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# An Expected Loss Model for FMEA under Periodic Monitoring of Failure Causes

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# FMEA에서 주기적인 고장원인 감시 하의 기대손실 모형

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In FMEA, occurrence and detectability are not related to only failure modes itself but also their causes. It is assumed that any failure occurs after at least one cause corresponding to failure occurs in advance. Occurrence of the failure mode is described by occurrence time of its cause and elapsed time to the actual failure. Under the periodic monitoring plan, the monitoring interval is another factor to determine the detectability and occurrence of each failure mode. When a failure cause occurs, the failure does not occur if the cause is identified and remedied before it actually occurs. Under this situation, we construct an economic model for prioritizing failure modes. The loss function is based on the unfulfilled mission period. We also provide an optimal monitoring plan with an illustrative example.

***Keywords:*** FMEA, RPN, Time Dependent Loss Model

## 1. Introduction

Correct evaluation of failure risk is an important part toward the efficiency of a firm's resource allocation. In industrial practices, firms have utilized FMEA (failure mode and effect analysis) as a means to estimate the risk of system failure and to provide an appropriate way of reducingits impact on the end customer. In conventional FMEA, criticality of failure risk is measured by the metric called the Risk Priority Number (RPN), which is a metric obtained by multiplying the ratings of severity, occurrence, and detectability of each failure. But the rating on each of the three components is usually based on the past experiences and intuition of the FMEA team. Many authors including Eubanks *et al*. (1997), Blivbamd *et al*. (2004), Bertolini *et al*. (2006), and Jeegadeshan *et al*. (2007) tried to find out improved methods that complement the conventional FMEA. Senol (2007) suggested a Poisson approach to determine the occurrence degree in failure mode and reliability analysis. Narayanagounder and Karuppusami (2009) and Sawhney *et al*. (2010) presented summary to previous efforts to improve the RPN prioritization method. Agung and Kwon (2010) suggested an expected loss model for improving risk prioritization in the conventional FMEA.

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In FMEA, the risk depends on the severity, detectability, and occurrence of failure modes and causes. The conventional FMEA and most of the previous works do not consider the role of time in evaluating the risk of failure. The failure with higher occurrence rating will occur at an earlier time than those with lower occurrence rating in the conventional FMEA. When we take time into consideration, it may be reasonable to assume that any failure can occur only after at least one of its causes has occurred in advance. Thus, a failure will not occur if its causes are detected and corrected before the failure itself occurs. In this situation, the periodic monitoring may be a good policy to prevent the system failure. The monitoring interval will affect detectability of failure causes.More frequent monitoring will prevent more effectively the failure from occurring. But monitoring activity itself requires cost and too frequent monitoring may not be cost effective. When deciding a monitoring policy, it will be desirable to consider both the expected loss due to the failure occurrence and the monitoring cost.

In this study, we extend the expected loss model of Agung and Kwon (2010) to the case where failure is dependent on time and the system is periodically monitored for preventing failure during its mission period. The loss due to each failure mode is assumed to depend on the remaining mission period of the system. If there are no failures during the mission period, there will be no losses at all. The risk of a failure is evaluated by the size of its resultant expected loss assuming that the elementary cost information is available or at least can be estimated. The optimal monitoring policy is determined on the basis of the expected loss and monitoring cost. This paper is organized as follows. In Section 2, the time distributions of a failure and its causes under periodic monitoring are derived assuming the Homogeneous Poisson Process (HPP) for failure and cause occurrence. In Section 3, the time dependent expected loss model is constructed for quadratic loss function. In Section 4, an optimal monitoring policy is presented and analyzed with an illustrative example. Section 5 concludes with possible future extensions for further studies.

## 2. The Failure Time Distribution

The occurrence of a failure may be described by the time elapsed from the beginning of system operation to the system failure. Every failure is supposed to have its own cause or causes that make it to occur. Occurrence of any failure comes after occurrence of one or more of its causes. Thus, the failure time is composed of two components; the cause occurrence time and its corresponding failure occurrence time.

