed Ansys CFX code to investigate the flow characteristic inside the jet pumps. Performances such as mass flow ratio, pressure ratio and efficiency were compared at three types of working condition. They concluded that the jet pump working at the given operating conditions with the highest mass flow ratio and maintaining the highest possible discharged pressure was optimum.

In the present work a numerical study has been performed for different primary and secondary fluids to find the performance and flow behavior of the jet pump. The effect of the area ratio and the throat length on the performance of the jet pump was also studied.

2. Numerical formulation

Generally CFD problems are non-linear and the solutions are obtained by iterative process to successively improve a solution, until convergence criteria is achieved. ANSYS-CFX utilizes Reynolds-averaged Navier Stokes (RANS) equations to deal with the turbulent flow and used in this work. Standard $k-\epsilon$ turbulence model was used for the analysis. First order upwind scheme was used for the discretization of the governing equations. The present work consisted of different geometries to be simulated and the residual criteria were met. The RMS residual target was set to 10^{-4} . Fluid flow through the jet pump was considered as incompressible and viscous.

Details of the major dimensions used in this problem are given in Table 1. The geometry for the simulations was selected from the article of Krishnamurthy [22]. In this work, three important parameters were used to describe and compare the performance of jet pump. The parameters are defined by:

$$\text{Mass flow ratio,} \quad M = \frac{Q_s}{Q_n} \quad (2)$$

$$\text{Head ratio, } N = \frac{P_d - P_s}{p_n - p_d} \quad (3)$$

$$\text{Efficiency, } \eta = M \cdot ? \cdot 100\% \quad (4)$$

Table 1 Specifications of the jet pump

Component	Details of geometry
Supply pipe	$L=154\text{mm}, d=50\text{mm}, t=5\text{mm}$
Driving nozzle	$d_i=50\text{mm}, d_e=17\text{mm}, \theta=36.5^\circ$
Setback distance	$S=22\text{mm}$
Suction pipe	$L=154\text{mm}, d=108\text{mm}$
Suction nozzle	$d_i=108\text{mm}, d_e=32\text{mm}, \alpha=55.6^\circ$
Mixing tube	$L_m=139\text{mm}, d_m=32\text{mm}$
Diffuser	$d_i=32\text{mm}, d_e=69\text{mm}, \beta=10^\circ$

Different components of the jet pump are shown in Fig. 3. Boundary conditions for the simulations were mass flow rates of the primary and secondary fluid inlets and static pressure at the outlet of the diffuser. As the pump handles only single phase fluids (primary and secondary fluids are liquids), there was no requirement to use two phase modelling. Wall boundary condition was applied for supply and suction pipes, suction and driving nozzles, mixing tube, diffuser, and delivery pipe walls. Pressure at the outlet was specified and the pressure at the face of the pressure outlet boundary was computed as an average of the specified pressure and the calculated interior pressures. Setting a wrong pressure value at the boundary may lead to boundary condition incompatibility [23]. A subsonic flow regime was considered and the simulations were performed for the mass flow ratios of 0.2, 0.4, 0.6, 0.8 and 1.0. The reference pressure and temperature were 10^5 N/m^2 and 25°C , respectively. Isothermal heat transfer model was used. High pressure primary fluid enters through the nozzle and the secondary fluid which is at lower pressure enters through the supply pipe. The mixture of the primary and secondary fluids exits the jet pump at the diffuser outlet (Fig. 3).

The meshing of the geometry was done using the CFX Mesh tool. An unstructured type mesh was generated. Grid sensitivity study which was suggested by Samad and Kim [24] was carried out in order to select an appropriate size of the grid for analysis. It is very important to determine an appropriate grid size that would optimize the solution time and accuracy. To study the effect of grid size, numerical simulations were carried out for different grid sizes. Effect of head ratio (N) on mass ratio (M) for the different number of grid elements is plotted in the Fig. 4. Number of mesh elements of 500,000 was selected for the study because this was taking comparative less time for the simulations and was matching well with the existing numerical results [3]. The result did not differ much by further increasing the number of elements. Mesh statistics is presented in Table 2.

