Robust Pilot-aided Frequency Offset Estimation Scheme for OFDM-based Broadcasting System with Cyclic Delay Diversity

Won-Jae Shin[1] and Young-Hwan You[1]
[1] Depart of Computer Engineering, Sejong University, Seoul, 143-747 - Korea
[e-mail: yhyou@sejong.ac.kr]
*Corresponding author: Young-Hwan You

Received July 2, 2013; revised September 16, 2013; accepted October 23, 2013; published December 27, 2013

Abstract

This paper proposes an improved carrier frequency offset (CFO) and sampling frequency offset (SFO) estimation scheme for orthogonal frequency division multiplexing (OFDM) based broadcasting system with cyclic delay diversity (CDD) antenna. By exploiting a periodic nature of channel transfer function, cyclic delay and pilot pattern with a maximum channel power are carefully chosen, which helps to enable a robust estimation of CFO and SFO against the frequency selectivity of the channel. As a performance measure, a closed-form expression for the achievable mean square error of the proposed scheme is derived and is verified through simulations using the parameters of the digital radio mondiale standard. The comparison results show that the proposed frequency estimator is shown to benefit from properly selected delay parameter and pilot pattern, with a performance better than the existing estimator.

Keywords: OFDM, cyclic delay diversity, carrier frequency offset, sampling frequency offset

This work is supported by Seoul R&BD Program (SS100009), and this work was supported by the Materials & Components development program funded by the Ministry of Trade, Industry & Energy (MOTIE, Korea).

http://dx.doi.org/10.3837/tiis.2013.12.006

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has found many applications in mobile radio communication systems. Because of its excellent characteristics, it enables high data rate transmissions over frequency-selective fading channels. OFDM modulation along with guard interval techniques allows the implementation of single frequency networks in digital broadcasting systems such as the digital audio broadcasting (DAB), digital radio mondiale (DRM), and terrestrial digital video broadcasting (DVB-T) [1]-[3]. DRM offers better sound quality and more reliable reception compared to analog AM and FM. The DRM consortium proposes to add a new VHF mode to the existing DRM modes [3], with a basic 100KHz bandwidth for the DRM robustness mode E, referred to as "DRM+" in this paper.

DRM+ will be used on FM frequency up to 174MHz [3]. The bandwidth used in the DRM+ system, approximately 100KHz, is much smaller than the coherence bandwidth of the channels in typical urban environment that is in the order of 1MHz [4], which results in a severe flat fading condition. Thus, the major challenge in FM band is to cope with severe flat fading conditions. One possible solution to combat flat fading is cyclic delay diversity (CDD) [5]-[11], which has been adopted as one of the multiple-input multiple-output (MIMO) diversity techniques in a wireless communication system. CDD transforms a multiple-input single-output (MISO) channel into an equivalent single-input single-output (SISO) channel with increased frequency-selectivity so that the available spatial diversity is transformed into additional frequency diversity. However, the increased frequency selectivity of channel characteristic can be a problem for post fast Fourier transform (FFT) estimation such as the channel estimation and frequency offset estimation [12]-[19]. Although they are proposed for general channel conditions, its accuracy heavily depends on frequency-selective multi-path distortions. As reported in [13]-[19], moreover, the post-FFT pilot-aided carrier frequency offset (CFO) and sampling frequency offset (SFO) estimation scheme is not anymore unbiased in the presence of large SFO over frequency-selective channels. The implementation complexity of synchronization mechanisms is an important topic from a practical point of view and there still remains the potential for a better estimator that is computationally efficient as well as is robust to the fading distortion.

This paper deals with how to determine the amount of cyclic delay and the pilot pattern for an improved CFO and SFO estimation in the OFDM-based DRM+ system with CDD transmit antennas. Cyclic delay and pilot pattern with a maximum channel power are chosen by exploiting a periodic nature of channel transfer function. The mean square error (MSE) of the proposed estimation scheme is numerically derived, and its simple expression is calculated. We show via simulations that such a design improves the robustness of the CFO and SFO estimation scheme with the increased frequency selectivity because of the use of CDD.

The rest of the paper is organized as follows. Section 2 introduces the signal model when the CDD is used in the DRM+ system. In Section 3, we give a brief overview of the conventional CFO and SFO estimation algorithm. A robust CFO and SFO estimation scheme is proposed and its performance is analyzed in Sections 4 and 5, respectively. In Section 6, we present simulation results verifying the MSE analysis and the effectiveness of the proposed scheme, while in section 7 some conclusions are drawn.

2. Signal Models

When the CDD scheme is adopted in OFDM systems, cyclic delayed replicas of the time domain data signal $x_l(n)$ over several transmit antennas are transmitted, thus the l-th OFDM signal of the t-th transmit antenna is

$$x_{l,t}(n) = \frac{1}{\sqrt{N_T}} x_l(n-\delta_t)_N, \quad t = 0,1,\cdots \qquad , \tag{1}$$

where N is the number of FFT (IFFT) points, $(\cdot)_N$ is the modulo-N operation, N_T is the number of transmit CDD antennas, and δ_t is the amount of cyclic delay at the t-th antenna. As discussed in many literatures [5]-[11], choosing

$$\delta_{t+1} = N / N_T + \delta_t, \quad t = 0,1,\cdots \qquad , \tag{2}$$

maximizes delay between transmit antennas, thereby N / N_T is the maximum possible cyclic delay.

Consider a discrete-time baseband OFDM system with N subcarrier and N_g guard interval (GI) samples. A frequency offset Δ_f between the transmit signal and the received signal can be introduced by inaccuracies of the receiver's local oscillator. After compensating the CFO with estimated CFO $\hat{\Delta}_f$ at the acquisition stage, hence, only a residual CFO $\Delta_c = \hat{\Delta}_f - \Delta_f$ and possible SFO Δ_s will remain during the data section of the frame. A frequency offset will appear as a phase shift $\phi(k)$ and is comprised of two parts: residual CFO Δ_c and SFO Δ_s. The former is the same for all subcarriers, while the latter contributes linearly with the subcarrier index k, i.e., $\phi(k) \approx \Delta_c + k\Delta_s$ [20]. Under the assumption of a quasi-stationary channel, the channel is constant during one OFDM symbol interval and inter-carrier interference (ICI) due to fading channel variations is negligible. At the receiver end, the GI is removed and the received symbol is demodulated using the FFT operation. Therefore, the FFT output $R_l(k)$ during the l-th period is given by [18]-[20]