Now let $X_{ki}$ be the time elapsed until the $i^{th}$ cause, i = 1, 2, ⋯, $n_k$ of failure mode k, k = 1, 2, ⋯, l occurs. Assume that each cause occurs at a constant rate $\lambda_{ki}$ over time. Then the probability density function of $X_{ki}$ will be

$$f_{X_{ki}}(x) = \lambda_{ki} e^{-\lambda_{ki} x},\ 0 < x \tag{1}$$

Next, let $Y_{ki}$ be the time elapsed until failure k occurs from the occurrence time point of its $i^{th}$ cause. Assume the occurrence rate of failure k due to its $i^{th}$ cause is constant over time, say $\mu_{ki}$. Then the probability density function of $Y_{ki}$ will be

$$f_{Y_{ki}}(y) = \mu_{ki} e^{-\mu_{ki} y},\ 0 < y. \tag{2}$$

Note that $X_{ki}$ and $Y_{ki}$ are assumed to be independent even if they are commonly related with the $i^{th}$ cause of failure mode k. The characteristic of the $i^{th}$ cause of failure mode k may affect both of the occurrence rates $\lambda_{ki}$ and $\mu_{ki}$.But it does not mean the occurrence times $X_{ki}$ and $Y_{ki}$ are statistically dependent.

Suppose the mission period of the system is (0, T] and the system is periodically monitored at an interval h = T/n as depicted in <Figure 1>. At each monitoring, every failure cause already occurred is assumed to be detected and corrected immediately.


**Figure 1.** Mission period and monitoring

Let X be the time to occurrence of the $i^{th}$ cause of the failure mode k from the time point of the most recent monitoring as depicted in <Figure 2>. Then, by the memoryless property of the exponential distribution, X will be exponentially distributed with failure rate $\lambda_{ki}$.

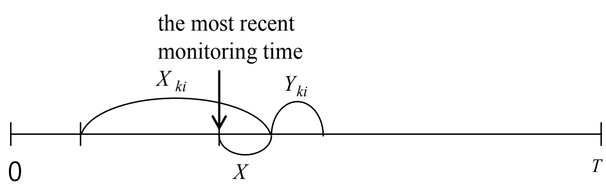


**Figure 2.** The occurrence of failure and its cause under periodic monitoring

Let $W_{ki}$ be the time elapsed until failure k occurs due to its $i^{th}$ cause from the beginning time point of the latest monitoring interval. Then

$$W_{ki} = X + Y_{ki} \tag{3}$$

and the distribution function can be obtained as

$$F_{W_{ki}}(w) = 1 - \frac{1}{\lambda_{ki} - \mu_{ki}} \left\{ \lambda_{ki} e^{-\mu_{ki} w} - \mu_{ki} e^{-\lambda_{ki} w} \right\}. \tag{4}$$

See <Appendix> for detailed derivation. The probability density function of $W_{ki}$ is

$$f_{W_{ki}}(w) = \frac{\lambda_{ki}\mu_{ki}}{\lambda_{ki}-\mu_{ki}}\left\{e^{-\mu_{ki}w} - e^{-\lambda_{ki}w}\right\}, \quad 0 < w. \tag{5}$$

Since there are $n_k$ causes for failure mode k, the time to occurrence of the failure k from the beginning time point of the latest monitoring interval is $W_k = \min\{W_{k1}, W_{k2}, \cdots, W_{kn_k}\}$. Assume i) the causes of failure mode k occur mutually independently and ii) the time to failure k due to the cause i = 1, 2, $\cdots$, $n_k$, i.e. $Y_{k1}, Y_{k2}, \cdots, Y_{kn_k}$ are mutually independent. Then $W_{k1}, W_{k2}, \cdots, W_{kn_k}$ will be mutually independent and we obtain

$$\begin{aligned} \Pr[W_k > w] &= \prod_{i=1}^{n_k} \Pr[W_{ki} > w] \\ &= \prod_{i=1}^{n_k} \frac{1}{\lambda_{ki}-\mu_{ki}}\left\{\lambda_{ki}e^{-\mu_{ki}w} - \mu_{ki}e^{-\lambda_{ki}w}\right\}, \end{aligned} \tag{6}$$

$$F_{W_k}(w) = 1 - \Pr[W_k > w]. \tag{7}$$

# 3. The Time Dependent Expected Loss

The risk of each failure mode may be reasonably evaluated by its resultant loss. The loss will be incurred if the system or process fails during its mission time duration (0, T]. If the system does not fail during (0, T], no loss is confronted. The loss function may be reasonably assumed to be a non-decreasing function of the length of the remaining mission time period. Under this assumption, the loss function may be constant, linearly increasing, or more severely increasing as the unfulfilled mission time period increases. A constant loss function implies that there is no need for preventive monitoring action, while a linear loss function presumes that delayed corrective action makes no difference with earlier action. These two types of loss function are somewhat unrealistic in the practical situations. Here, we are interested in the third case and take a quadratic loss function as a popular type, which seems to reflect the real situation better.

