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비-가우시안 잡음하의 적응 시스템을 위한 바이어스된 영-오차확률

Biased Zero-Error Probability for Adaptive Systems under Non-Gaussian Noise

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요 약

영-오차확률 성능 기준은 오차 샘플들이 직류 바이어스 잡음의 영향을 받을 때 적응 시스템에 사용되기에는 제약이 따른다. 이 논문에서는 바이어스 변수를 오차 분포에 도입하고 바이어스된 오차확률에서 오차를 0 으로 하여 새로운 성능 기준인 바이어스된 영-오차확률을 제안하였다. 또한, 확장 필터 구조를 기반으로 제안된 성능 기준을 최대화 함으로써 적응 알고리듬을 도출하였다. 통신 채널 등화에 대한 시뮬레이션 결과로부터 제안된 성능기준에 기반한 이 알고리듬이 강한 충격성 잡음과 직류-바이어스 잡음의 환경에서 동요 없이 오차 샘플들을 0 으로 집중시키는 성능을 보였다.

☞ 주제어 : 충격성, 직류-바이어스, 영-오차확률, 등화

ABSTRACT

The criterion of zero-error probability provides a limitation on error probability functions being used for adaptive systems when the error samples are shifted by the influence of DC-bias noise. In this paper, we employ a bias term in the error distribution and propose a new criterion of the biased zero-error probability with error being zero. Also, by maximizing the proposed criterion on expanded filter structures, a supervised adaptive algorithm has been derived. From the simulation results of supervised equalization, the algorithm based on the proposed criterion yielded zero-centered and highly concentrated error samples without disturbance in the environments of strong impulsive and DC-bias noise.

☞ keyword : impulsive, DC-bias, zero-error probability, equalization

1. INTRODUCTION

Communication channels are distorted by the incomplete channel conditions such as multipath fading and additive noise and induce intersymbol interference (ISI) that causes the communication systems to be not acceptable for reliable communications [1]. Particularly the impulsive noise that prevents equalizer algorithms from ISI cancellation commonly occurs in underwater communications [2], indoor communications [3], optical fiber communications and in-vehicle signal transmission [4] and digital TV systems [5].

Recently, to deal with ISI and impulsive noise problems simultaneously, a decision feedback approach based on error entropy concept has been studied [6]. For more enhanced performance, a zero-error probability function has been introduced as a performance criterion [7]. Maximization of the zero-error probability moves the error samples to concentrate on zero and has proven to yield superior performance in multipath fading and impulsive noise environments.

However, probability density functions (PDFs) of error samples in the work do not have any variables to control its mean value on the error-axis so that the criterion based on zero-error PDF has shown in this research to have limitations on ISI cancellation performance under non-Gaussian noise composed of impulsive and DC-bias noise.

To cope with the problems of non-Gasussian noise and ISI, in this paper, we propose a new criterion of the biased zero-error probability by employing a bias term. Also a supervised equalizer algorithm is derived by maximizing the proposed criterion based on the Gaussian kernel and proven

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to be superior in the environments of severe ISI and strong non-Gaussian noise.

2. BIASED ERROR PDF AND PROPOSED COST FUNCTION

The error sample of a given system is defined as $e_i = d_i - y_i$ where d_i is the desired symbol and y_i is the output at the symbol time i. Error distribution $f_E(e)$ reveals how concentrated error samples are and what is their mean value. Particularly the zero-error probability $f_E(0)$ shows how many error samples are gathered around zero on the error-axis and can be utilized as a performance criterion.

The zero-error probability criterion utilizes kernel density estimation method that is to estimate the probability density function of error as a random variable in a non-parametric way based on a finite error sample [8]. Given a set of M error symbols $E_M = \{e_1, e_2, ..., e_M\}$, the kernel density estimation method based on a zero-mean Gaussian kernel with standard deviation σ places a Gaussian kernel $G_\sigma(e - e_i)$ on each of the data points e_i. Then the M Gaussian kernels are summed as in (1) to make the kernel density estimate that can approximate the true density for continuous random variables.

$$f_E(e) \cong \frac{1}{M}\sum_{i=1}^{M} G_\sigma(e - e_i) \qquad (1)$$

In the construction of density functions at the receiver, smaller kernel sizes, on one hand, produce more accurate solutions but instable, on the other hand, wider ones bring more inaccurate solutions [9].

