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양자화와 오버플로우 비선형성을 가지는 이산시간 불확실 지연 특이시스템의 지연종속 강인 안정성

Delay-dependent Robust Stability of Discrete-time Uncertain Delayed Descriptor Systems using Quantization/overflow Nonlinearities

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Abstract - This paper considers the problem of robust stability for uncertain discrete-time interval time-varying delayed descriptor systems using any combinations of quantization and overflow nonlinearities. First, delay-dependent linear matrix inequality (LMI) condition for discrete-time descriptor systems with time-varying delay and quantization/overflow nonlinearities is presented by proper Lyapunov function. Second, it is shown that the obtained condition can be extended into descriptor systems with uncertainties such as norm-bounded parameter uncertainties and polytopic uncertainties by some useful lemmas. The proposed results can be applied to both descriptor systems and non-descriptor systems. Finally, numerical examples are shown to illustrate the effectiveness and less conservativeness.

Key Words : Nonlinear descriptor systems, Robust stability, Finite wordlength effect, Uncertain descriptor systems, Time-varying delay, LMI

1. Introduction

When designing a discrete-time system using computers or special-purpose hardware with fixed-point arithmetic, the occurrence of various nonlinearities due to finite wordlength, namely quantization and overflow are generated [1-5]. These nonlinearities may lead to instability. Therefore, the stability analysis of discrete systems using various combinations of quantization and overflow nonlinearities has drawn attentions [6-8]. Kandanvli and Kar [6] addressed the problem of global asymptotic stability of a class of uncertain discrete-time state-delayed systems employing saturation nonlinearities. Kandanvli and Kar [6] just considered saturation nonlinearities. Also, Kandanvli and Kar [7] presented the global asymptotic stability of a class of uncertain discrete-time state-delayed systems using combinations of quantization and overflow nonlinearities by LMI based criterion. The result [7] considered norm-bounded parameter uncertainty and time-invariant delay. Therefore, Kandanvli and Kar [8] extended the result [7] to time-varying delay systems. The result [8] proposed the global asymptotic stability of norm-bounded uncertain discrete-time time-varying state delayed systems using combinations of quantization and overflow nonlinearities on the basis of LMI criterion. However, Kandanvli and Kar [6-8] considered non-descriptor systems. One possible reason is the difficulty in managing the resultant constraints related to the descriptor matrix.

Recently, a great deal of attention has been devoted to the study of descriptor systems over the past decades. descriptor systems are referred to as descriptor systems, implicit systems, generalized state-space systems, or semi-state systems [9]. Therefore, the first aim is to extend the results of non-descriptor systems [6-8] into the stability analysis method of discrete uncertain delayed descriptor systems using quantization and overflow nonlinearities. Also, Kandanvli and Kar [6-8] considered norm-bounded parameter uncertainties. It is well-known that the norm-bounded uncertainty is more conservative than the polytopic uncertainty. Hence, the second objective is to consider the polytopic uncertainties in the discrete uncertain descriptor systems with interval time-varying delay and quantization/overflow nonlinearities. Furthermore, it is shown that the proposed stability analysis is a general method for both descriptor systems and non-descriptor systems.

In this paper, the delay-dependent robust stability conditions of discrete-time uncertain delayed descriptor systems using any combinations of quantization and overflow nonlinearities are presented by using LMI technique. Two uncertainties, norm-bounded uncertainty

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and polytopic uncertainty, are considered. The proposed stability conditions can be applied to both descriptor systems and non-descriptor systems. Finally, it is shown that the presented conditions are effective and less conservative by comparisons of previous results.

The notations are standard. I and 0 stand for the identity and the zero matrices with proper dimensions, respectively. The symmetric term in a symmetric matrix is denoted by $\ast$. $<X>$ denotes $X+X^T$.

