PERFORMANCE ANALYSIS OF COGNITIVE NETWORK WITH PRIMARY AND SECONDARY CHANNELS

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ABSTRACT. We consider a cognitive network with a primary and a secondary channel. Primary users have higher priority on the usage of the primary channel, and secondary users are allowed to opportunistically access the primary channel at times when the channel is not occupied by primary users. The secondary channel is dedicated only to secondary users. An analytical model is presented to obtain the performance of an opportunistic spectrum access using both the primary and secondary channels, and is validated by simulations.

1. Introduction

Reports of spectrum efficiency reveal that a considerable region of the spectrum remains unused. As a solution for such inefficient spectrum usage, opportunistic spectrum access (OSA) has been intensively studied. Under this system, a secondary user that does not have a license to use the spectrum is allowed to opportunistically occupy an idle spectrum band owned by a licensee that is termed by the primary user [1, 2]. Much attention has been paid to the quality-of-service of secondary users. Wang et al. [3] proposed two cognitive MAC schemes to support voice services in the presence of primary users. Lee [4] developed a simple approximate model for unslotted OSA networks under non-saturation conditions. Considering prioritization among secondary users, Lee [5] proposed an OSA scheme with channel reservation for high priority secondary users and buffering for low priority secondary users.

This paper considers a cognitive network with a primary and a secondary channel. Primary users have higher priority on the usage of the primary channel, and secondary users are allowed to opportunistically access the primary channel at times when the channel is not occupied by primary users. The secondary channel is dedicated only to secondary user. The main objective of
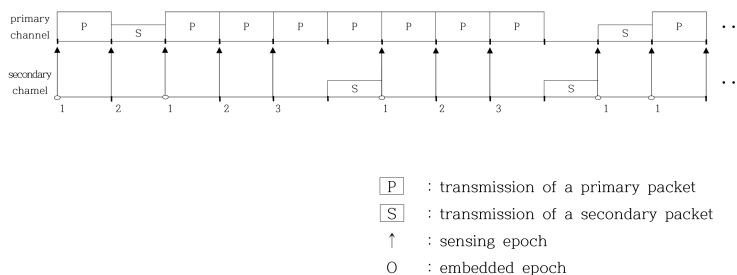
FIGURE 1. Network model with $D = 3$

this paper is to construct a mathematical model for a cognitive network with a single primary channel and a single secondary channel. By using a Markov renewal process, we analyze the mathematical model and obtain the performance of the cognitive network.

The outline of this paper is as follows: in Section 2, we describe the network model considered in this paper. Section 3 presents a mathematical model and analyzes the performance of a cognitive network using a Markov renewal process. Section 4 discusses some numerical results, and validates the results by simulations.

2. Network model

We consider a cognitive network with a single primary channel and a single secondary channel. The primary channel is shared by both primary users and secondary users, and the secondary channel is dedicated only to secondary users. Primary users have higher priority on the usage of the primary channel, and secondary users can opportunistically access the primary channel at times when the channel is not occupied by primary users. In this paper, we consider a single primary channel and a single secondary channel. The time is partitioned into slots, and in each time slot the primary channel is either occupied by primary users or not. The primary channel is at on-state if it is used by primary users, and at off-state otherwise. The occupancy of the primary channel is modeled as a discrete-time two-state Markov chain. The channel state takes the value 0 or 1 depending on whether the channel is at on-state or at off-state, respectively. It transits from on-state to off-state with probability α and stays in off-state with probability β.

We consider a secondary user seeking spectrum opportunities in the primary channel. Before sensing and accessing the primary channel, the secondary user first synchronizes with the slot structure of the primary channel. The OSA scheme considered in this paper is described as follows (see Figure 1): When a new secondary packet moves to the head-of-line position of the secondary user's buffer, the secondary user senses the occupancy of the primary channel. If the

channel is sensed to be idle, the secondary user can transmit the packet. Otherwise, the secondary user keeps the packet at the head-of-line position of the buffer, and senses the primary channel again at the beginning of the next slot. The secondary user can sense the primary channel for at most D consecutive time slots for a packet at the head-of-line position of the buffer, where $D \geq 1$. If a secondary packet at the head-of-line position has not been transmitted for D consecutive time slots, the secondary packet will be transmitted on the secondary channel in the next slot.

