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Drift Control for Multistory Moment Frames under Lateral Loading

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Abstract

The paper reports results of recent studies on the effects of column support conditions on the lateral displacements of moment frames at incipient collapse. The article presents a number of exercises in the plastic theory of structures that lead to useful design formulae. It has been shown that Drift Shifting (DS) is caused due to differences in the stiffnesses of adjoining columns, and that changes in drift ratios are more pronounced at first level column joints in both fixed as well as pinned base frames. In well proportioned moment frames, DS in the upper levels could be minimized, even reduced to zero. It has been demonstrated that DS can be eliminated in properly designed fixed and grade beam supported (GBS) moment frames. Several examples, including symbolic P-delta effects, have been provided to demonstrate the validity and the applications of the proposed ideas to the design and drift control of moment frames. The proposed methodology is exact within the bounds of the theoretical assumptions and is well suited for preliminary design and teaching purposes.

Keywords: Moment frames, Efficient plastic design, Boundary conditions, Lateral displacements, P-delta, Drift shifting

1. Introduction

Compliance with drift limits is one of the essential requirements of most contemporary building codes, e.g., ACI (2005), AISC (2005), Eurocode 3 (2011), etc. Almost all performance based plastic design methodologies, e.g., (Mazzolani, 1997; Priestly, 2007; Goel, 2008, 2010; Grigorian, 2011, 2012a, 2012b) emphasize the importance of drift control during elastic as well plastic phases of loading. Lateral drift in multi-story buildings is influenced by many factors. The scope of the current article is limited to the study of the effects of boundary support conditions on the lateral displacements of regular moment frames at incipient collapse. DS is the change in drift ratios of two consecutive levels in multi-story structures. Acceptable levels of DS commonly occur in multi level structures that have not been designed to withstand large inelastic displacements. DS and its effects are more pronounced in frames designed in accordance with plastic behavior than those designed on the basis of purely linear response. Plastic designs generally result in more slender structures and are more likely to experience larger lateral displacements (Hayman, 1961; Neal, 1963). While steel moment frames under lateral loading are most suitable for plastic design, they are also susceptible to unfavorable effects of DS, especially at incipient collapse. (Hamburger, 2009) have reported that a reduction in DS can help improve racking stability in all categories of moment frames. A survey of the literature over the past 60 years, starting with (Baker, 1964) including (Nethercot, 2001) up to (Wong, 2009) and (Schafer 2010) reveals that there is still scant information on practical methods of displacement analysis for sway frames at incipient collapse. Despite the encouraging surge of interest in machine oriented direct second-order Analysis, e.g., (Surovek, 2006) and (Ziemian, 2008), there are no short cut manual linear and/or non-linear methods of displacement analysis for such frames. However, recent studies suggest that it is possible to overcome the computational complexities associated with manual displacement analysis of certain classes of ductile systems at incipient collapse. Indeed, it is even possible to reduce the otherwise complicated task of non linear structural analysis to the study of one of the constituent column trees of the same structure under its own distribution of tributary forces. The paper attempts to show that the lateral displacements of well proportioned GBS moment frames could be smaller than identical frames with fixed and pinned boundary support conditions.

While seismic loading is not addressed in this article, its findings are highly applicable to performance based plastic design of earthquake resistant frames. The success of this approach may be attributed to the proper utilization of the design rules as well as an understanding of the elastic-plastic modes of response of ideally efficient moment frames at incipient collapse. An appreciation of the pertinent design criteria as well as the conditions of desirable performance is a priori to establishing the forthcoming arguments.

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1.1. Minimum design requirements

In the present context a properly designed moment frame is that for which the constituent elements have been selected in such a way that;

- the strong column-weak beam principle is observed for all columns (as required by major codes)
- boundary support and static equilibrium conditions are completely satisfied,
- the prescribed yield criteria are not violated anywhere within the structure,
- all beams and columns are stable throughout the loading history of the structure, all local, segmental and/or global buckling is prevented.
- gravity forces are either negligible or the frame has been designed not to develop beam type failure mechanisms (beam type mechanisms tend to increase lateral displacements),
- the global P-delta phenomenon has been taken into consideration,
- the prescribed drift ratios are not exceeded at incipient collapse,
- the entire structure and its supports have been designed to allow the unrestricted development of plastic hinges. Reduced beam sections (RBS) and/or added flange plates are acceptable options.
- all other code and jurisdictional requirements are satisfied.

A properly designed moment frame may or may not satisfy all conditions of frame efficiency described below. However the case of a generalized, properly designed frame that can be turned into an efficient system is briefly introduced in the following section.

1.2 Design efficiency

In its most general sense, design efficiency implies *engineering expediency* rather than mathematical optimization. However, the proposed methodologies have been formulated in such a way as to satisfy both the theoretical as well as the pertinent engineering requirements. In well proportioned frames the elastoplastic properties of the constituent elements are selected in such a way as to create of a structure of uniform response (Grigorian, 2012a, 2012d) where, members of similar groups such as beams, columns and braces of similar characteristics, e.g., equal lengths, share the same demand/capacity ratios regardless of their numbers within the group. In other words well proportioned frames are special classes of structures for which element demand-capacities as well as target drift ratios at incipient collapse are *enforced rather than investigated* (Grigorian, 2013a, 2013b, 2013c). Well proportioned frames lead to idealized designs and provide theoretical basis for fine-tuning as well as basic analytic studies. For instance, RBS technologies can be utilized to control and induce the sequential formation of plastic hinges at all beam ends. The class of efficient moment frames introduced in this paper, satisfy all conditions of the uniqueness theorem (Foulkes, 1953, 1954), (Grigorian, 1989, 2013c). An ideally efficient or well proportioned structural framework, both from practical as well as theoretical points of view is that for which most of the following criteria can be satisfied. These include, but are not limited to the conditions that;

- Structural irregularities are kept to a minimum and span lengths are within practical limits.
- The self weight of the structure is a practical minimum, i.e., the demand-capacity ratios of all members are selected as close to *unity* as possible.
- The variety of conforming shapes, sizes and connections is kept to a minimum for groups of similar elements, e.g. beams, columns and braces of any given level or subframe.
- The permissible inter-story drift ratio is kept relatively constant along the height of the structure. Uniform drift tends to help control the premature development of P-delta instabilities in multistory frames.
- The sequences of development of plastic hinges are controlled in such a way as to prevent the *premature formation* of plastic hinges at column supports and/or lower story girders. The early formation of plastic hinges at base level column supports and or lower level beams is *tantamount* to replacing their rotational restraints, by moment free hinges before the formation of plastic hinges in other *pre-designated* locations within the structure.

In the forthcoming sections first the *controlled* elastic-plastic response of a generalized regular moment frame under lateral forces is discussed and then attention is focused on the performance of efficient moment frames at incipient collapse.

