A study on Inventory Policy (s, S) in the Supply Chain Management with Uncertain Demand and Lead Time

Jae-Hyun Han* · Suk-Jae Jeong*
*School of Business, Kwangwoon University

불확실한 수요와 리드타임을 갖는 공급사슬에서 (s,S) 재고정책에 관한 연구

한 재 현* · 정 석 재*
*광운대학교 경영학부

Abstract

As customers' demands for diversified small-quantity products have been increased, there have been great efforts for a firm to respond to customers' demands flexibly and minimize the cost of inventory at the same time. To achieve that goal, in SCM perspective, many firms have tried to control the inventory efficiently. We present an mathematical model to determine the near optimal (s, S) policy of the supply chain, composed of multi suppliers, a warehouse and multi retailers. (s, S) policy is to order the quantity up to target inventory level when inventory level falls below the reorder point. But it is difficult to analyze inventory level because it is varied with stochastic demand of customers. To reflect stochastic demand of customers in our model, we do the analyses in the following order. First, the analysis of inventory in retailers is done at the mathematical model that we present. Then, the analysis of demand pattern in a warehouse is performed as the inventory of a warehouse is much effected by retailers' order. After that, the analysis of inventory in a warehouse is followed. Finally, the integrated mathematical model is presented. It is not easy to get the solution of the mathematical model, because it includes many stochastic factors. Thus, we get the solutions after the stochastic demand is approximated, then they are verified by the simulations.

Keywords : Inventory policy, Mathematical and simulation model, demand uncertainty, stochastic lead-time

1. Introduction

As customers' demands for diversified small-quantity products have been increased, there have been great efforts for a firm to respond to customers' demands flexibly and minimize the cost of inventory at the same time. It has been shown that, however, existing methods are not enough to solve issues emerging from a variety of customers' demands. To deal with these issues, the concept of supply chain management (SCM) has been emerged and considered as a solution for those.

† Corresponding author: Jeong, Suk-Jae. Kwangwoon University, School of Business
26 Kwangwoon-ro, Nowon-Gu, Seoul., M · P: 010-7714-7811, E-mail: sjjeong@kw.ac.kr
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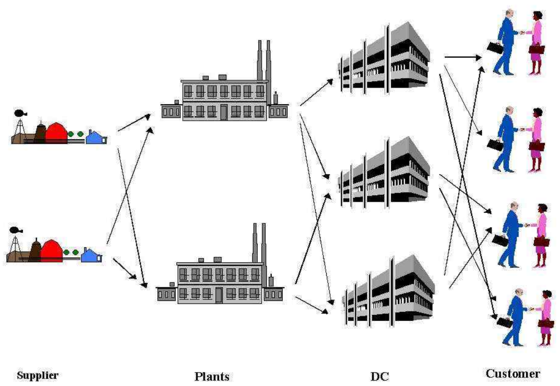
SCM is relatively a new concept and an approach to optimize entire supply chain by applying existing production methods that have been used by firms from suppliers to customers rather than optimizing a specific firm. In other words, SCM is an effort to optimize entire firms rather than a single firm.

Information sharing and effective inventory management are the key factors to decide whether SCM is successful. Effective inventory management, however, is much more important factor because of its nature.

Therefore, in this study, we propose a mathematical model that is adequate for establishing (s, S) policy. The model has the following supply chain network: a number of suppliers, one warehouse, and a number of retailers as shown in figure 1. The (s, S) policy is to order products as many as target inventory level (S) when inventory level falls down to the reorder level (s). But, it is a difficult to analyze since an inventory level varies with probabilistic customer's demand.



<Figure 1> General Structure of Supply Chain Network

Thus, the mathematical model presented in this paper, firstly, goes through a process of analyzing inventory of retailers. Secondly, it is followed by a process of analyzing the structures of demand in a warehouse since the inventory in a warehouse is affected by retailers' orders and then a process in analyzing inventory in warehouses is followed. After going through all the processes mentioned above, finally, an integrated mathematical model is presented. But, it is not easy to get solutions from our proposed model in that it has lots of probabilistic factors. To get solutions, we simplify the quantity of demand which is probabilistic and the results are tested by simulation. We find that it is applicable enough to simplify the quantity of demand because the results of the simulations show that there are no significant differences.

We review existing literatures in section 2. In section 3, the model is presented and explained. It is followed by a process of unfolding and solving the model in section 4. In section 5, the simulation model used in this study is examined and the results of the simulations are analyzed. We conclude and suggest future research direction in section 6.

2. Literature Review

The scope of this paper mainly covers the two areas: 1) Supply Chain Management, 2) Inventory Control. The following is the summary of existing literatures or cases on each issue.

