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GENERALIZED THERMO ELASTIC WAVES IN A CYLINDRICAL PANEL EMBEDDED ON ELASTIC MEDIUM

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ABSTRACT: In this paper the three dimensional wave propagation in a homogeneous isotropic thermo elastic cylindrical panel embedded in an elastic medium (Winkler model) is investigated in the context of the L-S (Lord-Shulman) theory of generalized thermo elasticity. The analysis is carried out by introducing three displacement functions so that the equations of motion are uncoupled and simplified**.** A Bessel function solution with complex arguments is then directly used for the case of complex Eigen values .This type of study is important for design of structures in atomic reactors, steam turbines, wave loading on submarine, the impact loading due to superfast train and jets and other devices operating at elevated temperature. In order to illustrate theoretical development, numerical solutions are obtained and presented graphically for a zinc material with the support of MATLAB.

1. INTRODUCTION

Cylindrical panel plays an important structural component in many engineering fields such as aerospace, civil, chemical, mechanical, naval and nuclear. The dynamical interaction between the cylindrical panel and solid foundation has great practical applications in modern engineering fields due to their static and dynamic behaviors will be affected by the surrounding media. Studies of propagation of elastic waves at an interface have long been of interest to researchers in the fields of geophysics, acoustics and nondestructive evaluation. Common to all these studies is the investigation of the degrees of interaction among the interfaces that manifest themselves in the forms of reflection and transmission agents and give rise to geometric dispersion. These interactions depend upon the mechanical properties, geometric arrangements, nature of the interfacial conditions and the loading conditions.

The analysis of thermally induced wave propagation of cylindrical panel embedded in an elastic medium is common place in the design of structures, atomic reactors, steam turbines, wave loading on submarine, the impact loading due to superfast train and jets and other devices operating at elevated temperature. In the field of nondestructive evaluation, laser-

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generated waves have attracted great attention owing to their potential application to noncontact and nondestructive evaluation of sheet materials. Moreover, it is well recognized that the investigation of the thermal effects on elastic wave propagation supported by elastic foundation has bearing on many seismological application. This study may be used in applications involving nondestructive testing (NDT), qualitative nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

The static analysis cannot predict the behavior of the material due to the thermal stresses changes very rapidly. Therefore in case of suddenly applied loading, thermal deformation and the role of inertia getting more important. This thermo elastic stress response being significant leads to the propagation of thermo elastic stress waves in solids. The theory of thermo elasticity is well established by Nowacki [1]. Lord and Shulman [2] and Green and Lindsay [3] modified the Fourier law and constitutive relations, so as to get hyperbolic equation for heat conduction by taking into account the time needed for acceleration of heat flow and relaxation of stresses. A special feature of the Green–Lindsay model is that it does not violate the classical Fourier's heat conduction law. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by X.Wang et.al [4]. Hallam and Ollerton [5] investigated the thermal stresses and deflections that occurred in a composite cylinder due to a uniform rise in temperature, experimentally and theoretically and compared the obtained results by a special application of the frozen stress technique of photo elasticity. Noda [6] have studied the thermal-induced interfacial cracking of magneto electro elastic materials under uniform heat flow. Chen et al [7] analyzed the point temperature solution for a penney-shapped crack in an infinite transversely isotropic thermo-piezo-elastic medium subjected to a concentrated thermal load applied arbitrarily at the crack surface using the generalized potential theory. Banerjee and Pao [8] investigated the propagation of plane harmonic waves in infinitely extended anisotropic solids by taking into account the thermal relaxation time. Sharma [9] investigated the three dimensional vibration analysis of a transversely isotropic thermo elastic cylindrical panel. Free vibrations of thin cylindrical shells having finite lengths with freely supported and clamped edges was discussed by Yu et.al[10]. An interesting problem in engineering is the static and dynamic analysis of plates and shell supported on elastic foundations [11].For isotropic cylindrical shell buried at a depth below the free surface of the ground, Wong et al. [12] gave its dynamic response from the point of view of three-dimensional elastic theory. Paliwal et al. [13] presented an clear investigation on the coupled free vibrations of isotropic circular cylindrical shell on Winkler and Pasternak foundations by employing a membrane theory. Upadhyay and Mishra [14] dealt with the non-axisymmetric dynamic behavior of buried orthotropic cylindrical shells excited by a combination of P-, SV and SH-waves. On natural frequencies of a transversely isotropic Cylindrical panel on a kerr foundation was discussed by Chen et al [15].

In this paper the three dimensional wave propagation in a homogeneous isotropic generalized thermo elastic cylindrical panel embedded in a elastic medium (Winkler model) is investigated in the context L.S theory of thermo elasticity. The analysis is carried out by introducing three displacement functions so that the equations of motion are uncoupled and simplified**.** A Bessel function solution with complex arguments is then directly used for the case of complex Eigen values. In order to illustrate theoretical development, numerical solutions are obtained and presented graphically for a zinc material.

2. THE GOVERNING EQUATIONS

Consider a cylindrical panel embedded on elastic medium as shown in Fig.1 of length L having inner and outer radius a and b with thickness h. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α. The deformation of the cylindrical panel in the direction r, θ, and z are defined by u, v and w. The cylindrical panel is assumed to be homogenous, isotropic and linearly elastic with Young's modulus E, poisson ratio ν and density ρ in an undisturbed state.