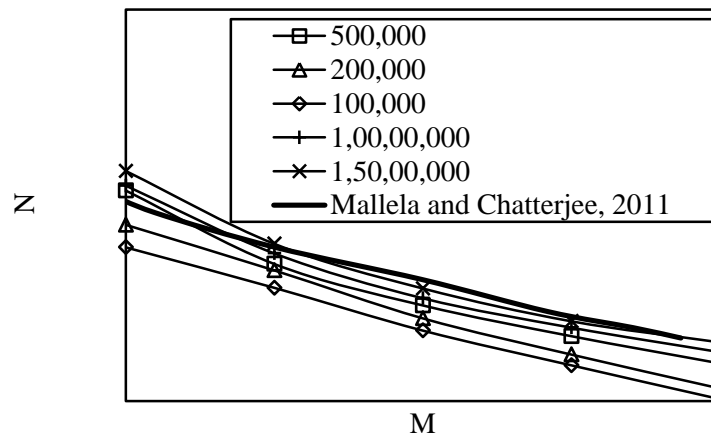


Fig. 4 Grid sensitivity study

Table 2 Statistics of mesh generated

Total number of nodes	150,000
Total number of tetrahedral	330,000
Total number of prisms	170,000
Total number of elements	500,000

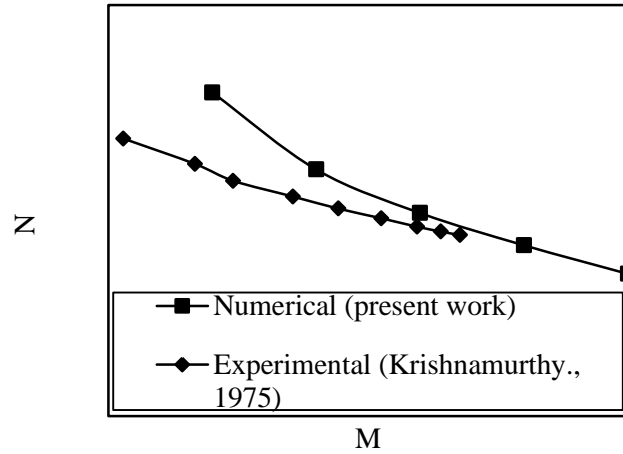
3. Results and Discussions

Water and diesel oil were used as the primary fluid and water, light and heavy oils were used as the secondary fluids. The effect of the area ratio and the throat length on the performance of the jet pump was studied by keeping water as the primary fluid and crude oil as the secondary fluid.

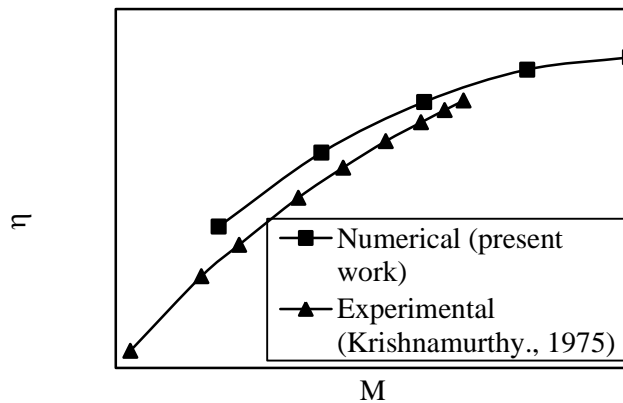
The numerical results of the base geometry (Table 1) were compared with the experimental results of Krishnamurthy [22] and are presented in figure 5. Trends of the numerical results match well with the experimental work for head ratio (N) with discharge ratio (M) (Fig. 5a) and efficiency (η) variations with discharge ratio (Fig. 5b) plots. However, the numerical results show an over estimation of the efficiency. This is because the experimental setup of Krishnamurthy [22] consisted a tee-joint which was not considered in the numerical work. The tee-joint was located at nozzle upstream pipe and the joint included a flow disturbance before fluid passes through the nozzle. Hence the jet pump produced less efficiency and head ratio. The work represents water as the primary and secondary fluids.