S.J. Seo and K.S. Kim (2000) defined the level of SCM, approach SCM with a simulation method and introduce simulators that have currently been developed. Hau L. Lee, V. Padmanabhan and Seungjin Whang (1997) take notice on distorted phenomenon when information of demand within supply chain network is transferred from the one side to the other sides and refer this distorted phenomenon as Bullwhip Effect. Noel P. Greis, John D. Kasarda (1997) pointed out that changes in firm's environments expedite changes in production structures, the importance of traditional supply chain network relationship, and the most importantly the role of logistics. Tom Davis (1993) examined the role of supply chain network management (SCNM) and suggests factors that should be considered in SCNM through a case study of Hewlett-Packard. Gerard P. Chchon and Paul H. Zipkin (1999) compared and analyzed the Echelon Inventory Game and the Local Inventory Game, and study a way to minimize the inventory cost of supply chain network. Ram Ganesham (1999) presented an inventory policy (s, Q) that is close to optimal in supply chain network with a number of suppliers, one warehouse, and a number of retailers. Scott Jordan (1988) studied the Just-in-time(JIT) by using Kanban and tries to minimize the overall level of inventory by allowing maximization of inventories among productions. Sita Bhaskaran (1998) pointed out increasingly competitive surroundings, higher needs of customers, a variety of products and quick response contribute to developments in lean production and supply chain network management. Soemon Takakuwa (1998), Soemon Takakuwa and Tsukasa Fujii (1999) introduced the process of developing a practical module using ARENA on shipment-inventory system. Dong Jin Kang and Sang Yong Yi (1985) considered an inventory model as one method of production management that maximizes service level for customers by providing products which are needed by customers in time with adequate inventory level kept. Woo Ju Kim (1997) integrated information system to comprehend inventory status more easily into a existing inventory system which has been studied. Moreover, he adds neural network to predict demand more accurately. Kyoo Sang Lee (1983) proposeed a mathematical model, sets up an optimal inventory policy, and keeps recording and maintaining by using computers to achieve scientific inventory management. In addition, he uses simulation assuming dynamic order interval from customers, the quantity of demand, and lead-times have probability distributions. Jung, Choong Y. (1985) analyzed structures of inventory under uncertainty in demand and lead-time. Chen, Derezner, Ryan, and Simchi-Levi (2000) discussed the importance of information sharing by considering the demand uncertainty and lead time in a two-stage supply chain system. Fransoo and Wounters (2000) measured the bullwhip effect on the basis of daily demand variability of convenience foods and proved that the bullwhip effect can be reduced by eliminating the amplification in demand variability.

Dejonckheere, Disney, Lambrecht, and Towill (2003) analyzed three forecasting methods with order-up-to replenishment policies to avoid bullwhip effect in supply chains. Disney and Towill (2003) developed an order policy to minimize bullwhip effect by controlling the inventory variance. Reyes (2005) presented an optimization model for a single period inventory problem in two-echelon supply chain.

3. Model Description

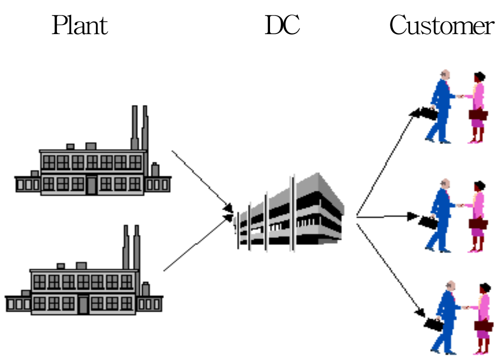
It is not an easy task to make a decision on an optimal inventory policy in a multi-echelon inventory system because of interactions of different levels of inventories. Despite the fact that lots of studies have presented solutions for a variety of issues in a multi-echelon inventory system until today, it is true that those studies have had limitations.

First, most of studies so far have provided solutions on branch-like structure even though the supply chain network which is considered for has tangled up network structure. Because most of earlier studies have dealt with a form of service, which distributes products that are supplied from suppliers to a number of customers, there are limits to apply solutions for a today's complex system like multi-echelon inventory system.

Secondly, existing studies have mainly considered how to distribute products. From an order to a delivery, there exists the process of transportation. Specially, transportation is closely connected with the lead-time, and thus it is vital to include it in calculating inventory cost.

To overcome aforementioned issues, this study assumes a 2 echelon inventory system like <Fig. 2> and targets stochastic lead-time supply chain network composed with two suppliers, one warehouse or distribution center, and three retailers.

The objective of this study is to find (s, S) policy for retailers and the warehouse of supply chain network like <Fig. 2> that is close to optimal under a service level constrain. It requires a complicated process to get an exact solution for (s, S) policy of supply chain network since it must consider many probabilistic factors. To solve that, in this paper, the first step is to get an approximate solution through a mathematical model, check secondly service ratio of backorder through simulations, and this will set up a standard as a reference for inventory policy decisions of real supply chain network in the future.



