Industrial Engineering
& Management Systems
Vol 12, No 1, March 2013, pp.23-29
ISSN 1598-7248 │ EISSN 2234-6473 │
http://dx.doi.org/10.7232/iems.2013.12.1.023

Inventory Models for Fresh Agriculture Products with Time-Varying Deterioration Rate

Yufu Ning*
School of Information Engineering, Shandong Youth University of Political Science, Jinan, China

Lixia Rong, Jianjun Liu
Department of Computer Science, Dezhou University, Dezhou, China

(Received: August 9, 2012 / Revised: December 1, 2012 / Accepted: March 5, 2013)

ABSTRACT

This paper presents inventory models for fresh agriculture products with time-varying deterioration rate. Due to the particularity of fresh agriculture products, the demand rate is a function that depends on sale price and freshness. The deterioration rate increases with time and is assumed to be a time-varying function. In the models, the inventory cycle may be constant or variable. The optimal solutions of models are discussed for different freshness and the deterioration rate. The results of experiments show that the profit depends on the freshness and deterioration rate of products. With the increasing inventory cycle, the sale price and profit increase at first and then start decreasing. Furthermore, when the inventory cycle is variable, the total profit is a binary function of the sale price and inventory cycle. There exist unique sale price and inventory cycle such that the profit is optimal. The results also show that the optimal sale price and inventory cycle depend on the freshness and the deterioration rate of fresh agriculture products.

Keywords: Inventory Model, Demand Rate, Time-Varying, Deterioration Rate, Fresh Agriculture Products

* Corresponding Author, E-mail: yufuning@163.com

1. INTRODUCTION

Fresh agriculture products such as vegetables, fruits, and living animals are perishable goods. The study of inventory systems for fresh agriculture products can learn from perishable goods inventory. In recent years, many researchers have studied the deteriorating items inventory model. Ghare and Schrader (1963) established the classical no-shortage inventory model with the constant rate of decay at first. Resh *et al.* (1976) and Donaldson (1977) studied a model with linearly time-varying demand.

Classical deterministic inventory models assume the demand rate to be either constant or time-dependent. However, for fresh agriculture product, the demand rate may be influenced by the price and freshness. Pal *et al.* (2006) studied the inventory model for deteriorating items with the demand rate dependent on displayed stock level and product price. A production planning model for a deteriorating rate with stochastic demand and consumer choice was presented by Lodree and Uqochukwu (2008). Chen and Dan (2009) studied the inventory model for fresh agriculture products when the demand depends on freshness and price.

In the study of deterministic inventory models, the inventory level depends on the demand rate and deterioration rate. Balkhi and Benkherouf (1996) presented a production lot size inventory model for deteriorating items and arbitrary production and demand rates. An inventory replenishment policy for deteriorating items with shortages and partial backlogging was studied by Wang (2002). He *et al.* (2010) proposed an optimal production-inventory model for deteriorating items with multiple-market demand. Some optimal inventory replenishment models for deteriorating items taking account of time discounting were developed by Chung and Lin (2001). Wu *et al.* (2006) presented an optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Some inventory models for perishable items with the inventory level dependent on demand rate were presented by Duan *et al.* (2012). Lee and Dye (2012) developed an inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. Alamri (2011) presented a unified general inventory model for integrated production of new items and remanufacturing of returned items for an infinite planning horizon. They considered a production environment that consisted of three shops. The first one was for remanufacturing returned items, the second was for manufacturing new items, while the third was for collecting returned items to be remanufactured in the first shop. The system was subject to joint production and remanufacturing options. Dye (2012) considered a deterministic economic order quantity (EOQ) model with generalized type demand, deterioration and unit purchase cost functions under two levels of trade credit policy. The objective was to find the optimal values of selling prices, replenishment number and replenishment scheme which maximized the total profit over the finite planning horizon. Sarkar (2012) dealt with an EOQ model for the finite replenishment rate where the demand and deterioration rate were both time-dependent. In this model, the retailers were allowed a trade-credit offer by the suppliers to buy more items with different discount rates on the purchasing costs. During the credit period, the retailers could earn more by selling their products. The interest on purchasing cost was charged for the delay of payment by the retailers. Some of the items might deteriorate in the course of time. Sarkar and Sarkar (2013a) dealt with an inventory model for deteriorating items with inventory dependent demand function. They considered the time-varying deterioration rate. Based on the demand and inventory, the model was considered for three possible cases. They established the necessary and sufficient conditions for each case to show the existence and uniqueness of the optimal solution. Furthermore, a simple solution algorithm has been proposed to obtain the optimal replenishment cycle time and ordering quantity such that the total profit per unit time was maximized. Sarkar and Sarkar (2013b) expanded an inventory model for deteriorating items with stock-dependent demand. This model provided the time-varying backlogging rate as well as the time-varying deterioration rate. The aim of this model was to determine the optimal cycle length of each product such that the expected total cost (holding, shortage, ordering, deterioration, and opportunity cost) was minimized. Moreover, the necessary and sufficient conditions were provided to show the existence and uniqueness of the optimal solution. Sarkar *et al.* (2013) developed an EOQ model for the finite production rate and deteriorating items with time-dependent increasing demand. The component cost and the selling price were considered at a continuous rate of time. The objective of this model was to maximize the total profit over the finite planning horizon. Gumasta *et al.* (2012) mapped the transportation model with the inventory model with time-varying demand and two types of customers for different perishable goods to simultaneously maximize the revenue and minimize the transportation and inventory cost, and hence maximizing the net profit.