2. System description

Consider the nonlinear discrete-time descriptor systems with interval time-varying delay

$$
\begin{aligned}
Ex(k+1) &= O\{Q(y(k))\} = f(y(k)) \\
&= [f_1(y_1(k))\ f_2(y_2(k))\ \cdots\ f_n(y_n(k))]^T \\
y(k) &= Ax(k) + A_d x(k-d(k)) = [y_1(k)\ y_2(k)\ \cdots\ y_n(k)]^T \\
x(k) &= \phi(k), \quad k = -d_2, -d_2+1, \ldots, 0
\end{aligned} \tag{1}
$$

where $x(k) \in R^n$ is the state vector, $y(k) \in R^n$, $\phi(t)$ is the initial value at time k, E is a descriptor(singular) matrix satisfying $rank(E) = r \le n$, and all system matrices have proper dimensions. $Q(\bullet)$ represents the quantization nonlinearities, $O(\bullet)$ denotes the overflow nonlinearities, and $f(\bullet)$ characterizes the composite nonlinear functions. $d(k)$ is a time-varying delay satisfying $0 < d_1 \le d(k) \le d_2 < \infty$. Here, d_1 and d_2 are known positive integers. In the event of $Q(\bullet)$ being either magnitude truncation or round off, $f(\bullet)$ turn out to be confined to the sector $[k_o, k_q]$, i.e.

$$
f_i(0) = 0, \quad k_o y_i(k)^2 \le f_i(y_i(k)) y_i(k) \le k_q y_i(k)^2, \quad i = 1, 2, \ldots, n \tag{2}
$$

$$
k_q = \begin{cases} 1 & \text{for } magnitude\ truncation \\ 2 & \text{for } \partial\ \ off \end{cases} \tag{3}
$$

$$
k_o = \begin{cases} 0 & \text{for } zeroing\ saturation \\ -\frac{1}{3} & \text{for } triangular \\ -1 & \text{for } two's\ complement \end{cases} \tag{4}
$$

The following Finsler's lemma is needed in the proof of main results.

Lemma 1.[10] Let $x \in R^n$, $Q = Q^T \in R^{n \times n}$ and $B \in R^{m \times n}$ such that $rank(B) < n$. The following statements are equivalent:

i) $x^T Q x < 0, \ \forall Bx = 0, \ x \ne 0$
ii) $B^{\perp T} Q B^{\perp} < 0$
iii) $\exists \mu \in R$: $Q - \mu B^T \mu < 0$
iv) $\exists X \in R^{n \times m}$: $Q + XB + B^T X^T < 0$

Here, $B^{\perp}$ is a basis for the null space of B.

Lemma 2.[11] Let Σ, Λ, and M be real matrices of appropriate dimensions with M satisfying $M = M^T$, then

$$
M + \Sigma F(k) \Lambda + \Lambda^T F(k)^T \Sigma^T < 0 \tag{5}
$$

for all $F(k)^T F(k) \le I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$
M + \epsilon^{-1} \Sigma \Sigma^T + \epsilon \Lambda^T \Lambda < 0. \tag{6}
$$

Definition 1.[7] The zero solution of the system described by (1)-(4) is globally asymptotically stable if the following holds:

i) it is stable in the sense of Lyapunov, i.e., for every $\mu > 0$ there exists a $\zeta = \zeta(\mu)$ such that $\|x(k)\| < \mu$ for all $k = 0, 1, 2, \ldots$, whenever $\|x(0)\| < \zeta$;
ii) it is attractive, i.e., $x(k) \to 0$ as $k \to \infty$.

3. Main results

In this section, a new delay-dependent Bounded Real Lemma (BRL) satisfying Definition 1 is derived using LMI approach. To get a globally asymptotic stability LMI condition for discrete delayed descriptor systems with quantization and overflow nonlinearities, we make use of proper Lyapunov function, Finsler's lemma, and LMI approach.

Theorem 1. The descriptor system described by (1)-(4) is globally asymptotically stable if there exist positive definite matrices P, Q_1, Q_2, Q_3, S_1, S_2, a positive definite diagonal matrix G, matrices M_j $(j = 1, 2, \ldots, 12)$, and X_k $(k = 1, 2, \ldots, 5)$ such that

