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A New Sensitivity-Based Reliability Calculation Algorithm in the Optimal Design of Electromagnetic Devices

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Abstract – A new reliability calculation method is proposed based on design sensitivity analysis by the finite element method for nonlinear performance constraints in the optimal design of electromagnetic devices. In the proposed method, the reliability of a given design is calculated by using the Monte Carlo simulation (MCS) method after approximating a constraint function to a linear one in the confidence interval with the help of its sensitivity information. The validity and numerical efficiency of the proposed sensitivity-assisted MCS method are investigated by comparing its numerical results with those obtained by using the conventional MCS method and the first-order reliability method for analytic functions and the TEAM Workshop Problem 22.

Keywords: Design sensitivity analysis, Finite element method, First-order reliability method, Monte Carlo simulation, Reliability index approach

1. Introduction

In an engineering optimization problem, a nominal design obtained by using conventional deterministic optimization method, which does not take into account the uncertainties in design variables, usually gives the best performance. However, in the practical problem, the uncertainties in design variables such as manufacturing tolerance, deviations of the material constant, and changes in operating conditions are inevitable. Consequently, the design variables will deviate from their nominal values due to the existence of uncertainty [1]. What's more, the uncertainties will move the optimal design to the infeasible region, as shown in Fig. 1, and then give a far worse performance than the original optimal design. Therefore, there is a strong increasing requirement to develop one algorithm for checking the reliability of a design [2].

The reliability of a design is defined as the probability of satisfying the constraint function, when the design variables deviate from their nominal values in predefined uncertain region. As shown in Fig. 1, the design A obtained from deterministic optimization algorithm can show better performance than any other designs in the feasible region. When there exist uncertainties in design variables, however, it has higher probability of turning to the design A* which does not satisfy the given constraints. On the contrary, the design B with a little worse performance, can guarantee all constraints even perturbed by uncertainties in design variables. In this sense, design B is considered as a more reliable design than design A.

For the quantitative evaluation of reliability R for a nominal design, several methods have been developed [3]-[5]: the reliability index approach (RIA), the performance measure approach (PMA), and the Monte Carlo simulation (MCS) method. In the RIA and the PMA, the reliability is calculated by solving an optimization problem where it is difficult to select a suitable initial searching point and step length at each iteration. The MCS method, a direct sampling-based approach, becomes computationally expensive especially when the constraint functions are to be calculated by using numerical methods such as the finite element method (FEM). It is because the MCS method requires as many sampling points as possible (like a few millions) to guarantee its accuracy [6]. To alleviate its expensive cost, the MCS method is normally applied to a meta-model [7] constructed by the Kriging or response surface method, however, until now, for reliability calculation, there is scarcely any researches about combination of the MCS and sensitivity analysis assisted by the FEM.

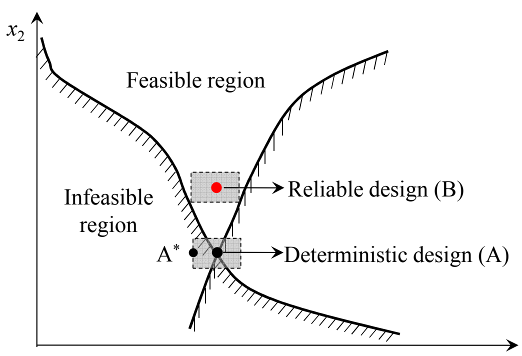

Fig. 1. Deterministic and reliable designs, where the gray rectangles denote uncertain regions.

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To overcome disadvantages of the conventional MCS and the RIA methods, in this paper, a new reliability calculation algorithm is developed by incorporating design sensitivity analysis with the MCS method. The suggested algorithm is especially useful to performance constraints in the electromagnetic applications. Its efficiency and validity are demonstrated through some examples.

2. Reliability Calculation Algorithms

Hereafter in this paper, the vectors $\mathbf{x}=[x_1, x_2, \ldots, x_{Nx}]^T$ and $\mathbf{d}=[d_1, d_2,\ldots,d_{Nd}]^T$ (T means transpose) will denote uncertain and deterministic design variables, respectively. The following assumptions are made to simplify the explanation of subsequent contents:

(1) the uncertain design variables are independent from each other and follow the Gaussian distribution, i.e. $\mathbf{x}\sim N(\boldsymbol{\mu}, \boldsymbol{\sigma})$ where $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are vectors of mean values and standard deviations of $\mathbf{x}$, respectively.