Let $M_k$ be the number of monitoring intervals until the failure mode k first occurs including the interval of failure occurrence. <Figure 3> depicts the case of $M_k = m$. Then the probability mass function of $M_k$ is given by

$$\Pr[M_k = m] = \left\{1 - F_{W_k}(h)\right\}^{m-1} F_{W_k}(h), \quad m = 1, 2, \cdots \tag{8}$$

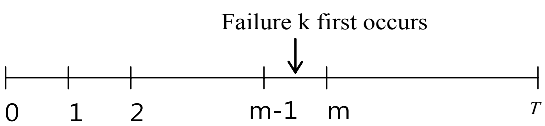


**Figure 3.** Occurrence of failure under periodic monitoring

Given $M_k = m$, the length of the remaining mission period will be $(n - m + 1)h - W'_k$ for $m \le n$ and 0 for $m > n$, where the probability density function of $W'_k$ is

$$f_{W'_k}(w) = \frac{f_{W_k}(w)}{F_{W_k}(h)}, \quad 0 \le w < h. \tag{9}$$

This situation is illustrated in <Figure 4>.

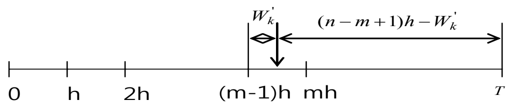


**Figure 4.** The remaining mission period

Now the loss function with $M_k = m$ will be obtained by

$$L_k = \begin{cases} \alpha_K\{(n-m+1)h - W'_k\}^2, & m \le n \\ 0, & m > n+1 \end{cases} \tag{10}$$

Thus, the conditional expected loss of failure k given $M_k = m$ can be obtained by

$$E[L_k | M_k = m] = \int_0^h \alpha_k\{(n-m+1)h - w\}^2 \frac{f_{W_k}(w)}{F_{W_k}(h)} dw. \tag{11}$$

Equation (11) can be rewritten as

$$E[L_k | M_k = m] = \alpha_k\{(n-m+1)^2h^2 - 2(n-m+1)hQ_1(h) + Q_2(h)\}, \tag{12}$$

where $Q_1(h) = h - \frac{1}{F_{W_k}(h)}\int_0^h F_{W_k}(w)dw$ and $Q_2(h) = h^2 - \frac{2}{F_{W_k}(h)}\int_0^h wF_{W_k}(w)dw$. The expected loss due to failure mode k can be obtained as

$$\begin{aligned} E[L_k] = {} & \alpha_k[(n+1)^2h^2 - 2(n+1)hQ_1(h) + Q_2(h)] \\ & \times\left\{1 - \left[1 - F_{W_k}(h)\right]^n\right\} \\ & - 2\alpha_k h[(n+1)h - Q_1(h)]S_1(h) \\ & + \alpha_k h^2\left[S_1(h) + \left(1 - F_{W_k}(h)\right)S_2(h)\right], \end{aligned} \tag{13}$$

where

$$\begin{aligned} S_1(h) &= \sum_{m=1}^{n} m[1 - F_{W_k}(h)]^{m-1}F_{W_k}(h) \\ &= \frac{1 - \left[1 - F_{W_k}(h)\right]^n}{F_{W_k}(h)} - n[1 - F_{W_k}(h)]^n \text{ and} \end{aligned}$$

$$\begin{aligned} S_2(h) &= \sum_{m=1}^{n} m(m-1)[1 - F_{W_k}(h)]^{m-2}F_{W_k}(h) \\ &= \frac{1}{[F_{W_k}(h)]^2}\left\{2 - [1 - F_{W_k}(h)]^n\left[2 + \frac{nF_{W_k}(h)\left(2 + (n-1)F_{W_k}(h)\right)}{1 - F_{W_k}(h)}\right]\right\}. \end{aligned}$$

See <Appendix> for detailed derivation.