Apparently, the deterioration rate increases with increasing time. In this paper, fresh agricultural products as a special category of perishable products are studied. The attenuation law of freshness is described with an exponential function, and the deterioration rate is assumed as a time-varying function. We present the inventory models for fresh agriculture products with a time-varying deterioration rate in order to provide the basis for the ordering of supermarkets.

The rest of the paper is organized as follows. Section 2 makes some assumptions and notations of fresh agriculture products inventory. In Section 3, the inventory models for fresh agriculture product with time-varying deterioration rate are presented. The models are solved and the results are analyzed in Section 4. Finally, Section 5 draws some conclusions and suggestions.

2. NOTATIONS AND ASSUMPTIONS

The notations used in this paper are given as below.

T : the length of the inventory cycle
Q : the optimal ordering quantity
A : the fixed cost per order
c : the inventory holding cost per unit per unit time
p : the sale price per unit
p_1 : the purchase price per unit
θ : the initial freshness of product
$I(t)$: the level of inventory at time t, $0 \le t \le T$
$\lambda(t)$: the deterioration rate at time t, $0 \le t \le T$

Before introducing the inventory models for fresh agriculture products with a time-varying deterioration rate, we make the following assumptions.

1) Consumers buy fresh agriculture products every day, shortage is not allowed, and the inventory cycle is T.
2) Fresh agriculture products are necessities. The demand of fresh agriculture products depends on their price and freshness. In this paper, $D = a - bp$ represents the inverse relationship of demand and price, $\theta(t) = \theta^t$ is the decay function of freshness with $0 < \theta < 1$, and $D(t)$ is the demand rate function,

$$D(t) = (a - bp)\theta^t,\ 0 \le t \le T. \quad (1)$$

3) Fresh agriculture products are perishable, the deterioration rate changes with time. The deterioration rate is $\lambda(t) = \alpha t (\alpha > 0)$ with $0 < \lambda(t) < 1$.

3. MODEL FORMULATION

By the assumptions in the previous section, the inventory level for fresh agriculture products with time-varying deterioration rate should satisfy the following differential equation:

$$\frac{dI(t)}{dt} = -(a-bp)\theta^t - \alpha t I(t),\ 0 \le t \le T. \tag{2}$$

Solving the differential equation with boundary condition $I(T)=0$, we have

$$I(t) = (a-bp)\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dx,\ 0 \le t \le T. \tag{3}$$

The maximum inventory level per cycle, in other words, the optimal ordering quantity is

$$Q = I(0) = (a-bp)\int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt.$$

The total number of inventory holding per cycle is

$$H = \int_0^T I(t)dt = (a-bp)\int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt.$$