(2) the linear or nonlinear constraints are expressed as:

$$\mathbf{g}(\mathbf{x},\mathbf{d})\geq 0, \quad \mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \text{ and } \mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U \tag{1}$$

where the vector $\mathbf{g}(g_1, g_2, \ldots, g_{Ng})$ consists of Ng constraint functions, the subscripts L and U stand for the lower and upper limits, respectively.

The design space is, as shown in Fig. 1, divided into feasible and infeasible regions by constraint surface. Thus, mathematically, the probability of a design ($\mathbf{x}$, $\mathbf{d}$) being safe against the kth constraint, $g_k(\mathbf{x}, \mathbf{d})\geq 0$, can be written as:

$$R = P\big(g_k(\mathbf{x},\mathbf{d})\geq 0\big) = \int_{g_k(\mathbf{x},\mathbf{d})\geq 0} \varphi_X(\mathbf{x})d\mathbf{x} \tag{2}$$

where $\varphi_X(\mathbf{x})$ is the joint probability density function of constraint function. It is very difficult to find analytic expression for (2) so that different reliability calculation methods are generated based on approximation for (2).

2.1 Conventional reliability calculation algorithms

A. Monte Carlo Simulation Method

In the MCS method, the uncertainties in design variables are represented by using the standard deviation of random numbers to be generated. For a given design ($\mathbf{x}$, $\mathbf{d}$), the reliability with respect to the kth constraint, g_k, will be computed, as shown in Fig. 2, as follows:

Step 1: Generate N random test points ($\boldsymbol{\xi}_j$, $\mathbf{d}$), (j=1,…,N) according to the statistical distribution of uncertain variables.

Step 2: For each test point ($\boldsymbol{\xi}_j$, $\mathbf{d}$), calculate constraint function value and check if it satisfies $g_k(\boldsymbol{\xi}_j, \mathbf{d})\geq 0$ or not.

Step 3: Evaluate the reliability of the design ($\mathbf{x}$, $\mathbf{d}$) by the following equation:

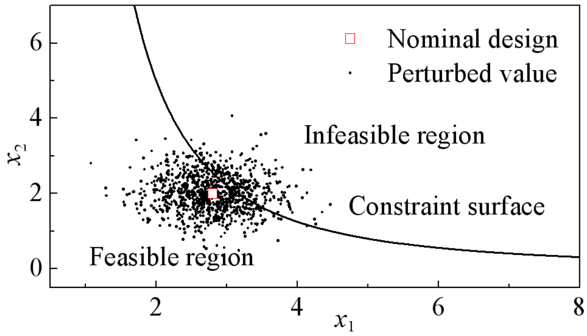


Fig. 2. The MCS method with 100,000 test points generated for a specified design $\mathbf{x}$=(2.8,2.0) and $\boldsymbol{\sigma}$=0.5.

$$R(g_k(\mathbf{x},\mathbf{d})) = n/N \tag{3}$$

where n is the number of test points satisfying the given constraint.

It is obvious that, from (3), the reliability will be more accurate as the number of trials N increases. This method is very flexible and simple to implement, and theoretically can be applied to most kinds of problems. However, it will be much more time-consuming as the number of trials increases especially when numerical analysis is needed. Although this method has tried to incorporate the sampling techniques such as the importance sampling [8] and the Latin hypercube sampling [9] to improve the efficiency, its application in the electrical engineering is still very limited.