2. The Theoretical Approach

A well proportioned or properly designed regular moment frame, such as that shown in Fig. 1(b), subjected to monotonically increasing lateral forces, $\overline{F}_i$ with an apex value $\overline{F}$, Fig. 1(a) in combination with relatively small constant gravity, $W_{i,j}$ and axial forces $P_{i,j}$, will fail *eventually* in accordance with the plastic collapse pattern of Fig. 1(c). The dashed line along $i = 0$ represents a continuous grade level beam that may be used to enhance the boundary conditions of rotationally non-restricted column supports. Here, $i = 1, 2\ldots m$ and $j = 1, 2\ldots n$, represent locations of story and column lines respectively. Because of the strong column-weak beam stipulation, which is the fundamental condition of the current presentation, the last intact segment of the frame, prior to total collapse, will be either a fixed base multistory column or a similar column with at least one outrigger beam as depicted in Figs. 3(b) or 3(c). In either case the structure is reduced to a statically determinate system with one more plastic hinge to form before becoming a mechanism. Once the location of the last plastic hinge is established, the maximum lateral displace-

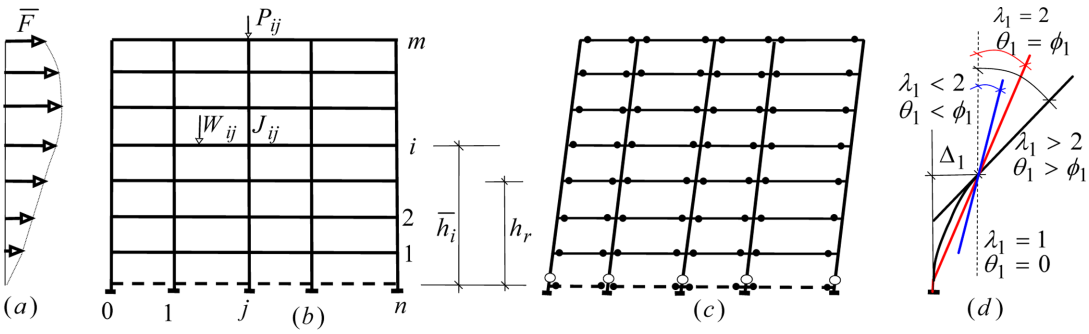

Figure 1. (a) Lateral loading, (b) Moment frame, (c) Failure pattern, (d) Effects of λ_1 on θ_1.

ments of the frame can be computed manually by the use of standard methods of structural analysis. The maximum lateral displacements of the moment frame at incipient collapse can be studied by computing the corresponding displacements of the last stable column at the onset of plastic failure. If the tributary shear force and the sum of the beam plastic moments acting on joint i,j of the last stable column, can be expressed as V_i and $\overline{M}_i^P = \overline{M}_{i,j}^P = \overline{M}_{i,j}^P + \overline{M}_{i,j+1}^P$ respectively then the corresponding free body diagram and the moment tree may be depicted as in Figs. 2(a) and 2(b). The full dots • indicate locations of plastic hinges at beam ends only. The hollow circles show locations of formation of plastic hinges at fixed base column feet without grade beams respectively. In Fig. 2(b) $M_m = \overline{M}_m^P$ and $M_0 = \overline{M}_0^P$ represent the total external joint moments acting on top of the uppermost and the base of the lowermost columns respectively. In the same figure, $M_{i,l}$ and $M_{i,u}$ are the lower and upper end moments respectively of olumn i located between joints i and i–1. $M_{s,u}^P$ is the plastic moment of resistance of the last standing beam at s^{th} level. L and I_S are the span length and the moment of inertia of the s^{th} level beam that may contain the very last plastic hinge forming within the frame. h_i and J_i stand for the height and moment of inertia of the i^{th} level column respectively.

2.1. The effects of the column over-strength factor λ

In a well proportioned moment frame under lateral loading, as illustrated in Figs. 2(c) and 3(g), the acting column tree end moments are equal (Grigorian, 2011), i.e.,

$$M_{i,l}=M_{i,u}=M_i=\frac{V_i h_i}{2} \quad \text{for } i>1 \tag{1}$$

$$\text{and} \quad N_1^P=\frac{V_1 h_1}{2(1+\lambda_1)}$$

where, λ_1 is the over-strength factor of the first level fixed base columns. Similarly if J_1 or J_m is known, then J_i can be determined from the proportionality relationships;

$$I_i=(M_i h_i+M_{i+1}h_{i+1})\frac{I_m}{M_m^P h_m} \quad \text{and} \tag{2}$$

$$J_i=\left(\frac{M_i h_i^2}{M_1 h_1^2}\right)J_1 \quad \text{or} \quad J_i=\left(\frac{M_i h_i^2}{M_m h_m^2}\right)J_m$$

The strong column-weak beam condition requires that;

$$N_i^P+N_{i+1}^P\geq\lambda_i\overline{M}_i \quad \text{for all } i>1 \tag{3}$$

$\lambda_i > 1$ is the column over-strength factor and serves two important purposes. Most commonly it is used to prevent soft story failure for levels 2 and above. In addition, it may be utilized to control the sequence of formation of the plastic hinges and/or to prevent the premature formation of the plastic hinges at the fixed column bases. Obviously, $\lambda_1 > \lambda$ can force the last sets of plastic hinges to form at the fixed column bases. And, if all columns of the frame,

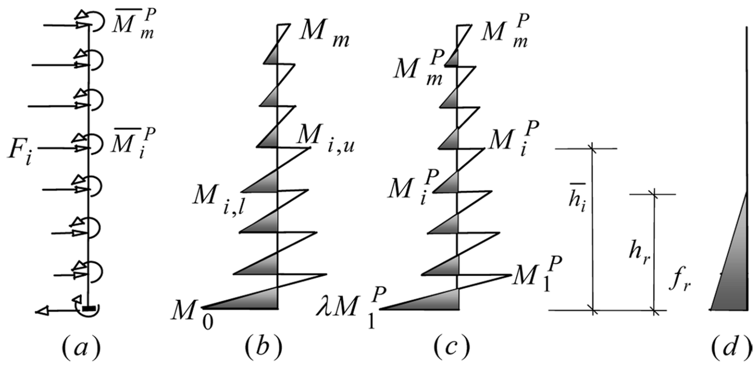

Figure 2. (a) Column tree, (b) Generic moment diagram, (c) Idealized moment distribution (d) Unit load moments.

except those at level 1, are to remain elastic in accordance with the pre-designated mechanism of Fig. 1(c), then the plastic moments of resistance of the column should be selected as; $N_1^P = \lambda_1 M_1^P$ for $i = 1$ and $N_i^P = \lambda_i M_i^P$ for all other i. However, since N_i^P never reaches $\lambda_i M_i^P$ for $i > 1$, the corresponding λ_i does not appear in the forthcoming computations.