$$R_l(k) \;= \alpha(\phi(k)) H_l(k) X_l(k) e^{j2\pi\phi(k)(lN_u+N_g)/N} + I_l(k) + W_l(k), k \in [-N/2, N/2], \tag{3}$$

where $\alpha(\phi(k)) = sin(\pi\phi(k)) / (N sin(\pi\phi(k)/N))$ is the self-distortion coefficient of each subcarrier due the frequency offsets Δ_c and Δ_s, $X_l(k)$ are the complex numbers placed on the k-th data subcarrier of the l-th OFDM symbol, $N_u = N + N_g$ denotes the OFDM symbol length after GI insertion, $H_l(k)$ is the channel's frequency response with zero-mean and variance σ_H^2, $W_l(k)$ is is a zero-mean complex Gaussian noise with variance σ_W^2 during the l-th symbol period, and $I_l(k)$ is the ICI term introduced by frequency offset. In the presence of small CFO and SFO, $\alpha(\phi(k)) \approx 1$ and the ICI contribution will be omitted in the following discussion, since its power is very small compared with the additive noise power [20]. In (3),

$H_l(k)$ denotes the equivalent channel transfer function (CTF), which appears to be a superposition of CTFs from all transmit antennas with the corresponding phase shifts

$$H_l(k) = \frac{1}{\sqrt{N_T}} \sum_{t=0}^{N_T-1} H_{l,t}(k) e^{j2\pi k\delta_t/N}, \tag{4}$$

where $H_{l,t}(k)$ is the frequency channel response from the t -th transmit antenna with zero-mean and variance σ_H^2 .

3. Conventional Joint CFO and SFO Estimation Scheme

A post-FFT pilot-assisted joint CFO and SFO estimation scheme considered in [17]-[19] is revisited in this section. Since there is no frequency reference cell (FRC) in the DRM+ system, which is mainly used for frequency-offset estimation as studied in [21], a gain reference cell (GRC) can be a possible candidate as a pilot. Carrier indices for a time-frequency lattice-type GRCs are defined as [3]

$$\mathrm{S} = \left\{ k \mid k = 2 + \frac{D_f}{D_t}(s)_{D_t} + pD_f, 0 \le s \le N_s - 1 \right\}, k \in [-N/2, N/2], \tag{5}$$

where p is integer-valued, N_s is the number of symbol per frame, and D_f and D_t are the periodicity of the GRC pattern in the frequency and time directions, respectively. Here, we assume that the number of elements in S, equivalently, the number of pilots in one OFDM symbol, is N_p . The pilot subcarriers with length N_p are divided into two sets of S_0 and S_1. Let S_0 and S_1 denote the set of $N_p/2$ indices in the left half $k \in [-N/2, 0)$ and the right half $k \in (0, N/2]$ of the spectrum, respectively.

By using the consecutive GRCs as pilot symbols, we can effortlessly apply the basic concept in [14]-[19] to the DRM+ system. Then, a cumulative correlation on S_0 and S_1 is represented by

$$\begin{aligned} \Omega_i(g) &= \sum_{k\in \mathrm{S}_i} R_l^*(k + gD_f/D_t) R_{l+D_t}(k + gD_f/D_t) \\ &\approx \sum_{k\in \mathrm{S}_i} E_s \,|H_l(k + gD_f/D_t)|^2\, e^{j2\pi\phi(k)D_t\rho} + \sum_{k\in \mathrm{S}_i} \hat{W}_l(k),\ 0 \le g < D_t, i = 0,1 \end{aligned} \tag{6}$$

wth

$$\begin{aligned} \hat{W}_l(k) &= H_l^*(k) X_l^*(k) W_{l+D_t}(k) e^{-j2\pi\phi(k)D_t(lN_u+N_g)/N} \\ &+ H_{l+D_t}(k) X_{l+D_t}(k) W_l^*(k) e^{j2\pi\phi(k)D_t((l+D_t)N_u+N_g)/N} + W_l^*(k) W_{l+D_t}(k), \end{aligned} \tag{7}$$

where $E_s = |X_l(k)|^2$ and $\rho = (N + N_g)/N$. For simple description, we have assumed that $H_l(k) \approx H_{l+D_t}(k)$ in (6). As originally suggested in [17]-[19], the estimate of Δ_c and Δ_s is respectively computed as

$$\hat{\Delta}_c = \frac{1}{2\pi D_t \rho} \frac{arg\{\Omega_0(g)\} + arg\{\Omega_1(g)\}}{2} \tag{8}$$

and

$$\hat{\Delta}_s = \frac{1}{2\pi D_t \rho} \frac{arg\{\Omega_1(g)\} - arg\{\Omega_0(g)\}}{k_{N_p/2+1}(g) - k_1(g)}, 0 \le g < D_t, \tag{9}$$

where $arg\{\cdot\}$ denotes the argument of a complex number and $k_{iN_p/2+1}(g)$ stands for the first subcarrier index in $\mathbb{S}_i$ ($i = 0,1$), depending on the g -th GRC pattern. Note that the performance of (8) and (9) heavily depends on the index g of comb-type GRC patterns because of the periodic nature of the CTF in the frequency direction. Hence, one can expect that a proper choice of g will enhance the performance of (8) and (9). In doing so, we consider the issue of selecting the cyclic delay and pilot pattern with a maximum channel power in the following section.