3. Mathematical analysis

We assume perfect spectrum sensing. We also assume that secondary user operates under saturation conditions, where it always has at least one packet awaiting transmission. Since the saturation condition means that the system has reached its maximum traffic handling capacity, the saturation performance is a fundamental performance defined as the limit reached by the system performance as the offered load increases.

To analyze the performance of our network system, we employ the method of embedded Markov chain. We first consider a set of time points embedded at the beginning of the first slot after a head-of-line packet leaves the buffer. Let $\{t_n, n = 1, 2, \cdots\}$ be a sequence of embedded points and $X(t_n)$ be the state of the primary channel at the nth embedded point t_n. We define

$$\tau_n \equiv t_{n+1} - t_n. \tag{1}$$

Then, the process $\{(\tau_n, X(t_n)), n = 1, 2, \cdots\}$ is a Markov renewal process whose behavior is governed by the semi-Markov kernel

$$q_{ij}(k) \equiv P\{X(t_{n+1}) = j, \tau_n = k | X(t_n) = i\} \tag{2}$$

for $i = 0, 1$, $j = 0, 1$, and $k = 1, 2, \cdots, D + 1$. The kernel is given by

$$q_{00}(k) = \begin{cases} 0, & k = 1, \\ (1-\alpha)^{k-2}\alpha(1-\beta), & k = 2, 3, \cdots, D, \\ (1-\alpha)^{D-1}\left[(1-\alpha)^2 + \alpha(1-\beta)\right], & k = D+1, \end{cases}$$

$$q_{01}(k) = \begin{cases} 0, & k = 1, \\ (1-\alpha)^{k-2}\alpha\beta, & k = 2, 3, \cdots, D, \\ (1-\alpha)^{D-1}\left[(1-\alpha)\alpha + \alpha\beta\right], & k = D+1, \end{cases}$$

$$q_{10}(k) = \begin{cases} 1-\beta, & k = 1, \\ 0, & k = 2, 3, \cdots, D+1, \end{cases}$$

$$q_{11}(k) = \begin{cases} \beta, & k = 1, \\ 0, & k = 2, 3, \cdots, D+1. \end{cases}$$

Thus,

$$p_{ij} \equiv \sum_{k=1}^{D+1} q_{ij}(k), \quad i = 0, 1, \quad j = 0, 1, \tag{3}$$

is the transition probability from state i to state j for the embedded Markov chain $\{X(t_n), n = 1, 2, \cdots\}$, given by

$$p_{00} = (1 - \beta) - (\alpha - \beta)(1 - \alpha)^D, \tag{4}$$
$$p_{01} = \beta + (\alpha - \beta)(1 - \alpha)^D, \tag{5}$$
$$p_{10} = 1 - \beta, \tag{6}$$
$$p_{11} = \beta. \tag{7}$$

Note that the conditional probability mass function

$$h_i(k) \equiv P\{\tau_n = k | X(t_n) = i\}, \quad i = 0, 1, \quad k = 1, 2, \cdots, D + 1,$$

for τ_n, given that $X(t_n) = i$, is obtained as

$$h_0(k) = \begin{cases} 0, & k = 1, \\ (1 - \alpha)^{k-2}\alpha, & k = 2, 3, \cdots, D, \\ (1 - \alpha)^{D-1}, & k = D + 1, \end{cases} \tag{8}$$

$$h_1(k) = \begin{cases} 1, & k = 1, \\ 0, & k = 2, 3, \cdots, D + 1. \end{cases} \tag{9}$$