B. Reliability Index Approach

In this approach, the $\mathbf{x}$-design space is transformed into a normalized $\mathbf{u}$-design space, as shown in Fig. 3, so that a given design ($\mathbf{x}$, $\mathbf{d}$) may correspond to the origin of the $\mathbf{u}$-design space. For the ith independent Gaussian stochastic variable $x_i\sim N(\mu_i, \sigma_i)$, the transformation is given as:

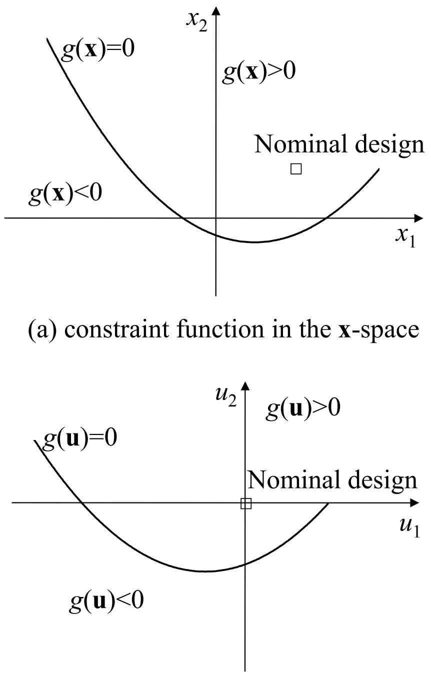
$$u_i = (x_i - \mu_i)/\sigma_i, \quad i = 1,\cdots, Nx \tag{4}$$

where u_i is the ith normalized stochastic variable with a zero mean value and unit standard deviation, $u_i\sim N(0, 1)$. For the independent non-Gaussian and dependent stochastic variables, other transformations such as Rosenblatt and Nataf transformations can be used [10-11].

The most probable point of failure (MPPF), $\mathbf{u}^*$, is defined, as shown in Fig. 4, in the normalized design space as the closest point on the constraint surface to the origin [12]. The MPPF can be found through an optimization problem:

$$\begin{aligned} &\text{minimize} \quad \|\mathbf{u}\| \\ &\text{subject to} \quad \mathbf{g}(\mathbf{u})=0 \end{aligned} \tag{5}$$

The reliability index (β) defined as the distance from the origin to the MPPF, as shown in Fig. 4 is calculated as:



(a) constraint function in the **x**-space

(b) constraint function in the normalized **u**-space

Fig. 3. Transformation from the **x**-design space to the normalized **u**-design space.

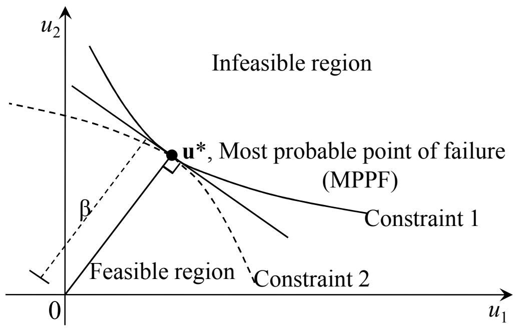


Fig. 4. The reliability index approach.

$$\beta=\sqrt{\sum_{i=1}^{Nx}(u_i^*)^2}. \tag{6}$$

Based on the reliability index, the reliability R of a design ($\mathbf{x}$, $\mathbf{d}$) for the kth constraint $g_k(\mathbf{x},\mathbf{d}) \geq 0$ is calculated as:

$$R = 1-\Phi\left(-\beta\right) \tag{7}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

The reliability calculated from (7) can be considered as the true one for the linear constraint functions. For the non-linear constraint function, however, (7) is expected to give reliability for the linearly approximated constraint functions in the vicinity of the MPPF instead of the original non-linear ones as shown in Fig. 4. Because of the assumption of linearity, the above method is also known as the first-order reliability method (FORM) [13]. This method, for this reason, is expected to have big error as the constraint functions become strongly nonlinear although it is accurate for linear constraint functions. For example, for the nonlinear constraints 1 and 2, shown in Fig. 4, which physically have different reliabilities, the RIA gives same reliabilities by finding the same MPPF and the reliability index [8]. Another serious drawback of this method is related with the solution of (5), i.e. searching for the MPPF. In general, the MPPF is found by using optimization algorithm such as the steepest decent algorithm and the conjugate gradient method with an initial point on the constraint surface. This procedure, however, has many difficulties related with the selection of the initial point and proper step length at each iteration. The iterative method, therefore, becomes much more time-consuming and often fails to find the true MPPF so that its accuracy becomes worse than the expected one.