<Figure 2> Target Supply Chain Network

4. Model Analysis

The model proposed in this study is to find an approximate optimal solution for a reorder point and a target inventory of (s, S) policy in the warehouse and retailers under the constraint in customer's service ratio of supply chain network model.

Therefore, the design of the model needs the following three processes: (1) inventory analysis of each retailer, (2) demand structure analysis in the warehouse, (3) inventory analysis in the warehouse. Each analysis is ultimately integrated to minimize inventory in supply chain network.

The model is composed of one product with a unit price of V, N_r retailers, one warehouse or distribution center, n independent supplier. The quantity of demand at each retailer(r), is assumed to follow average λ_r's Poisson distribution. And if an inventory level is below reorder point(s_r), then retailers immediately order as much as $(S_r - i_r)$ to the warehouse. i_r refers to inventory level at the time of reorder. The lead-time from the warehouse to a retailer and from suppliers to the warehouse is L_r and L_w, respectively. It is assumed to follow normal distribution. If the ordered quantity is not satisfied, then it is remained as backorder.

Inventory at the warehouse, however, is decreased due to the orders from the retailers. Whenever the inventory level is below reorder point (s_w), there is an order as much as $(S_w - i_w)$ to suppliers. Likewise, i_w is a current inventory level at the warehouse when the order is occurred. This order is made to n independent suppliers who are pats of the supply chain network at the same time. Thus, the quantity of the order is $(S_w - i_w)/n$ to each supplier.

Because inventory level (i) in the particular warehouse and retailers is decided by an order which is occurred at random from retailers, the quantity of an order ($S - i$) is also probabilistic. It is, therefore, rational to use expected value in the analysis. For an easier analysis, it is assumed that expected quantity of an order in the warehouse is an integer multiple of that of retailers.

Moreover, the lead-time from a supplier to the warehouse is stochastic and it is assumed if orders are not fulfilled for all the warehouse and retailers, then order will not occur again.

If an order occurs again which results in orders more than twice, then it will lead to too many orders and high level of inventory. Thus, in case of unfulfilled orders outstanding, it is logical not to order again. The unfulfilled orders from a retailer to the warehouse is dealt with backorders as well. After transporting suppliable quantity to each customer among demands occurred during this time, the rest remains as a backorder. Available quantity of backorder between a retailer and the warehouse is up to the service level of total demand.

For example, suppose the current inventory level is 50 and the quantity of an order is 100. If the service level is 100%, then 100 will be transported and 50 will be the backorder. But, if the service level is 95%, then 95 will be transported and 45 will be the backorder.

In this study, r and w represent subscript for a retailer and the warehouse, respectively. We define notations as follows:

· N_r = the number of retailers

· n = the number of suppliers

· s_r, s_w = a reorder point in a retailer and the warehouse, respectively

· S_r, S_w = a target inventory in a retailer and the warehouse, respectively

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· D_r, D_W = daily demand in each retailer and the warehouse

· μ_{D_r}, μ_{D_w} = average demand in each retailer and warehouse

· R_r, R_W= expected yearly demand in each retailer and warehouse ($R_i = 360\mu_{D_i}$), respectively

· L_r, L_W = lead-time from the warehouse to a retailer and from suppliers to the warehouse, respectively

· μ_{L_r}, μ_{L_w} = average lead-time in each retailer and the warehouse, respectively

· Y_{L_r}, Y_{L_w} = demand during the lead-time in each retailer and the warehouse, respectively

· μ_{Y_r}, μ_{Y_w} = average demand during the lead-time in each retailer and the warehouse, respectively

· ρ_r, ρ_W = service level for each retailer and the warehouse, respectively

4.1 Inventory analysis in retailers

Although we assume N_r retailers in this study, for convenience in analysis, we examine inventory in one retailer case and show that it is applicable to all retailers independently.

As mentioned previously, when the level of inventory in a retailer below the reorder (s_r) level, a retailer makes an order which is amount to quantity of subtracting current inventory from target inventory, i.e. $S_r - i_r$. The warehouse, then, transport products as many as order quantity to a retailer. It takes L_r as the lead-time. If the current level of inventory is not enough to satisfy the amount ordered, then that order becomes a backorder and it depends on the service level. The shortage during this delivery period can be expressed as $\mathrm{E}[\,I(i_r < D_r)(D_r - i_r)]\rho_r$. s_r should satisfy the condition of $\mathrm{ES}_r \leq \mathrm{E}[\,I(i_r < D_r)(D_r - i_r)]\rho_r$. $I_r(i_r < D_r)$ is a binary variable that has the value of 0 or 1 depending on whether the condition in parenthesis is satisfied. In summary, a backorder will occur only if demand from a retailer is more than current inventory.