We introduce the service completion time, which is one of important performance measures when we consider the quality of service of secondary users. The service completion time of a secondary packet is defined as the time period needed for the packet to be successfully transmitted after it is positioned in the head-of-line position of the buffer for the first time, which depends on the state of the primary channel when it moves to the head-of-line position. The probability π_i, $i = 0, 1$, that the state of the primary channel is i when a secondary packet moves to the head-of-line position of the buffer is given by

$$\pi_0 = \frac{p_{10}}{p_{01} + p_{10}} = \frac{1 - \beta}{1 + (\alpha - \beta)(1 - \alpha)^D}, \tag{10}$$

$$\pi_1 = \frac{p_{01}}{p_{01} + p_{10}} = \frac{\beta + (\alpha - \beta)(1 - \alpha)^D}{1 + (\alpha - \beta)(1 - \alpha)^D}. \tag{11}$$

Given that the state of the primary channel is i when a secondary packet moves to the head-of-line position of the buffer, the conditional probability that the service completion time of the packet is k is exactly the same as the probability $h_i(k)$. Thus, unconditioning the state of the primary channel, we can obtain the probability distribution for service completion time C of secondary packets. The probability $P\{C = k\}$ that the service completion time of a secondary

packet is k is given by

$$P\{C = k\} = \pi_0 h_0(k) + \pi_1 h_1(k)$$

$$= \begin{cases} \frac{\beta + (\alpha - \beta)(1-\alpha)^D}{1 + (\alpha - \beta)(1-\alpha)^D}, & k = 1, \\[2mm] \frac{(1-\beta)(1-\alpha)^{k-2}\alpha}{1 + (\alpha - \beta)(1-\alpha)^D}, & k = 2, 3, \cdots, D, \\[2mm] \frac{(1-\beta)(1-\alpha)^{D-1}}{1 + (\alpha - \beta)(1-\alpha)^D}, & k = D + 1. \end{cases} \qquad (12)$$

The average service completion time of secondary packets is obtained as

$$E[C] = 1 + \frac{1}{\alpha} \cdot \frac{(1-\beta)\left[1 - (1-\alpha)^D\right]}{1 + (\alpha - \beta)(1-\alpha)^D}. \qquad (13)$$

Finally, we can determine the saturation throughput U_p (respectively U_s) of the secondary user over the primary (respectively secondary) channel, defined as the number of secondary packets successfully transmitted on the primary (respectively secondary) channel per a slot. Then, the saturation throughput U_p and U_s are obtained by

$$U_p = \frac{1 - P\{C = D + 1\}}{E[C]} = \frac{1 + (\alpha - \beta)(1-\alpha)^D - (1-\beta)(1-\alpha)^{D-1}}{1 + (\alpha - \beta)(1-\alpha)^D + (1-\beta)\frac{1-(1-\alpha)^D}{\alpha}},$$

$$U_s = \frac{P\{C = D + 1\}}{E[C]} = \frac{(1-\beta)(1-\alpha)^{D-1}}{1 + (\alpha - \beta)(1-\alpha)^D + (1-\beta)\frac{1-(1-\alpha)^D}{\alpha}}.$$

The total throughput U of the secondary user, defined as the number of secondary packets successfully transmitted per a slot, is given by

$$U = U_p + U_s = \frac{1 + (\alpha - \beta)(1-\alpha)^D}{1 + (\alpha - \beta)(1-\alpha)^D + (1-\beta)\frac{1-(1-\alpha)^D}{\alpha}}. \qquad (14)$$

4. Numerical examples

We present analytical and simulation results to evaluate the performance of the proposed cognitive OSA scheme under different occupancy statistics of primary channel. In case 1, the state of the primary channel remains unchanged with a large probability 0.8. This corresponds to bursty traffic arrivals in the primary network [6]. In case 2, the primary channel is equally likely to change the state or remain at the current state. In case 3, the state of the primary channel remains unchanged with a small probability 0.2. In all three cases, the probability that the primary channel is available is same as 0.05. We vary the value D from 1 to 15. Note that all parameters in this section are given in dimensionless units and can be modified to reflect other situations.