$$
\Phi = \begin{bmatrix}
\Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 & \Phi_6 & d_2 M_1^T & d_1 M_7^T \\
\ast & \Phi_7 & \Phi_8 & \Phi_9 & -A_d^T X_5^T & \Phi_{10} & d_2 M_2^T & d_1 M_8^T \\
\ast & \ast & \Phi_{11} & \Phi_{12} & -M_{11} & X_3 - M_{12} & d_2 M_3^T & d_1 M_9^T \\
\ast & \ast & \ast & \Phi_{13} & -M_5 & X_4 - M_6 & d_2 M_4^T & d_1 M_{10}^T \\
\ast & \ast & \ast & \ast & \Phi_{14} & \Phi_{15} & d_2 M_5^T & d_1 M_{11}^T \\
\ast & \ast & \ast & \ast & \ast & \Phi_{16} & d_2 M_6^T & d_1 M_{12}^T \\
\ast & \ast & \ast & \ast & \ast & \ast & -d_2 S_1 & 0 \\
\ast & \ast & \ast & \ast & \ast & \ast & \ast & -d_1 (S_2 - S_1)
\end{bmatrix} < 0 \tag{7}
$$

where

$$
\begin{aligned}
\Phi_1 &= -E^T P E + d^{\ast} Q_1 + Q_2 + Q_3 + d_{12} S_1 + d_1 S_2 + <M_1> + <M_7> \\
\Phi_2 &= M_2 + M_8 - X_1 A_d - A^T X_2^T \\
\Phi_3 &= M_3 - M_7^T + M_9 - A^T X_3^T
\end{aligned}
$$

$$\Phi_4 = -M_1^T + M_4 + M_{10} - A^T X_4^T$$
$$\Phi_5 = M_5 + M_{11} - A^T X_5^T - d_{12} S_1 - d_1 S_2$$
$$\Phi_6 = M_6 + M_{12} + X_1 - A^T X_6^T, \quad \Phi_7 = -Q_1 - < X_2 A_d >$$
$$\Phi_8 = - M_8^T - A_d^T X_3^T, \; \Phi_9 = -M_2^T - A_d^T X_4^T$$
$$\Phi_{10} = X_2 - A_d^T X_6^T, \; \Phi_{11} = -Q_2 - < M_9 >$$
$$\Phi_{12} = - M_3^T - M_{10}, \; \Phi_{13} = - Q_3 - < M_4 >$$
$$\Phi_{14} = P - 2G + d_{12} S_1 + d_1 S_2$$
$$\Phi_{15} = X_5 + (k_q + k_o) G, \; \Phi_{16} = < X_5 > - 2 k_q k_o G$$
$$d_{12} = d_2 - d_1, \quad d^* = d_{12} + 1, \; \eta(k) = f(y(k)) - x(k).$$

Proof. Choose a Lyapunov functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \tag{8}$$

where

$$V_1(k) = x(k)^T E^T P E x(k)$$
$$V_2(k) = \sum_{i=k-d(k)}^{k-1} x(i)^T Q_1 x(i) + \sum_{(j=-d_2+2)}^{-d_{1+1}} \sum_{(l=k+j-1)}^{k-1} x(l)^T Q_1 x(l)$$
$$V_3(k) = \sum_{i=k-d_1}^{k-1} x(i)^T Q_2 x(i) + \sum_{i=k-d_2}^{k-1} x(i)^T Q_3 x(i)$$
$$V_4(k) = \sum_{(\beta=-d_{2+1})}^{=d_1} \sum_{(j=k-1+\beta)}^{k-1} \eta(j)^T S_1 \eta(j) + \sum_{(\beta=-d_{1+1})}^{0} \sum_{(j=k-1+\beta)}^{k-1} \eta(j)^T S_2 \eta(j)$$

where $\eta(k) = f(y(k)) - x(k)$ and positive-definite matrices P, Q_1, Q_2, Q_3, S_1, and S_2 are to be determined. Taking the forward difference $\Delta V(k) = V(k+1) - V(k)$ along the trajectories of system (1) yields

$$\Delta V_1(k) = f(y(k))^T P f(y(k)) - x(k)^T P x(k) \tag{9}$$

$$\Delta V_2(k) \le d^* x(k)^T Q_1 x(k) - x(k-d(k))^T Q_1 x(k-d(k))$$

$$\Delta V_3(k) = x(k)^T Q_2 x(k) - x(k-d_1)^T Q_2 x(k-d_1) + x(k)^T Q_3 x(k) - x(k-d_2)^T Q_3 x(k-d_2)$$