We also mention previously that daily demand form a retailer follows Poisson (λ_r) distribution. Assume the lead-time follow the probability of $P(L_r = t_r) = P_{t_r}$ and the conditional probability of demand at $L_r = t_r$ follow Poisson ($\lambda_r t_r$) ($t_r = 1, 2, \cdots, m$, only). Because a backorder will only occur when the demand exceeds s_r , expected shortage (ES_{t_r}) at each delivery period can be expressed as equation (1).

$$\mathrm{ES}_{t_r}(s_r) = \sum_{i=s_r}^{\infty}(i - s_r)\ e^{-\lambda_r t_r}(\lambda_r t_r)^i / i! \tag{1}$$

And equation (1) can be calculated as equation (2).

$$\lambda_r t_r - \sum_{i=0}^{s_r-1} i\ e^{-\lambda_r t_r}(\lambda_r t_r)^i / i! - s_r \times \left(1 - \sum_{i=0}^{s_r-1} e^{-\lambda_r t_r}(\lambda_r t_r)^i / i!\right) \tag{2}$$

From equation (2), we can see that an average backorder is relatively smaller than an average inventory quantity. Thus, predicted expected shortage during overall lead-time period can be shown as equation (3) by using equation (1) and (2).

$$\mathrm{ES}_r = \sum_{t_r=1}^{m} \mathrm{ES}_{t_r}(s_r) P_{t_r} \tag{3}$$

4.2 Demand Structure in Warehouse

Demand structure in the warehouse has the characteristic that it depends on orders from retailers. Thus, the orders from the warehouse to a supplier should be large enough to satisfy retailer's orders.

Let us assume the warehouse get orders from one retailer. If the retailer's expected yearly demand is R_r($R_r = 360\mu_{D_r}$), then one-time expected order quantity becomes $\mathrm{E}[I(i_r < s_r)(S_r - i_r)]$. The total number of expected yearly orders becomes expected yearly demand divided by expected order quantity, that is, $R_r / \mathrm{E}[I(i_r < s_r)(S_r - i_r)]$. $I_r(i_r < s_r)$ is also a binary variable with the value of 0 or 1.

Whenever there is an order, the retailer will ask the quantity of $E[I(i_r < s_r)(S_r - i_r)]$ and yearly order quantity in the warehouse becomes R_r. If we assume 360 day basis for a year, daily average demand in the warehouse is $R_r/360$ and let it be μ_{D_w}, then we can see that $\mu_{D_w} = \mu_{D_r}$.

In reality, however, supply chain network has multiple retailers like the model used in this study, and the warehouse should deal with all retailers′ orders. Therefore, a daily average quantity of the demand at the warehouse is equal to the sum of all retailers′ daily average demands as expressed in equation (4).

$$\mu_{D_w} = \sum \mu_{D_r} \quad (4)$$

Retailers′ demand used in this study is assumed to follow Poisson distribution with average λ_r. and there are N_r retailers which mean that $\lambda_W = \sum \lambda_r = N_r \times \lambda_r$. It also implies average demand in the warehouse follows Poisson distribution with average λ_W.

In this case, it has been known that if a number of retailers increase infinitely, demand in the warehouse follows Poisson distribution. It will be a good way to approach this issue by assuming demand in the warehouse follows Poisson distribution in case of more than 20 retailers. It is not an issue to generally assume Poisson distribution since it is not a strict restriction though it is a very important fact.

4.3 Inventory Analysis in the Warehouse

Inventory in the warehouse is very similar to that of a retailer except two things. One is that the warehouse must consider retailers′ expected order quantity, $E[I(i_r < s_r)(S_r - i_r)]$. It is because an order from the warehouse depends on orders occurred from retailers. $I(i_r < s_r)$ is, of course, a binary variable with the value of 0 or 1. The other is that an order from the warehouse is made to multiple suppliers.

Like retailers, given the service level ρ_W, if order quantities, $E[I(i_r < s_r)(S_r - i_r)]$, from retailers to the warehouse are EQ_r, then unsatisfied order quantity will be $EQ_r \rho_W$. Thus, if let predictable shortage be ES_W, then s_W and S_W in the warehouse should satisfy the condition of $ES_W \le E[I(i_W < EQ_r)(EQ_r - i_W)]\rho_W$.

We mention that daily demand, D_W, follows Poisson distribution with average λ_W in the previous section. Let L_W^i be suppliers′ ith($i = 1, 2, \cdots, n$) lead-time. The warehouse orders the quantity of $E[I(i_W < s_W)(S_W - i_W)]/n$ to n suppliers at the same time. L_W^i will depend on the fastest arrival order among them. That is, $L_W = \mathrm{Min}(L_W^1, \ldots, L_W^n)$. Let $H(t)$ be each L_W^i′s conditional probability mass function. Then, a solution can be obtained as the following equation, $P(L_W \le t) = 1 - [1 - H(t)]^n$.