The total sale number per cycle is

$$S = \int_0^T (a-bp)\theta^t dt.$$

Therefore, the total profit per cycle is

$$\begin{aligned} C(p,T) &= pS - A - p_1Q - cH \\ &= p(a-bp)\int_0^T \theta^t dt - A - p_1(a-bp)\int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt \\ &\quad - c(a-bp)\int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt. \end{aligned} \tag{4}$$

3.1 Model for Constant Inventory Cycle

If T is constant, the formula (4) is equivalent to

$$\begin{aligned} C(p) &= p(a-bp)\int_0^T \theta^t dt - A - p_1(a-bp) \\ &\quad \int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt - c(a-bp)\int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt. \end{aligned} \tag{5}$$

Theorem 1. If T is constant, there exists a unique p^* such that $C(p^*)$ is the maximum of $C(p)$.

Proof. It is obvious that $C(p)$ is a univariate function of p, then

$$\frac{dC(p)}{dp} = (a-2bp)\int_0^T \theta^t dt + p_1 b \int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt + cb\int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt.$$

Since

$$\frac{d^2C(p)}{dp^2} = -2b\int_0^T \theta^t dt < 0,$$

formula (5) has a maximum. Let $\frac{dC(p)}{dp}=0$, there exists a unique p^* maximizing $C(p)$ with

$$p^* = \frac{a}{2b} + \frac{\ln\theta}{2(\theta^T-1)}\left(p_1\int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt + c\int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt\right). \tag{6}$$

The proof is complete.

3.2 Model for Variable Inventory Cycle

Theorem 2. If T is variable, there exists a unique (p^*, T^*) such that $C(p^*, T^*)$ is the maximum of $C(p,T)$.

Proof. If T is variable, let

$$\frac{\partial C(p,T)}{\partial p} = (a-2bp)\int_0^T \theta^t dt + p_1 b\int_0^T \theta^t e^{\frac{\alpha}{2}t^2}dt + cb \int_0^T\int_t^T \theta^x e^{\frac{\alpha}{2}(x^2-t^2)}dxdt = 0,$$

and

$$\frac{\partial C(p,T)}{\partial T} = (a-bp)\theta^T(p - p_1 e^{\frac{\alpha}{2}T^2} - c\int_0^T e^{\frac{\alpha}{2}(T^2-t^2)}dt) = 0,$$

then we can get the stable point (p_0, T_0).

Let H' be the Hessian matrix of $C(p,T)$. Since

$$\frac{\partial C^2(p,T)}{\partial p^2}|_{(p_0,T_0)} = -2b\int_0^{T_0}\theta^t dt < 0,$$

$$\frac{\partial C^2(p,T)}{\partial T^2}|_{(p_0,T_0)} = (a-bp_0)\theta^{T_0}[p_0 ln\theta - p_1 e^{\frac{\alpha}{2}T_0^2}(ln\theta + \alpha T_0) - c((ln\theta+\alpha T_0)\int_0^{T_0} e^{\frac{\alpha}{2}(T_0^2-t^2)}dt + 1)] < 0,$$

and

$$\frac{\partial C^2(p,T)}{\partial p\partial T}|_{(p_0,T_0)} = (a-2bp_0)\theta^{T_0} + p_1 b\theta^{T_0}e^{\frac{\alpha}{2}T_0^2} + cb\theta^{T_0}\int_0^{T_0} e^{\frac{\alpha}{2}(T_0^2-t^2)}dt,$$

we have

$$[H']|_{(p_0,T_0)} > 0,$$

which implies that H' is negative definite, and there exists a unique vector $(p^*, T^*) = (p_0, T_0)$ such that maximizing $C(p,T)$. The proof is complete.

4. MODEL SOLUTION AND ANALYSIS

In this section, we will solve the models and analyze the optimal when parameters are varying.

4.1 Solving Model for Constant Inventory Cycle

When inventory cycle T is constant, it is easy to obtain the optimal ordering quantity

$$Q^* = (a - bp^*)\int_0^T \theta^t e^{\frac{\alpha}{2}t^2}\, dt. \quad (7)$$

Tables 1–5 illustrate the results with different freshness and deterioration rate vector when inventory cycle T is 8, 10, 15, 20 or 25. Set a = 50, b = 10, and $c = 0.1$. Because the purchase price is different with the changing of freshness, we assume $p_1 = \theta^2$.

