International Journal of Advanced Smart Convergence Vol.2 No.1 12-17 (2013)
DOI 10.7236/IJASC.2013.2.1.012

IJASC 13-1-3

Nonlinear Characteristics of Fuzzy Scatter Partition-Based Fuzzy Inference System

Keon-Jun Park[1], Wei Huang[2], C. Yu[3], and Yong K. Kim[1]*

[1]Dept. of Information and Communication Engineering, Wonkwang University,
460, Iksandae-ro, Iksan, Chonbuk, 570-749 South Korea
[2]School of Computer and Communication engineering, Tianjin University of Technology, Tianjin 300191, China
[3]Dept. of Electrical and Computer Engineering, North Carolina State University, Raeigh, NC27405, USA
bird75@wonkwang.ac.kr, huangwabc@163.com,ykim@wonkwang.ac.kr

Abstract

This paper introduces the fuzzy scatter partition-based fuzzy inference system to construct the model for nonlinear process to analyze nonlinear characteristics. The fuzzy rules of fuzzy inference systems are generated by partitioning the input space in the scatter form using Fuzzy C-Means (FCM) clustering algorithm. The premise parameters of the rules are determined by membership matrix by means of FCM clustering algorithm. The consequence part of the rules is represented in the form of polynomial functions and the parameters of the consequence part are estimated by least square errors. The proposed model is evaluated with the performance using the data widely used in nonlinear process. Finally, this paper shows that the proposed model has the good result for high-dimension nonlinear process.

Key words: Fuzzy Scatter Partition, Fuzzy Inference Systems, Fuzzy C-Means Clustering Algorithm, Rule Generation, Nonlinear Process.

1. INTRODUCTION

Fuzzy sets and fuzzy modeling proposed by Zadeh [1] have been studied to deal with complex, ill-defined, and uncertain systems in many other avenues [2, 3, 4]. Linguistic modeling [5] and fuzzy relation equation-based approach [6] were proposed as primordial identification methods for fuzzy models. The general class of Sugeno-Takagi models [7] gave rise to more sophisticated rule-based systems. In fuzzy modeling, the structure and parameter identification are usually concerned. The designers find it difficult to develop adequate fuzzy rules and membership functions to reflect the essence of the data. The generation of fuzzy rules has the problem that the number of fuzzy rules exponentially increases.

Manuscript received : Jan. 23, 2013 / revised : Apr. 13, 2013
Corresponding Author: ykim@wonkwang.ac.kr
Tel: +82-63-850-6695, Fax: +82-63-850-6695
Dept. of Information and Communication Engineering, Wonkwang University, Korea

In this paper, we introduce a fuzzy scatter partition-based fuzzy inference system. Fuzzy partition of input space realized with FCM clustering [8] help determine the fuzzy rules of fuzzy model. The premise part of the rules is realized with the aid of the scatter partition of input space generated by FCM clustering algorithms. The number of the partition of input space equals the number of clusters and the individual partitioned spaces describe the rules. The consequence part of the rules is represented by polynomial functions. For simulating nonlinear process, the proposed model is evaluated with numerical experimentation. Finally, this paper shows that the proposed model has the good result for high-dimension nonlinear process.

2. DESIGN OF FUZZY SCATTER PARTITION-BASED FUZZY INFERENCE SYSTEM

2.1. Premise Identification

The premise part of the FIS is developed by means of the fuzzy c-means clustering algorithm [8]. This algorithm divides the input space by the clusters and each partitioned local space represents the fuzzy rules. Therefore, the number of clusters is equal to the number of rules. This algorithm is aimed at the formation of 'c' clusters (relations) in $\mathbf{R}^n$.