Let ES_{t_W} be expected shortage when the most efficient lead-time t_W is occurred. Then, it can be solved as the following equation (5).

$$ES_{t_r}(s_W) = \sum_{i=s_W}^{\infty} (i - s_W)\, e^{-\lambda_W t_W} (\lambda_W t_W)^i / i! \quad (5)$$

Equation (5) also can be expressed as the equation (6).

$$\lambda_W t_W - \sum_{i=0}^{s_W - 1} i\, e^{-\lambda_W t_W} (\lambda_W t_W)^i / i! - s_W \times \left(1 - \sum_{i=0}^{s_W - 1} e^{-\lambda_W t_W} (\lambda_W t_W)^i / i!\right) \quad (6)$$

Finally, expected shortage ES_W during the lead-time can be calculated as the equation (7).

$$ES_W = \sum_{t_W = 1}^{I} ES_{t_W}(s_W) P_{t_W} \quad (7)$$

Here, P_{t_W} is the probability of $L_W = t_W$.

4.4 Overall Inventory Cost Model

Expected total annual cost can be represented as equation (8):

$$TC = c_o + c_h + c_s \tag{8}$$

c_o is ordering cost, c_h is inventory carrying cost , and c_s is cost of shortage. And, each cost can be expressed separatedly as follows:
First, ordering cost can be solved by the following equation.

$$c_o = (a + bn) R_w / \ \mathrm{E}[\, I(i_w < s_w)(S_w - i_w)]$$
$$+ N_r A_r R_r / \ \mathrm{E}[\, I(i_r < s_r)(S_r - i_r)]$$

$a + bn$ is related with set-up cost. We can see that as the number of suppliers increase, ordering cost increases. In addition, $R_i / \ \mathrm{E}[\, I(i_i < s_i)(S_i - i_i)]$ (i = r, w) is the value that is calculated as dividing yearly expected demand by expected order. Through this, we can get the number of yearly orders.

Inventory carrying cost is calculated as follows:

$$c_h = (\ \mathrm{E}[\, I(i_w < s_w)(S_w - i_w)]/2 + s_w - \mu_{Y_w})\, vh$$
$$+ (\ \mathrm{E}[\, I(i_r < s_r)(S_r - i_r)]/2 + s_r - \mu_{Y_r})\, vhN_r$$

$\mathrm{E}[\, I(i_i < s_i)(S_i - i_i)]/2$ and $s_i - \mu_{Y_i}$(i=r, w) refer to a cycle between orders occurred in the warehouse and retailers and safety stock, respectively and h represents a symbol for yearly inventory carrying cost per unit.
Lastly, the following equation is for the cost of shortage.

$$c_s = R_w \overline{S_w}(x_w) / \ \mathrm{E}[\, I(i_w < s_w)(S_w - i_w)]\pi$$
$$+ R_r \overline{S_r}(x_r) / \ \mathrm{E}[\, I(i_r < s_r)(S_r - i_r)]\pi N_r$$

$\overline{S_i}(x_i)$($i = r, w$) is possible expected inventory shortage at each cycle and π is the cost of out of stock per unit. It is explained in the previous section how to get $\overline{S_i}(x_i)$.

Transportation cost is not considered in this study. Since transportation causes shortage transportation cost is included in the cost of out of stock.

Finally, the objective in this model is to find S_w, s_w, S_r, s_r that minimize TC under the following constrains:

$$\mathrm{ES}_w \le \ \mathrm{E}[\, I(i_w < \mathrm{EQ}_r)(\mathrm{EQ}_r - i_w)]\rho_w \tag{9}$$
$$\mathrm{ES}_r \le \ \mathrm{E}[\, I(i_r < D_r)(D_r - i_r)]\rho_r \tag{10}$$
$$\mathrm{E}[I(i_w < s_w)(S_w - i_w)]$$
$$= k\,\mathrm{E}[I(i_r < s_r)(S_r - i_r)] \tag{11}$$
$$s_w, s_r \ge 1 \tag{12}$$

k can be any positive number. But, the expected value is used in equation (11), any number that is close to a positive is acceptable.