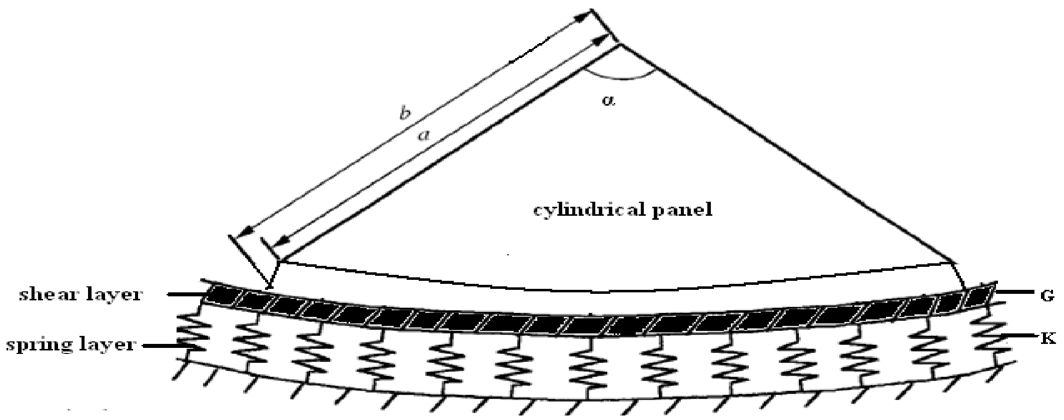

Fig.1 Geometry of the problem

In cylindrical coordinate the three dimensional stress equation of motion, strain displacement relation and heat conduction in the absence of body force for a linearly elastic medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt} \tag{1a}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rzz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt} \tag{1b}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{r\theta} = \rho w_{,tt} \tag{1c}$$

$$\kappa\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}\right) - \rho C_v\left(T_{,t} + \tau_0 T_{,tt}\right) = \beta T_0\left(\frac{\partial}{\partial t} + \delta_{1k}\tau_0\frac{\partial^2}{\partial t^2}\right) \times\left(u_{,r} + r^{-1}\left(u + v_{,\theta}\right) + w_{,z}\right) \tag{1d}$$

where ρ is the mass density, c_v is the specific heat capacity, $\kappa = K/\rho c_v$ is the diffusity, K is the thermal conductivity, T_0 is the reference temperature.

$$\sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} - \beta(T + \tau_1\delta_{2k}T_{,t}) \tag{2a}$$

$$\sigma_{\theta\theta} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{\theta\theta} - \beta(T + \tau_1\delta_{2k}T_{,t}) \tag{2b}$$

$$\sigma_{zz} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{zz} - \beta(T + \tau_1\delta_{2k}T_{,t}) \tag{2c}$$

e_{ij} are the strain components, β is the thermal stress coefficients, T is the temperature, t is the time, λ and μ are Lame' constants. τ_0 and τ_1 are the thermal relaxation times and the comma notation is used for spatial derivatives. Here the δ_{ij} is the Kronecker delta function. In addition, $k = 1$ for L.S theory and $k = 2$ for G.L theory. The thermal relaxation times τ_0 and τ_1 satisfies the inequalities $\tau_0 \geq \tau_1 \geq 0$ for G.L theory only. The strain e_{ij} are related to the displacements are given by

$$\sigma_{r\theta} = \mu\gamma_{r\theta} \quad \sigma_{rz} = \mu\gamma_{rz} \quad \sigma_{\theta z} = \mu\gamma_{\theta z} \quad e_{rr} = \frac{\partial u}{\partial r} \quad e_{\theta\theta} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} \tag{3}$$

$$e_{zz} = \frac{\partial w}{\partial z} \quad \gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta} \quad \gamma_{rz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \quad \gamma_{z\theta} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta} \tag{4}$$

where u, v, w are displacements along radial, circumferential and axial directions respectively. $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ are the normal stress components and $\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{zr}$ are the shear stress components, $e_{rr}, e_{\theta\theta}, e_{zz}$ are normal strain components and $e_{r\theta}, e_{\theta z}, e_{zr}$ are shear strain components.

Substituting the equations (2), (3) and (4) in equations (1), gives the following three displacement equations of motion

$$\left(\lambda+2\mu\right)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + \mu r^{-2}u_{,\theta\theta} + \mu u_{,zz} + r^{-1}\left(\lambda+\mu\right)v_{,r\theta} - r^{-2}\left(\lambda+3\mu\right)v_{,\theta}$$
$$+\left(\lambda+\mu\right)w_{,rz} - \beta\left(T_{,r} + \tau_1\delta_{2k}T_{,rt}\right) = \rho u_{,tt}$$

$$\mu\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-2}\left(\lambda+2\mu\right)v_{,\theta\theta} + \mu v_{,zz} + r^{-2}\left(\lambda+3\mu\right)u_{,\theta} + r^{-1}\left(\lambda+\mu\right)u_{,r\theta}$$
$$+r^{-1}\left(\lambda+\mu\right)w_{,\theta z} - \beta\left(T_{,\theta} + \tau_1\delta_{2k}T_{,\theta t}\right) = \rho v_{,tt},$$