2.2 Proposed sensitivity-assisted monte carlo simulation (S-MCS) method

In general, the uncertainties in design variables are not quite big so that test points in the MCS method are generated in a relatively small range around the nominal design. As an example, if the uncertainties are related with the position of a point or the length of a line which defines the shape of a device, modern manufacturing techniques will confine uncertainties (or the standard deviations from the nominal values) within few or few ten microns while their design ranges are few or few ten centimeters.

In view of the above fact, with main consideration of improvement of numerical efficiency for reliability analysis in the electromagnetic problem, the proposed reliability calculation method approximates the constraint function values at the test point using the sensitivity of the constraint function at the nominal value ($\mathbf{x}$).

For a given design ($\mathbf{x}$,$\mathbf{d}$), a test point ($\boldsymbol{\xi}$,$\mathbf{d}$) in the MCS method can be represented as follows:

$$(\boldsymbol{\xi},\mathbf{d})=(\mathbf{x}\pm\Delta\mathbf{x},\mathbf{d}), \quad -k\boldsymbol{\sigma}\leq\Delta\mathbf{x}\leq k\boldsymbol{\sigma} \tag{8}$$

where k is a specified confidence level. Then the ith constraint function $g_i(\boldsymbol{\xi},\mathbf{d})$ is approximated as:

$$g_i(\boldsymbol{\xi},\mathbf{d})\cong g_i(\mathbf{x},\mathbf{d})+\nabla g_i(\mathbf{x},\mathbf{d})\cdot(\boldsymbol{\xi}-\mathbf{x}) \tag{9}$$

where the gradient vector of constraint function with respect to uncertain design variables is defined as follows:

$$\nabla g_i(\mathbf{x},\mathbf{d})=\left\{\frac{\partial g_i}{\partial x_1},\frac{\partial g_i}{\partial x_2},\cdots,\frac{\partial g_i}{\partial x_{Nx}}\right\}^T \tag{10}$$

For the geometric constraints, the gradient vector can be calculated analytically. When the constraint function is related with performance analysis, the gradient vector can be computed by using sensitivity analysis.

In the FEM analysis, main formulations of the first-order sensitivity analysis by the adjoint variable method are listed as follows [14]:

$$[\mathbf{K}][\mathbf{A}] = \{\mathbf{Q}\} \tag{11}$$

$$\nabla g_i(\mathbf{x},\mathbf{d}) = \left.\frac{\partial g_i}{\partial[\mathbf{x}]^T}\right|_{\mathbf{A}=C} - [\boldsymbol{\lambda}]^T \cdot \left\{ \frac{\partial[\mathbf{K}]}{\partial[\mathbf{x}]^T}[\tilde{\mathbf{A}}] - \frac{\partial\{\mathbf{Q}\}}{\partial[\mathbf{x}]^T} \right\} \tag{12}$$

$$[\mathbf{K}][\boldsymbol{\lambda}] = \partial g_i / \partial[\mathbf{A}] \tag{13}$$

where [**K**] is the system matrix, [**A**] is the magnetic vector potential, {**Q**} is the forcing vector, **[Ã]** is the converged solution of (11), and **[λ]** is the adjoint variable vector.

Once the gradient vector of the performance function is computed from (11) - (13), the nonlinear constraint can be treated as an explicit analytic function. The reliability evaluation, therefore, can be performed very efficiently by incorporating the sensitivity analysis with the MCS method, which results in a sensitivity-assisted MCS (S-MCS) method.

3. Numerical Examples

Two analytic test problems with nonlinear geometric constraints and the TEAM problem 22 are taken to check the efficiency and validity of the proposed S-MCS algorithm by comparing with the conventional MCS and the FORM methods.