2.2. The Generalized displacement function

Assuming that the column is kinematically stable until the formation of the last plastic hinge, which may occur either at the ends of the s^{th} level beam or at the fixed column feet, then the generalized elastic-plastic lateral displacements of joint i of any such column, regardless of the boundary support conditions, may be computed by performing the virtual work summation, using Figs. 2(c) and 2(d), over the height of the column;

$$l \times \Delta_i = \sum_{r=1} f_r \vartheta + \int (\overline{M}/EI)_r f_r ds \tag{4}$$

where, f_r is the virtual bending moment at joint r due to the unit load applied in the same sense and location as the desired displacement. Since at the onset of collapse, the last plastic hinge associated with rotation ϑ is just forming, it may be set to zero to compute the desired displacement (Hayman, 1971; Horne, 1979). Considering the generalized distribution of bending moments of Fig. 2(b) and the distribution of the virtual moments of Fig. 2(d), the required displacement equation of the subject column at incipient collapse may be formulated as;

$$\Delta_i = \frac{1}{6E} \sum_{r=1}^{i} \frac{h_r}{J_r} [(M_{r-1,l}^P - 2M_{r,u}^P) f_r \tag{5}$$

$$-(M_{r,u}^P - 2M_{r-1,l}^P) f_{r-1}] \frac{1}{U_r} + \left(\frac{mhLM_{s,u}^P}{6EI_s U_s} \right)$$

where $f_r = 0$ for $\overline{h}_i < h_r$ and $f_r = \overline{h}_i - h_r$ for $\overline{h}_i \geq h_r$. $U_r = 1 - P_i/P_{cr,i}$ and $U_s = 1 - P_s/P_{cr,s}$ are the moment magnifying or capacity reduction factors due to the P-delta phenomenon (Horne, 1965). $P_{cr,i}$ is the critical load associated with the segment i of the same column. Eq. (5) is ideally suited for manual as well as spreadsheet computations and may be utilized to compute Δ_i for any combination of sequence of formation of plastic hinges and boundary support conditions, provided that the entire column remains kinematically stable until and up to the onset of plastic collapse. However, in using Eq. (5), the following boundary conditions should be observed;

For fixed base columns; $\lambda > 1$ and $I_{s=0} = \infty$, for pinned base supports; $M_{0,l}^P = 0$, $\infty > I_s > 0$, $I_0 = 0$ and $s \geq 1$ and

for GBS columns; $\lambda > 1$, $\infty > I_0 > 0$ and $s = 0$. Informative accounts of manual methods of displacement analysis at incipient collapse may be found in (Beedle, 1958) and (Hayman, 1961, 1971).

2.2.1. Example1. Verification test 1

Derive the tip displacement of an upright cantilever subjected to forces V and P at its free end. **Solution:** $i = r = 1$, $h_1 = h$, $J_1 = J$, $M_{1,u}^P = f_1 = 0$, $M_{0,l}^P = -Fh$, $f_0 = h$ and $U_1 = U = 1 - P/P_{cr,1}$, where $P_{cr,1} = 3EJ/h^2$. Eq. (5) gives, as expected; $\Delta = Fh^3/3EJU$.

3. Fixed Base Column Supports

The validity of Eq. (5) has been verified by long hand as well as computer analysis. Since the majority of multi-level moment frames subjected to lateral loading are designed with fixed column supports, attention is focused first on fixed boundary support condition and then on pinned and GBS columns. The proceeding presentation is greatly simplified, without loss of generality, by selecting mathematically expedient and traceable structural models and loading functions.

3.1. Lateral displacement of well proportioned moment frames with fixed base columns

Consider the lateral displacements of a well designed, $m \times n$, moment frame that satisfies both the minimum weight as well as efficient frame design conditions. For an efficient or well proportioned moment frame; $J_{i,0} = J_{i,n} = J_i$ and $J_{i,j} = 2J_i$ for all other j. Similarly; $N_{i,0}^P = N_{1,n}^P = N_i^P$, $F_{i,0} = F_{i,n} = \overline{F}_i/2n$, and, $N_{i,j}^P = 2N_i^P$ and $F_{i,j} = F_i/n$ for all other j. $I_{i,j} = I_i$ and $M_{i,j}^P = M_i^P$. The plastic demand-capacity ratios of a well proportioned interior column tree of a fixed base moment frame may be expressed (Grigorian, 2012a) as;

$$\overline{M}_1 = \left[\frac{V_1 h_1}{(\lambda_1 + 1)} + \frac{V_{i+1} h_{i+1}}{2} \right] \text{ for } i = 1 \text{ and} \tag{6}$$

$$\overline{M}_i = \frac{1}{2} [V_i h_i + V_{i+1} h_{i+1}] \text{ for all other } i$$

It may be noted that $V_i h_i = 4n M_i^P$ is the plastic racking moment of the imaginary subframe of level $m \geq i > 1$. Substituting for $M_{0,l}^P = \lambda_1 M_1^P$, $M_{1,u}^P = M_1^P$ and $M_{r,u}^P = M_{r-1,l}^P = M_r^P$ for $r > 1$ in Eq. (5), it gives;

$$\Delta_i = \frac{M_1^P h_1 [(2\lambda_1 - 1) f_0 + (\lambda - 2) f_1]}{6EJ_1 U_1} \tag{7}$$

$$+ \frac{1}{6E} \sum_{r=2}^{i} \frac{M_r^P h_r}{J_r U_r} (f_{r-1} - f_r)$$

Note that because of implementation of the strong column-weak beam condition, λ_i, which is the inherent column over strength factor for the upper level columns, ($i > 1$) does not appear in Eqs. (5) and (7). In fixed base moment frames, the drift angle due to racking effects is generally a maximum near the base and a minimum near the top (Taranath, 1998; Grigorian, 2012c). This implies that it would be prudent to prevent the possibility of premature formation of plastic hinges at the feet of the base level columns. It has been shown (Grigorian, 2012e) that in order to prevent early formation of plastic hinges at column feet, the over strength factor λ_1 for base level columns, should be selected in accordance with the rule of minimum relative stiffness;

$$\lambda_1 \geq \frac{3\rho_1+1}{3\rho_1} \tag{8}$$

where, $\rho_1 = I_1h_1/LJ_1$. Similar sets of closed form solutions describing the response of efficient moment frames with pinned and GBS boundary conditions are presented under separate headings below.

3.1.1. Example 2. Verification test 2

Verify the accuracy of the maximum plastic displacements of the 3 story fixed base moment frame with column forces and properties summarized in Table1 below. Let $U_i = 1$, $F = 133.446$ kN, $h = 3.048$ *m*, $J = 2348.02 \times 10^6$ mm^4 and $E = 200{,}000$ MPa. The computations of the basic quantities required to complete the solution are presented in Appendix1. The corresponding plastic moment distribution is shown in Fig. 3(f).

Solution: Next, substituting from Table 1 into Eq. (7), it gives;

$$\Delta_3 = \left[\frac{Fh\times1.5h[(2\times2-1)(7h/2)+(2-2)2h]}{1} + \right.$$

$$\left.\frac{5Fh\times h(2h-h)}{6\times(10/27)} + \frac{Fh\times h(h-0)}{2\times(2/9)}\right]\frac{1}{6EJ} = \frac{81Fh^3}{24EJ} = 27.165 \text{ mm},$$

$$\Delta_2 = \left[\frac{Fh\times1.5h[(2\times2-1)(5h/2)+(2-2)2h]}{1} + \right.$$

$$\left.\frac{5Fh\times h(h-0)}{6\times(10/27)}\right]\frac{1}{6EJ} = \frac{54Fh^3}{24EJ} = 18.110 \text{ mm},$$

$$\Delta_1 = \left[\frac{Fh\times1.5h[(2\times2-1)(3h/2)+(2-2)10]}{1}\right]\frac{1}{6EJ} =$$

$$\frac{27Fh^3}{24EJ} = 9.055 \text{ mm}.$$

The results of verification test 2 are in complete agreement with computer generated data reported in the last two columns of Table 1.