Consider the set $\mathbf{X}$, which consists of N data points treated as vectors located in some n-dimensional Euclidean space. In clustering we assign patterns $\mathbf{x}_p \in \mathbf{X}$ into c clusters, which are represented by its prototypes $\mathbf{v}_i \in \mathbf{R}^n$. The assignment to individual clusters is expressed in terms of the partition matrix $\mathbf{U} = [u_{ip}]$ where

$$\sum_{i=1}^{c} u_{ip} = 1, \quad 0 < \sum_{p=1}^{N} u_{ip} < N, \quad 1 \le p \le N, \quad 1 \le i \le c \tag{1}$$

The objective function Q guiding the clustering is expressed as a sum of the distances of individual data from the prototypes $\mathbf{v}_1, \mathbf{v}_2, \ldots,$ and $\mathbf{v}_c$,

$$Q = \sum_{i=1}^{c} \sum_{p=1}^{N} u_{ip}^{m_i} \left\| \mathbf{x}_p - \mathbf{v}_i \right\|^2 . \tag{2}$$

Here $\|\cdot\|$ denotes the Euclidean distance; 'm' stands for a fuzzification coefficient, m_i>1.0. The resulting partition matrix is denoted by $\mathbf{U} = [u_{ip}]$.

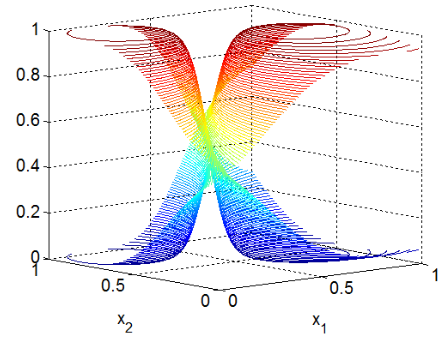
The minimization of Q is realized through successive iterations by adjusting both the prototypes and entries of the partition matrix, that is min $Q(\mathbf{U}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_c)$. The corresponding formulas used in an iterative fashion read as follows.

$$\mathbf{v}_i = \frac{\sum_{p=1}^{N} u_{ip}^{m_i} \mathbf{x}_p}{\sum_{p=1}^{N} u_{ip}^{m_i}} . \tag{3}$$

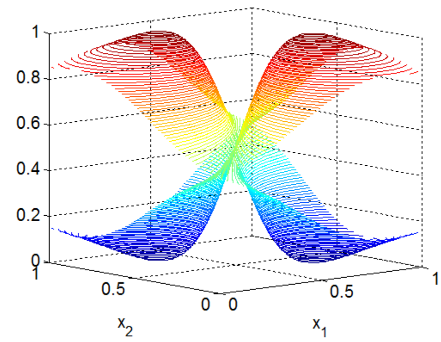
$$u_{ip} = \frac{1}{\sum_{j=1}^{c} \left(\frac{\left\| \mathbf{x}_p - \mathbf{v}_i \right\|}{\left\| \mathbf{x}_p - \mathbf{v}_j \right\|} \right)^{\frac{2}{m_i - 1}}} . \tag{4}$$

Figure 1 shows the membership matrix according to fuzzification coefficient. The shape of membership grade is affected by the values of the fuzzification coefficient. This means that the shape of membership grade becomes sharper as the value of coefficient is larger.

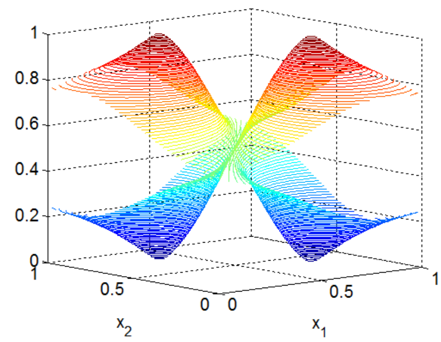
The resulting partitioned local spaces by means of FCM clustering algorithm represent the fuzzy rules of system. Figure 2 visualizes the example of the fuzzy partitioned spaces of input space with five clusters.