$$\left(\lambda+2\mu\right)w_{,zz} + \mu\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + \left(\lambda+\mu\right)u_{,rz} + r^{-1}\left(\lambda+\mu\right)v_{,\theta z}$$
$$+r^{-1}\left(\lambda+\mu\right)u_{,z} - \beta\left(T_{,z} + \tau_1\delta_{2k}T_{,zt}\right) = \rho w_{,tt}$$

$$\rho c_v \kappa\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}\right) = \rho c_v\left[T_{,t} + \tau_0 T_{,tt}\right] + \beta T_0\left(\frac{\partial}{\partial t} + \tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right) \times\left[u_{,r} + r^{-1}(u + v_{,\theta}) + w_{,z}\right] \tag{5}$$

To solve equation (5), we take the displacement potential as

$$u = \frac{1}{r}\psi,_{\theta} - \phi,_{r} \qquad v = -\frac{1}{r}\phi,_{\theta} - \psi,_{\varpi} \qquad w = -\chi,_{z} \tag{6}$$

Using Eqs (6) in Eqs (5), we find that ϕ, χ, T satisfies the equations

$$((\lambda+2\mu)\nabla^2{}_1 + \mu\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\phi - (\lambda+\mu)\frac{\partial^2\chi}{\partial z^2}^2 = \beta\ (T + \tau_1\delta_{2k}T_{,t}) \tag{7a}$$

$$(\mu\nabla_1{}^2 + (\lambda+2\mu)\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2})\chi - (\lambda+\mu)\nabla_1{}^2\phi = \beta(T + \tau_1\delta_{2k}T_{,t}) \tag{7b}$$

$$(\nabla_1{}^2 + \frac{\partial^2}{\partial z^2} - \frac{\rho}{\mu}\frac{\partial^2}{\partial t^2})\psi = 0 \tag{7c}$$

$$\nabla_1{}^2 T + \frac{\partial^2 T}{\partial z^2} - \frac{\rho C_V i\omega\eta_1 T}{K} = \frac{\beta T_0(i\omega)\eta_2}{K}(\nabla_1{}^2\phi + \frac{\partial^2\chi}{\partial z^2}) \tag{7d}$$

where $\eta_0 = 1 + i\omega\tau_0$, $\eta_1 = 1 + i\omega\delta_{2k}\tau_1$, $\eta_2 = 1 + i\omega\delta_{1k}\tau_0$.

Equation (7c) in ψ gives a purely transverse wave, which is not affected by temperature. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$.

3. SOLUTION TO THE PROBLEM

The equation (7) is coupled partial differential equations of the three displacement components. To uncouple equation (7), we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [9]

$$\begin{aligned}
\psi(r,\theta,z,t) &= \overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t} \\
\phi(r,\theta,z,t) &= \overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \\
\chi(r,\theta,z,t) &= \overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \\
T(r,\theta,z,t) &= \overline{T}(r,\theta,z,t)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}
\end{aligned} \tag{8}$$

Where m is the circumferential mode and n is the axial mode, ω is the angular frequency of the cylindrical panel motion. By introducing the dimensionless quantities

$$r' = \frac{r}{R} \qquad z' = \frac{z}{L} \qquad \overline{T} = \frac{T}{T_0} \qquad \delta = \frac{n\pi}{\alpha} \qquad t_L = \frac{m\pi R}{L} \qquad \overline{\lambda} = \frac{\lambda}{\mu}$$

$$\in_4 = \frac{1}{2+\overline{\lambda}}\ C_1^{\ 2} = \frac{\lambda+2\mu}{\rho}\ \omega^2 = \frac{\Omega^2 R^2}{C^2{}_1}$$

After substituting equation (9) in (8), we obtain the following system of equations

$$(\nabla_2^2 + k_1^{\ 2})\overline{\psi} = 0 \tag{9a}$$

$$(\nabla_2^2 + g_1)\overline{\phi} + g_2\overline{\chi} - g_4\overline{T} = 0 \tag{9b}$$

$$(\nabla_2^2 + g_3)\overline{\chi} + (1+\overline{\lambda})\nabla_2^2\overline{\phi} + (2+\overline{\lambda})g_4\overline{T} = 0 \tag{9c}$$

$$(\nabla_2^{\ 2} - t_L^{\ 2} + \in_2 \varpi^2 - i\in_3)\overline{T} + i\in_1 \varpi\nabla_2^{\ 2}\overline{\phi} - i\in_1 \varpi t_L^{\ 2}\overline{\chi} = 0 \tag{9d}$$

where

$$\nabla_2^2 = \frac{\partial^2}{\partial r^2}\frac{1}{r}\frac{\partial}{\partial r} - \frac{\delta^2}{r^2},\ \ \in_1 = \frac{T_0 R\beta^2}{\rho^2 C_V C_1 K},\ \ \in_2 = \frac{C_1^{\ 2}}{C_V K},\ \ \in_3 = \frac{C_1 R}{K}$$

$$g_1 = (2+\overline{\lambda})(t_L^2 - \omega^2),\ \ g_2 = \in_4 (1+\overline{\lambda})t_L^{\ 2},$$