3.2. Inter-story drift control for fixed base column supports

Inter-story DS is generally caused by the differences in the relative stifnesses of the members of vertically adjacent sub-frames. The effects of DS on the lateral displacements of moment tree at incipient collapse may best be studied by formulating the response of a regular frame with $h_i = h$ and $U_i = U$ under a triangular distribution of lateral forces defined by the force function $F_i = F \times i/m$. For equal heights $(f_{r-1} - f_r) = h_r$, $f_0 = mh$ and $f_1 = (m-1)h$. Eq. (7) yields;

$$\Delta_i = \frac{M_1^P h^2[(2\lambda_1-1)m+(\lambda_1-2)(m-1)]}{6EJ_1U_1} + \frac{h^2}{6EU}\sum_{r=2}^{i}\frac{M_r^P}{J_r} \tag{9}$$

The bending moment distribution of the subject moment tree under triangular distribution of lateral forces can be expressed (Grigorian, 2013) as;

$$M_{1,u}^P = M_1^P = \frac{(m+1)Fh_1}{2(\lambda_1+1)} \quad \text{for } m = 1 \tag{10}$$

and $M_{0,l}^P = M_0^P = \lambda M_1^P$ for $m = 0$

$$M_{i,l}^P = M_{i+1,u}^P = \left[\frac{m(m+1)-i(i-1)}{4m}\right]Fh \tag{11}$$

for $m \geq i > 1$

An effective way to reduce inter-story DS is to select the properties of the elements of the structure in accordance with member specific rules of proportionality. This may be achieved by imposing uniform demand-capacity ratios for members of similar groups such as beams and columns of the same sub system. E.g., the variation of the moments of inertia I_i of the beams and J_i of the columns may be related to their bending moments through Eq. (2) above. This causes the members of the frame to behave as part of a structure of uniform response (Grigorian, 2013). However, if the demand-capacity ratios of the columns are the same, then the summation term of Eq. (9) may be simplified as;

$$\sum_{r=2}^{i}\frac{M_r^P}{J_r} = \sum_{r=2}^{i}\frac{M_m^P}{J_m} = (i-1)\frac{M_m^P}{J_m} = \frac{Fh(i-1)}{2J_m} \tag{12}$$

Eq. (9) may now be expressed in its simplest form as;

$$\Delta_i = \frac{Fh^3(m+1)[(2\lambda_1-1)m+(\lambda_1-2)(m-1)]}{12(\lambda+1)EJ_1U_1} + \frac{Fh^3(i-1)}{12EU_mJ_m} \tag{13}$$

3.3. Drift control and DS due to fixed boundary support conditions

Eq. (13) suggests that while the drift function $\phi_i = \phi_2$

Table 1. Loading and properties, fixed base frame

i	F_i	V_i	h_i	M_i	$\overline{M}_i^P$	J_i	λ_i	$f_{r,i=3}$	$f_{r,i=2}$	$f_{r,i=1}$	Δ_i	δ_i
3	F	F	h	$Fh/2$	$Fh/2$	$2J/9$	1.1	0	0	0	27.165	9.055
2	$2F/3$	$5F/3$	h	$5Fh/6$	$4Fh/3$	$10J/27$	1.1	h	0	0	18.110	9.055
1	$F/3$	$2F$	$3h/2$	Fh	$11Fh/6$	J	2.0	$2h$	h	0	9.055	9.055
0	0	-	-	λFh	-	-	-	$7h/2$	$5h/2$	$3h/2$	0	0

may be constant for $m \geq i \geq 1$, it could become discontinuous at $i = 1$, e.g. the blue and black curves, Fig. 1(c). The differential drift angle at $i = 1$ is caused by the differences in the rigidities of the two ends of the fixed based column. DS or the difference between the drift ratio and the top end rotation of the base level column can be compared by computing the drift angle ϕ_1 and the tip rotation θ_1 of the base level column, thus;

$$\phi_1 = (2\lambda_1 - 1)\frac{M_1^P h_1}{6EJ_1 U_1}, \tag{14}$$

$$\theta_1 = (\lambda_1 - 1)\frac{M_1^P h_1}{2EJ_1 U_1} \quad \text{and} \quad (\theta_1 + \psi_2) = \phi_2$$

where, ψ_2 is the local or inter story drift angle of the second level column (Grigorian, 2012e). The effects of λ_1 on θ_1 are depicted in Fig. 1(d). For the sake of clarity assume $U_i = 1$ throughout the forthcoming discussions. It may be instructive to note that, contrary to common belief, the over strength factor λ not only effects the sequence of formation of the plastic hinges, but also the magnitude of the plastic displacements. It may be seen from the group of Eqs. (14) that the maximum column rotation θ_1 at the onset of failure is comparable in magnitude with the corresponding drift angle ϕ_1, and as illustrated in Fig. 1(d). This causes the drift angle of the upper levels to increase from ψ_2 to $\psi_2 + \theta_1$, and implies that the relative rigidities of the support level fixed base column control the drift angles of the entire structure during elastic as well as plastic phases of bending. In other words, the stiffer the base level column, the more flexible and probably the more economical the upper level columns. Note that θ_1 is always larger than zero for all values of $\lambda > 1$ and $J_1 \neq \infty$. Zero shifting, which is an ideal condition occurs at $\lambda_1 = \lambda_{ideal} = 2$, i.e., when $\theta_1 = \phi_1$, otherwise, θ_1 is always larger than ϕ_1 for all values of $\lambda_1 > 2$. The purpose of this section is to generate enough data in order to verify the validity and the accuracy of the proposed formulations. The best way to achieve this is to compare the results of the Eq. (5) and/or (13) against that of the ideal drift profile. The most ideal drift profile is a straight line or combinations of straight lines with $\lambda_1 < 2$, that satisfy the target displacement limits at specific load levels, i.e.,

$$\Delta_i = \phi_1 h_1 + (\psi_2 + \theta_1)(\bar{h}_i - h_1) = \Delta_1 + (\bar{h}_i - h_1)\bar{\phi} \tag{15}$$

The case of an ideal drift profile is further examined in the following example.

3.3.1. Example 3. Drift control for a fixed base frame

Determine the column moments of inertia J_i of an $m = 10$ story, regular, efficient moment frame with $F = 133.446$ kN, $\phi_1 = 0.0025 < 0.0033$ rad, $h_i = 3.048$ *m.*, $U_i = 1$ and $\lambda_1 = 2.5$, provided that the maximum inter-story drift ratio ϕ_i does not exceed $\bar{\phi} = 0.0033$ rad. **Solution:** Anticipating higher drift ratios for the upper levels, since $\lambda_1 = 2.5 > \lambda_{ideal}$, the required drift ratio for the base level column may now be selected as say $\phi_1 = 0.0025 < 0.0033$ radians. Using Eqs. (10) and (14) to estimate an initial value for the first level column moment of inertia J_1, it gives; $M_{10}^P = (10+1) \times 133.446 \times 3.048/2(2.5+1) = 639.168$ kN-m, $J_1 = (2 \times 2.5-1) \times 639.168 \times 3.048 \times 10^9/6 \times 200000 \times 0.0025 = 2597.5788 \times 10^6$ mm^4 and $\theta_1 = (2.5-1) \times 639.168 \times 3.048 \times 10^3/2 \times 200000 \times 2597.5788 \times 10^6 = 0.002813$ radians. Next, if the drift ratios of the upper stories are to be maximized such that $\phi_i = \phi_2 = \bar{\phi} = 0.0033$ rad. then the local drift angles of the upper levels would be given by; $\psi_i = \psi_2 = \phi - \theta_1 = 0.0033-0.002813 = 0.000487$ rad.