$$\begin{aligned} \Delta V_4(k) = {} & d_{12} \eta(k)^T S_1 \eta(k) + \sum_{j=k-d_1}^{k-1} \eta(j)^T S_1 \eta(j) \\ & - \sum_{j=k-d_2}^{k-1} \eta(j)^T S_1 \eta(j) + d_1 \eta(k)^T S_2 \eta(k) - \sum_{j=k-d_1}^{k-1} \eta(j)^T S_2 \eta(j) \\ = {} & d_{12} \eta(k)^T S_1 \eta(k) + \sum_{j=k-d_2}^{k-1} \eta(j)^T S_1 \eta(j) + d_1 \eta(k)^T S_2 \eta(k) \\ & - \sum_{j=k-d_1}^{k-1} \eta(j)^T (S_2 - S_1) \eta(j) \end{aligned}$$

By finite sum inequality like Lemma 1 of [12], the last some terms of the right hand side in $\Delta V_4(k)$ can be changed to get the delay-dependent BRL. It is clear that

$$\begin{bmatrix} S_1 & Y_1 \\ Y_1^T & Y_1^T S_1^{-1} Y_1 \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} S_2 - S_1 & Y_2 \\ Y_2^T & Y_2^T (S_2 - S_1)^{-1} Y_2 \end{bmatrix} < 0 \tag{11}$$

where $Y_1 = [M_1 \; M_2 \; M_3 \; M_4 \; M_5 \; M_6]$ and $Y_2 = [M_7 \; M_8 \; M_9 \; M_{10} \; M_{11} \; M_{12}]$. Hence,

$$\sum_{j=k-d_2}^{k-1} \begin{bmatrix} \eta(j) \\ \zeta(k) \end{bmatrix}^T \begin{bmatrix} S_1 & Y_1 \\ Y_1^T & Y_1^T S_1^{-1} Y_1 \end{bmatrix} \begin{bmatrix} \eta(j) \\ \zeta(k) \end{bmatrix} \ge 0 \tag{12}$$

$$\sum_{j=k-d_2}^{k-1} \begin{bmatrix} \eta(j) \\ \zeta(k) \end{bmatrix}^T \begin{bmatrix} S_2 - S_1 & Y_2 \\ Y_2^T & Y_2^T (S_2 - S_1)^{-1} Y_2 \end{bmatrix} \begin{bmatrix} \eta(j) \\ \zeta(k) \end{bmatrix} \ge 0. \tag{13}$$

After some manipulations of (12) and (13), the following relations can be obtained

$$- \sum_{j=k-d_2}^{k-1} \eta(j)^T S_1 \eta(j) \le \zeta(k)^T \Pi_1 \zeta(k) + d_2 \zeta(k)^T Y_1^T S_1^{-1} Y_1 \zeta(k) \tag{14}$$

$$\begin{aligned} & - \sum_{j=k-d_1}^{k-1} \eta(j)^T (S_2 - S_1) \eta(j) \\ & \le \zeta(k)^T \Pi_2 \zeta(k) + d_1 \zeta(k)^T Y_2^T (S_2 - S_{1)}^{-1} Y_2 \zeta(k) \end{aligned} \tag{15}$$

where

$$\Pi_1 = \begin{bmatrix} < M_1 > & M_2 & M_3 - M_1^T + M_4 & M_5 & M_6 \\ * & 0 & 0 & -M_2^T & 0 & 0 \\ * & * & 0 & -M_3^T & 0 & 0 \\ * & * & * & -< M_4 > & -M_5 & -M_6 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$$\Pi_2 = \begin{bmatrix} < M_7 > & M_8 - M_7^T + M_9 & M_{10} & M_{11} & M_{12} \\ * & 0 & -M_8^T & 0 & 0 & 0 \\ * & * & -< M_9 > & -M_{10} & -M_{11} & -M_{12} \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}$$

$$\zeta(k) = \left[x(k)^T \; x(k-d(k))^T \; x(k-d_1)^T \; x(k-d_2)^T \; f(y(k))^T \; y(k)^T \right]^T$$