The numerical simulations were carried out to determine the performance of a jet pump when the density (γ) and viscosity (μ) of the primary fluids are different from those of the secondary fluids. The primary fluids used for the study was water and diesel oil. The secondary fluid or the reservoir fluid was considered to be water, light oil and heavy oil. The properties of the different fluids are given in Table 3. The different configurations of the jet pumps used for the study are shown in Table 4.

M vs N was plotted in Fig. 6 for the water as primary fluid. The secondary fluid viscosities (μ) were 1, 29.4, 150 and 350 cSt. It can be seen that the maximum head for a particular secondary fluid reduces as the viscosity and density increases. Figure 7 is a plot of M versus the η of the jet pump. The efficiency of the jet pump increases with the mass ratio up to a certain point and then it is seen to have a decreasing trend. This trend of the efficiency curve is completely in agreement with literatures which show that the jet pumps reaches its peak performance when the mass flow ratio is maintained in the range of 0.2 to 1.



a) M vs N



b) M vs η

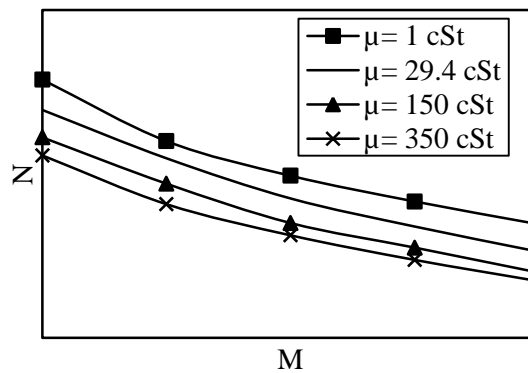
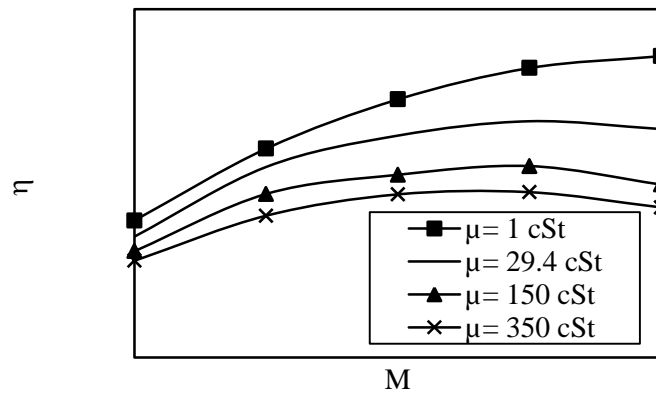
Fig. 5 Validation

Table 3 Properties of different fluids used for study

Fluid	γ	μ
Water	1000	1
Diesel oil	800	76
Light crude oil	850	29.4
Crude oil	900	150
Heavy Crude oil	960	350

Table 4 Details of different jet pump configurations used for study

Configurations	Variable values
1	$R=0.35, L_m/d_m=7.5$
2	$R=0.5, L_m/d_m=7.5$
3	$R=0.65, L_m/d_m=7.5$
4	$R=0.35, L_m/d_m=11$
5	$R=0.5, L_m/d_m=11$
6	$R=0.65, L_m/d_m=11$
7	$R=0.35, L_m/d_m=14.5$
8	$R=0.5, L_m/d_m=14.5$
9	$R=0.65, L_m/d_m=14.5$

**Fig. 6** Variation of head ratio with mass flow ratio at different viscosities of secondary fluid (Primary fluid: water)**Fig. 7** Variation of efficiency with mass flow ratio at different viscosities of secondary fluid (Primary fluid: water)

Diesel oil which is having viscosity of 76 cSt is the most common light oil primary fluid. Simulations were carried out for diesel oil as primary fluid and the secondary fluids ($\mu=1, 29.4, 150$ and 350 cSt.). Figure 8 shows that the maximum head achieved for a particular secondary fluid reduces as the viscosity and density of the secondary fluid increases. In Fig. 9, it can be observed that the efficiency of the jet pump increases with the mass flow ratio up to a certain point and then it becomes almost constant. This trend of the efficiency curve is completely in agreement with literatures which show that the jet pumps reaches its peak performance when the mass flow ratio is maintained between 0.2-1.