As shown in Tables 1–5, if the inventory cycle and freshness are the same, with the decreasing of deterioration rate, the sale price is decreasing while the profit is increasing. As a result, the sale price and profit depend on the deterioration rate of product. The deterioration rate is smaller, the lost value is smaller, and the profit is higher.

Table 1. The results when θ and α changing and $T = 8$

θ	α	p^*	Q^*	C^*
0.90	0.01	2.71	175.55	185.55
0.90	0.005	2.70	130.09	185.37
0.90	0.001	2.69	126.00	190.40
0.85	0.01	2.80	106.56	164.73
0.85	0.005	2.79	102.96	169.01
0.85	0.001	2.78	100.26	172.20
0.80	0.01	2.85	85.96	148.59
0.80	0.005	2.83	83.53	151.34
0.80	0.001	2.82	81.71	153.42
0.75	0.01	2.85	71.06	132.89
0.75	0.005	2.84	69.38	134.69
0.75	0.001	2.83	68.11	136.04
0.70	0.01	2.83	59.90	118.61
0.70	0.005	2.82	58.71	119.78
0.70	0.001	2.82	57.81	120.65

Table 2. The results when θ and α changing and $T = 10$

θ	α	p^*	Q^*	C^*
0.90	0.01	2.80	155.71	189.26
0.90	0.005	2.78	146.58	201.69
0.90	0.001	2.76	140.04	210.59
0.85	0.01	2.90	116.37	172.87
0.85	0.005	2.87	110.98	180.20
0.85	0.001	2.86	107.06	185.49
0.80	0.01	2.93	91.05	153.72
0.80	0.005	2.91	87.65	158.19
0.80	0.001	2.89	85.17	161.43
0.75	0.01	2.91	73.86	135.95
0.75	0.005	2.89	71.62	138.67
0.75	0.001	2.88	69.98	140.66
0.70	0.01	2.87	61.46	120.38
0.70	0.005	2.86	59.96	122.02
0.70	0.001	2.85	58.84	123.23

Table 3. The results when θ and α changing and $T = 15$

θ	α	p^*	Q^*	C^*
0.90	0.01	3.09	191.98	177.01
0.90	0.005	3.00	172.02	214.25
0.90	0.001	2.94	158.82	238.13
0.85	0.01	3.14	130.28	168.63
0.85	0.005	3.06	120.55	187.70
0.85	0.001	3.01	113.96	200.02
0.80	0.01	3.08	97.45	152.04
0.80	0.005	3.02	91.92	162.02
0.80	0.001	2.99	88.15	168.54
0.75	0.01	3.00	77.03	135.43
0.75	0.005	2.96	73.69	140.61
0.75	0.001	2.94	71.41	144.06
0.70	0.01	2.92	63.00	120.30
0.70	0.005	2.90	60.96	122.98
0.70	0.001	2.88	59.54	124.80

Table 4. The results when θ and α changing and $T = 20$

θ	α	p^*	Q^*	C^*
0.90	0.01	3.49	208.16	117.51
0.90	0.005	3.22	182.94	199.51
0.90	0.001	3.10	164.71	246.01
0.85	0.01	3.37	136.22	143.81
0.85	0.005	3.19	123.73	181.58
0.85	0.001	3.10	115.35	202.65
0.80	0.01	3.19	100.55	142.80
0.80	0.005	3.09	93.27	159.78
0.80	0.001	3.03	88.70	169.45
0.75	0.01	3.04	78.39	132.36
0.75	0.005	2.99	74.26	139.89
0.75	0.001	2.96	71.65	144.38
0.70	0.01	2.94	63.51	119.38
0.70	0.005	2.91	61.17	122.77
0.70	0.001	2.89	59.63	124.91