From the (9), (14), and (15), we have

$$\Delta V(k) \le \zeta(k)^T \left(\Omega_1 + d_2 Y_1^T S_1^{-1} Y_1 + d_1 Y_2^T (S_2 - S_1)^{-1} Y_2 \right) \zeta(k) - 2\beta < 0 \tag{16}$$

where, $\beta = [k_q y(k) - f(y(k))]^T G [f(y(k)) - k_o y(k)] \ge 0$.
Using Lemma 1,
$\zeta(k)^T [\Omega_1 + d_2 Y_1^T S_1^{-1} Y_1 + d_1 Y_2^T (S_2 - S_1)^{-1} Y_{2]} \zeta(k) < 0$in (16) is equivalent to

$$\Omega_1 + d_2 Y_1^T S_1^{-1} Y_1 + d_1 Y_2^T (S_2 - S_1)^{-1} Y_2 + X \Omega_2 + \Omega_2^T X^T < 0 \tag{17}$$

where,

$$\Omega_1 = \begin{bmatrix} \Psi_1 & M_2 + M_8 & \Psi_2 & \Psi_3 & \Psi_4 & M_6 + M_{12} \\ * & -Q_1 & -M_8^T & -M_2^T & 0 & 0 \\ * & * & -Q_2 - < M_9 > & -M_3^T - M_{10} & -M_{11} & -M_{12} \\ * & * & * & -Q_3 - < M_4 > & -M_5 & -M_6 \\ * & * & * & * & \Psi_5 & (k_q + k_o) G \\ * & * & * & * & * & -2 k_q k_o G \end{bmatrix}$$

$$\Psi_{1} = -E^T PE + d^* Q_1 + Q_2 + Q_3 + d_{12} S_1 + d_1 S_2 + <M_1> + <M_7>$$
$$\Psi_2 = M_3 - M_7^T + M_9,\ \Psi_3 = -M_1^T + M_4 + M_{10}$$
$$\Psi_5 = P - 2G + d_{12} S_1 + d_1 S_2,\ \Omega_2 = [-A\ -A_d\ 0\ 0\ 0\ I]$$
$$X = \left[X_1^T\ X_2^T\ X_3^T\ X_4^T\ X_5^T\ X_6^T\right]^T.$$

Therefore, if the condition (17) is satisfied, then $\Delta V(k) < 0$ is also satisfied. Using Schur complements, the condition (17) is changed to (7). ■

Now, we consider a descriptor system (1) with polytopic uncertainty. The system matrices are supposed to be uncertain and unknown but belonging to a known convex compact set of

$$\Xi = (A, A_d) \in \Theta \tag{18}$$

where

$$\Theta = \left\{ \Xi(\lambda) = \sum_{i=1}^{N} \lambda_i \Xi_i,\ \sum_{i=1}^{N} \lambda_i = 1 \geq 0 \right\}$$

where $\Xi_i = (A_i, A_{di}) \in \Theta,\ i = 1, \ldots, N$, which denotes the ith vertex of the polyhedral domain Ξ. In the following Theorem 2, we consider the delayed discrete descriptor system with polytopic uncertainties and quantization/overflow nonlinearities using parameter-dependent Lyapunov function.

Theorem 2. The descriptor system described by (1)-(4) with polytopic uncertainty (18) is robust stable in spite of polytopic uncertainties, time-varying delay, and quantization/overflow nonlinearities if there exist positive definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, S_{1i}, S_{2i}, a positive definite diagonal matrix G, matrices $M_j (j = 1, 2, \ldots, 12)$, and $X_k\ (k = 1, 2, \ldots, 5)$ such that for $i = 1, \ldots, N$