Since, $J_{10} = M_{10}^P h_{10}/2E\psi_{10} = 203.372 \times 3.048 \times 10^9/6 \times 200000 \times 0.000487 = 1060.7082 \times 10^6$ mm^4, then Eqs. (2) and (11) may be combined to give;

$$J_i = \left[\frac{10(10+1) - i(i-1)}{4 \times 10}\right]\left(\frac{133.446 \times 3.048}{203.372}\right)J_{10} \tag{16}$$

$$J_{10} = \left[\frac{110 - i(i-1)}{20}\right] \times 1060.7082 \times 10^6\ mm^4$$

The drift line Eq. (15) gives; $\Delta_{10} = [\phi_1 + (m-1)\phi_1]h = [0.0025+(10-1) \times 0.0033] \times 3.048 \times 10^3 = 98.146$ mm. The proposed drift Eq. (13) based on material properties of the moment tree yields;

$$\Delta_{10} =$$

$$\frac{133.446 \times 3.048^3(10+1)[(2\times2.5-1)\times10+(2.5-2)(10-1)]10^{12}}{12(2.5+1)\times200000\times2597.5788\times10^6}$$

$$+\ \frac{133.446\times3.048^3(10-1)\times10^{12}}{12\times200000\times1060.7082\times10^6} = 98.143\ \text{mm}$$

This implies that Eq. (5) and its special case, Eq. (13), are exact within the bounds of the theoretical assumptions.

4. Pinned Base Column Supports

Consider the lateral displacements of a properly designed, efficient, pinned base moment frame such as that shown in Fig. 3(b), with the last set of plastic hinges forming at the ends of the s^{th} level beam. The object of this exercise is to study the effects of pinned column supports on the roof level displacements at incipient collapse. The corresponding plastic bending moment and the unit load moment distributions are depicted in Figs. 3(d) and 3(e), respectively. The generalized displacement Eq. (5) may now be customized for pin supported columns as;

$$\Delta_i = \frac{1}{6E}\sum_{r=1}^{s}\frac{h_r M_r^P}{J_r U_r}(f_r - f_{r-1}) + \tag{17}$$

$$\frac{1}{6E}\sum_{r=s+1}^{i}\frac{h_r M_r^P}{J_r U_r}(f_{r-1} - f_r) + \frac{f_{s,u} L M_{s,u}^P}{6EUI_{s,u}}$$

Furthermore, if the distribution of the lateral forces is defined by $F_i = F \times i/m$, then for the unit virtual load

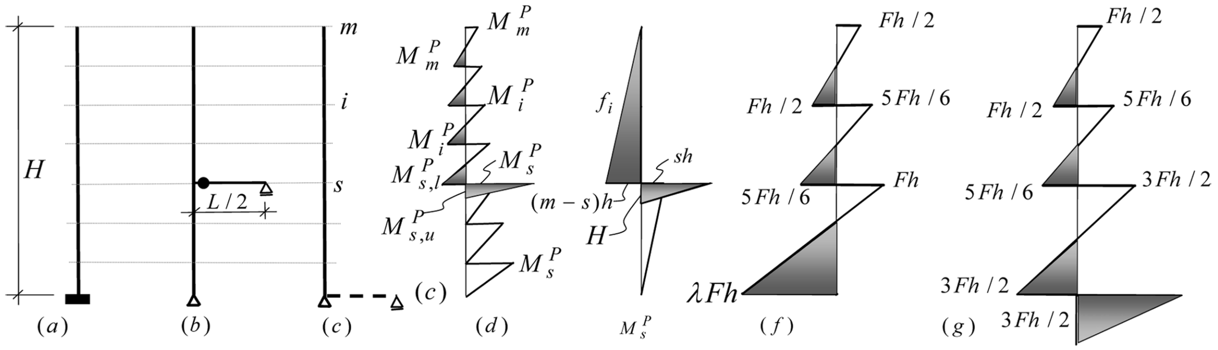

Figure 3. (a) Fixed base column, (b) Pinned base column and last stable beam, (c) GBS column, (d) Moment tree for case b, (e) Unit load moments for case b, (f) Moment tree of example 2, (g) Moment tree of example 7.

applied at the roof level; $f_{s,u} = H$, $(f_{r-1}-f_r) = h_r$ for $r > s$, $(2f_r + f_{r-1})=(3r-1)h_r$ for $r \le s$ and Eq. (17) becomes;

$$\Delta_i=\frac{1}{6E}\sum_{r=1}^{s}\frac{(3r-1)h_r^2M_r^P}{J_rU_R}+\frac{1}{6E}\sum_{r=s+1}^{i}\frac{h_r^2M_r^P}{J_rU_r}+\frac{HLM_{s,u}^P}{6EUI_{s,u}} \quad (18)$$

In pin supported moment frames, the drift angle due to racking effects is also a maximum near the base and a minimum near the top. Therefore, it would be prudent to try to prevent the premature formation of plastic hinges at the ends of the lower level girders. Eq. (18) provides a simple means of observing the effects of formation of plastic hinges at the ends of the first level column beams just before the structure becomes a mechanism. For instance if $i = m$, $s = 1$ and $h_i = h$ for $i > 1$ and $U_i = U$ for all i, then Eq. (18) yields the corresponding roof level maximum displacement as;

$$\Delta_m=\frac{2h_1^2M_1^P}{6EJ_1U}+\frac{1}{6E}\sum_{r=2}^{m}\frac{h_r^2M_r^P}{J_rU_r}+\frac{[h_1-(m-1)h]LM_{1,u}^P}{6EUI_{1,u}} \quad (19)$$

4.1. Example4. Verification test 3

Derive the lateral displacement formula of a pin supported portal frame subjected to a roof level forces V and column axial loads P. **Solution:** Putting $m = s = r = i = 1$ in Eq. (19) and rearranging, it results in the well known formula (Klienlogel, 1952; Davison, 1993; Grigorian, 1993).

$$\delta_1=\frac{M_1^Ph_1^2}{6EJ_1U_1}\left[2+\frac{1}{\rho_1}\right] \text{ and } \theta_1=\frac{M_1^PL}{6EI_1U_1} \quad (20)$$

4.2. Inter-story drift control for pinned base column supports

The plastic bending moment distributions of the pinned base moment tree can be expressed as;

$$M_{s,u}^P=\left[\frac{m(m+1)-s(s-1)}{4m}\right]Fh \quad (21)$$

for $s \ge 1$

$$M_{i,l}^P=M_{i+1,u}^P=\left[\frac{m(m+1)-i(i-1)}{4m}\right]Fh \quad (22)$$

for $m \ge i \ge s$

$$M_{i,l}^P=M_{i+1,u}^P=\left[\frac{m(m+1)-i(i-1)}{2m}\right]Fh \quad (23)$$

for $s \ge i \ge 1$, $M1_{0,l}^P=M_0^P=0$

The variations of the beam and column moments of inertia I_i and J_i in the range $m \ge i \ge 1$ are the same as those given by Eq. (2). However for reduced DS due to pinned base boundaries I_1 and J_1 may best be selected as;

$$J_1=\frac{2M_1^Ph_1}{M_m^Ph_m}J_m\,,\ I_{1,u}=(M_1^P+M_2^P)\frac{I_m}{M_m^P} \quad (24)$$

and $\left[\frac{M_1^P+M_2^P}{I_{1,u}}\right]=\frac{M_1^P}{I_1}$

Obviously a similar set of similar equations can be derived for closed loop modules resting on top of a supporting fixed base portal frame.