The zero-error probability $f_E(0)$ becomes

$$f_E(0) \cong \frac{1}{M}\sum_{i=1}^{M} G_\sigma(e_i) \qquad (2)$$

Maximization of $f_E(0)$ moves the error samples e_i close to zero by removing the negative effects from outliers caused by intersymbol interference (ISI) and impulsive noise [7].

On the other hand, the error distribution $f_E(e)$ does not contain any variable to move its mean value on the error-axis so that the criterion $f_E(0)$ has a limit when the error samples are shifted by some influences such as DC-bias noise. Based on this analysis, we employ the biased error (BE) PDF $f_{BE}(e)$ with bias term τ as

$$f_{BE}(e) = f_E(e+\tau) \cong \frac{1}{M}\sum_{i=1}^{M} G_\sigma((e+\tau) - e_i) \qquad (3)$$

In this paper, instead of dealing with $f_E(0)$, we propose a new criterion of the biased zero-error PDF $f_{BE}(e=0)$ as

$$f_{BE}(0) = f_E(\tau) = \frac{1}{M}\sum_{i=1}^{M} G_\sigma(e_i - \tau) \qquad (4)$$

For simplicity, we let

$$e_{i,biased} = e_i - \tau \qquad (5)$$

Then we have

$$f_E(\tau) = \frac{1}{M}\sum_{i=1}^{M} G_\sigma(e_{i,biased}) \qquad (6)$$

The maximization of the proposed criterion $f_E(\tau)$ forces the error PDF to be shifted by the amount of the bias τ as well as most of the error values become zero. So according to the amount of influence of DC-bias noise the bias τ is to be estimated in adaptive algorithms based on the proposed cost function.

3. ADAPTIVE ALGORITHMS BASED ON THE BIASED ZERO-ERROR PROBABILITY

For a tapped delay line (TDL) structure with system weights $\mathbf{W} = [w_0, w_1, ..., w_{N-1}]^T$ and system input $\mathbf{X}_i = [x_i, x_{i-1}, ..., x_{i-N+1}]^T$, output y_i is defined as

$$y_i = \mathbf{W}^T \mathbf{X}_i \quad (7)$$

On the other hand, we can extend the system by adding another weight element w_N to $\mathbf{W}$ and a constant b to $\mathbf{X}_i$ as

$$\mathbf{W}_{ext} = [w_0, w_1, w_2, ..., w_N]^T \quad (8)$$

$$\mathbf{X}_{i,ext} = [x_i, x_{i-1}, ..., x_{i-N+1}, b]^T \quad (9)$$

Then the extended system output can be expressed as

$$y_{i,ext} = \mathbf{W}_{ext}^T \mathbf{X}_{i,ext} \quad (10)$$

Then we can define the extended system error $e_{i,ext}$ as

$$e_{i,ext} = d_i - y_{i,ext} \quad (11)$$

It is desirable that we verify the relationships between the system error e_i and the extended system error $e_{i,ext}$. From (7) and (10), we have

$$y_{i,ext} = y_i + w_N \cdot b \quad (12)$$

By inserting (12) into (11), we obtain

$$\begin{aligned} e_{i,ext} &= d_i - y_i - w_N \cdot b \\ &= e_i - w_N \cdot b \end{aligned} \quad (13)$$