$$g_3 = (\omega^2 - \in_4 t_L^{\ 2}),\ \ g_4 = \frac{\beta T_0 R^2 \eta_1}{\lambda+2\mu},\ \ g_5 = \in_1 \omega$$

$$\begin{vmatrix} (\nabla_2^{\ 2} + g_1) & -g_2 & g_4 \\ (1+\overline{\lambda})\nabla_2^{\ 2} & (\nabla_2^{\ 2} + g_3) & (2+\overline{\lambda})g_4 \\ ig_5\nabla_2^{\ 2} & -ig_5 t_L^{\ 2} & (\nabla_2^{\ 2} - t_L^{\ 2} + \in_2 \omega^2 - i\omega\in_3) \end{vmatrix}(\overline{\phi},\overline{\chi},\overline{T}) = 0 \tag{10}$$

Equation (10), on simplification reduces to the following differential equation:

$$\nabla_2^6 + A\nabla_2^4 + B\nabla_2^2 + C = 0 \tag{11}$$

where,

$$A = -g_1 + g_2(1+\overline{\lambda}) + g_3 - g_4 g_5 i t_L^{\ 2} + \in_2 \omega^2 - i\in_3 \varpi$$

$$B = -g_1 g_3 - g_1 g_4 g_5 i - g_2 g_4 g_5 i(2+\overline{\lambda}) + t_l^{\ 2}(g_1 - g_2 - g_3) + g_4 g_5 i t_L^{\ 2} + g_2\omega^2 \in_2 (1+\overline{\lambda})$$
$$-g_2 i\in_3 \omega(1+\overline{\lambda}) - g_2 t_L^{\ 2}\overline{\lambda} + g_3\omega(\omega\in_2 - i\in_3) + g_1\omega(i\in_3 - \omega\in_2)$$

$$C = g_1 g_3(t_L^{\ 2} + i\in_3 \omega - \in_2 \omega^2) + i g_3 g_4 g_5 t_L^{\ 2}(2+\overline{\lambda})$$

The solution of equation (11) for the coupled theory of thermo elasticity $(\tau_1 = \tau_0 = 0)$ are obtained as

$$\begin{aligned}\overline{\phi}(r) &= \sum_{i=1}^{3}(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))\\ \overline{\chi}(r) &= \sum_{i=1}^{3} d_i(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))\\ \overline{T}(r) &= \sum_{i=1}^{3} e_i(A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r))\end{aligned} \tag{12}$$

Here, $(\alpha_i r)^2$ are the non-zero roots of the algebraic equation

$$(\alpha_i r)^6 - A(\alpha_i r)^4 + B(\alpha_i r)^2 - C = 0$$

The arbitrary constant d_i and e_i is obtained from

$$d_i = \left[\frac{\left(1+\overline{\lambda}\right)\delta_i^2 - (2+\overline{\lambda})\delta_i^2 - g_1}{g_2(2+\overline{\lambda}) - \delta_i^2 - g_3}\right]$$

$$e_i = \left(\frac{\lambda + 2\overline{\mu}}{\beta T_0 R^2}\right)\left[\frac{\varepsilon_4\delta_i^2 + \left(\varepsilon_4\left(g_1+g_3\right) + \varepsilon_4(1+\overline{\lambda})g_2\right)\delta_i^2 + \varepsilon_4 g_1 g_3 + \delta_i^2) - g_1 g_3}{\varepsilon_4 g_3 + \varepsilon_4\delta_i^2 - g_2}\right] \tag{13}$$

Eq. (9a) is a Bessel equations with its possible solutions are

$$\overline{\psi} = \begin{cases} A_4 J_\delta(k_1 r) + B_4 Y_\delta(k_1 r), & k_1^2 > 0\\ A_4 r^\delta + B_4 r^{-\delta}, & k_1^2 = 0\\ A_4 I_\delta(k_1' r) + B_4 K_\delta(k_1' r), & k_1^2 < 0\end{cases} \tag{14}$$

where $k_1'^2 = -k_1^2$ and J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. $A_{i,} B_i$ $i = 1,2,3,4$ are the arbitrary constants. Generally $k_1^2 \neq 0$, so that the situation $k_1^2 \neq 0$ is will not be discussed in the following. For convenience, we consider the case of $k_1^2 > 0$ and the derivation for the case of $k_1^2 < 0$ is similar.

The solution of equation (9a) is

$$\overline{\psi}(r) = A_4 J_\delta(k_1 r) + B_4 Y_\delta(k_1 r) \tag{15}$$

where $k_1^2 = (2+\overline{\lambda})\omega^2 - t_L{}^2$.