3.1 Analytic test problems with geometric constraints

A. Mathematic Example 1

For a test problem with two independent uncertain design variables $\mathbf{x}=[x_1,x_2]^T$ following Gaussian distribution $\mathbf{x}\sim N(\boldsymbol{\mu}, \boldsymbol{\sigma})$, the three nonlinear geometric constraints are defined, as shown in Fig. 5, as follows:

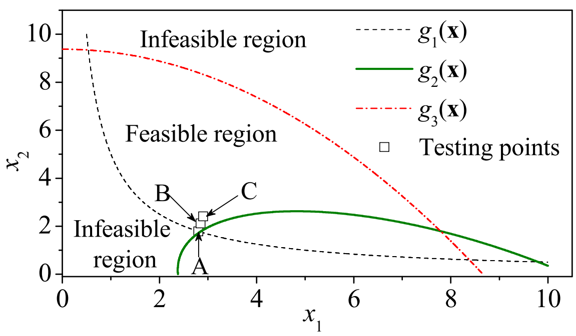


Fig. 5. Geometric constraints of analytic test problem 1.

$$g_1(\mathbf{x}) = \frac{x_1 x_2}{5} - 1 \geq 0 \tag{14-a}$$

$$g_2(\mathbf{x}) = \frac{(x_1 + x_2 - 5)^2}{30} + \frac{(x_1 - x_2 - 12)^2}{120} - 1 \geq 0 \tag{14-b}$$

$$g_3(\mathbf{x}) = \frac{80}{x_1^2 + 8x_2 + 5} - 1 \geq 0. \tag{14-c}$$

The corresponding gradient vectors of each constraint function with respect to uncertain design variables are driven as follows:

$$\nabla g_1(\mathbf{x}) = \left\{ \frac{\partial g_1}{\partial x_1}, \frac{\partial g_1}{\partial x_2} \right\}^T = \left\{ \frac{x_2}{5}, \frac{x_1}{5} \right\}^T \tag{15-a}$$

$$\nabla g_2(\mathbf{x}) = \left\{ \frac{\partial g_2}{\partial x_1}, \frac{\partial g_2}{\partial x_2} \right\}^T = \left\{ \frac{5x_1 + 3x_2 - 32}{60}, \frac{3x_1 + 5x_2 - 8}{60} \right\}^T \tag{15-b}$$

$$\nabla g_3(\mathbf{x}) = \left\{ \frac{\partial g_3}{\partial x_1}, \frac{\partial g_3}{\partial x_2} \right\}^T = \left\{ \frac{-160x_1}{(x_1^2 + 8x_2 + 5)^2}, \frac{-640}{(x_1^2 + 8x_2 + 5)^2} \right\}^T \tag{15-c}$$

With the help of (15), the reliability for a given design can be calculated easily by using the proposed S-MCS method.

Three different designs, A, B, and C are selected as shown in Fig. 5. For each design, the reliability is calculated by using the MCS, the S-MCS, and the FORM methods. In the FORM, the MPPF is found through solving (5) by particle swarm optimization method. In the reliability calculation, the following conditions are assumed:

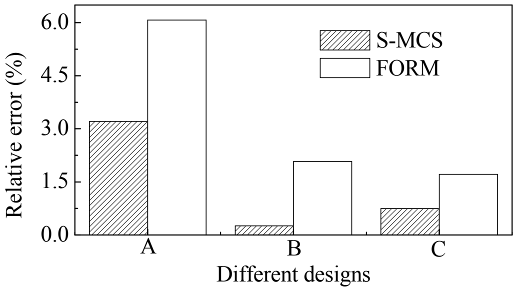
(1) the standard deviations for x_1 and x_2 are fixed to 0.3,
(2) in the MCS and the S-MCS methods, one million trials are generated with a confidence level of 95%.

The reliabilities and relative errors for constraints $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$ are compared in Table 1 and Fig. 6, respectively. The relative errors are computed by taking the reliability of the MCS method as a reference value. From the comparisons, it can be seen that the S-MCS gives higher accuracies than the FORM for constraint $g_1(\mathbf{x})$. Even in the worst case for constraint $g_2(\mathbf{x})$, the S-MCS method can give a reliability as accurate as the FORM. Therefore, the S-MCS without any optimization strategy is more effective than the FORM and the conventional MCS methods.