The effects of area ratio on the efficiency at different L_m/d_m ratio of the jet pumps are plotted in Fig. 10. The efficiency of the pump increases with R . Another noticeable feature is that the increase in the efficiency with the throat length. The role of the mixing tube in the jet pumps is to mix the primary and secondary fluids properly before the combined flow reaches the diffuser. Previous research [3] indicates that the jet pump will perform well if the mixing tube length is 5-15 times the mixing tube diameter. Based on our study, it was found that the maximum efficiency was obtained at $L_m/d_m=14.5$ and $R=0.65$. Efficiency can be increased by further increase in R and L_m/d_m .

The Fig. 11 shows the turbulence effects once the primary and secondary fluids meet. Turbulence length scale which is a physical quantity related to the size of the large eddies containing energy in turbulent flows decreases gradually through the throat section. The energy transfers from the motive stream to the propelled stream quickly. In water-crude oil flow, the turbulent energy is high at the throat entrance. This results in a quick and more uniform mixing of the primary and secondary fluid. But the turbulence when the secondary fluid is water is lesser and this makes the mixing process slower. Hence a longer mixing tube is needed when the viscosity of the secondary fluid increases.

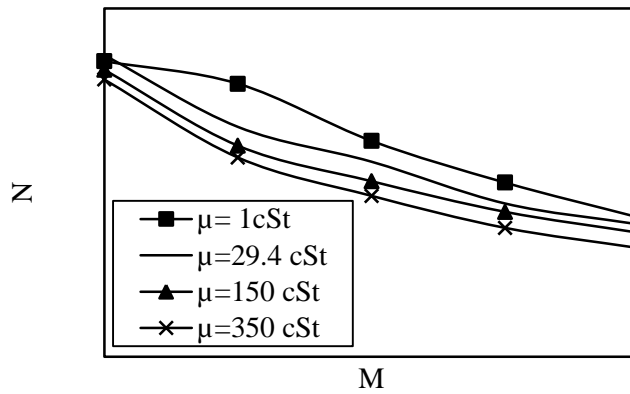


Fig. 8 Variation of head ratio with mass flow ratio at different viscosities of secondary fluid (Primary fluid: diesel oil, $\mu= 76$ cSt)

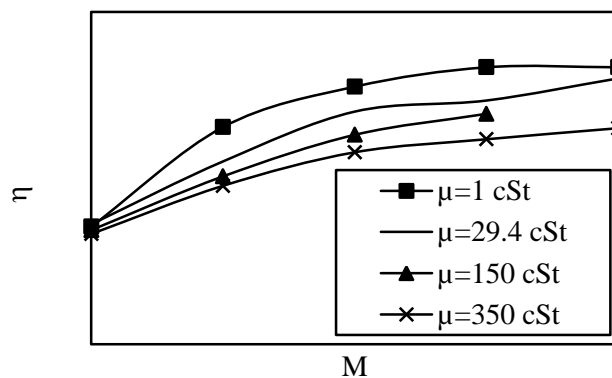


Fig. 9 Variation of efficiency with mass flow ratio at different viscosities of secondary fluid (Primary fluid: diesel oil, $\mu= 76$ cSt)