Comparing (12) and (5) and replacing $e_{i,biased}$ with $e_{i,ext}$, that is, $e_{i,ext} = e_{i,biased}$, we can observe that the bias τ can be controllable as $\tau = w_L \cdot b$. This indicates that the proposed criterion $f_E(\tau)$ can be applied to the extended system. Then the criterion for the extended system, $f_{E,ext}(\tau)$ becomes

$$f_{E,ext}(\tau) = \frac{1}{M} \sum_{i=1}^{M} G_\sigma(e_{i,ext}) \quad (14)$$

For the maximization of the criterion $f_{E,ext}(\tau)$, we have the gradient of $f_{E,ext}(\tau)$ at the sample time k as follows.

$$\frac{\partial f_{E,ext}(\tau)}{\partial \mathbf{W}_{ext}} = \frac{1}{\sigma^2 M} \sum_{i=k-M+1}^{k} -e_{i,ext} \cdot G_\sigma(e_{i,ext}) \cdot \frac{\partial e_{i,ext}}{\partial \mathbf{W}_{ext}} \quad (15)$$

Using $\frac{\partial e_{i,ext}}{\partial \mathbf{W}_{ext}} = -\frac{\partial y_{i,ext}}{\partial \mathbf{W}_{ext}}$, the gradient becomes

$$\begin{aligned} \frac{\partial f_{E,ext}(\tau)}{\partial \mathbf{W}_{ext}} &= \frac{1}{\sigma^2 M} \sum_{i=k-M+1}^{k} e_{i,ext} \cdot G_\sigma(e_{i,ext}) \cdot \frac{\partial y_{i,ext}}{\partial \mathbf{W}_{ext}} \\ &= \frac{1}{\sigma^2 M} \sum_{i=k-M+1}^{k} e_{i,ext} \cdot G_\sigma(e_{i,ext}) \cdot \frac{\partial y_{i,ext}}{\partial \mathbf{W}_{ext}} \\ &= \frac{1}{\sigma^2 M} \sum_{i=k-M+1}^{k} e_{i,ext} \cdot G_\sigma(e_{i,ext}) \cdot \mathbf{X}_{i,ext} \end{aligned} \quad (16)$$

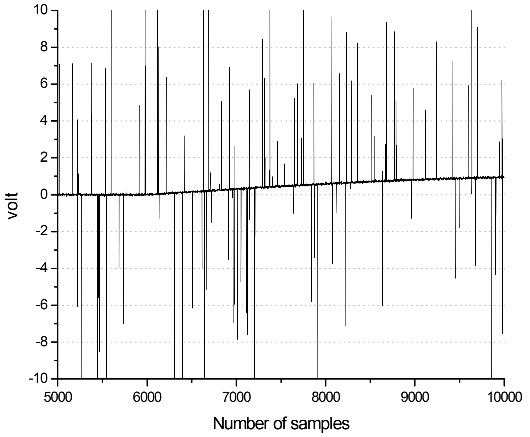
By the steepest ascent method with (15), the update algorithm for the extended weight vector can be summarized as

$$\mathbf{W}_{k+1,ext} = \mathbf{W}_{k,ext} + \frac{\mu}{\sigma^2 M} \sum_{i=k-M+1}^{k} e_{i,ext} \cdot G_\sigma(e_{i,ext}) \cdot \mathbf{X}_{i,ext} \quad (17)$$

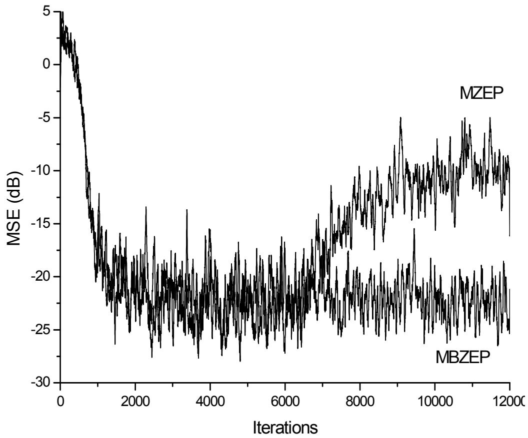
For convenience's sake, this proposed algorithm (17) for supervised systems with (8) and (9) will be referred to in this paper as MBZEP algorithm.