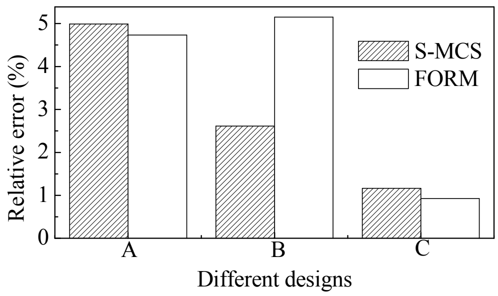
Table 1. Reliabilities of different designs

Constraint [a]	Designs	Calculation methods		
		MCS	S-MCS	FORM
g_1	A(2.80, 1.78)	0.47748	0.49282	0.50648
	B(2.85, 2.11)	0.85723	0.85499	0.83944
	C(2.90, 2.40)	0.98623	0.97886	0.96937
g_2	A(2.80, 1.78)	0.58705	0.55777	0.55929
	B(2.85, 2.11)	0.80599	0.78495	0.76448
	C(2.90, 2.40)	0.93853	0.92758	0.92983

[a] For all cases, the reliabilities for constraint g_3 are 1.0.



(a) relative error of constraint $g_1(\mathbf{x})$



(b) relative error of constraint $g_2(\mathbf{x})$

Fig. 6. Relative errors of reliability calculation.

B. Mathematic Example 2

A strongly nonlinear function with two uncertain design variables ($0 \le x_1, x_2 \le 10$) is shown as follows:

$$g(\mathbf{x}) = -1 + (s-6)^2 + (s-6)^3 - 0.6(s-6)^4 + t \ge 0 \qquad (16)$$

where s=0.9063x_1+0.4226x_2 and t=0.4226x_1-0.9063x_2. The designs D and E as marked in Fig. 7 are selected to make the further validation of the proposed S-MCS method.

It is obvious that the function in (16) is much more complex than that in (14). The calculation result under a standard deviation of $\boldsymbol{\sigma}$=0.3 is given in Table 2. In this case, both the S-MCS and the FORM show a lower accuracy. It can be concluded that the S-MCS method is inadequate for problems with a stronger nonlinearity. For the accuracy improvement of the S-MCS method, the higher-order sensitivity analysis should be studied in the future research.

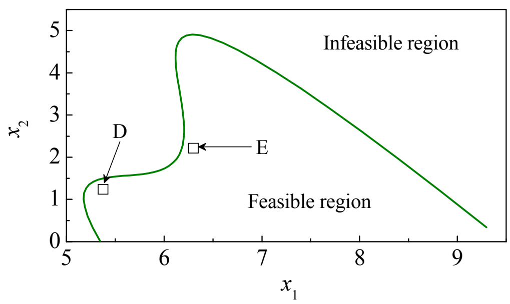


Fig. 7. Geometric constraint of analytic test problem 2.

Table 2. Reliabilities of different designs

Designs	MCS	S-MCS	FORM
D(5.376,1.236)	0.62252	0.76778	0.75984
E(6.300,2.210)	0.68705	0.65249	0.65280

3.2 TEAM problem 22

TEAM problem 22, shown in Fig. 8, is a problem of the superconducting magnetic energy storage (SMES) system. In this problem, there are two design targets: 1) the stored magnetic energy in the system should be as close as to E_0= 180 MJ, and 2) the stray magnetic field B_{stray}, which is evaluated at 22 equidistant sampling points on the line a and b as shown in Fig. 8, should be as small as possible. In order to achieve these targets, the geometric parameters of inside and outside coils, and current densities (J_1, J_2) should be optimized [15]. The objective function to be minimized is defined as follows:

$$f = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|E - E_0|}{E_0} \qquad (17\text{-a})$$

$$B_{stray}^2 = \frac{1}{22}\sum_{i=1}^{22} B_{stray,i}^2 \qquad (17\text{-b})$$

where $B_{stray,i}$ is the magnetic flux density on the ith sampling point and the reference stray field is B_{norm}=3 mT.

There are two critical constraints, as shown in Fig. 9, to keep the superconducting coils from quenching as follows:

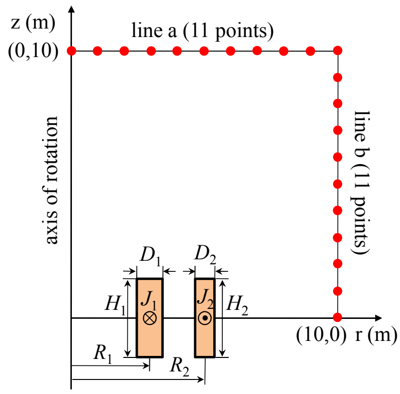


Fig. 8. Configuration of the TEAM problem 22.