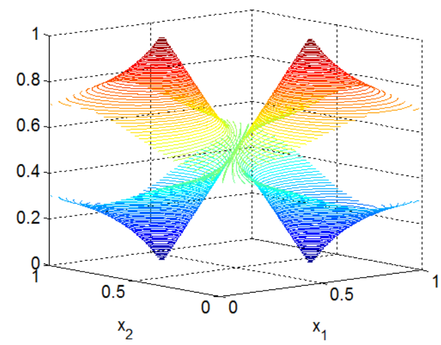
(a) $m = 1.5$



(b) $m = 2.0$

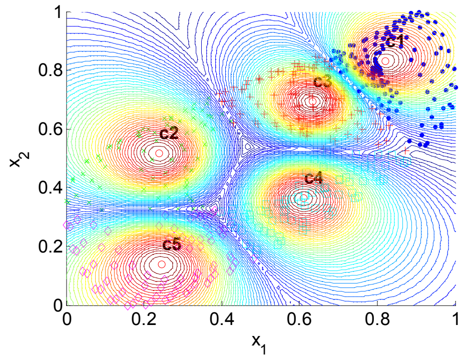


(c) $m = 2.5$

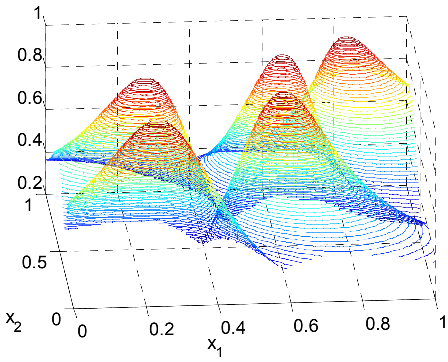


(d) $m = 3.0$

Fig. 1. Membership matrix according to fuzzification coefficient.



(a) Partitioned local spaces



(b) Membership grade

Fig. 2. Fuzzy scatter partition of input space.

2.2 Consequence Identification

The identification of the conclusion parts of the rules deals with a selection of their structure that is followed by the determination of the respective parameters of the local functions occurring there. The conclusion is expressed as follows.

$$R^j : If\ \ x_1\ and \cdots and\, x_d\ is\ F_j \quad Then\, y_j = f(x_1,\ldots,x_d)\,. \tag{5}$$

Type 1 (Simplified Inference):

$$f = a_{j0}\,. \tag{6}$$

Type 2 (Linear Inference):

$$f = a_{j0} + \sum\nolimits_{k=1}^{d} a_{jk} x_k\,. \tag{7}$$

Type 3 (Quadratic Inference):

$$f = a_{j0} + \sum\nolimits_{k=1}^{d} a_{jk} x_k + \sum\nolimits_{k=1}^{d} a_{j,(d+k)} x_k + \sum\nolimits_{k=1}^{d} \sum\nolimits_{l=k+1}^{d} a_{jz} x_k x_l \tag{8}$$

Type 4 (Modified Quadratic Inference):

$$f = a_{j0} + \sum\nolimits_{k=1}^{d} a_{jk} x_k + \sum\nolimits_{k=1}^{d} \sum\nolimits_{l=k+1}^{d} a_{jz} x_k x_l \tag{9}$$

Where R^j is the j-th rule, x_k represents the input variables, F_j is a memberships obtained using FCM clustering algorithm, **a**'s are coefficient of polynomial function, z is the number of combinations of input variables.

The calculations of the numeric output of the model, based on the activation (matching) levels of the rules there, are carried out in the well-known format

$$y^* = \frac{\sum_{j=1}^{n} w_{jp} y_j}{\sum_{j=1}^{n} w_{jp}} = \sum_{j=1}^{n} \hat{w}_{jp} y_j\,. \tag{10}$$

Here, as the normalized value of w_{jp}, we use an abbreviated notation to describe an activation level $\hat{w}_{jp}$, which values are determined by the partition matrix **U**;

$$w_{jp} = u_{ip}\,. \tag{11}$$

Therefore, the inferred output value y* can be expressed as

$$y^* = \sum_{j=1}^{n} w_{jp} y_j\,. \tag{12}$$

If the input variables of the premise and parameters are given in consequence parameter identification, the optimal consequence parameters that minimize the assumed performance index can be determined.