## 4. The Optimal Monitoring Interval

Once a failure occurs, it takes much time and cost for remedy. But the cause is detected before the failure actually occurs, it usually does not take much time and cost for correction. When the cause is detected before the corresponding failure occurs, immediate corrective action is assumed to be taken without any loss.If we set the monitoring interval shorter, we can reduce the occurrence of failure and eventually the expected loss due to the failure. On the other hand, however, the monitoring cost will increase. Since there are l failure modes, the expected total cost with the monitoring interval $h = T/n$ will be

$$\mathrm{ETC} = \sum_{k=1}^{l} E(L_k) + nC, \tag{14}$$

where C is the unit inspection cost per monitoring. Here, the optimization problem is to decide n that minimizes the expected total cost. We cannot get the optimal value of h or n in a closed form. But the numerical solution can be obtained for given parameters using an appropriate computer program.

***Example*** **:** *Suppose that the system under study has only three failure modes with two possible causes for each. The distribution and cost parameters are given as <Table 1>. The mission period is assumed to be 100 years.*

**Table 1.** The distribution and cost parameters

| Failure Mode | Cause | λ | μ | $\alpha_k$ |
|---|---|---|---|---|
| 1 | 1 | 0.0010 | 0.03 | 40 |
| | 2 | 0.0020 | 0.035 | |
| 2 | 1 | 0.0015 | 0.015 | 20 |
| | 2 | 0.0025 | 0.025 | |
| 3 | 1 | 0.0030 | 0.01 | 10 |
| | 2 | 0.0035 | 0.02 | |

*<Figure 5> depicts ETC versus n for the monitoring inspection cost C = \$500 thousand, \$1,000 thousand, and \$2000 thousand. The optimal value n* of n is that which minimizes ETC. As can be seen in Figure 1, if C takes smaller value, n* becomes larger, which implies more frequent inspection for monitoring. This is consistent with our intuition. More specifically, n* is 28, 19, and 12 for C = \$500 thousand, \$1,000 thousand, and \$2000 thousand, respectively.*

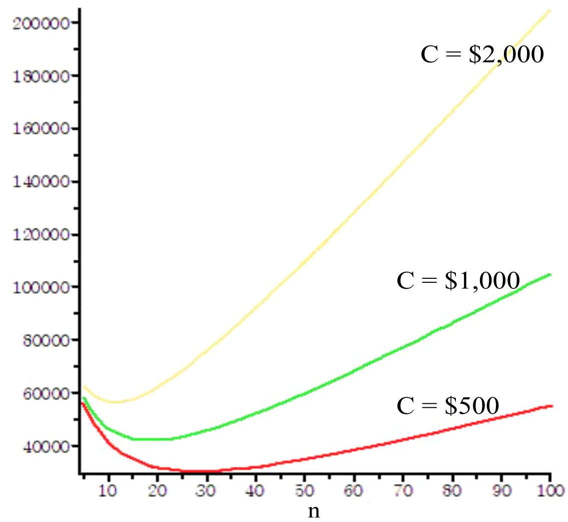


**Figure 5.** Graph of ETC versus n

## 5. Conclusion

The optimal monitoring inspection interval is obtained when failure occurs over the time span. For occurrences of failures and their causes, a homogeneous Poisson process is assumed. The system is assumed to be periodically monitored for preventing failure during its mission period. The loss due to each failure is assumed to depend on the remaining mission period of the system. If there are no failures during the mission period, there will be no losses at all. The risk of a failure is evaluated by the size of its resultant expected loss assuming that the elementary cost information is available or at least can be estimated. The optimal monitoring policy is determined on the basis of the expected loss and monitoring cost.

A numerical example shows that more frequent inspection is required if the cost of monitoring inspection becomes smaller. The opposite is true if the inspection cost become larger. Also, larger losses due to failures require frequent inspection. The results are consistent with reasonable intuition.

The study can be extended to the case of non-homogeneous Poisson process for failure and cause occurrence. And other types of monitoring policy may also be considered such as a non uniform monitoring interval.