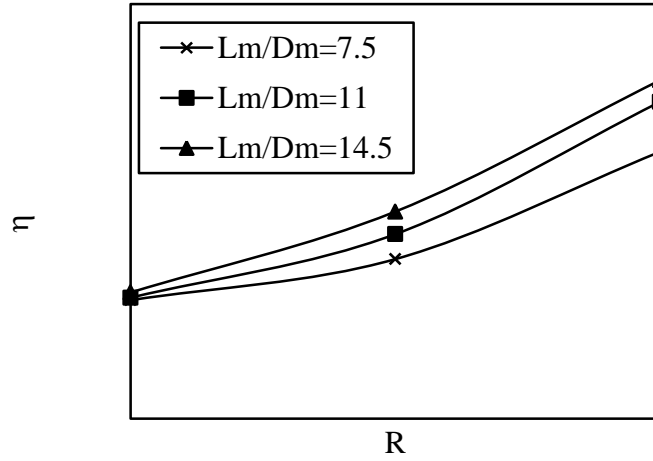


Fig. 10 Effect of area ratio on efficiency at different L_m/d_m ratio

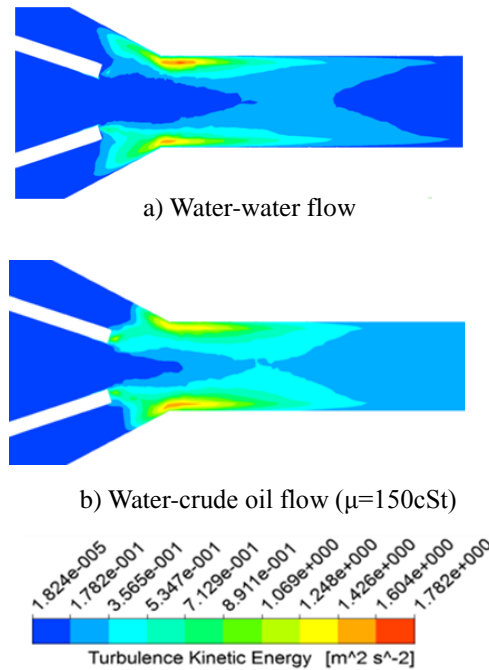


Fig. 11 Turbulence contour at throat entrance for water-water and water-heavy oil ($\mu=150\text{cSt}$) flow.

4. Conclusions

The effects of different fluids and geometric parameters on the jet pump performance were studied by numerical simulations. The maximum suction head for a secondary fluid reduced as the viscosity and density increases. The efficiency of the jet pump increased with the mass flow ratio increase upto 20 to 30% further increasing mass flow rate decreases the efficiency. The mass flow ratio between 0.2-1 produced maximum efficiency. The light oil produced lesser efficiency than the water as primary fluid. The major losses arise due to incomplete mixing before the combined fluid reaches the diffuser. Increasing the mixing tube length can compensate the losses occurring during the mixing of the two fluids. Longer mixing tubes is favorable only at higher discharge ratios but at lower discharge ratios, the usage of longer mixing tubes leads to higher skin friction losses. An area ratio of 0.65 and $L_m/d_m=14.5$ produced maximum efficiency.

Nomenclature

Abbreviations

AL	Artificial lift
ESP	Electric submersible pumps
PCP	Progressive cavity pump
RANS	Reynolds averaged Navier Stokes
RMS	Root mean square
SRP	Sucker rod pump

Notations

d	Diameter [mm]
L	Length [mm]
M	Mass flow ratio, $(\frac{Q_s}{Q_n})\frac{Q_s}{Q_n}$
N	Head ratio, $(\frac{P_d - P_s}{P_n - P_d})$
P	Pressure [N/m ²]
ΔP	Pressure drop, $(P_{in} - P_{out})$ [N/m ²]
Q	Volume flow rate [m ³ /s]
R	Area ratio,

$$\frac{c/s \text{ area of the nozzle}}{c/s \text{ area of the throat}}$$

R_v	Viscosity ratio $\frac{\mu_s}{\mu_n}$
S	Setback distance [mm]
t	Wall thickness [mm]
α	Inner cone angle of suction nozzle [°]
β	Inner cone angle of diffuser [°]
γ	Specific gravity [kg/m ³]
η	Efficiency, (M*N)
μ	Viscosity [cSt]
θ	Inner cone angle of driving nozzle [°]
ρ	Density [kg/m ³]

Subscripts

d	Diffuser outlet
in	Inlet
l	Light oil
m	Mixing tube
n	Driving nozzle
out	Outlet
p	Reservoir or secondary fluid
s	Suction nozzle

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