4. SIMULATION RESULTS IN CHANNELS WITH NON-GAUSSIAN NOISE

In this section the performance of the proposed algorithm in (17) is compared with the existing MZEP algorithm in [7] for fading channels with non-Gaussian noise composed of impulsive and time-varying DC-bias noise. The transmitted symbol points are $d_i = \{-3,-1,1,3\}$ and the first channel model H_1 is the same as in [7] for fair comparison. The second channel H_2 is from underwater channel data actually acquired from a shallow-water communication experiments [10] as described below in z-transform.



(Fig. 1) Impulsive noise with time-varying DC bias.



(Fig. 2) MSE performance for H1 with impulsive and time-varying DC bias noise.

$$H_1(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \quad (18)$$

$$H_2(z) = 0.798z^{-4} + 0.543z^{-6} + 0.259z^{-8} \quad (19)$$

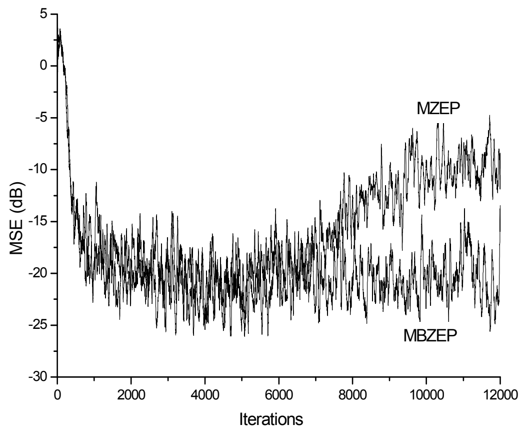
The impulsive noise model in this section is the same as the one used in [7]. The distribution of the impulsive noise n_{Im} is $f_{IM}(n_{\text{Im}}) = (1-\varepsilon)/\sigma_1\sqrt{2\pi} \cdot \exp[-n_{\text{Im}}{}^2/2\sigma_1^2] + \varepsilon/\sigma_2\sqrt{2\pi} \cdot$ $\cdot \exp[-n_{\text{Im}}{}^2/2\sigma_2^2]$ where $\varepsilon = 0.03$, $\sigma_{IN}^2 = 50$ is impulse noise variance, $\sigma_1^2 = 0.001$ is the variance of background white Gaussian noise, and $\sigma_2^2 = \sigma_1^2 + \sigma_{IN}^2$.

Time-varying DC-bias noise $n_{DC,k}$ for the non-Gaussian noise in this simulation is added to the impulsive noise as $n_k = n_{\text{Im},k} + n_{DC,k}$. The time-varying DC-bias is generated as $n_{DC,k} = \sin(2\pi f_0 k)$ with $f_0 = 1/20000$ and added at $k = 6000$ after convergence. The parameter f_0 is selected for being observable as shown in Fig. 1.

The equalizer length $N = 11$ and the constant for $\mathbf{X}_{i,ext}$ isset $b = 2$. The number of data-block size $M = 4$. The step-size is 0.004 for H_1 and 0.01 for H_2. And the kernel size σ is 0.7 for H_1 and 1.0 for H_2. All these parameters are applied to both algorithms under comparison.