4. BOUNDARY CONDITION AND FREQUENCY EQUATIONS

In this section we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at $r = a, b$

$$u = \left(-\bar{\phi}' - \frac{\delta \bar{\psi}'}{r} \right) \sin(m\pi z)\sin(\delta\theta) e^{i\omega t}$$

$$v = \left(-\bar{\psi}' - \frac{\delta \bar{\phi}'}{r} \right) \sin(m\pi z)\cos(\delta\theta) e^{i\omega t} \tag{16}$$

$$w = \bar{\chi} t_L \cos(m\pi z)\sin(\delta\theta) e^{i\omega t}$$

$$T = \bar{T} \sin(m\pi z)\cos(\delta\theta) e^{i\omega t}$$

$$\bar{\sigma}_{rr} = \left[\left(2+\bar{\lambda}\right)\delta\left(\frac{\bar{\psi}'}{r} - \frac{\bar{\psi}}{r^2} \right) + \left(2+\bar{\lambda}\right)\left(\frac{1}{r}\bar{\varphi}' + (\alpha_i^2 - \frac{\delta^2}{r^2}\bar{\varphi}) \right) + \bar{\lambda}\left(\frac{\delta}{r^2}\bar{\psi} - \frac{1}{r}\bar{\phi}' - \frac{\delta^2}{r^2}\bar{\varphi} - \frac{\delta}{r}\bar{\psi}' - t_L^2\bar{\chi} \right) \right]$$

$$\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{r\theta} = 2\left(\frac{1}{r}\bar{\psi} + (\alpha_i^2 - \frac{\delta^2}{r^2})\bar{\psi} - \frac{2\delta}{r}\bar{\varphi}' + \frac{2\delta}{r^2}\bar{\varphi} + \frac{\bar{\psi}'}{r} - \frac{\delta^2}{r^2}\bar{\psi} \right) \sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{rz} = 2t_L\left(-\bar{\varphi}' - \frac{\delta}{r}\bar{\psi} + \bar{\chi}' \right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t}$$

where prime denotes the differentiation with respect to r $\bar{u}_i = u_i/R, (i = r, \theta, z)$ are three non – dimensional displacements and $\bar{\sigma}_{rr} = \sigma_{rr}/\mu, \bar{\sigma}_{r\theta} = \sigma_{r\theta}/\mu, \bar{\sigma}_{rz} = \sigma_{rz}/\mu$ are three non-dimensional stresses. For the purpose of comparison, we first consider the uncoupled free vibration of isotropic cylindrical panel. In this case both convex and concave surface of the panel are traction free

$$\sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = 0, T_{,r} = 0 \qquad (r = a, b) \tag{17}$$

$$\left| E_{ij}^1 \right| = 0 \qquad i, j = 1, 2.....8 \tag{18}$$

$$E_{11}^{1} = (2+\overline{\lambda})\left((\delta J_{\delta}(\alpha_1 t_1)/t_1^{\,2} - \frac{\alpha_1}{t_1} J_{\delta+1}(\alpha_1 t_1)) - ((\alpha_1 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_1 t_1)/t_1^{\,2} \right)$$

$$+\overline{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_1 t_1)/t_1^{\,2} - \frac{\alpha_1}{t_1} J_{\delta+1}(\alpha_1 t_1) \right) + \overline{\lambda} d_1 t_L^{\,2} J_{\delta}(\alpha_1 t_1) - \beta T_0 R^2 e_1 \overline{\lambda}$$

$$E_{13}^{1} = (2+\overline{\lambda})\left((\delta J_{\delta}(\alpha_2 t_1)/t_1^{\,2} - \frac{\alpha_2}{t_2} J_{\delta+1}(\alpha_2 t_1)) - ((\alpha_2 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_2 t_1)/t_1^{\,2} \right)$$

$$+\overline{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_2 t_1)/t_1^{\,2} - \frac{\alpha_2}{t_1} J_{\delta+1}(\alpha_2 t_1) \right) + \overline{\lambda} d_2 t_L^{\,2} J_{\delta}(\alpha_2 t_1) - \beta T_0 R^2 e_2 \overline{\lambda}$$

$$E_{15}^{1} = (2+\overline{\lambda})\left((\delta J_{\delta}(\alpha_3 t_1)/t_1^{\,2} - \frac{\alpha_2}{t_2} J_{\delta+1}(\alpha_3 t_1)) - ((\alpha_3 t_1)^2 R^2 - \delta^2) J_{\delta}(\alpha_3 t_1)/t_1^{\,2} \right)$$

$$+\overline{\lambda}\left(\delta(\delta-1) J_{\delta}(\alpha_3 t_1)/t_1^{\,2} - \frac{\alpha_2}{t_1} J_{\delta+1}(\alpha_3 t_1) \right) + \overline{\lambda} d_3 t_L^{\,2} J_{\delta}(\alpha_3 t_1) - \beta T_0 R^2 e_3 \overline{\lambda}$$

$$E_{17}^{1} = (2+\overline{\lambda})\left((\frac{k_1 \delta}{t_1} J_{\delta+1}(k_1 t_1) - \delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{\,2} \right)$$

$$+\overline{\lambda}\left(\delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{\,2} - \frac{k_1 \delta}{t_1} J_{\delta+1}(k_1 t_1) \right)$$

$$E_{21}^{1} = 2\delta\left((\alpha_1/t_1) J_{\delta+1}(\alpha_1 t_1) - \delta(\delta-1) J_{\delta}(\alpha_1 t_1) \right)$$

$$E_{23}^{1} = 2\delta\left((\alpha_2/t_1) J_{\delta+1}(\alpha_2 t_1) - \delta(\delta-1) J_{\delta}(\alpha_2 t_1) \right)$$