4.3. Drift control and DS due to pinned support conditions

The DS phenomenon for pin based columns may best be studied by rearranging Eq. (14c) as;

$$\Delta_m=\frac{M_1^Ph_1^2}{6EJ_1U}\left[2+\frac{1}{\rho}\right]+\sum_{r=2}^{m}\frac{h^2M_r^P}{6EJ_rU}+\frac{(m-1)hLM_1^P}{6EUI_1} \quad (25)$$

$$= \delta_1+\sum_{r=2}^{m}\delta_r+(m-1)\theta_1h$$

It is evident that the magnitude of Δ_m is affected appreciably by the DS angle θ_1. DS is usually most pronounced at the top end of base level columns. DS for pin supported moment frames can also be studied by computing the drift angle ϕ_1 and the tip rotation θ_1 of the base level column of a single bay pinned base portal frame under a lateral force V and axial column loads P as expressed by

Table 2. Loading and properties, pinned base frame

i	F_i	V_i	h_i	M_i	$\overline{M}_i^P$	J_i	λ_i	$f_{r,i=3}$	$f_{r,i=2}$	$f_{r,i=1}$	Δ_i	δ_i
3	F	F	h	$Fh/2$	$Fh/2$	$J/18$	1.1	0	0	0	19.56	5.59
2	$2F/3$	$5F/3$	h	$5Fh/6$	$4Fh/3$	$5J/54$	1.1	h	0	0	13.97	5.59
1	$F/3$	$2F$	$3h/2$	$3Fh$	$23Fh/6$	J	1.1	$2h$	h	0	8.38	8.38
B	-	-	$3h/2$	$3Fh$	-	-	1.0	$7h/2$	$5h/2$	$3h/2$	-	-

Eq. (20). It may be observed that θ_1 is always smaller than ϕ_1 for pin supported columns and that the best case scenario corresponds to the unlikely condition $J_1 = \infty$. In conclusion, in order to reduce the effects of DS on the lateral displacements of the upper level columns I_1 should be selected so that $\phi_2 = (\theta_i + \psi_2)$ is a practical minimum.

4.3.1. Example5. Verification test 4

Verify the accuracy of the computer generated displacements of the 3 story moment tree of Fig. 3(b), with pinned supports, as detailed in Table 2. Assume that the last plastic hinge forms at the first level beam ends. Given; $J_1 = J = 8455.0588 \times 10^6$ mm^4, $I_3 = J/6$, $P = 0$ and $L = 2h = 6.096\ m$. **Solution:** The group of Eqs. (21) through (24), give; $J_3 = [(Fh/2)h / 2(3Fh)(3h/2)]J = J/18$, $J_2 = [(5Fh/6)h / 2(3Fh)(3h/2)]J = 5J/54$, $I_{1,u} = [3Fh+5Fh/6]I_3 / (Fh/2) = 23/3\ I_3$, and $I_1 = 3Fh\times(23I_3/3) / (3Fh+5Fh/6) = 6I_3 = 8455.1587 \times 10^6$ mm^4. Substituting from Table 2 into Eq. (25), it gives;

$$\Delta_3 = \left[\frac{27Fh^3}{2J} + \frac{9Fh^3}{J} + \frac{9Fh^3}{J} + \frac{7Fh^3}{2I_3}\right]\frac{1}{6E}$$

$$= \left[\frac{31.5}{J} + \frac{3.5\times6}{J}\right]\frac{Fh^3}{6E} = \frac{105Fh^3}{12EJ} = 19.558 \text{ mm}$$

4.3.2. Example6. DS in a pin supported portal frame

Study the plastic DS phenomenon associated with the pinned base moment frame of example 5 above. **Solution:** Observing that $I_1 = J$ and substituting from Table 2 into Eqs. (25), it gives;

$$\delta_1 = \frac{3Fh\times(3h/2)}{6E}\left[\frac{2(3h/2)}{J} + \frac{2h}{I_1}\right] = \frac{45Fh^3}{12EJ} = 8.38 \text{ mm},$$

$\theta_1 = 3Fh\times2h / 6EJ = 0.000733$ radians and $\phi_1 = \delta_1/h_1 = 8.38/4572 = 0.001833$ rad. This gives the relevant drift shift as $\phi_1 - \theta_1 = 0.0011$ rad. However, since I_i and J_i were selected in such a way as to enforce a uniform drift defined by $\phi_1 = \phi_1 = \phi_3$ and $\theta_1 = \theta_1 = \theta_3$, then the effects of the first level DS were automatically normalized by enforcing $\phi_i - \theta_i =$ constant along the height of the frame.

5. Grade Beam Supported Columns

While plastic DS can not be avoided for pinned and fixed boundary conditions, GBS systems offer practical means of reducing, even eliminating differential changes between the drift ratios of first and second as well as other story level junctions. The governing roof level displacement equation for the particular case under study may be formulated by substituting $s = 0$ in Eqs. (17) or (18), thus;

$$\Delta_m = \sum_{r=1}^{m}\frac{h_r^2 M_r^P}{6EJ_r U_r} + \frac{LM_1^P}{6EUI_1}\sum_{r=1}^{m} h_r = \sum_{r=1}^{1}(\delta_r + \theta_1\times h_r) \tag{26}$$

$\theta_i = \theta_1$ is a particular property of well proportioned/efficient GBS frames for all column segments, i.e., $\theta_{i+1} - \theta_i = 0$ for all i. Therefore, DS is zero along the height of the structure. The variations of the beam and column moments of inertia I_i and J_i in the range $m \geq i > 1$ are the same as those given by Eq. (2). However, in order to achieve zero DS, the boundary elements I_1 and J_1 should be selected in accordance with the following rules of proportionality;

$$J_1 = \frac{M_1^P h_1}{M_m^P h_m} J_m \text{ and } I_0 = \frac{M_1^P h_1}{M_m^P h_m} I_m \tag{27}$$

5.1. Example7. Drift control for a GBS moment frame

Compute the section properties of the 3 story GBS moment tree of Table 3 in such a way that the entire structure rotates through a constant drift angle; $\phi \leq 0.0022$ rad. at incipient collapse. The corresponding plastic moment distribution is shown in Fig. 3(g). Eq. (26) indicates that the solution to an imaginary single story single bay GBS moment frame may be expressed as;