5. Simulation Experiments

5.1 Simulation Model

5.1.1 Set-up for Simulation Model

The aim of simulation is to check the difference between a backorder in retailers and the warehouse and a reorder point and a target inventory which is set by an analytic method. As we can see from the aforementioned formulas, however, they include many probabilistic factors and have an indicator variable like $\mathrm{E}[\, I(i_i < s_i)(S_i - i_i)]$(i = r, w) that has only values when the condition is satisfied. Thus, it is not easy to get a solution.
To overcome this, we simplify it as $\mathrm{E}[\, I(i_i < s_i)(S_i - i_i)] = S_i - s_i = Q_i$ (i = r, w) and assume there will be a no order after an order is made until the products are delivered. But, since orders are made to multiple suppliers at the same time for the warehouse we let an order be made possible as long as some of the orders are delivered. The value obtained from above is applied to the simulation model to get an average backorder

quantity and check the performance level about it. The performance level can be obtained from the following formular;

$$100 \times \left| \frac{\sim \text{ulation result} - \text{mathematical model result}}{\sim \text{ulation result}} \right|$$

Simulation is performed total 10 years with 360 days as 1 year. The first year is warm-up period and the remaining 9 years′ average is used in performance evaluation. And we use ARENA, widely used simulation language, for simulation.

5.1.2 Construction of Simulation Model

<Table 1> shows inventory related cost and supply chain structure of the simulation model. Parameters that used for simulation are as follows:

<Table 1> Inventory Related Cost and Supply Chain Structure

Number of Retailers	3
Number of Suppliers	2
Price of Product(v)	\$25
Set-up Parameter(Retailer)	\$10
Set-up Parameter(Warehouse)	\$10 + \$5 n (n is number of suppliers.)
Inventory Holding Cost	\$2
Stockout Cost	\$5
Interval between orders (Retailer)	Exponential distribution with average 1

<Table 2> Detailed Construction of Simulation Model

Experiments #	ρ_W(%)	ρ_r(%)	L_W	L_r
1	100	100	H	H
2	100	95	H	H
3	100	100	L	H
4	100	95	L	H
5	100	100	H	L
6	100	95	H	L
7	100	100	L	L
8	100	95	L	L
9	95	100	H	H
10	95	95	H	H
11	95	100	L	H
12	95	95	L	H
13	95	100	H	L
14	95	95	H	L
15	95	100	L	L
16	95	95	L	L

- Daily demand in a retailer(D_r) : Poisson(10)
- Lead-time from the warehouse to a retailer(L_r) : $N(2, 1^2)$, $N(2, 2^2)$
- Lead-time from a supplier to the warehouse(L_w) : $N(3, 1^2)$, $N(3, 2^2)$
- Service level at a retailer (ρ_r) : 100%, 95%
- Service level at the warehouse (ρ_w) : 100%, 95%

The following <Table 2> is details about each simulation model. The simulation model is built by combining parameters that previously explained and there are total 16 models. H in <Table 2> refers to a relatively higher value among parameters that belong to the model. For example, H of ρ_w, ρ_r, L_w, L_r is 100%, 100%, $N(3, 2^2)$, $N(2, 2^2)$, respectively. L, of course, refers to the lower value.

5.2 Experimental Results

The experiment is comprised of two parts. At first, a solution from the mathematical model should be obtained. It is followed by building adequate simulation model and getting a solution from the simulation.

Because we simplify $Q = S - s$ to solve the mathematical model the initial value of order quantity Q is obtained from EOQ model and a reorder point at this moment is set to 0 and applied to get a solution by repetitions. And, if there is no clear improvement in getting a solution, search process is stopped.

5.2.1 Result at Retailers

5.2.1.1 Result from the Mathematical Model

Stage 1

$$Q_1 = \sqrt{2AD/h} = \sqrt{2 \times 10 \times 10 \times 360/2} = 190$$

$$\int_{s_r^*}^{\infty} f(x)\,dx = 2 \times 190 / (5 \times 360 \times 10) = 0.21$$

Here, $f_{Y_{L_r}}(x)$ is the function of demand quantity occurring from retailers during the lead-time. We

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can see that $s_r^* = 29$ from the equations above.

Stage 2

$$ES_r = 0.7$$

$$Q_2 = \sqrt{2D(A+\pi\overline{S}(x))/h}$$

$$= \sqrt{2\times10\times[10\times360+5\times0.7]/2} = 190$$

Since there is no difference in values between Q_1 and Q_2 search process is stopped. At that moment, reorder point (s_r^*) is 29, and order quantity is 190, and thus, reminding of $Q=S-s$, target inventory (S_r^*) should be 219. The expected shortage at this time, that is, backorder is 0.054 which imply there is almost no shortage.

5.2.1.2 The Result of the Simulation

According to the results of the model, if the service level is set to 100%, i.e. all demand is satisfied, then, $S_r^*=219$ and $s_r^*=29$. The backorder at that moment is 0.054, which implies that there will be no backorder. Because the quantity of the products are an integer, 0.054 becomes 0. But, we simply display calculated errors as in the mathematical solution. ΔR is the value obtained from the performance formula which is mentioned in the section 5.1.1. In other words, $\Delta R = \frac{\text{simulation result} - 0.054}{\text{simulation result}}$ is got from the formula, percentage error is the value of $\Delta R\times100$. Because the model has total 3 retailers, the average of 3 retailers is used as the backorder. The following <Table 3> is the summary of the results of the simulations.