$$E_{25}^{1} = 2\delta\left((\alpha_3/t_1) J_{\delta+1}(\alpha_3 t_1) - \delta(\delta-1) J_{\delta}(\alpha_3 t_1) \right)$$

$$E_{27}^{1} = (k_1 t_1)^2 R^2 J_{\delta}(k_1 t_1) - 2\delta(\delta-1) J_{\delta}(k_1 t_1)/t_1^{\,2} + k_1/t_1 J_{\delta+1}(k_1 t_1)$$

$$E_{31}^{1} = -t_L(1+d_1)\left(\delta/t_1 J_{\delta}(\alpha_1 t_1) - \alpha_1 J_{\delta+1}(\alpha_1 t_1) \right)$$

$$E_{35}^{1} = -t_L(1+d_3)\left(\delta/t_1 J_{\delta}(\alpha_3 t_1) - \alpha_2 J_{\delta+1}(\alpha_3 t_1) \right) \quad E_{37}^{1} = -t_L(\delta/t_1) J_{\delta}(k_1 t_1)$$

$$E_{41}^{1} = e_1[(\delta/t_1) J_{\delta}(\alpha_1 t_1) - (\alpha_1) J_{\delta+1}(\alpha_1 t_1)]$$

$$E_{43}^{1} = e_2[(\delta/t_1) J_{\delta}(\alpha_2 t_1) - (\alpha_2) J_{\delta+1}(\alpha_2 t_1)]$$

$$E_{45}^{1} = e_3[(\delta/t_1) J_{\delta}(\alpha_3 t_1) - (\alpha_3) J_{\delta+1}(\alpha_3 t_1)]$$

$$E_{47}^{1} = 0$$

In which $t_1 = a/R = 1 - t^*/2$, $t_2 = b/R = 1 + t^*/2$ and $t^* = b - a/R$ is the thickness -to-mean radius ratio of the panel. Obviously $E_{ij}\left(j = 2,4,6,8\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i = 1,3,5,7\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i=5,6,7,8\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i=1,2,3,4\right)$ with t_2.

Now we consider the coupled free vibration problem. Allowing for the effect of the surrounded elastic medium, which is treated as the Pasternak model , the boundary conditions at the inner and outer surfaces *r* = *a*,*b* can consider as follows

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0 \qquad (r=a) \tag{19}$$

$$\sigma_{rr}=-Ku+G\Delta u_{,r} \qquad \sigma_{r\theta}=\sigma_{rz}=0 \qquad (r=b) \tag{20}$$

where $\Delta=\partial^2/\partial z^2+\left(1/r^2\right)\partial^2/\partial\theta^2$, K is the foundation modulus and G is the shear modulus of the foundation. It is mentioned here that the elastic medium can be modeled as Winkler type by setting G=0 in equation (20).From Eq. (20), and the results obtained in the preceding section, we get the coupled free vibration frequency equation as follows:

$$\left|E_{ij}^2\right|=0 \qquad i,j=1,2.. \tag{21}$$

$$E_{ij}^2=E_{ij}^1\left(i=1,2,3,4,6,7,8;\ j=1,2....8\right)$$

$$E_{51}^2=E_{51}^1-p\delta J_\delta\left(\alpha_1 t_2\right)/t_2$$

$$E_{52}^2=E_{52}^1-p\delta Y_\delta\left(\alpha_1 t_2\right)/t_2$$

$$E_{53}^2=E_{53}^1-p\left(\delta/t_2 J_\delta\left(\alpha_2 t_2\right)-t_2 J_{\delta+1}\left(\alpha_2 t_2\right)\right)$$

$$E_{54}^2=E_{54}^1-p\left(\delta/t_2 Y_\delta\left(\alpha_2 t_2\right)-t_2 Y_{\delta+1}\left(\alpha_2 t_2\right)\right)$$

$$E_{55}^2=E_{55}^1-p\left(\delta/t_2 J_\delta\left(\alpha_3 t_2\right)-t_2 J_{\delta+1}\left(\alpha_3 t_2\right)\right)$$

$$E_{56}^2=E_{56}^1-p\left(\delta/t_2 Y_\delta\left(\alpha_3 t_2\right)-t_2 Y_{\delta+1}\left(\alpha_3 t_2\right)\right)$$

where

$$p=p_1+p_2\left(t_L^2+n^2/t_2^2\right) \qquad \textit{where} \quad p_1=KR/\mu \quad \textit{and} \quad p_2=G/R\mu .$$

5. NUMERICAL RESULTS AND DISCUSSION

The coupled free wave propagation in a simply supported homogenous isotropic thermo elastic cylindrical panel embedded in a Winkler type of elastic medium is numerically solved for Zinc material by setting $p_2=0$ and Winkler elastic modulus $K=1.5\times10^7$.For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha = 2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave. The frequency equation for a closed cylindrical shell can be obtained by setting $\delta = l\left(l = 1, 2, 3.....\right)$ where l is the circumferential wave number in equations (18). The material properties of a Zinc is

$$\rho = 7.14\times10^{3} kgm^{-3} \qquad T_0 = 296K \qquad K = 1.24\times10^{2} Wm^{-1}\deg^{-1}$$

$$\mu = 0.508\times10^{11} Nm^{-2} \qquad \beta = 5.75\times10^{6} Nm^{-2}\deg^{-1}$$

$$\in_1 = 0.0221 \qquad \lambda = 0.385\times10^{11} Nm^{-2} \text{ and } C_v = 3.9\times10^{2} J\,kg^{-1}\deg^{-1}$$