4. Proposed Joint CFO and SFO Estimation Scheme

In order to improve the performance of the CFO and SFO estimation scheme when the CDD is adopted in the DRM+ system, an efficient frequency estimation scheme by using the periodic nature of overall CTF $H_l(k)$ and multiple uniformly distributed comb-type pilots referred to as GRC is suggested in this section. For simplicity, it is assumed that $\delta_0 = 0$ and cyclic delay between adjacent antennas is equi-distant. The power of equivalent CTF in (4) can be expressed as

$$\begin{aligned} |H_l(k)|^2 &= \frac{1}{N_T}\sum_{t=0}^{N_T-1}|H_{l,t}(k)|^2 \\ &+ \frac{2}{N_T}\sum_{t_1=0}^{N_T-1}\sum_{t_2>t_1}^{N_T-1}|H_{l,t_1}(k)||H_{l,t_2}(k)|cos(2\pi k(\delta_{t_2} - \delta_{t_1})/N + \theta_{t_1,t_2}), \end{aligned} \tag{10}$$

where θ_{t_1,t_2} is the phase of $H_{l,t_1}(k)H^*_{l,t_2}(k)$. By using the fact that the DRM+ system suffers from severe flat fading conditions [4], the channels $\{H_{l,t}(k)\}$ are assumed to be frequency-flat and one can build the channel $H_l(k)$ that is periodic with a period of

$$H_P = \frac{N}{\delta_{t+1} - \delta_t}, t = 0,1,\cdots \quad . \tag{11}$$

As we can see in (10), the channel magnitude of the received signal heavily depends on the cyclic delay in the second term on the right-hand side when the system parameters N_p, D_f, and N are fixed. If we choose $\delta_0 = 0$ and $\delta_1 = N/2$ as in (2) in the case of two-transmit antenna, one can find that the CTFs in (10) are same at all GRC positions.

In order to improve the accuracy of the frequency estimation scheme, first, one can find a minimum cyclic delay $D_\delta = \delta_{t+1} - \delta_t$ as

$$D_\delta = \frac{N}{H_P} = \frac{N}{D_f}, \tag{12}$$

which meets with the condition $H_l(k) \approx H_l(k + D_f)$. Then, we suggest to use an integer multiple of D_δ

$$mD_\delta = m\frac{N}{D_f}, \quad 1 \le m < D_f, \tag{13}$$

as a candidate of cyclic delay between neighboring antennas. Note that an integer-valued m is a fundamental design parameter that deals with the amount of cyclic delay and the receiver performance, depending on the number of CDD antennas. Since the increase in m results in the decrease in the periodicity of the channel in (13), substituting (13) into (11) yields the periodicity of the channel $H_{P,m} = D_f / m$, which still guarantees periodically flat-fading condition $H_l(k) \approx H_l(k + D_f)$. From the above discussion, the transmit-antenna specific cyclic delays between adjacent antennas are set as equidistant delays

$$\delta_{t+1} - \delta_t = mD_\delta,\ 1 \le m < D_f, t = 0,2,\cdots \qquad , \tag{14}$$

where $mD_\delta \ge N_g + 1$ should be guaranteed [8].

The amount of cyclic delay is chosen as (13) such that the channel in all pilot positions is periodically flat-fading and the number of distinguishable channel coefficient is maximized. For a possible index of GRC positions $g \in \{0,1,\cdots$, a careful selection of m in (13) leads to the maximization of the number of distinguishable channel coefficient $H_l(k + gD_f / D_t)$ at $k \in \mathrm{S}$. In order to maximize the number of distinguishable channel coefficient denoted by N_{max}, the periodicity of the channel $H_{P,m} = D_f / m$ should not be reduced to lowest terms, i.e.,

$$\frac{D_f}{m} \ne \frac{n \cdot D'_f}{n \cdot m'}, \quad n = 2,3,4,\cdots \tag{15}$$

where D'_f and m' are obtained by reduction of a fraction to the lowest terms. If $H_{P,m} = D_f / m$ is reduced to lowest terms such that $H_{P,m} = D'_f / m'$, one can see that an integer value D'_f is a multiple of $H_{P,m}$ and $H_l(k) \approx H_l(k+D'_f)$, which says that the channel $H_l(k)$ is periodically flat every D'_f subcarriers. If $H_{P,m}$ is not reduced to lowest terms, $H_l(k) \approx H_l(k+D_f)$. Since $D_f > D'_f$, one can expect that the number of distinguishable channel coefficient N_{max} becomes smaller.

When the cyclic delay is appropriately chosen in accordance with (14) and (15), one GRC pattern which has the maximum channel power is eventually selected. By sensing all possible channel powers of the GRCs, the estimated GRC position with maximum channel power can be obtained. Since pilot symbols are known at the receiver, the estimated GRC position can be practically estimated by

$$\hat{g} = \underset{0 \le g < D_t}{arg\ max} \left\{ \sum_{k \in S} | R_l(k + gD_f / D_t) / X_l(k + gD_f / D_t) |^2 \right\}. \tag{16}$$

Based on the selected GRC position according to (16), the proposed estimation scheme can be simply constructed by

$$\hat{\Delta}_c = \frac{1}{2\pi D_t \rho} \frac{arg\{\Omega_0(\hat{g})\} + arg\{\Omega_1(\hat{g})\}}{2} \tag{17}$$

and

$$\hat{\Delta}_s = \frac{1}{2\pi D_t \rho} \frac{arg\{\Omega_1(\hat{g})\} - arg\{\Omega_0(\hat{g})\}}{k_{N_p/2+1}(\hat{g}) - k_1(\hat{g})}, \tag{18}$$

which are in a form identical to (8) and (9) with g replaced by $\hat{g}$, respectively. In DRM mode E, four GRC patterns are defined as in (5) and the number of available pilots differs from GRC patterns. In order to provide a framework for a fair comparison, we assume that N_p is even-numbered. A close observation of (5) obviously indicates that there are two possible pilot configurations: (C1) $N_p = 12$ when $(s)_{D_t} = 0,3$ (C2) $N_p = 14$ when $(s)_{D_t} = 1,2$.

5. MSE Analysis

For simplicity, we consider the CDD scheme employing $N_T = 2$ transmit antennas. Since most of OFDM systems use the powerful channel coding scheme and there is a certain amount of channel correlation between transmit antennas in practical deployments, the transmit antennas with $N_t > 2$ leads only to a small performance improvement [5]-[7].

