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RK- Methods for Robot Application problems

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Abstract

The significance, is to introduce a novel way to employ the improved Runge-Kutta fifth order five stage method, here after called as Modified IRK(5,5) method, for system of second order robot arm problem and variations in angles at the joints in which parameters governing with two degrees of freedom which requires lesser number of function evaluations per time step as compared to the existing ones, in order to save time and spaceAn ultimate aim of this present paper is to solve application problem such as robot arm and initial value problems by applying Runge-Kutta fifth order five stage numerical techniques. The calculated output for robot arm coincides with exact solution which is found to be better, suitable and feasible for solving real time problems.

Keywords: Runge-Kutta(5,5) Numerical Methods; Robot Arm; Initial Value Problems: Ordinary Differential Equations.

1. INTRODUCTION

Differential equations are one of the most significant and widely employed techniques in mathematical modeling. But, not many differential equations have an analytic solution and even if there is one, usually it is extremely difficult to obtain and it is not very practical. Therefore, numerical methods are truly a crucial part of solving differential equations which cannot be neglected. Among the models using differential equations (DEs), ordinary differential equations are frequently used to describe various physical problems, for example, robot arm, motions of the planet in a gravity field like the Kepler problem, the simple pendulum, electrical circuits and chemical kinetics problems. Taha [1,2] discussed about the dynamics of robot arm problems and presented its results. This is due to its nature of extending accuracy in order to determine the approximate solutions and its flexibility. Murugesan et al. [3] discussed about the parameters governing the arm model of a robot control problem through RK Butcher algorithm. Gopal et al. [4] provided exact solution of the system of equations representing the arm model of a robot using RK Butcher algorithm. Ponalagusamy and Senthilkumar [5] investigated and presented numerical solution for robot arm problem through RK sixth order algorithm including of stability polynomial. An efficient RK (4,5) pair was discussed by Boghachi and Shampine [5]for linear and non-linear problems.

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2. MATHEMATICAL PROBLEM: ROBOT ARM

Taha [1,2] discussed dynamics of robot arm problem which can be represented as

$$T = A(Q)\ddot{Q} + B(Q,\dot{Q}) + C(Q)\,. \tag{1}$$

where $A(Q)$ represents coupled inertia matrix, $B(Q,\dot{Q})$ is the matrix of coriolis and centrifugal forces. $C(Q)$ is the

gravity matrix, and T denotes the input torques applied at various joints.By considering lumped equivalent massless links, for a robot with two degrees of freedom, i.e. it means point load or in this case, the mass concentrated at the end of the links, dynamics
are represented by

$$\left.\begin{aligned} T_1 &= D_{11}\ddot{q}_1 + D_{12}\ddot{q}_{21} + D_{122}(\ddot{q}_2)^2 + D_{112}(\dot{q}_1\dot{q}_2) + D_1, \\ T_2 &= D_{21}\ddot{q}_1 + D_{22}\ddot{q}_2 + D_{211}(\dot{q}_1)^2 + D_2, \end{aligned}\right\} \quad (2)$$

Where, $D_{11} = (M_1 + M_2)d_2^2 + 2M_2 d_1 d_2 \cos(q_2)$,

$D_{12} = D_{21} = M_2 d_2^2 + M_2 d_1 d_2 \cos(q_2)$,

$D_{22} = M_2 d_2^2$, $D_{112} = -2M_2 d_1 d_2 \sin(q_2)$,

$D_{122} = D_{211} = -M_2 d_1 d_2 \sin(q_2)$,

$D_1 = [(M_1 + M_2)d_1 \sin(q_1) + M_2 d_2 \sin(q_1 + q_2)]g$ and

$D_2 = [M_2 d_2 \sin(q_1 + q_2)]g$.