Table 3. Loading and properties, grade beam supported frame

i	F_i	V_i	h_i	M_i	$\overline{M}_i$	J_i	I_i	$f_{r,i=3}$	$f_{r,i=2}$	$f_{r,i=1}$	Δ_i	δ_i	ϕ_i
3	F	F	h	$Fh/2$	$Fh/2$	$2J/9$	$2I/9$	0	0	0	24.659	7.043	ϕ
2	$2F/3$	$5F/3$	h	$5Fh/6$	$4Fh/3$	$10J/27$	$16I/27$	h	0	0	17.616	7.043	ϕ
1	$F/3$	$2F$	$3h/2$	$3Fh/2$	$7Fh/3$	J	$37I/27$	$2h$	h	0	10.564	10.564	ϕ
B	-	-	0	$3Fh/2$	$3Fh/2$	-	I	$7h/2$	$5h/2$	$3h/2$	-	-	

$$J_1 = \frac{M_1^P h_1}{6E\phi_1 U_1}\left[1+\frac{1}{\rho_1}\right] \text{ and } \theta_1 = \frac{M_1^P L}{6EI_1 U_1} \tag{28}$$

It follows therefore that $J_1 = 2(3Fh/2)(3h/2)/6E \times 0.9 \times 0.0022 = 2348.02 \times 10^6$ mm^4 and that $I_3 = J_3 = J_1/4.5$ and $I = J = J/4.5$. Since the plastic moments of resistance and moments of inertias of the members of the frame were selected in accordance with the same demand-response ratio then the structure would collapse through simultaneous formation of plastic hinges at all beam ends and would deform in accordance with a linearly varying drift profile defined by;

$$\Delta_i = \phi_1 \times \bar{h}_i \tag{29}$$

The numerical results of Eq. (29), as verified by computer analysis have been summarized in Table 3.

6. Comparison of Boundary Effects between Fixed, Pinned and Grade Supported Frames

The relative merits and characteristics of fixed, pinned and grade beam supported moment frames e briefly discussed in the following two sections

6.1. Comparison of boundary effects between fixed and grade supported frames

A comparison of the lateral displacements of examples 2 and 7 reveals that the lateral displacements of well proportioned GBS moment frames, $\Delta_{3,grade} = 24.659$ mm, could be smaller than those of identical frames with fixed boundary supports, $\Delta_{3,fix} = 27.165$ mm. In order to formally compare the effects of the three sets of boundary support conditions on the same structure, consider the performance of a single bay, m story efficient moment frame with uniform drift $\phi_i = \phi$ above level one, under any distribution of monotonically increasing lateral forces. Let for simplicity sake $h_i = h$, $h_1 = h_1$, $V_1 = V$ and $U_i = U$ for both sets of support conditions. First, consider the case of a GBS frame with $\rho_1 = 1$ and constant drift $\phi_i = \phi_1 = \phi$ along the height of the entire frame. Using Eq. (28) the magnitude of ϕ_1 and the roof level displacement, in terms of $J_{1,grade}$ and V may be expressed as;

$$\phi = \frac{Vh_1^2}{24EJ_{1,grade}U_1}\left[1+\frac{1}{\rho_1}\right] \tag{30}$$
$$\text{and } \Delta_m = \phi[h_1+(m-1)h]$$

Next, if the same frame is to be supported on fixed column supports, then the corresponding roof level displacement may be computed by substituting $\psi = \phi$ in Eq. (15), i.e.;

$$\Delta_m = \phi_1 h_1 + (\phi+\theta_1)(m-1)h \tag{31}$$

Eliminating Δ_m from Eqs. (30) and (31), and substituting for ϕ_1 and θ_1 from Eq. (14), it gives;

$$J_{1,fix} = \left[(2\lambda_1-1)+3(\lambda_1-1)(m-1)\frac{h}{h_1}\right]\frac{2J_{1,grade}}{(\lambda_1+1)(1+1/\rho_1)} \tag{32}$$

as the equivalent moment of inertia of the first floor columns for reduced DS. Naturally, as the number of stories increase, so do the magnitudes of the base level column section inertias to counteract the effects of the DS. In order to further study the relationship between $J_{1,fix}$ and $J_{1,grade}$, consider the applications of Eq. (32) to a single bay portal frame. For $m = 1$, Eq. (32) reduces to;

$$\frac{J_{1,fix}}{J_{1,grade}} = \frac{2(2\lambda_1-1)}{(\lambda_1+1)(1+1/\rho_{1,grade})} \tag{33}$$

A graphic interpretation of Eq. (33) in terms of the main variables λ and ρ, and the limiting vales of λ_{min} is presented in Fig. 4. The solid lines to the right and above the

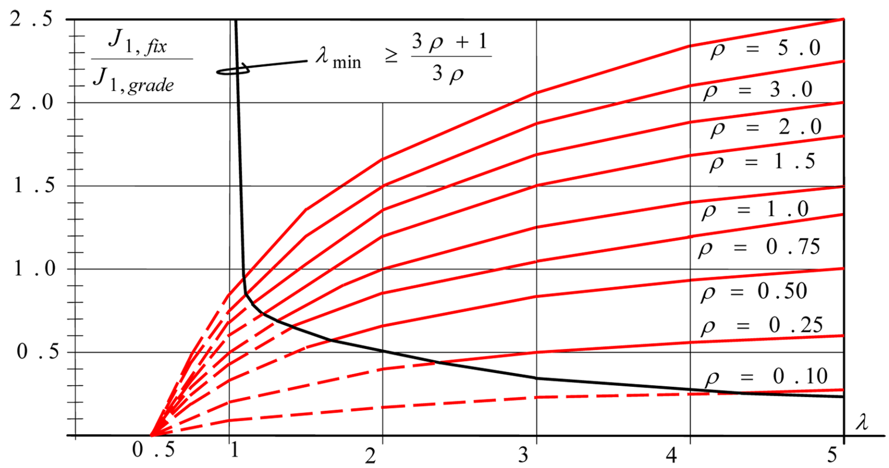

Figure 4. Variations of ($J_{1,fix}/J_{1,grade}$) in terms of ρ and λ.

λ_{min} curve represent the permissible combinations of λ and ρ that can be used to determine $J_{1,fix}$ in terms of $J_{1,grade}$. It may be observed that for $\rho_1 > 0.5$, $J_{1,fix}$ is consistently larger than $J_{1,grade}$. This implies that as far as DS is concerned, the GBS system may actually result in more desirable, even more economical structure than fixed bas supported frames.

6.2. Comparison of boundary effects between pinned and grade supported frames

Next, compare the response of a pin supported frame with constant drift $\phi_i = \phi$ with that of an equivalent grade beam supported system. Eq. (31) may be used in conjunction with Eqs. (20) to determine the required magnitude of $J_{1,pin}$ in terms of $J_{1,grade}$ as

$$J_{1,pin} = \left[2 + \frac{1}{\rho_1} + \frac{(m-1)}{\rho_{1,pin}}\frac{h}{h_1}\right]\frac{2J_{1,grade}}{(1+1/\rho_{1,grade})} \tag{34}$$

The relationship between $J_{1,pin}$ and $J_{1,grade}$ may be further studied by putting $m = 1$ in Eq. (34), thus;

$$J_{1,pin} = \frac{2(2+1/\rho_{1,pin})}{(1+1/\rho_{1,grade})}J_{1,grade} \tag{35}$$

Note that because of doubly symmetry both the grade beam and the pin supported portal frames may be treated as statically determinate structures and that their displacements at incipient collapse are independent of the corresponding column over strength factors.