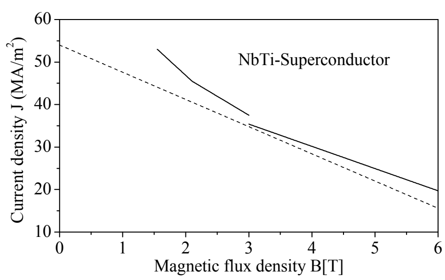


Fig. 9. Quenching curve of the superconductor and the dashed line is a linear approximation used in this paper.

$$g_i(\mathbf{x},\mathbf{d}) = 54 - |J_i| - 6.4|B_{m,i}(\mathbf{x},\mathbf{d})| \geq 0, \quad i=1,2 \tag{18}$$

where $B_{m,i}$ is maximum magnetic flux density in the ith coil. In the proposed sensitivity-based reliability calculation method, the constraints are approximated for the uncertain variables **x** as follows:

$$g_i(\boldsymbol{\xi},\mathbf{d}) \cong g_i(\mathbf{x}_0,\mathbf{d}) + \left.\frac{\partial g_i}{\partial \mathbf{x}}\right|_{(\mathbf{x}_0,\mathbf{d})} \cdot (\boldsymbol{\xi} - \mathbf{x}_0), \quad i=1,2. \tag{19}$$

The sensitivity vector of the maximum magnetic flux density with respect to the uncertain variables is calculated with the help of the FEM as follows:

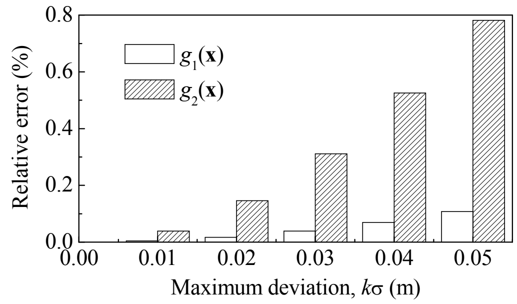
$$\frac{d|B_{m,i}|}{d[\mathbf{x}]^T} = \left.\frac{\partial |B_{m,i}|}{\partial [\mathbf{x}]^T}\right|_{A=C} + [\boldsymbol{\lambda}]_i^T \cdot \frac{\partial \{\mathbf{Q}\}}{\partial [\mathbf{x}]^T}, \quad i=1,2 \tag{20-a}$$

$$[\mathbf{K}][\boldsymbol{\lambda}]_i = \partial |B_{m,i}| / \partial [\mathbf{A}], \quad i=1,2 \tag{20-b}$$

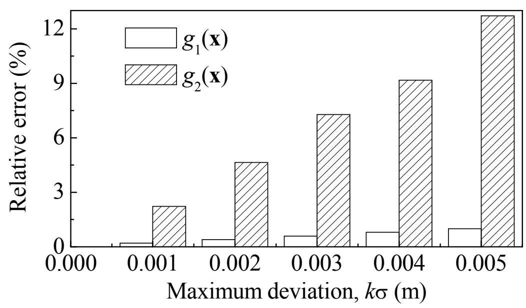
where the symbols have the same meaning as in (11)~(13). For all the reliability analysis, the test designs in the MCS and the S-MCS are set 10,000, and k in (8) is 1.96.

A. Uncertainty is Considered in Geometric Variables

From the published papers, several optimal designs are collected in Table 3. For design [15], the approximated constraint functions by the first-order sensitivity are compared with their target values. The corresponding relative errors are shown in Fig. 10 (a) and (b) when the maximum deviations of R2 and H2 change, respectively. The maximum relative errors are around 0.8% and 12%, respectively. Due to the advanced manufacturing technology, the maximum deviation may be smaller than the listed ones in Fig. 10, in other words, the accuracy of the first-order sensitivity approximation is allowable and practical for the real application.



(a) when the maximum deviation of R_2 is changed.



(b) when the maximum deviation of H_2 is changed.

Fig. 10. Constraint approximation using the first-order sensitivity.