Table 5. The results when θ and α changing and $T = 24$

θ	α	p^*	Q^*	C^*
0.90	0.01	3.98	187.00	43.98
0.90	0.005	3.41	184.78	174.90
0.90	0.001	3.18	165.37	245.16
0.85	0.01	3.58	136.48	113.71
0.85	0.005	3.27	124.66	173.31
0.85	0.001	3.14	115.42	202.08
0.80	0.01	3.26	102.27	133.61
0.80	0.005	3.12	93.77	157.43
0.80	0.001	3.05	88.78	169.27
0.75	0.01	3.06	79.09	130.01
0.75	0.005	3.00	74.44	139.31
0.75	0.001	2.96	71.68	144.33
0.70	0.01	2.94	63.70	118.86
0.70	0.005	2.91	61.22	122.64
0.70	0.001	2.89	59.65	124.90

When the inventory cycle and deterioration rate are constant with the decreasing of deterioration rate, the sale price and profit increase at first and then start decreasing. From these results, we can conclude that the supplier must set appropriate prices depending on the deterioration rate of product in order to obtain the maximum profit.

In order to illustrate the relationship between the freshness, the deterioration rate and the inventory cycle of products and profit better, Figures 1–3 present the changes in profit with freshness when the inventory cycles are different. As shown in the figures, when the deterioration rates are the same, with the increasing of freshness, the profit is increasing generally. But profit also depends on the deterioration rate and inventory cycle. If the deterioration rate is small, the profit increases with increasing inventory cycle. When the deterioration rate is bigger, with the increasing inventory cycle, the profit decreasing, and the profit decreasing with the increasing of freshness. Therefore, the inventory cycle of products depends on the deterioration rate.

In the inventory model of fresh agriculture products, the price is also an important factor. Figures 4–6 show the relationship between the freshness, the deterioration rate and the inventory cycle of products and sale price. The figures show that the sale price is increasing while the inventory cycle is increasing when deterioration rates are the same. If the inventory cycle is smaller, with the increasing of freshness, the sale price increases at first and then starts decreasing. With the increasing of deterioration rate, the growth rate of sale price increases while the freshness and inventory cycle are increasing.

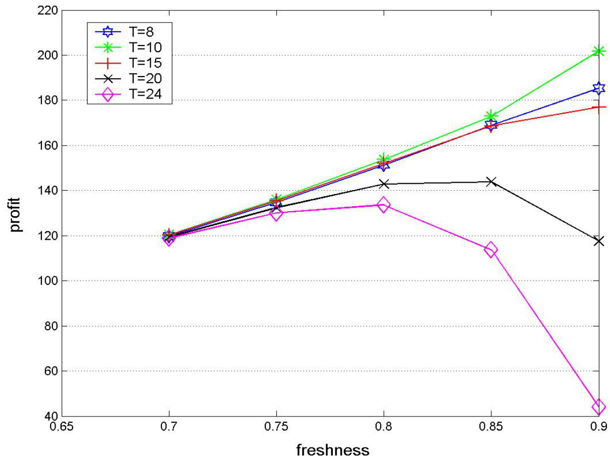

Figure 1. The profit when θ and T changing and $\alpha = 0.01$.

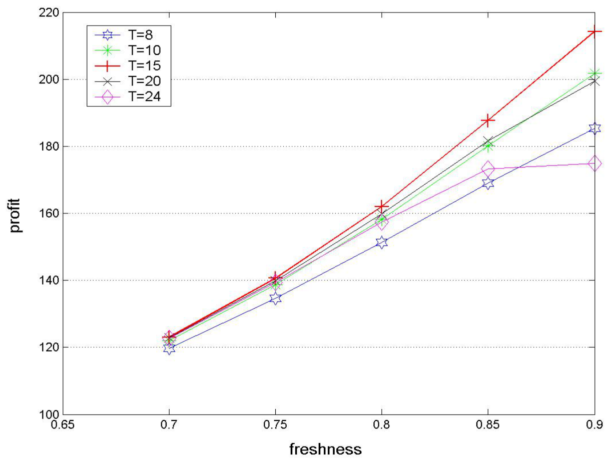

Figure 2. The profit when θ and T changing and $\alpha = 0.005$.