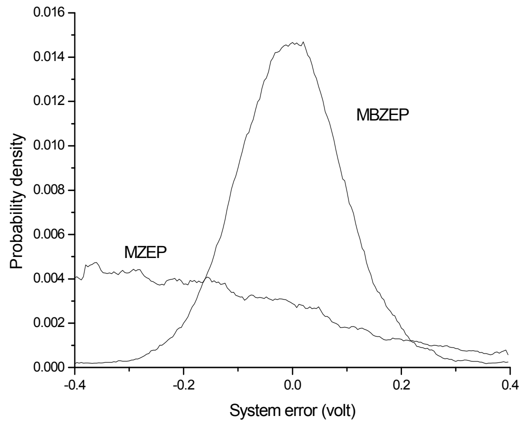
In Fig. 1, The convergence performance of MSE for H_1 is shown. Both algorithms converge at about 2000 samples to around -23 dB of steady state MSE even under strong impulsive noise. Right after the DC-bias noise is added to the channel output, however, MZEP algorithm starts to diverge yielding gradually increasing MSE as the added DC-bias noise increases. Apparently the proposed MBZEP algorithm shows no perturbations at all. As the DC-bias reaches 1 volt at around 12000 iterations, the MSE performance difference leads to about 13 dB. In the shallow-water communication channel H_2, we observe similar results as shown in Fig. 2. After reaching the steady state MSE around -22 dB at the same rapid speed even in the channel condition of severe distortion and impulsive noise, MZEP shows growing MSE in proportion to the amplitude of DC-bias noise from the iteration number 6000. But the proposed MBZEP algorithm keeps the same steady state as in H_1.

The superior immunity against non-Gaussian noise can be proved from the perspective of error distribution as shown in Fig. 4 for H_1 and in Fig. 5 for H_2. In Fig. 4, The proposed MBZEP algorithm produces zero-centered and highly concentrated error samples, but the mean of error samples of MZEP algorithm is spread and shifted to the left centered around -0.4. This means most error samples are not zero but have negative values, that is, most output of MZEP algorithm are positively biased from their corresponding



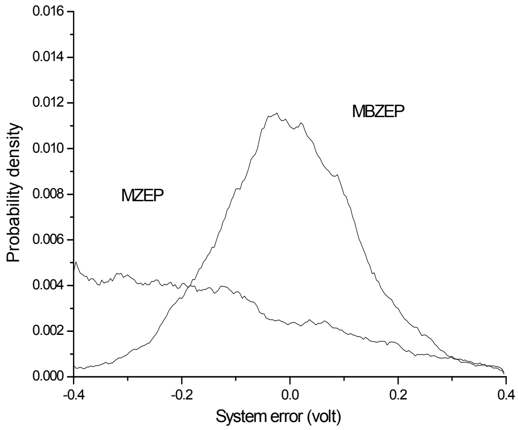
(Fig. 3) MSE performance for H2 with impulsive and time-varying DC bias noise.

symbol points owing to the positive DC-bias noise at about the iteration number 12000 as shown in Fig. 1. The highly



(Fig. 4) Error distribution for H1 with impulsive noise and slowly varying DC bias.

dispersed distribution of MZEP algorithm indicates that MZEP algorithm stricken with DC-bias noise loses the merits of ISI-cancellation performance and/or immunity to impulsive noise. In the severer channel H_2, we observe the disadvantage of MZEP algorithm more apparently. The error distribution of MZEP is more dispersed and shifted to the negative region of error-axis larger than in the case of H_1. This implies that in severer channels, MZEP algorithm



(Fig. 5) Error distribution for H2 with impulsive noise and slowly varying DC bias.

suffers more performance degradation due to DC-bias noise. On the other hand, the proposed MBZEP algorithm in both channel conditions yields zero-centered and highly concentrated error samples without disturbance in the environments of non-Gaussian noise such as impulsive and DC-bias noise.

5. CONCLUSION

Since the error distribution does not have any variable to move its mean value on the error-axis, the MZEP criterion based on zero-error probability has a limit on the cancellation of ISI when the error samples are influenced by DC-bias noise. To cope with the problems of ISI and non-Gasussian noise, in this paper, we have proposed a new criterion of the biased zero-error probability by employing a bias term and letting error be zero.

The proposed algorithm derived from maximizing the biased zero-error probability based on the Gaussian kernel has the forces of shifting error samples and concentrating them on zero on the error-axis in the environments of severe ISI and strong non-Gaussian noise.

From the simulation results of supervised equalization, we conclude that MZEP algorithm under DC-bias noise loses the merits of ISI-cancellation performance or immunity to impulsive noise, but the proposed algorithm yields zero-centered and highly concentrated error samples without disturbance in the environments of non-Gaussian noise such as impulsive and DC-bias noise.

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