$$PI = \frac{1}{m}\sum_{p=1}^{m}\left(y_p - y_p^*\right)^2\,. \tag{13}$$

The minimal value produced by the least-squares method is governed by the following expression:

$$\hat{\mathbf{a}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y}\,. \tag{14}$$

3. Experimental Studies

We discuss numerical example in order to evaluate the advantages and the effectiveness of the proposed approach. This time series data (296 input-output pairs) coming from the gas furnace nonlinear process has been intensively studied in the previous literature [9]. The delayed terms of methane gas flow rate u(t) and carbon dioxide density y(t) are used as input variables organized in a vector format as [u(t-3), u(t-2), u(t-1), y(t-3), y(t-2), y(t-1)]. y(t) is the output variable. The first part of the data set (consisting of 148 pairs) was used for training purposes. The remaining part of the series serves as a testing data set. We consider the MSE as a performance index.

We construct the model for a two-dimensional system by configuring 2-input 1-output system using u(t-3) and y(t-1) as inputs and six-dimensional system using all inputs.

Table 1 and Table 2 summarize the performance index for training and testing data by setting the number of clusters and inference type. Here, PI and E_PI stand for the performance index for the training data set and the testing data set, respectively.

From the Table 1 and Table 2, we selected the best model with five rules (clusters) with quadratic inference that exhibits PI = 0.015 and E_PI = 0.299 for two-dimension system and five rules (clusters) with linear inference that exhibits PI = 0.012 and E_PI = 0.181 for six-dimension system.

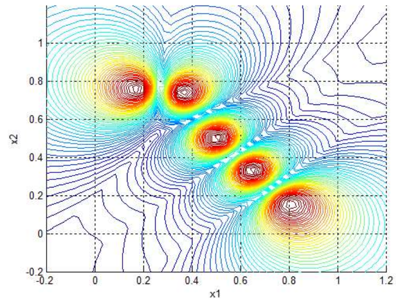
Table 1. Performance of the proposed model (Two-dimension)

No. of Clusters	Type	PI	E_PI
2	Type 1	2.268	2.418
	Type 2	0.022	0.338
	Type 3	0.021	0.331
	Type 4	0.021	0.332
3	Type 1	1.018	1.473
	Type 2	0.021	0.347
	Type 3	0.020	0.320
	Type 4	0.021	0.340
5	Type 1	0.664	1.232
	Type 2	0.018	0.316
	Type 3	0.015	0.299
	Type 4	0.016	0.299
10	Type 1	0.491	0.970
	Type 2	0.015	0.315
	Type 3	0.013	0.339
	Type 4	0.014	0.312

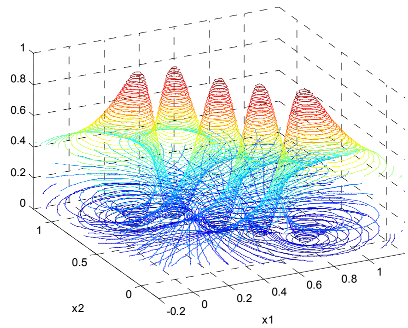
Table 2. Performance of the proposed model (Six-dimension)

No. of Clusters	Type	PI	E_PI
2	Type 1	2.192	3.190
	Type 2	0.015	0.188
	Type 3	0.008	0.206
	Type 4	0.009	0.248
3	Type 1	1.452	2.246
	Type 2	0.014	0.203
	Type 3	0.006	0.422
	Type 4	0.005	0.319
5	Type 1	1.085	2.243
	Type 2	0.012	0.181
	Type 3	0.001	6.191
	Type 4	1.707E-16	1332.582
10	Type 1	0.901	2.131
	Type 2	0.008	0.241
	Type 3	1.345E-19	6.752
	Type 4	2.355E-18	77.600

Fig.3 shows fuzzy-partitioned input spaces and membership matrix using FCM clustering algorithm in two-dimensional input space for the selected model.