When the amount of cyclic delay is chosen according to (14) and one GRC pattern $\hat{g}$ which has the maximum channel power is selected as in (16), the maximum of the channel can be obtained by

$$\sum_{k \in S} | H_l(k + \hat{g}D_f / D_t) |^2 \le \frac{1}{2} \sum_{k \in S} \left\{ | H_{l,0}(k + \hat{g}D_f / D_t) | + | H_{l,1}(k + \hat{g}D_f / D_t) | \right\}^2, \tag{19}$$

which corresponds to when $cos(2\pi k(\delta_{t_2} - \delta_{t_1})/N + \theta_{t_1,t_2}) = 1$ in (10). Based on assuming that $\{H_{l,t}(k)\}$ are frequency-flat, thereby we have from (19)

$$\begin{aligned}\Omega_i(\hat{g}) &= E_s S(N_p/2)\mathrm{H}_l(k)e^{j\psi(k_{iN_p/2+1}(\hat{g}))} + \sum_{k\in S_i}\hat{W}_l(k) \\ &= E_s S(N_p/2)\mathrm{H}_l(k)e^{j\psi(k_{iN_p/2+1}(\hat{g}))}\left[1+\sum_{k\in S_i}\mathrm{W}_l(k)\right],\ i=0,1,\end{aligned} \tag{20}$$

where

$$S(N_p/2) = sin(\pi(N_p/2)D_f\delta_1/N)/sin(\pi D_f\delta_1/N), \tag{21}$$

$$\mathrm{H}_l(k) = \{|H_{l,0}(k+\hat{g}D_f/D_t)| + |H_{l,1}(k+\hat{g}D_f/D_t)|\}^2/2, \tag{22}$$

$$\psi(k_{iN_p/2+1}(\hat{g})) = 2\pi\Delta_c D_t\rho + \pi\Delta_s[k_{iN_p/2+1}(\hat{g}) + k_{(i+1)N_p/2}(\hat{g})]D_t\rho, \tag{23}$$

and

$$\mathrm{W}_l(k) = \frac{\hat{W}_l(k)}{E_s S(N_p/2)\mathrm{H}_l(k)}e^{-j\psi(k_{iN_p/2+1}(\hat{g}))}. \tag{24}$$

From (20), one can remark that at high SNR's

$$arg\{\Omega_i(\hat{g})\} \approx \psi(k_{iN_p/2+1}(\hat{g})) + \sum_{k\in S_i}\mathrm{W}_l^Q(k), i=0,1, \tag{25}$$

where x^Q denotes the imaginary part of x. Then, the error of the SFO estimate takes the expression

$$\varepsilon_s(\hat{g}) = \frac{1}{\pi[k_{N_p/2+1}(\hat{g}) + k_{N_p}(\hat{g}) - k_1(\hat{g}) - k_{N_p/2}(\hat{g})]D_t\rho}\left[\sum_{k\in S_1}\mathrm{W}_l^Q(k) - \sum_{k\in S_0}\mathrm{W}_l^Q(k)\right], \tag{26}$$

which depends on the estimated GRC $\hat{g}$. Since GRCs are uniformly distributed as defined in (5), one can easily see that $k_{N_p/2+1}(\hat{g}) - k_1(\hat{g}) = k_{N_p}(\hat{g}) - k_{N_p/2}(\hat{g})$ for all $\hat{g}$'s. Based on $E\{\mathrm{W}_l^Q(k)\} = 0$, (26) is simplified into

$$\varepsilon_s(\hat{g}) = \frac{1}{2\pi[k_{N_p/2+1}(\hat{g}) - k_1(\hat{g})]D_t\rho}\left[\sum_{k\in S_1}\mathrm{W}_l^Q(k) - \sum_{k\in S_0}\mathrm{W}_l^Q(k)\right], \tag{27}$$

which leads to

$$E\{|\varepsilon_s(\hat{g})|^2\} = \left(\frac{1}{2\pi[k_{N_p/2+1}(\hat{g}) - k_1(\hat{g})]D_t\rho}\right)^2 \left[E\{\sum_{k\in S_1}|W_l^Q(k)|^2\} + E\{\sum_{k\in S_0}|W_l^Q(k)|^2\}\right]. \quad (28)$$

Based on an approximation at high SNR, the product of two noise terms is negligible. After some straightforward calculations, the noise variance can be obtained by

$$E\left\{\sum_{k\in S_i}|W_l^Q(k)|^2\right\} \approx \frac{\sigma_W^2 N_p/2}{(S(N_p/2))^2 E_s} E\left\{\frac{1}{H_l(k)}\right\},\ i = 0,1. \quad (29)$$

Since $H_{l,0}(k)$ and $H_{l,1}(k)$ are statistically independent complex Gaussian-distributed random variables, $x = |H_{l,0}(k)| + |H_{l,1}(k)|$ is a sum of two independent Rayleigh random variables, whose probability density function takes the expression [22]

$$f(x) = \frac{x^3}{3\sigma_H^4} e^{-x^2/\sqrt{3}\sigma_H^2}. \quad (30)$$

Hence we have from (30)

$$E\left\{\frac{1}{H_l(k)}\right\} = \frac{1}{\sqrt{3}\sigma_H^2}. \quad (31)$$

Substituting (31) into (29) yields the MSE expression

$$E\{|\varepsilon_s(\hat{g})|^2\} \approx \frac{N_p}{4\sqrt{3}\pi^2[k_{N_p/2+1}(\hat{g}) - k_1(\hat{g})]^2 D_t^2\rho^2(S(N_p/2))^2\bar{\gamma}}, \quad (32)$$

where $\bar{\gamma} = \sigma_H^2 E_s/\sigma_W^2$ is the average SNR. The final MSE expression can be obtained by averaging (32) over $\hat{g}$ since N_p and $k_{N_p/2+1}(\hat{g}) - k_1(\hat{g})$ is different according to the estimated $\hat{g}$. Similarly, the error of CFO estimate can be expressed as

$$\varepsilon_c = \frac{1}{4\pi D_t\rho}\left[\sum_{k\in S_0}W_l^Q(k) + \sum_{k\in S_1}W_l^Q(k)\right] + \frac{\Delta_s}{4}[k_1(\hat{g}) + k_{N_p/2}(\hat{g}) + k_{N_p/2+1}(\hat{g}) + k_{N_p}(\hat{g})]. \quad (33)$$

Based on the fact that $k_{N_p/2+1}(\hat{g}) - k_1(\hat{g}) = k_{N_p}(\hat{g}) - k_{N_p/2}(\hat{g})$, it is found that the second term on the right hand side of (33) is zero. Similar to (32), the MSE of (33) is directly given out without providing the detailed derivation as

$$E\{|\varepsilon_c|^2\} \approx \frac{N_p}{16\sqrt{3}\pi^2 D_t^2 \rho^2 (S(N_p/2))^2 \bar{\gamma}}. \quad (34)$$

6. Simulation Results

In order to verify the usefulness of the proposed estimation scheme and the accuracy of MSE analysis, simulations are performed in the DRM robustness mode E, considering $N=213$, $N_g=24$, $D_f=16$, $D_t=4$, $BW=96$ KHz, and center frequency is 90MHz [3]. **Table 1** shows the channel models used in our simulations [3]. For antenna configuration, the antennas are placed such that their CTFs can be considered as uncorrelated. The CFO Δ_c is set to of the subcarrier spacing 2% and $\Delta_s=20$ ppm. For a fair comparison, we assume that there are two possible pilot configurations: (C1) $N_p=12$ when $(s)_{D_t}=0,3$ (C2) $N_p=14$ when $(s)_{D_t}=1,2$.