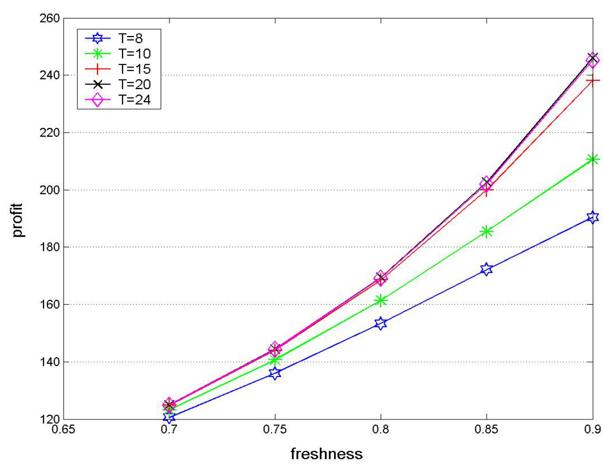

Figure 3. The profit when θ and T changing and $\alpha = 0.001$.

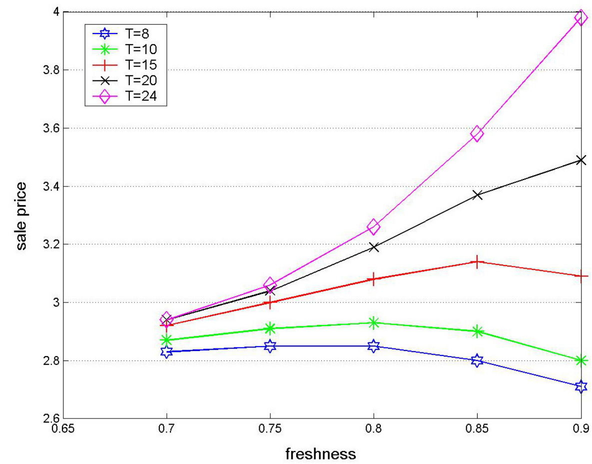

Figure 4. The sale price when θ and T changing and $\alpha = 0.01$.

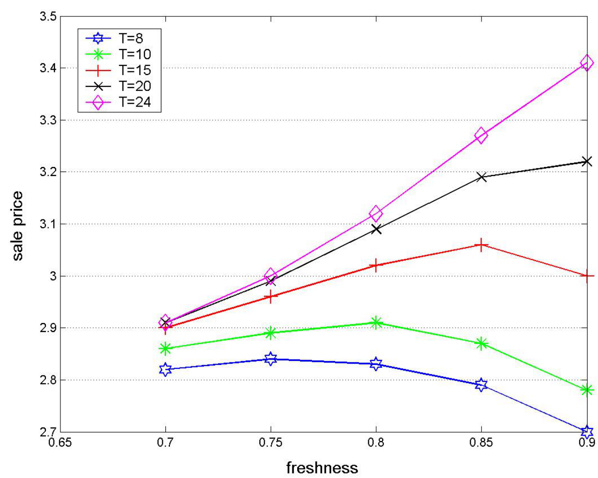

Figure 5. The sale price when θ and T changing and $\alpha = 0.005$.

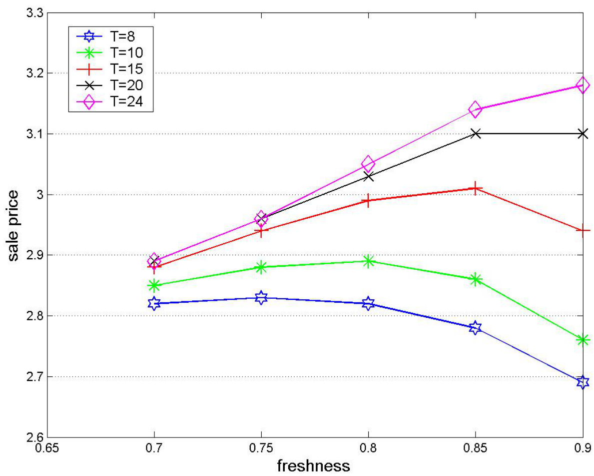

Figure 6. The sale price when θ and T changing and $\alpha = 0.001$.

In summary, in the case where the freshness and the deterioration rate are the same, if the inventory cycle increases, the sale price increases, and the profit increases at first but then starts decreasing. Thus, in order to obtain the higher profit, the supplier must select the inventory cycle and the sale price based on the freshness and deterioration rate of products.