Taking geometric variables $\mathbf{x}=[R_2, H_2, D_2]^T$ as uncertain variables with $\boldsymbol{\sigma}=[0.0153,0.01,0.01]^T$ m, Table 4 shows calculation results of optimal designs in Table 3. Obviously, the S-MCS shows good agreement with the MCS method.

B. Uncertainty is Considered in Physical Parameters

Here, the uncertain physical parameters $\boldsymbol{J}=[J_1, J_2]^T$ follow Gaussian distributions with the standard deviation $\boldsymbol{\sigma}=[0.179,0.179]^T$ MA/m^2, respectively.

Table 5 compares the reliabilities of five test designs from the MCS and the S-MCS, where the nominal current density is $\boldsymbol{J}_0=[16.78, -15.51]^T$ MA/m^2. For the constraint $g_1(\mathbf{J}, \mathbf{d})$, the test designs have different reliabilities from 0.5 to 0.9. From the viewpoint of accuracy for reliabilities, it is found that, with all test designs, the S-MCS gives almost same values with the conventional MCS. On the other hand, if comparing the computational cost, the proposed method requires just three times of the FEM calls (once for performance, twice for sensitivity calculations) while the MCS takes 10,000 times of the FEM analysis. It means the proposed method is much more efficient than the conventional MCS.

Table 6 compares reliabilities calculated for optimal designs in Table 3, where the corresponding current densities are taken as uncertain ones with $\boldsymbol{\sigma}=[0.179,0.179]^T$ MA/m^2. The optimal design [15], however, is definitely

Table 3. Optimal designs selected from published papers

Ref.	R_1 [m]	$H_1/2$ [m]	D_1 [m]	R_2 [m]	$H_2/2$ [m]	D_2 [m]	J_1 [MA/m^2]	J_2 [MA/m^2]
[15]	2.0	0.8	0.27	3.08	0.239	0.394	22.5	-22.5
[16]	2.0	0.8	0.27	3.05	0.246	0.400	22.5	-22.5
[17]	1.32	1.07	0.59	1.80	1.480	0.250	16.78	-15.51
[18]	1.296	1.089	0.583	1.80	1.513	0.195	16.695	-18.91

Table 4. Reliabilities for constraint $g_2 \geq 0$

Designs	[15]	[16]	[17]	[18]
MCS	0.98074	0.72312	1.00	0.83752
S-MCS	0.98054	0.72001	1.00	0.83853

Table 5. Reliabilities comparison for constraint $g_1 \geq 0$

(D_2,$H_2/2$)	MCS(A)	S-MCS(B)	Relative error δ [a]
P_1(0.244,1.490)	0.5068	0.5066	3.946E-2
P_2(0.248,1.490)	0.6763	0.6765	2.957E-2
P_3(0.250,1.500)	0.7644	0.7778	1.7530
P_4(0.253,1.482)	0.8759	0.8619	1.5983
P_5(0.251,1.500)	0.9298	0.9288	1.075E-1
FEM calls	10,000	3	-

[a] δ=|A-B|/A×100% and other design variables are same as [17].

Table 6. Reliabilities of designs in Table 3

Designs	[15]	[16]	[17]	[18]
f	0.088	0.122	0.29356	6.79E-3
$R(g_1 \geq 0)$	1.00	1.00	0.72415	0.48363
$R(g_2 \geq 0)$	0.99925	0.82744	1.00	1.00

better than [16] because the former gives a better objective value and a higher reliability at the same time. Between the optimal designs [17] and [18], neither of them can be said a superior design because of the confliction between the objective value and the reliability level.

4. Conclusion

In order to guarantee a reliable solution with uncertain design variables in the optimal design of electromagnetic devices, a new reliability calculation algorithm is proposed. By incorporating the sensitivity analysis with the finite element method and the Monte Carlo simulation, the proposed S-MCS algorithm gives numerically efficient reliability for a given design especially when the constraint function is related with a numerical performance analysis.

Due to the high numerical efficiency and accuracy, the proposed algorithm is expected to be widely applied in the area of the reliability-based design optimization. However, the application of the first-order function approximation will make the S-MCS insufficient when problems under consideration involve bigger uncertainties or the performance function is strongly nonlinear. In the subsequent research, the higher-order sensitivity analysis will be studied.

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