Figs. 1 and **2** depict the MSE of the CFO and SFO estimation schemes with respect to m defined in (14), respectively, when $N_T=2$. In order to provide a framework for a fair comparison, the mobile speed is not taken into account in **Table 1**, i.e., $v=0$ km/h. As shown in these figures, we can observe that the performance of the proposed CFO and SFO estimation scheme depends on the amount of cyclic delay, while the performance of the conventional one is insensitive to m. Since one can see from (16) that $D'_f=2$ for $m=8$, $D'_f=4$ for $m=4$, $D'_f=8$ for $m=2$ and 6, $D'_f=16$ for other m's, the number of distinguishable channel coefficient $\{H_l(k+4g)\}$ at subcarrier $k\in\mathbb{S}$ ($g=0,1,2,3$) is $N_{max}=1$ for $m=4$ and 8, $N_{max}=2$ for $m=2$ and 6, and $N_{max}=4$ for others. Therefore, the performance when $m=3$, 5, and 7 is better than the other values of m's, which says that there are m's that give the minimum MSE regardless of SNRs. Interestingly, the performance of the proposed scheme when $m=4$ is nearly the same as that of the conventional scheme. This mainly stems from the fact that the channel $H_l(k)$ when $m=4$ is periodically flat every $D'_f=4$ subcarriers in accordance with (15), which enables the channel power for all g's in (16) to be identical. As expected, the CFO and SFO estimation scheme endowed with cyclic delay (14) and (15) is capable of robustly estimating the frequency offset in the presence of increased frequency selectivity of the channel caused by the CDD. **Fig. 3** shows the MSE of the CFO estimation scheme with respect to m when $N_T=3$ and $N_T=4$ are used. These results are qualitatively similar to those reported in **Figs. 1** and **2**, indicating that m's that give the minimum MSE are also independent of N_T.

Figs. 4 and **5** present the MSE of the CFO and SFO estimation schemes, respectively, when $N_T=2$. As discussed in **Figs. 1 and 2**, three representative cases $m=2,3,4$ are considered in the proposed scheme for the purpose of justifying the choice of m for varying SNR. In the case of the conventional scheme, the MSE curves only when $m=2$ are plotted because its performance is independent of m, as confirmed in Figs. 1 and 2. Also, we assumed that $v=0$ km/h. When the channel is frequency-flat as in CM1, the use of the criterion (12) and a careful selection of $m=3$ that satisfies (15) dramatically improve the performance of (8) and (9), which provides a benefit to the DRM+ receiver with CDD. More importantly, we observe that the analytic curves (32) and (34) are very close to the simulated curves when $m = 3$, or equivalently $N_{max} = 4$. This is due to the fact that the maximum channel power in (19) is assumed to be obtained when the number of distinguishable channel coefficient is as large as possible in deriving (32) and (34). When the frequency selectivity is increased as in CM2, on the other hand, the performance gap between the conventional and proposed schemes becomes further smaller, and the MSE performance in the frequency-selective channel is slightly worse than that in CM1. The reason for this is that the frequency-flat assumption used in (20) is not valid in CM2. As confirmed in **Figs. 1 and 2**, the results in **Figs. 4 and 5** justify why we choose $m = 3$ in the proposed approach, which is used in the following example.

Fig. 6 shows the MSE of the CFO estimation schemes when $m = 3$, taking into consideration the mobile speed defined in **Table 1**. The presence of v causes a severe performance degradation for all frequency estimators at high SNR. This primarily arises from the impact of the time selectivity of the channel, which is one of the main disadvantages for frequency estimation in OFDM. Nevertheless, the proposed scheme still outperforms the conventional scheme even for the fast fading.

7. Conclusion

In this paper, the problem of frequency detection for OFDM-based FM systems employing with CDD transmit antennas has been considered. The robust CFO and SFO estimation scheme was derived by properly selecting the amount of cyclic delay and a pilot pattern. To verify the effectiveness of the frequency estimation scheme, the MSE performance of the proposed scheme was theoretically analyzed. It was verified by performance analysis and computer simulations that the proposed CFO and SFO estimation scheme endowed with appropriately chosen cyclic delay and pilot pattern is robust to the frequency selectivity of the channel, when compared to the conventional scheme.

References

[1] ETSI ETS 300 401, “Radio broadcasting systems: digital audio broadcasting (DAB) to mobile, portable and fixed receivers," ETSI, Tech. Rep., February 1995.
[2] ETSI ETS 300 744, “Digital video broadcasting (DVB): frame structure, channel coding and modulation for digital terrestrial television (DVB-T)," ETSI, Tech. Rep., March 1997.
[3] ETSI ES 201 980 V3.1.1, “Digital radio mondiale (DRM) - system specification," August 2009.
[4] A. Steil, F. Schad, M. Feilen, M. Kohler, J. Lehnert, E. Hedrich, and G. Kilian, “Digitising VHF FM sound broadcasting with DRM+ (DRM mode E)," in *Proc. of International Symposium on Broadband Multimedia Systems and Broadcasting 2009*, pp. 1-6, 2009. doi:10.1109/ISBMSB.2009.5133727
[5] A. Lodhi, F. Said, M. Dohler, and A. Aghvami, “Performance comparison of space-time block coded and cyclic delay diversity MC-CDMA systems,” *IEEE Wireless Commun*., vol.12, no.2, pp.38-45, Apr. 2005. doi:10.1109/MWC.2005.1421927
[6] A. Dammann, R. Raulefs, G. Auer, and G. Bauch, “Comparison of space-time block coding and cyclic delay diversity for a broadband mobile radio air interface," in *Proc. of Int. Symp. Wireless Pers. Multimedia Commun*., Oct. 2003.
[7] Y. Zhang, J. Cosmas, K.-K. Loo, M. Bard, and R. D. Bari, “Analysis of cyclic delay diversity on DVB-H systems over spatially correlated channel," *IEEE Trans. Broadcasting*, vol. 53, no. 1, pp. 247-255, March 2007. doi:10.1109/TBC.2007.891702
[8] G. Bauch and J.S.Malik, “Cyclic delay diversity with bit-interleaved coded modulation in orthogonal frequency division multiple access,” *IEEE Trans. Wireless Commun*., vol.5, no.8,