4.2 Solving Model for Variable Inventory Cycle

According to the conclusions of the previous section, the supplier must select the inventory cycle and the sale price opportunely to obtain higher profit. The results are listed in Table 6 when freshness and deterioration rate coefficient change, where $a = 50$, $b = 10$, $c = 0.1$ and $p_1 = \theta^2$. With the decreasing deterioration rate, the inventory cycle and maximum profit are increasing. In fact, the inventory cycle depends on the deterioration rate of products. Figures 7, 8 illustrate the conclusion better.

Table 6. The results when θ and α changing

θ	α	T^*	p^*	Q^*	C^*
0.90	0.01	10.15	2.81	157.07	189.67
0.90	0.005	12.75	2.90	162.87	212.81
0.90	0.001	18.5	3.05	163.75	245.00
0.85	0.01	10.9	2.94	119.79	174.31
0.85	0.005	13.55	3.01	118.77	187.58
0.85	0.001	19.5	3.09	115.30	202.62
0.80	0.01	11.25	2.97	93.25	154.21
0.80	0.005	13.9	3.00	91.38	162.06
0.80	0.001	20	3.03	88.70	169.45
0.75	0.01	11.5	2.95	75.18	136.52
0.75	0.005	14.5	2.96	73.60	140.63
0.75	0.001	20.3	2.96	71.65	144.37
0.70	0.01	11.7	2.90	62.22	120.70
0.70	0.005	15	2.90	60.96	122.98
0.70	0.001	20.5	2.89	59.64	124.91

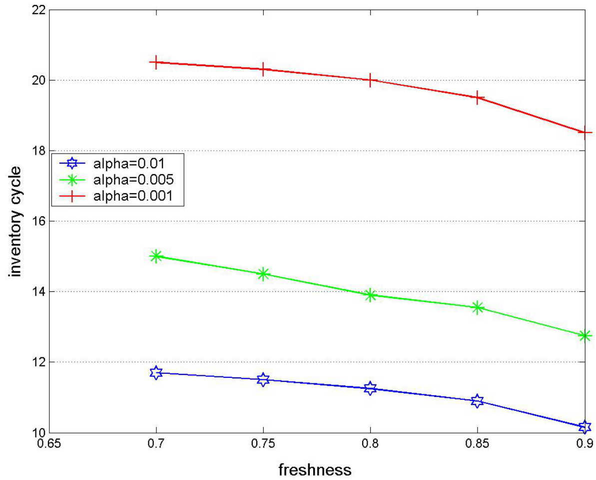

Figure 7. The inventory cycle when θ and α changing.

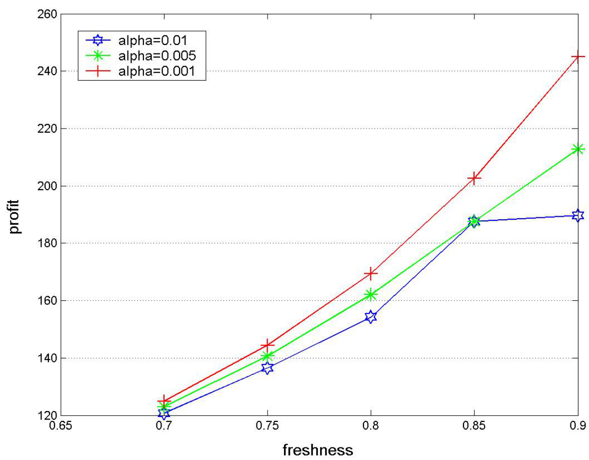

Figure 8. The profit when θ and α changing.

5. CONCLUSION

This paper presented inventory models for fresh agriculture products with a time-varying deterioration rate. In the actual situation of the problem, the demand rate depended on the sale price and freshness, and the deterioration rate was a time-varying function. When the inventory cycle was constant, the total profit was a univariate function of the sale price per unit. There existed a unique sale price satisfying the function which made it possible to obtain the optimal profit. The results of experiments showed that the profit depended on the freshness and deterioration rate of products. With the increasing inventory cycle, the sale price and profit increased at first and then started decreasing. Therefore, the selection of inventory cycle was important. Furthermore, when the inventory cycle was variable, the total profit was a binary function of the sale price and inventory cycle. There existed unique sale price and inventory cycle such that the profit was optimal. The results of our experiments led us to conclude that the optimal inventory cycle and sale price were based on the freshness and deterioration rate of products.