Shown in Figure 2 and Figure 3 are the average service completion time and the saturation throughput under different occupancy statistics of primary channel. We see from the figures that although the average traffic load of the
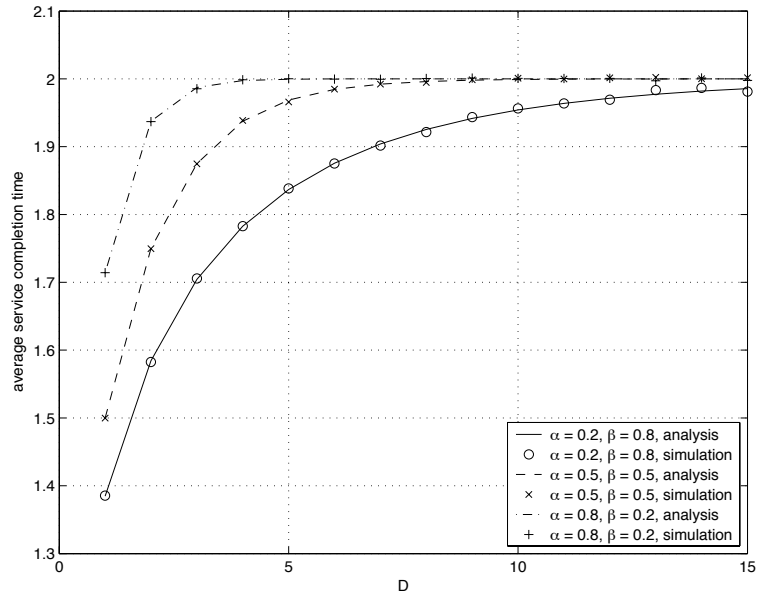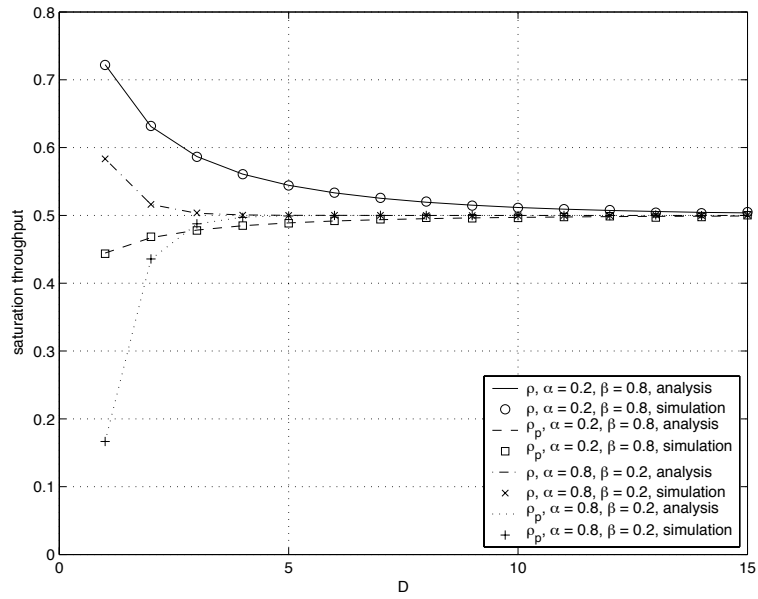
FIGURE 2. Average service completion time

FIGURE 3. Saturation throughput

primary channel is the same in all cases, different traffic statistics of the primary channel lead to different performance of secondary packets. The proposed OSA scheme is the most effective when it is overlayed over a primary channel with bursty traffic. As expected, as D increases, the average service completion time of secondary packets increases to 2, the reciprocal of the probability the primary channel is available (see Figure 2). The saturation throughput U_s over secondary channel, which is the difference between U and U_p in Figure 2, decreases to 0, as D increases.

5. Conclusions

We have developed an analytical model to evaluate performance of an OSA scheme using primary and secondary channels, and derived the performance such as service completion time and saturation throughput. Numerical examples are given to show the performance of the OSA scheme.

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