pp.2092-2100, Aug. 2006. doi:10.1109/TWC.2006.1687724
[9] A. Dammann, "On the influence of cyclic delay diversity and doppler diversity on the channel characteristics in OFDM systems," in *Proc. of ICC'07*, pp.4179-4184, Jun. 2007. doi:10.1109/ICC.2007.689
[10] H. Chang and L. Wang, "Achieving arbitrary multiplexing rates for MIMO-OFDM systems by hybrid cyclic delay diversity," *IEEE Trans. Commun.*, vol. 61, no. 3, pp. 1072-1079, March 2013. doi:10.1109/TCOMM.2013.020413.120239
[11] Q. Yan, Kah C. Teh, Kwok H. Li, and Y. Hu, "Multi-relay-selection scheme with cyclic delay diversity," *IEEE Communications Letters*, vol. 17, no. 2, pp. 349-352, February 2013. doi:10.1109/LCOMM.2012.121912.122493
[12] A. Auer, "Channel estimation for OFDM with cyclic delay diversity," in *Proc. of PIMRC'04*, pp.1792-1796, Sep. 2004. doi:10.1109/PIMRC.2004.1368308
[13] W. Huang, C. Li, and H. Li, "Optimal pilot sequence design for channel estimation in CDD-OFDM systems," *IEEE Trans. Wireless Commun*., vol. 11, no. 11, pp. 4006-4016, November 2012. doi:10.1109/TWC.2012.092412.112033
[14] J. Lei and T.-S Ng, "A consistent OFDM carrier frequency offset estimator based on distinctively spaced pilot tones," *IEEE Trans. Wireless Commun*., vol.3, no.2, pp.588-599, Mar. 2004. doi:10.1109/TWC.2004.825350
[15] M. Sandell, D. McNamara, and S. Parker, "Analysis of frequency-offset tracking in MIMO OFDM systems," *IEEE Trans. Commun*., vol. 54, no. 8, pp. 1481-1491, August 2006. doi:10.1109/TCOMM.2006.878841
[16] K. Nikitopoulos and A. Polydoros, "Inter-frame, fine frequency/phase synchronization for simple space-time-coded OFDM receivers," *IEEE Trans. Wireless Commun*., vol. 6, no. 10, pp. 3510-3514, October 2007. doi:10.1109/TWC.2007.060027
[17] M. Speth, S. A. Fechtel, G. Fock, and H. Meyr, "Optimum receiver design for OFDM-based broadband transmission-Part II: a case study," *IEEE Trans. Commun*., vol. 49, pp. 571-578, April 2001. doi: 10.1109/26.917759
[18] K. Shi, E. Serpedin, and P. Ciblat, "Decision-directed fine synchronization in OFDM systems," *IEEE Trans. Commun.*, vol.53, no.3, pp.408-412, Mar. 2005. doi:10.1109/TCOMM.2005.843463
[19] M. Morelli and M. Moretti, "Fine carrier and sampling frequency synchronization in OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1514-1524, April 2010. doi:10.1109/TWC.2010.04.091058
[20] M. Speth, S. A. Fechtel, G. Fock, and H. Meyr, "Optimum receiver design for wireless broad-band systems using OFDM-Part I," *IEEE Trans. Commun.,* vol. 47, pp. 1668-1677, November 1999. doi:10.1109/26.803501
[21] W. J. Shin, J. W. Seo, and Y. H. You, "MSE analysis of sampling frequency offset estimation scheme for OFDM-based digital radio mondiale (DRM) systems," *Wireless Personal Communication*, vol. 71, no. 2, pp. 1271-1281, July 2013. doi:10.1007/s11277-012-0873-2
[22] J. Hu, N. C. Beaulieu, "Accurate simple closed-form approximations to Rayleigh sum distributions and densities," *IEEE Commun. Lett.,* vol.9, no.2, pp.109-111, Feb. 2005. doi:10.1109/LCOMM.2005.02003

Table 1. DRM channel profiles

No	Profile Name	Speed v (km/h)	Maximum delay (μs)
CM1	Urban	60	3.0
CM2	Terrain obstructed	60	16.0
CM3	Hilly terrain	100	82.7

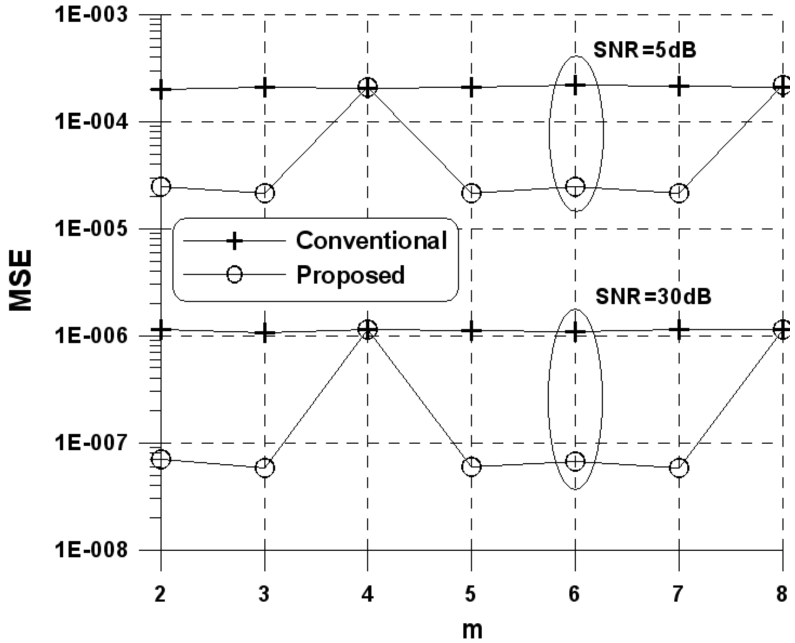

Fig. 1. MSE of the CFO estimators versus m in CM1 when $N_T = 2$.