ACKNOWLEDGMENTS

This paper was supported by Shandong Provincial Social Science Programming Research Project (No. ZR2010BL009).

REFERENCES

Alamri, A. A. (2011), Theory and methodology on the global optimal solution to a General Reverse Logistics Inventory Model for deteriorating items and time-varying rates, *Computers and Industrial Engineering*, **60**(2), 236-247.

Balkhi, Z. T. and Benkherouf, L. (1996), A production lot size inventory model for deteriorating items and arbitrary production and demand rates, *European Journal of Operational Research*, **92**(2), 302-309.

Chen, J. and Dan, B. (2009), Fresh agricultural product supply chain coordination under the physical loss-control, *Systems Engineering - Theory and Practice*, **29**, 54-62.

Chung, K. J. and Lin, C. N. (2001), Optimal inventory replenishment models for deteriorating items taking account of time discounting, *Computers and Operations Research*, **28**(1), 67-83.

Donaldson, W. A. (1977), Inventory replenishment policy for a linear trend in demand: an analytic solution, *Operational Research Quarterly*, **28**(3), 663-670.

Duan, Y. R., Li, G. P., Tien, J. M., and Huo, J. Z. (2012), Inventory models for perishable items with inventory level dependent demand rate, *Applied Mathematical Modelling*, **36**(10), 5015-5028.

Dye, C. Y. (2012), A finite horizon deteriorating inventory model with two-phase pricing and time-varying demand and cost under trade credit financing using particle swarm optimization, *Swarm and Evolutionary Computation*, **5**, 37-53

Ghare, P. M. and Schrader, G. F. (1963), A model for exponentially decaying inventories, *Journal of Industrial Engineering*, **14**(5), 238-243.

Gumasta, K., Chan, F. T., and Tiwari, M. K. (2012), An incorporated inventory transport system with two types of customers for multiple perishable goods, *International Journal Production Economics*, **139** (2), 678-686.

He, Y., Wang, S. Y., and Lai, K. K. (2010), An optimal production-inventory model for deteriorating items with multiple-market demand, *European Journal of Operational Research*, **203**(3), 593-600.

Lee, Y. P. and Dye, C. Y. (2012), An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate, *Computers and Industrial Engineering*, **63**(2), 474-482.

Lodree, E. J. and Uzochukwu, B. M. (2008), Production planning for a deteriorating rate with stochastic demand and consumer choice, *International Journal of Production Economics*, **116**(2), 219-232.

Pal, A. K., Bhunia, A. K., and Mukherjee, R. N. (2006), Optimal lot size model for deteriorating items with demand rate dependent on displayed stock level (DSL) and partial backordering, *European Journal of Operational Reasearch*, **175**(2), 977-991.

Resh, M., Friedman, M., and Barbosa, L. C. (1976), On a general solution of the deterministic lot size problem with time-proportional demand, *Operational Research*, **24**(4), 718-725.

Sarkar, B. (2012), An EOQ model with delay in payments and time-varying deterioration rate, *Mathematical and Computer Modelling*, **55**(3/4), 367-377.

Sarkar, B. and Sarkar, S. (2013a), Variable deterioration and demand: an inventory model, *Economic Modelling*, **31**, 548-556.

Sarkar, B. and Sarkar, S. (2013b), An improved inventory model with partial backlogging, time-varying deterioration and stock-dependent demand, *Economic Modelling*, **30**, 924-932.

Sarkar, B., Saren, S., and Wee, H. M. (2013), An inventory model with variable demand, component cost and selling price for deteriorating items, *Economic Modelling*, **30**, 306-310.

Wang, S. P. (2002), An inventory replenishment policy for deteriorating items with shortages and partial backlogging, *Computers and Operations Research*, **29**(14), 2043-2051.

Wu, K. S., Ouyang, L. Y., and Yang, C. T. (2006), An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging, *International Journal Production Economics*, **101**(2), 369-384.