As we can see from the <Table 3>, the percentage error is very close to 100% in most of the cases. But, as we saw in the mathematical derivation, the backorder is 0.05. From that, we expect there will be no backorder. The result of the simulation, however, shows that the backorder is close to 1, which cause the big percentage errors. Therefore, it will do no harm to think that there is no big difference in the backorder between the mathematical model and the result of simulation in the case of the retailers.

5.2.2 The Result in the Warehouse

5.2.2.1 A Mathematical Solution

An average demand quantity in the warehouse is the sum of average demand quantity of retailers as we mentioned earlier. Thus, we get a solution by letting $\lambda_W = 3\times10 = 30$.

<Table 3> The Result in Retailers

Experiment #	ΔR	Percentage Error
1	0.947	94.7%
2	0.946	94.6%
3	0.951	95.1%
4	0.936	93.6%
5	0.962	96.2%
6	0.962	96.2%
7	0.964	96.4%
8	0.962	96.2%
9	0.947	94.7%
10	0.947	94.7%
11	0.949	94.9%
12	0.948	94.8%
13	0.967	96.7%
14	0.967	96.7%
15	0.954	95.4%
16	0.947	94.7%

Stage 1

$$Q_1 = \sqrt{2(a+bn)D/h}$$

$$= \sqrt{2\times(10+5\times2)[30\times360]/2} = 465$$

$$\int_{s_w^*}^{\infty} f_{Y_{L_w}}(x)\,dx = 2\times465/(5\times30\times360) = 0.0172$$

$f_{L_w}(x)$ is the function of demand quantity occurred in the warehouse during the lead-time. From the equations above, we can see that $s_W^* = 110$.

Stage 2

$$ES_W = 20.075$$

$$Q_2 = \sqrt{2\times(10+5\times2)\times[30\times360+5\times20.075]/2}$$

$$= 467$$

$$\int_{s_w^*}^{\infty} f_{Y_{L_w}}(x)\,dx = 2\times467/(5\times30\times360) = 0.0173$$

Because there is no big difference in values between Q_1 and Q_2 and the value of reorder point (s^*_w) is equal to 110 which shows no change, the process is stopped. At this moment, reorder point (s^*_w) is 110 and order quantity is 467. Therefore, a target inventory (S^*_w) becomes 577.
Expected shortage at this time, in other words, the backorder is 20.075. We expect about 20 backorder quantities.

5.2.2.2 The Result of Simulation

A mathematical solution in the warehouse is obtained just like the case of retailers. In the case of the warehouse, we let average daily demand quantity be $\lambda_w = N_r \times \lambda_r$ and calculate S_w, s_w, and a backorder.

When $\lambda_w = 30$, The result is that $S^*_w = 577$ and $s^*_w = 110$ obtained in the previous section. At this moment, service level was assumed to be 100%, as well. Expected backorder quantity is 20.075, which is about 20. Like the case of retailers, the formula for performance is $\Delta W = \left| \frac{\text{simulation result} - 20.075}{\text{simulation result}} \right|$, a percentage error is calculated as $\Delta W \times 100$. The results are shown in the <Table 4>.

We assume the order quantity from the warehouse to the suppliers is divided by 2, which is the number of the suppliers. Thus, each supplier will get the same amount of order quantity at the same time. The minimum amount of time is considered as the lead-time about the order. The order quantity is expressed as $E[I(i_w < s_w)(S_w - i_w)]/2$, and reorder can be made as long as the products from an order out of all orders are arrived.

According to <Table 4>, the percentage error varies from the minimum 2.2% to the maximum 46.8%. This is because, firstly, we cannot obtain accurate average daily demand quantity in the warehouse in that average daily demand quantity in the warehouse is calculated as the sum of retailers' average daily demand quantity. Secondly, this is the result from the fact that orders to the suppliers from the warehouse is calculated as the overall order $E[I(i_w < s_w)(S_w - i_w)]$ divided by the number of suppliers(n). This yields the same quantity of an order to each supplier. In other words, orders are distributed in a number of ways and lead-time varies, which cause the inventory level in the warehouse to be insufficient to satisfy the order quantity at the right time.

Even if that is the case, it is reasonable enough to apply the process to the given model since the difference in theoretically calculated value is the maximum 15.