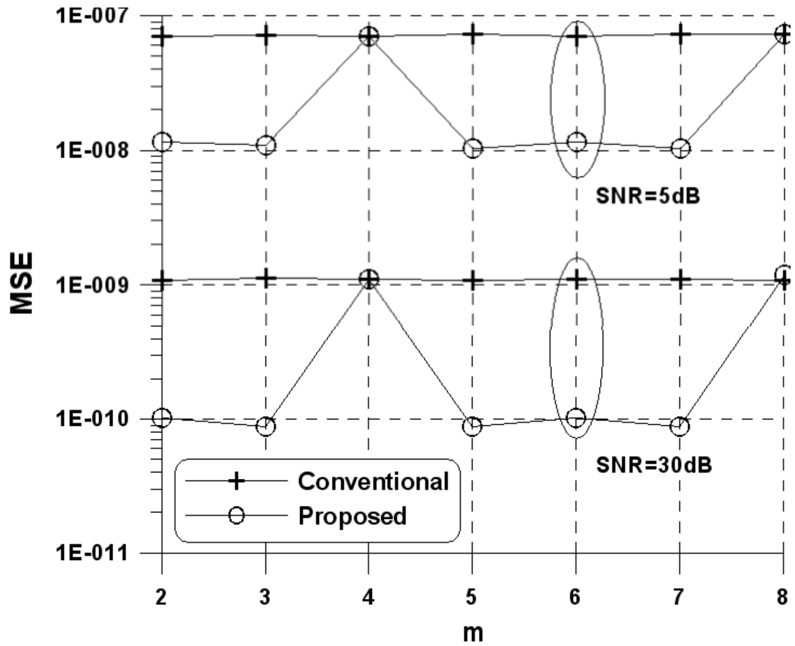

Fig. 2. MSE of the SFO estimators versus m in CM1 when $N_T = 2$

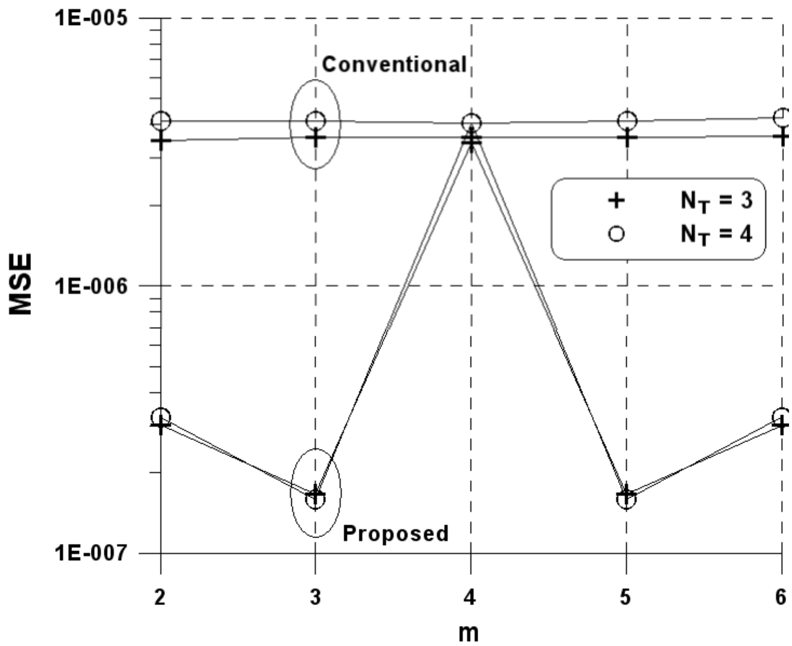

Fig. 3. MSE of the CFO estimators versus m in CM1 when SNR = 25dB, N_T = 3 and 4 are used.