The values of robot parameters used are M_1= 2kg, M_2 = 5kg, $d_1 = d_2 = 1$. For problem of set point regulation, the state vectors are represented by

$$X = (X_1, X_2, X_3, X_4)^T = (q_1 - q_{1d}, \dot{q}_1, q_2 - q_{2d}, \dot{q}_2)^T, \quad (3)$$

where q_1 and q_2 are the angles at joints 1 and 2 respectively, and q_{1d} and q_{2d} are constants. Thus, equation (3) may be expressed in state space representation as,

$$\left.\begin{aligned} \dot{e}_1 &= x_2, \\ \dot{x}_2 &= \frac{D_{22}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2) \\ \dot{e}_3 &= x_4, \\ \dot{x}_4 &= \frac{-D_{12}}{d}(D_{122}X_2^2 + D_{112}X_2X_4 + D_1 + T_1) - \frac{D_{12}}{d}(D_{211}X_4^2 + D_2 + T_2) \end{aligned}\right\} \quad (4)$$

Here the robot is simply a double inverted
pendulum and the Lagrangian approach is used to develop the equations. By selecting suitable parameters, non-linear equation (4) of the two-link robot-arm model may be reduced to the following system of linear equation:

$$\left.\begin{aligned} \dot{e}_1 &= x_2, \ \dot{x}_2 = B_{10}T_1 - A_{11}x_2 - A_{10}e_1, \\ \dot{e}_3 &= x_4 \ ; \ \dot{x}_4 = B_{20}^2 T_2 - A_{21}^2 x_4 - A_{20}^2 e_3. \end{aligned}\right\} \quad (5)$$

From (5), one can obtain the system of second order linear equations:

$\ddot{x}_1 = -A_{11}\dot{x}_1 - A_{10}x_1 + B_{10}T_1$; $\ddot{x}_3 = -A_{21}^2\dot{x}_3 - A_{20}^2 x_3 + B_{20}^2 T_2$,

where values of the parameters of joint-1 given by

$A_{10} = 0.1730$, $A_{11} = -0.2140$, $B_{10} = 0.00265$,and parameters of joint-2 given by,

A_{20}= 0.0438, A_{21} = 0.3610, B_{20} = 0.0967.

With T_1 = r (constant) and $T_2 = \lambda$ (constant), it is now possible to find the complementary functions of equation (5) because the nature of the roots of auxiliary equations (A. Es) of (5) is unpredictable. Due to this reason and for the sake of simplicity, we take $T_1 = T_2 = 1$. Using $q_1 = q_2 = 0$, $q_{1d} = q_{2d} = 1$ and $\dot{q}_1 = \dot{q}_2 = 0$, initial conditions are given by$e_1(0) = e_3(0) = -1$ and$e_2(0) = e_4(0) = 0$

3. Numerical Results for Robot Arm

The discrete and analytical solutions of the robot arm model problem have been calculated for different time periods of (t) using the equations (5) and y_{n+1}. (See tables 1-4).

Table 1. Solution for equation (5) of $e_1(t)$

Time	RK(5,5) Type-I
0.000	-1.0000000e+000
1.010	-8.9212351e-001
1.020	-8.8992838e-001
1.030	-8.8771047e-001
1.040	-8.8569489e-001

Table 2. Solution for equation (5) of $e_2(t)$

Time	RK(5,5) Type-I
0.000	-3.2999999e-010
1.010	2.1837542e-001
1.020	2.2065166e-001
1.030	2.2292895e-001
1.040	2.2520726e-001

Table 3. Solution for equation (5) of $e_3(t)$

Time	RK(5,5) Type-I
0.000	-1.1104170e+000
1.010	-9.9345516e-001
1.020	-9.9230708e-001
1.030	-9.9115920e-001
1.040	-9.9001151e-001

Table 4. Solution for equation (5) of $e_4(t)$

Time	RK(5,5)Type-I
0.000	0.0000000e+000
1.010	1.0661335e-002
1.020	1.0779674e-002
1.030	1.0858462e-002
1.040	1.0956831e-002

4. CONCLUSION

Runge-Kutta fifth order five stage numerical techniques are employed for robot arm problem and initial value problems. The computed output for robot arm and IVPs coincides with exact solution which is found to be better, suitable and feasible for solving real time problems.

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