7. Conclusions

There is still scant information on simple methods to study the lateral displacements of sway frames at incipient collapse. The studies carried out in this research program provided some useful results for the preliminary design of moment frames. Such studies may also be used for higher educational purposes. It was demonstrated that otherwise complicated task of computing maximum lateral displacements for well designed moment frames at incipient collapse can be reduced to the use of simple formulae for pinned, fixed and GBS boundary conditions. It was shown that pinned and fixed boundary support conditions are more likely to cause kinks or DS at the first level junctions of multilevel frames than similar GBS structures, and that drift shifting tends to increase the lateral displacements of the upper levels of the frame by the same order of magnitude as the racking drift ratios of the upper level sub frames. It was also demonstrated that GBS boundary conditions offer practical means of elimination and/or reduction of DS along the height of the frame as well as preventing the premature formation of plastic hinges at column supports. The proposed solutions satisfy the prescribed yield criteria, equilibrium and boundary support conditions and as such, they can not be far from minimum weight design.

In the interim, several parametric as well as numerical examples were presented to demonstrate the applications of the proposed formulae. It was suggested that the findings of the present article are particularly applicable to performance based plastic design of wind and earthquake resistant moment frames. It was also shown that the over strength factors not only affect the sequence of formation of plastic hinges, but can also influence the magnitude of the plastic displacements. The proposed formulations are exact within the bounds of the theoretical assumptions and accurately predict the results of long hand and computer generated solutions. The proposed methodology lends itself well to manual as well as spreadsheet computations. A similar approach can be employed to study DS in braced frameworks and similar structures.

Appendix 1, Basic computations for example 2

Acting moments $M_i = V_i h_i / 2$ for $i > 1$ and $N_1^P = V_1 \times h_1 / 2(1+\lambda) = (2F) \times 3h / 2(2+1) = Fh$ were computed using Eq. (1). Column sectional inertias; $J_2 = (M_2^P h_2^2 / M_1^P h_1^2)$, $J_1 = (5Fh/6) \times h^2 J / (Fh) \times (1.5h)^2 = 10J/27$ and $J_3 = (Fh/2) \times h^2 J / (Fh) \times (1.5h)^2 = 2J/9$ were computed using Eq. (2).

References

ACI 318-05. (2005). Building code requirements and commentary, Farmington Hills, Michigan: US.

AISC. (2005). Seismic provisions for structural steel buildings, Chicago, Illinois: US.

Baker, J. F., Horne, M. R. and Hayman, J. (1964). The steel skeleton. Vols. 1 and 2, Cambridge University Press, UK.

Beedle, L. S. (1958). Plastic design of steel frames. John Wiley & Sons, US.

Davison, B. and Owens, G. (1993). The Steel Designers Manual, 7th Edn, Wiley-Blackwell, UK.

Foulkes, J. (1953). “Minimum weight design and the theory of plastic collapse.” Q. Appl. Math., 10, pp. 347~358. 13, UK.

Foulkes, J. (1954). “The minimum-weight design of structural frames.” *Proc. R. Soc. London Ser. A*, 223, pp. 482~494, UK.

Goel, S. C. and Chao, S. H. (2008). Performance-based plastic design, ICC, US.

Goel, S. C., Liao, W. C., Bayat, M. R. and Chao, S. H. (2010). “Performance-based plastic design (PBPD) method for earthquake resistant structures: An overview.” *Struct. Des. Tall Spec. Build.*, 19(1-2), pp. 115~137.

Grigorian, M. (1993). “On the lateral response of regular high-rise frames.” *Struct Des. Tall Build.* 2(3), pp. 233~252.

Gardner, L. and Nethercot, D. (2011). Designers guide to Eurocode 3. Design of Steel Structures, 2nd edition. ICE, UK.

Grigorian, M. and Grigorian, C. (2011). “Performance control for seismic design of moment frames.” *J. Constr. Steel Res.*, 67(7), pp. 1106~1114.

Grigorian, M. and Grigorian, C. (2012a). “Performance control: A new elastic-plastic design procedure for earthquake

resisting moment frames." *J. Struct. Eng. ASCE*, 138(6).
Grigorian, M. and Grigorian, C. (2012b). "A new performance based design approach for moment resisting frames." *Can J. Civil Eng.* 39, pp. 1~11.
Grigorian, M. and Grigorian, C. (2012c). "Recent developments in plastic design analysis of steel moment frames." *J. Construct. Steel Res.*, 2012, 03:011.
Grigorian, M. and Grigorian, C. (2012d). "An Introduction to the Methodology of Earthquake Resistant Structures of Uniform Response." *Buildings*, 2, pp. 107~125.
Grigorian, M. and Grigorian, C. (2012e). "Lateral displacements of moment frames at incipient collapse." *Eng. Struct.*, pp. 174~185.
Grigorian, M. (2013a). "An introduction to performance control for moment frames of uniform response under lateral loading." *Asian J. Civ. Eng.*, 14(1), pp. 123~143.
Grigorian, M. and Grigorian, C. (2013b). "An overview on performance control and efficient design of lateral resisting moment frames." *Int. J. High-rise Buildings*, 2(2), pp. 141~152.
Grigorian, M. and Kaveh, A. (2013c). "A Practical weight optimization for moment frames under combined loading," *Int. J. Optim. Civil Eng.*, 3(2), pp. 289~312.
Hayman, J. (1961). "On the estimation of deflections in elastic plastic frames", *Proc. Inst. Civ. Eng.*, 19, 89.
Hayman, J. (1971). Plastic design of fames, Cambridge University Press, UK.
Hamburger, R. O., Krawinkler, H., Malley, J. O. and Adan, S. M. (2009). "Seismic design of steel special moment frames: A guide for practicing engineers." NEHRP Seismic Design Technical Brief No. 2. US.
Horne, M. R. (1979). Plastic theory of structures, Pergamon Press, UK.
Horne, M. R. and Merchant, W. (1965). The stability of frames. Pergamon Press.
Klienlogel, A. (1952). Rigid frame formulas. Fredrick Ungar Publishing. New York.
Mazzolani, M. (1997). "Plastic design of seismic resistant steel frames." *Earthquake Eng. Struct. Dynam*, 26(2), pp. 167~169.
Neal, B. G. (1963). The plastic methods of structural analysis. Chapman and Hall, London.
Nethercot, D. A. (2001). Limit state design of structural steelwork, Spon press. UK.
Priestly, M. J. N., Calvi, G. M. and Kowalsky, M. J. (2007). Displacement based seismic design of structures, IUSS, Italy.
Surovek, A. E., Camotim, D. A., Hajjar, J., Teh, L, White, D. W. and Ziemian, R. D. (2006). "Direct second-order analysis for the design of steel structures." Proc. American Society of Civil Engineers Structures Congress, St. Louis, MO, US.
Schafer, D., Eichler, B. and Amlung, L. (2010). Modern plastic design for steel structures, EU Publications, LUX.
Taranath, B. S. (1998). Steel, concrete and composite design of tall buildings, 2nd ed. Mc Graw Hill, US.
Wong, M. B. (2009). Plastic Analysis and Design of Steel structures, Elsevier, UK.
Ziemian, R. D., Seo, D. W. and McGuire, W. (2008). "On the inelastic strength of beam-columns under biaxial bending." Proc. American Institute of Steel Construction National Steel Construction Conference and Structural Stability Research Council Technical Session, Nashville, TN, US.