$$\Gamma_i = \begin{bmatrix} \Gamma_{1i} & \Gamma_{2i} & \Gamma_{3i} & \Gamma_{4i} & \Gamma_{5i} & \Gamma_{6i} & d_2 M_1^T & d_1 M_7^T \\ * & \Gamma_{7i} & \Gamma_{8i} & \Gamma_{9i} - A_{di}^T X_5^T & & \Gamma_{10i} & d_2 M_2^T & d_1 M_8^T \\ * & * & \Gamma_{11i} & \Phi_{12} & -M_{11} & X_3 - M_{12} & d_2 M_3^T & d_1 M_9^T \\ * & * & * & \Gamma_{13i} & -M_5 & X_4 - M_6 & d_2 M_4^T & d_1 M_{10}^T \\ * & * & * & * & \Gamma_{14i} & \Phi_{15} & d_2 M_5^T & d_1 M_{11}^T \\ * & * & * & * & * & \Phi_{16} & d_2 M_6^T & d_1 M_{12}^T \\ * & * & * & * & * & * & -d_2 S_{1i} & 0 \\ * & * & * & * & * & * & * & -d_1 (S_{2i} - S_{1i}) \end{bmatrix} < 0 \tag{19}$$

where

$$\Gamma_{1i} = -E^T P_i E + d^* Q_{1i} + Q_{2i} + Q_{3i} + d_{12} S_{1i} + d_1 S_{2i} + <M_1> + <M_7>$$
$$\Gamma_{2i} = M_2 + M_8 - X_1 A_{di} - A_i^T X_2^T$$
$$\Gamma_{3i} = M_3 - M_7^T + M_9 - A_i^T X_3^T$$
$$\Gamma_{4i} = -M_1^T + M_4 + M_{10} - A_i^T X_4^T$$
$$\Gamma_{5i} = M_5 + M_{11} - A_i^T X_5^T - d_{12} S_{1i} - d_1 S_{2i}$$
$$\Gamma_{6i} = M_6 + M_{12} + X_1 - A_i^T X_6^T$$
$$\Gamma_{7i} = -Q_{1i} - <X_2 A_{di}>,\quad \Gamma_{8i} = -M_8^T - A_{di}^T X_3^T$$
$$\Gamma_{9i} = -M_2^T - A_d^T X_4^T,\ \Gamma_{10i} = X_2 - A_{di}^T X_6^T$$
$$\Gamma_{11i} = -Q_{2i} - <M_9>,\ \Phi_{12} = -M_3^T - M_{10}$$
$$\Gamma_{13i} = -Q_{3i} - <M_4>,\ \Gamma_{14i} = P_i - 2G + d_{12} S_{1i} + d_1 S_{2i}$$
$$\Phi_{15} = X_5 + (k_q + k_o) G,\ \Phi_{16} = <X_5> - 2 k_q k_o G$$
$$d_{12} = d_2 - d_1,\ d^* = d_{12} + 1,\ \eta(k) = f(y(k)) - x(k).$$

Proof. The result can be directly derived by the proof procedures of Theorem 1. When we apply the polytopic uncertainty (18) to the system, we can get $\sum_{i=1}^{N} \lambda_i \Gamma_i < 0$. Therefore, if $\Gamma_i < 0$ holds, $\sum_{i=1}^{N} \lambda_i \Gamma_i < 0$ is guaranteed. ■

Next, we will show that the proposed method can be extended to discrete delayed descriptor systems with parameter uncertainties and quantization/overflow nonlinearities. Consider the following discrete delayed uncertain descriptor systems with same dimensions of the descriptor system (1)

$$\begin{aligned} Ex(k+1) &= O\{Q(y(k))\} = f(y(k)) \\ &= [f_1(y_1(k))\ f_2(y_2(k))\ \ldots\ f_n(y_n(k))] \end{aligned} \tag{20}$$
$$y(k) = (A + \Delta A(k)) x(k) + (A_d + \Delta A(k)) x(k - d(k))$$
$$x(k) = \phi(k),\quad k = -d_2, -d_2 + 1, \ldots, 0$$

where the uncertain parameters are defined as

$$\Delta A(k) = HF(k) D,\quad \Delta A_{d(k)} = H_d F_d(k) D_{d.} \tag{21}$$

Here, H, H_d, D, and D_d are known real constant matrices with appropriate dimensions. $F(k)$ and $F_d(k)$ are unknown real matrices which satisfy