The roots of the algebraic equation (11) were calculated using a combination of Birge-Vita method and Newton-Raphson method. In the present case simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. This combination has overcome the difficulties in finding the roots of the algebraic equations of the governing equations. Here the values of the thermal relaxation times are taken [10] as $t_0 = 0.75\times10^{-13}\sec$ and $t_1 = 0.5\times10^{-13}\sec$ .Because the roots of the algebraic equation [11] are complex for all values of wave number, therefore the waves are attenuated in space. We can write $c^{-1} = v^{-1} + i\omega^{-1}q$, so that $k = R + iq$,where $R = \omega/v$ and the wave speed (v) and the attenuation coefficient (q) are real numbers.

Table 1. Comparison of non-dimensional frequencies between the Lord-Schulman Theory (L -S) and Classical Theory (CT) of thermo-elasticity for symmetric and anti symmetric modes of thermally insulated cylindrical panel for $p_1 = 0.005$

Wave number	Anti symmetric mode		Symmetric mode	
	LS	CT	LS	CT
0.1	0.3335	0.0541	0.2255	0.1564
0.2	0.5337	0.1174	0.5073	0.2444
0.6	0.8292	0.1994	0.5941	0.3487
1.2	1.1408	0.2964	0.6303	0.6584
1.8	1.4579	0.4051	0.7070	0.7551
2.4	1.7707	0.6478	1.2007	0.9038

A comparison is made for the the symmetric and anti symmetric modes non-dimensional frequencies among the Lord-Schulman Theory (L - S) and Classical Theory (CT) for $p_1 = 0.005, 0.05$ of the thermally insulated cylindrical panel in Tables 1 and 2, respectively. From these tables, it is clear that as the sequential number of the wave number increases, the non dimensional frequencies also increases for both the symmetric and anti symmetric modes. Also, it is clear that the non dimensional frequency exhibits higher amplitudes for the LS theory compared with the CT for both $p_1 = 0.005$ and $p_1 = 0.05$ due to the combined effect of thermal relaxation times and damping of the foundation.

Table 2. Comparison of non-dimensional frequencies between the Lord-Schulman Theory (L-S) and Classical Theory (CT) for symmetric and anti symmetric modes of thermally insulated cylindrical panel with wave number for $p_1 = 0.05$.

Wave number	Anti symmetric mode		Symmetric mode	
	LS	CT	LS	CT
0.1	0.4781	0.0532	0.3084	0.2259
0.2	0.5747	0.1801	0.5130	0.2702
0.6	0.8492	0.2063	0.6220	0.3950
1.2	1.5391	0.3967	0.7295	0.6837
1.8	1.7853	0.5010	0.7752	0.8129
2.4	1.9288	0.7400	1.6349	0.9727

In Fig.2 the variation of attenuation coefficient with respect to circumferential wave number of cylindrical shell is discussed for CT and LS in first mode. The magnitude of the attenuation coefficient increases monotonically to attain maximum value in $0.4 \le \delta \le 0.8$ for both CT and LS and slashes down to became asymptotically linear in the remaining range of circumferential wave number. The variation of attenuation coefficient with respect to circumferential wave number of cylindrical shell is discussed for second mode in Fig.3, here the attenuation coefficient attain maximum value in $0 \le \delta \le 0.4$ for both CT and LS theories and slashes down to become linear due relaxation times. From Fig.2 and Fig.3 it is clear that the attenuation profiles exhibits high amplitude for LS theory compared with CT due to the combine effect of thermal relaxation times and damping effect of foundation.

Fig.4 reveals that the variation of non dimensional frequency with the foundation parameter p_1 for first and second mode without thermal effect. By comparing with the classical thin shell theory from Yu [10], it is clear that the exact one agree well with the classical thin shell theory (CTST) for the first and second mode. This is identical to the well-known property of CTST for the uncoupled problem. However, for the thinner panel, when the effect of the foundation is obvious, the frequency of CTST will become smaller than the exact one. From the comparison of the dispersion curves in Fig.4 it is quite clear that due to the damping effect of the foundation on outer sides of the panel the non dimensional frequency vary significantly and become steady for $p_1 \geq 0.5$. The dispersion curves become more smoothen in this case than those in the absence of foundation parameter because of the shock absorption nature of foundation.