<Table 4> The Result of Simulation in the Warehouse

Experiment #	ΔW	Percentage Error
1	0.431	43.1%
2	0.244	24.4%
3	0.439	43.9%
4	0.245	24.5%
5	0.348	34.8%
6	0.348	34.8%
7	0.343	34.3%
8	0.060	6.0%
9	0.468	46.8%
10	0.468	46.8%
11	0.322	32.2%
12	0.199	19.9%
13	0.030	3.0%
14	0.022	2.2%
15	0.193	19.3%
16	0.033	3.3%

The results of simulations in the warehouse and retailers as in the <Table 3> and <Table 4> show that performance is not good. Specially, in the case of retailers, performance is close to 100% so that it seems the process of the mathematical solution is not suitable. Retailers' case has, however, 0 backorder and the backorder from the simulation is 1. This results in high percentage errors. If we consider the difference in terms of the number of the products, then it makes almost no difference.

The simplification of $Q = S - s$ to facilitate handling formulas in the case of retailers shows very good results.

Compared to the performance of retailers, performance in the warehouse shows a lot more balanced values. In spite of this, it is true that the variability in an expected order quantity of the warehouse is wider than that of retailers. As it was pointed out when we discuss the result of the warehouse, we will get more accurate results by studying further on 1) Increase in the number of retailers, 2) allocation of order quantity to suppliers.
If a number of retailers are increased, then daily demand quantity for the warehouse becomes close to reality so that more accurate results can be obtained from the mathematical derivation.

In addition, allocation of order quantity has a big impact on the lead-time from the suppliers to the warehouse. Actually, there is a big difference in values when all the ordered products are arrived, i.e. the case with maximum value as the lead-time among all lead-times and the case with minimum value as the lead time. But, simplification in mathematical derivation leads to big errors in results because it does not reflect this phenomenon. The predictable expected backorder quantity from the result of simulation dose not deviate much in the worst case from the theoretical value of 20 to 35 which is not that bad of a result. And, except this case, the result of the simulation shows that predictable expected backorder quantity is within around 20, which means that it is trustworthy to get the result even if there is simplification in decisions of a reorder point and a target inventory level.

6. Conclusion

Firms have paid more attentions on supply chain network management since it helps firms to adjust to fast changing corporate environments and satisfy detailed customers' demands. It has been known that the success of supply chain network management in its nature depends on efficient inventory management.

Thus, this study builds a mathematical model for (s, S) policy of supply chain network under probabilistic environments. But, we find that it is not an easy task to find an optimal reorder point and a target inventory level due to random change in demand and the difficulty in predicting order quantity. Thus, under some assumptions, we firstly try to get approximate solutions of a reorder point and an inventory level. Secondly, solutions obtained from the model are applied to the simulation model to get the answers for backorder quantity.

According to the result of the simulations, there seems a big difference in percentage errors. In the case of retailers and the warehouse, however, the errors are maximum 1 and 15, respectively which implies that there is no big differences with real value. Therefore, we believe (s, S) policy of supply chain network which determines a reorder point and a target inventory is similar to that of a simplification method used in this paper.

This study, of course, has some limitations. First, average demand in the warehouse is simply calculated as the sum of average demands of retailers. Fundamentally, this causes big errors. If there is a better way to measure, then it will be a great help in understanding the trend of demand of downstream firms for supply chain network and make it easier to make suitable (s, S) policy.

Secondly, in case of multiple suppliers, this study assumes order quantity is divided by the number of suppliers, and thus an order is made to the suppliers simultaneously. This may incur additional ordering cost from unnecessary orders and the different lead-time by orders. Actually, there is a big difference in a backorder between (1) the case which an order is made to the suppliers only when all the ordered products are arrived and (2) the case which an order is made to the suppliers as long as some of the ordered products are arrived. Thus, we need to examine a suitable policy throughly when there are multiple suppliers because it will affect the overall cost of the inventory.

Lastly, this study deals with only one product. But, in reality, supply chain network handles a variety of products. Because each product has a unique demand structure and an order trend, it is possible for a firm to set a policy for each product. But, a firm may choose not to pay attention to some products in inventory management due to timing or policy. In this case, it will create gaps in the overall cost of the inventory or an backorder quantity. This requires an applicable policy that includes significance factors in a supply chain network that deals with multiple products.

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저 자 소 개

한 재 현

고려대학교 동양사학과 학사 University of Chicago 금융수학 석사 George Washington 대학 재무관리 박사, 현재 광운대학교 경영대학 경영학부 교수
관심분야: 기업재무, 재무적 공급사슬

주소: 서울시 노원구 광운로 20 광운대학교 경영대학 경영학부

정 석 재

한국해양대학교 물류시스템공학과 학사, 연세대학교에서 산업공학 석사 및 박사취득, 현재 광운대학교 경영대학 경영학부 교수
관심분야는 공급사슬 관리와 스케줄링이 주 전공이며 관심분야는 SCM, 친환경공급사슬, 가치기반 공급사슬 등

주소: 서울시 노원구 광운로 20 광운대학교 경영대학 경영학부