$$F(k)^T F(k) \leq I,\quad F_d(k)^T F_d(k) \leq I. \tag{22}$$

Theorem 3. The descriptor system described by (20) with parameter uncertainties (21) is robust stable in spite of parameter uncertainties, time-varying delay, and quantization/overflow nonlinearities if there exist positive definite matrices P, Q_1, Q_2, Q_3, S_1, S_2, a positive definite diagonal matrix G, positive real constants ϵ, ϵ_d, matrices $M_j\ (j = 1, 2, \ldots, 12)$, and $X_k\ (k = 1, 2, \ldots, 5)$ satisfying

$$\left[\begin{matrix} \tilde{\Phi}_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 & \Phi_6 \\ * & \tilde{\Phi}_7 & \Phi_8 & \Phi_9 & -A_d^T X_5^T & \Phi_{10} \\ * & * & \Phi_{11} & \Phi_{12} & -M_{11} & X_3 - M_{12} \\ * & * & * & \Phi_{13} & -M_5 & X_4 - M_6 \\ * & * & * & * & \Phi_{14} & \Phi_{15} \\ * & * & * & * & * & \Phi_{16} \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * I \end{matrix}\right.$$

$$\begin{array}{cccc} d_2M_1^T & d_1M_7^T & -X_1H & -X_1H_d \\ d_2M_2^T & d_1M_8^T & -X_2H & -X_2H_d \\ d_2M_3^T & d_1M_9^T & -X_3H & -X_3H_d \\ d_2M_4^T & d_1M_{10}^T & -X_4H & -X_4H_d \\ d_2M_5^T & d_1M_{11}^T & -X_5H & -X_5H_d \\ d_2M_6^T & d_1M_{12}^T & -X_6H & -X_6H_d \\ -d_2S_1 & 0 & 0 & 0 \\ * & -d_1(S_2-S_1) & 0 & 0 \\ * & * & -\epsilon I & 0 \\ * & * & * & -\epsilon_d I \end{array}\Bigg] < 0 \qquad (23)$$

where $\tilde{\Phi}_1 = \Phi_1 + \epsilon D^T D$, and $\tilde{\Phi}_7 = \Phi_7 + \epsilon_d D_d^T D_d$.

Proof. By similar procedures of Theorem 1, the following condition can be obtained

$$\Phi + \begin{bmatrix} -<X_1\Delta A> & -\Delta A^T X_2^T & -\Delta A^T X_3^T & -\Delta A^T X_4^T & -\Delta A^T X_5^T & -\Delta A^T X_6^T & 0 & 0 \\ * & -<X_2\Delta A_d> & -\Delta A_d^T X_3^T & -\Delta A_d^T X_4^T & -\Delta A_d^T X_5^T & -\Delta A_d^T X_6^T & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix} < 0. \qquad (24)$$

The condition (24) can be written as

$$\Phi + \Sigma F(k)\Lambda + \Sigma_d F_d(k)\Lambda_d < 0 \qquad (25)$$

where

$$\Sigma = \left[-H^TX_1^T - H^TX_2^T - H^TX_3^T - H^TX_4^T - H^TX_5^T - H^TX_6^T \; 0 \; 0\right]^T$$
$$\Sigma_d = \left[-H_d^TX_1^T - H_d^TX_2^T - H_d^TX_3^T - H_d^TX_4^T - H_d^TX_5^T - H_d^TX_6^T \; 0 \; 0\right]^T$$
$$\Lambda = [D \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$
$$\Lambda_d = [0 \quad D_d \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T.$$

By Lemma 2, (25) is equivalent to

$$\Phi + \epsilon^{-1}\Sigma\Sigma^T + \epsilon\Lambda^T\Lambda + \epsilon_d^{-1}\Sigma_d\Sigma_d^T + \epsilon_d\Lambda_d^T\Lambda < 0 \qquad (26)$$

where, $\epsilon > 0$ and $\epsilon_d > 0$. Using Schur complement, (26) can be expressed as (23). This completes the proof. ■

Remark 1. The stability analysis method in [6-8] cannot be applied to the descriptor systems. However, the proposed Theorems can be applied to both descriptor systems and non-descriptor systems. In the case of $E=I$, the problems of non-descriptor systems [6-8] can be solved directly from the Theorem 1-3. Therefore, the proposed methods are more general than [6-8].

Remark 2. The proposed stability conditions in Theorem 1-3 are linear in terms of all finding variables. Thus, the conditions can be easily solved by LMI Toolbox [13].