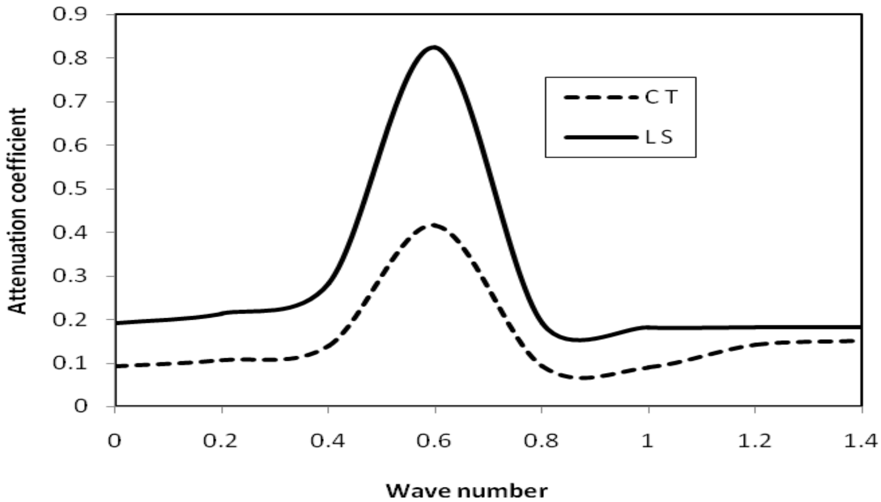

Fig.2. Variation of attenuation coefficient of cylindrical shell with wave number on elastic foundation $\left(\nu = 0.3, n = 1, K = 1.5 \times 10^7, p_2 = 0\right)$

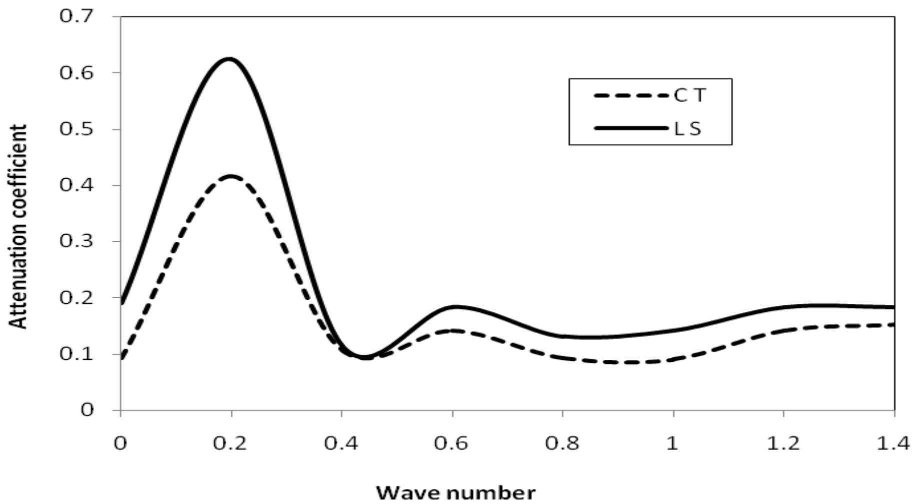

Fig.3. Variation of attenuation coefficient of cylindrical shell with wave number on elastic foundation $\left(\nu = 0.3, n = 2, K = 1.5 \times 10^7, p_2 = 0\right)$

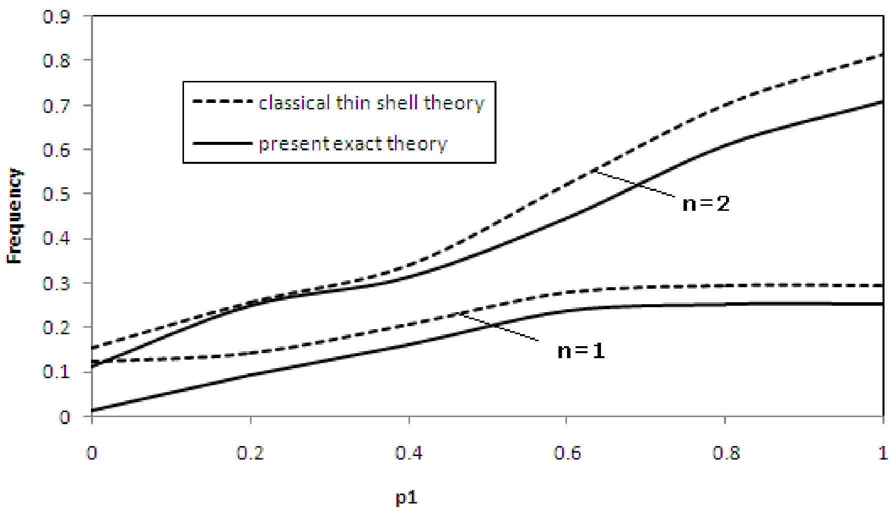

Fig.4. Variation of the foundation parameter p_1 versus Non-dimensional frequency $\left(\nu = 0.3, mR/L = 0.4, p_2 = 0\right)$.

6. CONCLUSION

The three dimensional wave propagation of a homogeneous isotropic generalized thermo elastic cylindrical panel embedded on the Winkler type of elastic foundation has been considered for this paper. For this problem, the governing equations of three dimensional linear theory of generalized thermo elasticity have been employed in the context of Lord Shulman theory and solved by Bessel function with complex argument. The effect of the attenuation coefficient against wave number and the foundation parameter p_1 on the natural frequencies of a closed Zinc cylindrical shell is investigated and the results are presented as dispersion curves. The table values show the impact of foundation on the cylindrical panel.

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