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# <Appendix 1> Proof of (4)

Since $X_{ki}$ and $Y_{ki}$ are mutually independent, their joint probability density function is given by

$$f_{X_{ki},Y_{ki}}(x, y) = \lambda_{ki}\mu_{ki}e^{-\lambda_{ki}x-\mu_{ki}y},\ 0 < x,\ 0 < y. \tag{A1}$$

If we change variables as $W_{ki} = X_{ki} + Y_{ki}$ and $X_{ki} = X_{ki}$, the joint probability density function of $X_{ki}$ and $W_{ki}$ will be

$$f_{X_{ki},W_{ki}}(x,w) = \lambda_{ki}\mu_{ki}e^{-\lambda_{ki}x-\mu_{ki}(w-x)} = \lambda_{ki}\mu_{ki}e^{-(\lambda_{ki}-\mu_{ki})x-\mu_{ki}w},\ 0 < x < w. \tag{A2}$$

The probability density function of $W_{ki}$ is obtained by integrating (A2) with x over its range, that is,

$$f_{W_{ki}}(w) = \int_0^W \lambda_{ki}\mu_{ki}e^{-(\lambda_{ki}-\mu_{ki})x-\mu_{ki}W}dx = \frac{\lambda_{ki}\mu_{ki}}{\lambda_{ki}-\mu_{ki}}\left\{e^{-\mu_{ki}w}-e^{-\lambda_{ki}w}\right\},\ 0 < w.$$

Thus, we get Equation (4).

# <Appendix 2> Proof of (13)

$$\begin{aligned}
E[L_k] &= E\left[E[L_k|M_k]\right] \\
&= \sum_{m=1}^{n}\alpha_k\left\{(n-m+1)^2h^2-2(n-m+1)hQ_1(h)+Q_2(h)\right\}\Pr[M_k=m] \\
&= \sum_{m-1}^{n}\alpha_k\left\{[(n+1)^2h^2-2(n+1)hQ_1(h)+Q_2(h)]-2[(n+1)h-Q_1(h)]hm+h^2m^2\right\}\times\left\{1-F_{W_k}(h)\right\}^{m-1}F_{W_k}(h) \\
&= \alpha_k[(n+1)^2h^2-2(n+1)hQ_1(h)+Q_2(h)]\times\sum_{m=1}^{n}[1-F_{W_k}(h)]^{m-1}F_{W_k}(h) \\
&\quad -2\alpha_k[(n+1)h-Q_1(h)]h\sum_{m=1}^{n}m[1-F_{W_k}(h)]^{m-1}F_{W_k}(h)+\alpha_kh^2\sum_{m=1}^{n}m^2[1-F_{W_k}(h)]^{m-1}F_{W_k}(h)
\end{aligned}$$

Put $F_{W_k}(h) = p$ and $S_0 = \sum_{m=1}^{n}[1-p]^m p$. Then it can be easily shown that

$$S_0 = (1-p)[1-(1-p)^n].$$

Taking the derivative of $S_0$ with respect to p, we obtain

$$-\sum_{m=1}^{n}m(1-p)^{m-1}p+\sum_{m=1}^{n}(1-p)^{m-1} = -[1-(1-p)^n]+n(1-p)^n.$$

Thus, we get

$$S_1 = \sum_{m=1}^{n}m(1-p)^{m-1}p = \frac{1}{p}[1-(1-p)^n]-n(1-p)^n.$$

Similarly, $S_2 = \sum_{m=1}^{n} m(m-1)(1-p)^{m-2} p = \frac{1}{p^2}\left\{2-(1-p)^n\left[2+\frac{np(2+(n-1)p)}{1-p}\right]\right\}$ and $\sum_{m=1}^{n} m^2 (1-p)^{m-1} p = S_1 + (1-p)S_2$.

By replacing p with $F_{W_k}(h)$, we obtain

$$S_1(h) = \sum_{m=1}^{n} m[1-F_{W_k}(h)]^{m-1} F_{W_k}(h) = \frac{1-[1-F_{W_k}(h)]^n}{F_{W_k}(h)} - n[1-F_{W_k}(h)]^n$$

and $S_2(h) = \sum_{m=1}^{n} m(m-1)[1-F_{W_k}(h)]^{m-2} F_{W_k}(h) = \frac{1}{[F_{W_k}(h)]^2}\left\{2-[1-F_{W_k}(h)]^n\left[2+\frac{nF_{W_k}(h)\left(2+(n-1)F_{W_k}(h)\right)}{1-F_{W_k}(h)}\right]\right\}$

And finally, we obtain equation (13).