4. Numerical examples

In this section, numerical examples are given to illustrate the effectiveness of the proposed method. The robust stability analysis for discrete-time descriptor systems with polytopic uncertainties and quantization/overflow nonlinearities is presented in Example 1. To show that the proposed Theorems are general including both descriptor systems and non-descriptor systems, comparisons between the existing paper and Theorems for parameter uncertainty are illustrated by Example 2.

Example 1. Consider the modified descriptor system (1)-(4) in [8] with sector $[k_o, k_q]$ and polytopic uncertainty (18)

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0.8 & 0 \\ 0.05+\delta_1 & 0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1+\delta_2 \end{bmatrix}, \qquad (27)$$
$$\Delta_1(t)| \le 0.5, \quad |\Delta_2(t)| \le 0.5$$

The stability conditions [6-8] cannot be applied to the descriptor system (27) because of descriptor system matrix. According to Theorem 2, we can get the upper bound of time delay in Table 1 for different sectors when $d_1 = 2$. According to the selection of sectors, the upper bound of time-varying delay is different. In order to apply Theorem 2 to non-descriptor systems, we take $E=I$ in (27). Then, the d_2 can be calculated in Table 2 when $d_1 = 5$ and $E=I$. The existing results [6-8] cannot be applied to the polytopic systems.

Example 2. To consider the norm-bounded parameter uncertainty in Theorem 3, we treat the following example.

$$E = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}, \quad H = H_d = \alpha \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}$$
$$D = [0.01 \; 0], \quad D_d = [0 \; 0.01]$$

where, α is a maximum uncertainty bound. To get the upper bound of time-varying delay bound of α according to the different sectors, we take $d_1 = 2$ and $\alpha = 10$ in Table 3. Also, we consider same example of [8] to compare the existing result. We can get the maximum uncertainty bound in Table 4 when $E=I$, $d_1 = 2$, and $d_2 = 8$. The maximum value of α is 1 in [8], while we obtain 38 in the same sector. In other sectors, we obtain the maximum uncertainty bounds in Table 4, while the method [8] cannot be obtained with any α. In other words, the condition in [8] fails as an asymptotic stability in other sectors and given any uncertainty bound α. Also, delay-independent condition in [6] fails as an asymptotic stability in this example. It is clear that the presented condition of this paper gives better results than that the previous result [8]. Also, the method of [8] cannot be applied to the descriptor system. Therefore, the proposed conditions can be applied to both descriptor and non-descriptor systems.

Table 1 The upper bound, d_2, for different sectors when $d_1=2$.

$[k_o\, k_q]$	$[0\quad 1]$	$[0\quad 2]$	$[-1/3\quad 1]$	$[-1/3\quad 2]$
d_2	64	7	23	4

Table 2 The upper bound, d_2, for different sectors when $d_1=5$ and $E=I$.

$[k_o\, k_q]$	$[0\quad 1]$	$[0\quad 2]$	$[-1/3\quad 1]$	$[-1/3\quad 2]$
d_2	67	10	26	7

Table 3 The upper bound, d_2, for different sectors when $d_1=2$ and $\alpha=10$.

$[k_o\, k_q]$	$[0\quad 1]$	$[0\quad 2]$	$[-1/3\quad 1]$	$[-1/3\quad 2]$
d_2	20	18	18	16

Table 4 The maximum uncertainty bound of α for different sectors when $d_1=2$ and $E=I$.

$[k_o\quad k_q]$	$[0\quad 1]$	$[0\quad 2]$	$[-1/3\quad 1]$	$[-1/3\quad 2]$
Theorem 1 in [8]	1	-	-	-
Theorem 3 (proposed method)	38	21	29	18

5. Conclusion

Delay-dependent robust stability conditions for discrete-time uncertain delayed descriptor systems using various combinations of quantization and overflow nonlinearities have been presented by LMI technique. Moreover, we have considered two kinds of uncertainties such as parameter uncertainty and polytopic uncertainty. The proposed robust stability conditions have been established without the decomposition of system matrices. Finally, the conditions have been illustrated by numerical examples to show the general tests and the effectiveness for both descriptor systems and non-descriptor systems.

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