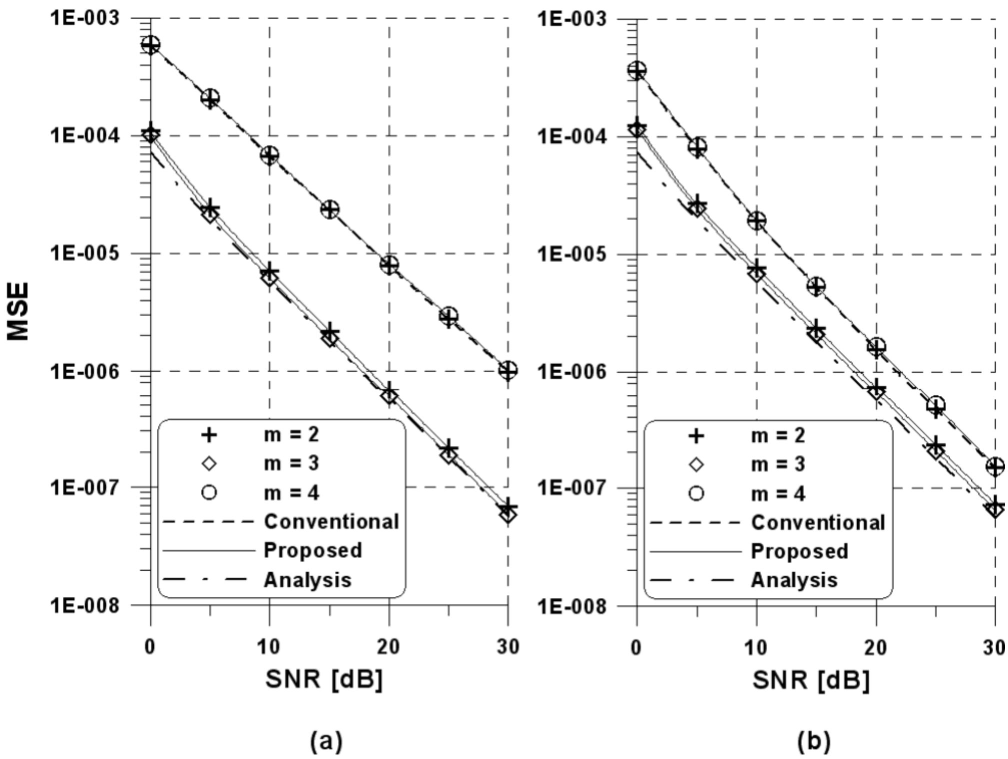

Fig. 4. MSE of the CFO estimators when N_T = 2: (a) CM1 (b) CM2

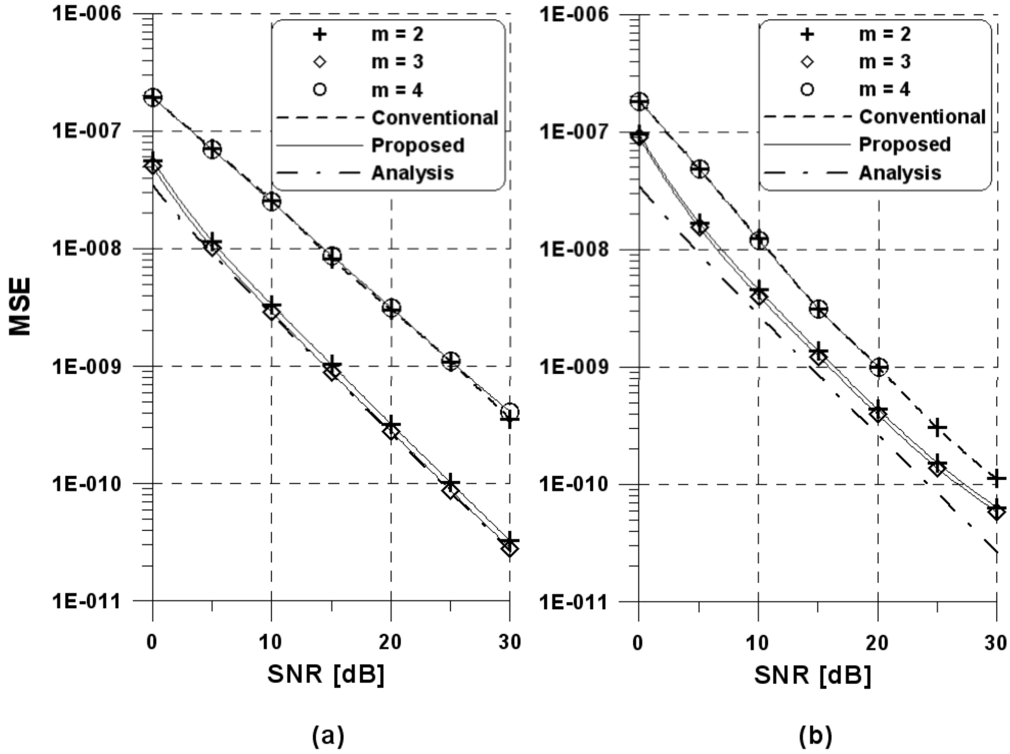

Fig. 5. MSE of the SFO estimators when N_T = 2: (a) CM1 (b) CM2.

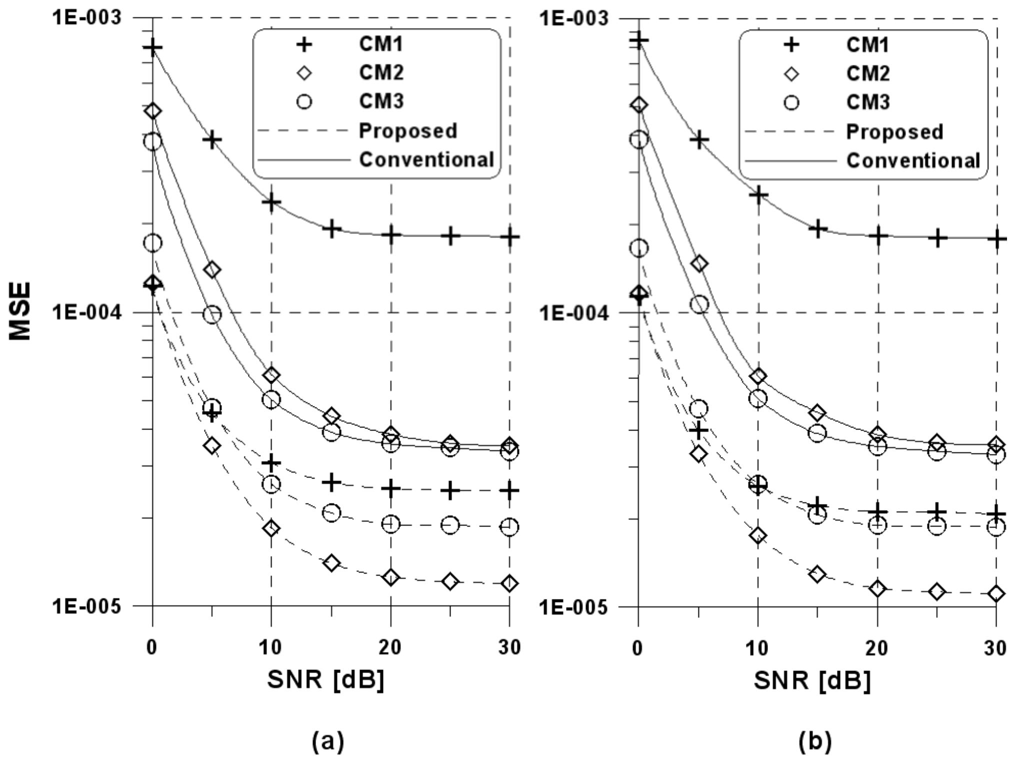

Fig. 6. MSE of the CFO estimators in CM1~CM3: (a) N_T = 3 (b) N_T = 4.

Won-Jae Shin received the B.S. and M.S. degrees in School of Computer Engineering from Sejong University, Seoul, Korea, in 2007 and 2009, respec-tively. He is currently working toward the Ph.D. degree in the School of Computer Engineering from Sejong University, Seoul, Korea. His research interests are in the areas of wireless communications systems design, multicarrier transceivers, and system architecture for realizing next-generation communications systems.

Young-Hwan You received the B.S., M.S., and Ph.D. degrees in electronic engineering from Yonsei University, Seoul, Korea, in 1993, 1995, and 1999, respectively. From 1999 to 2002 he had been a senior researcher at the wireless PAN technology project office, Korea Electronics Technology Institute (KETI), Korea. Since 2002 he has been a professor of the Department of Computer Engineering, Sejong University, Seoul, Korea. His research interests are in the areas of wireless communications systems design, spread spectrum transceivers, and system architecture for realizing advanced digital wireless communications systems, especially, for wireless OFDM.