(a) Partitioned input spaces

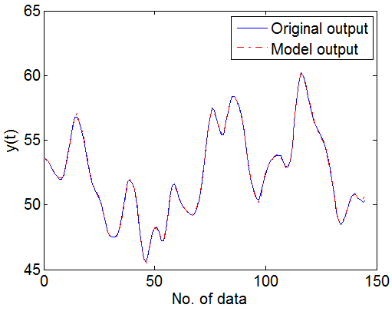


(b) membership matrix

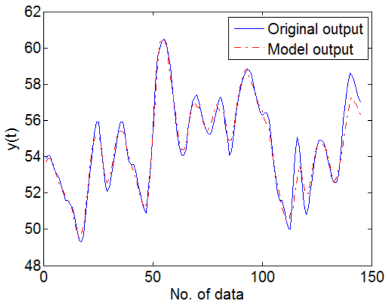
Fig. 3. Partitioned input spaces and membership matrix using FCM clustering algorithm (5 clusters)

Figure 4 and Figure 5 depict the original and model outputs of training and testing data for the selected model. This figure shows that the model outputs are approximately the same for original output.

Figure 6 and Figure 7 show the resulting predicting errors for training and testing data for the selected model. The differences between predicting errors for training and testing data in six-dimension system have more small values.

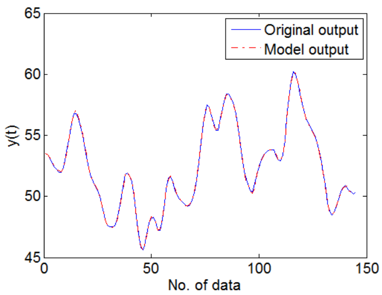


(a) Training data

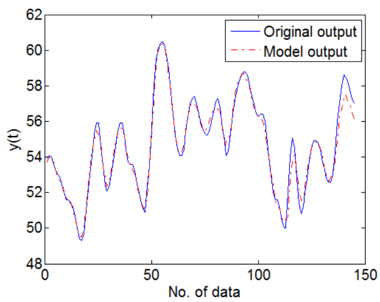


(b) Testing data

Fig. 4. Model outputs (5 clusters, quadratic inference) for two-dimension system.

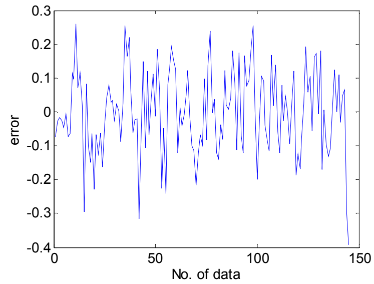


(a) Training data

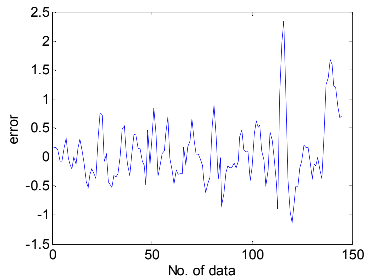


(b) Testing data

Fig. 5. Model outputs (5 clusters, linear inference) for six-dimension system.

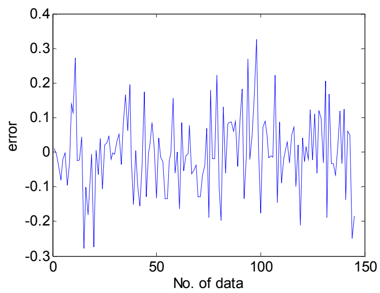


(a) Training data

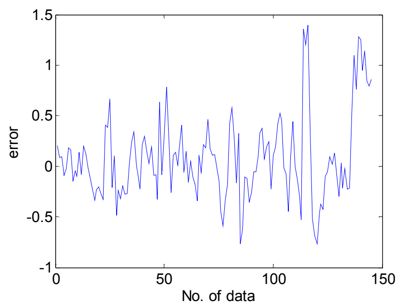


(b) Testing data

Fig. 6. Predicting errors (5 clusters, quadratic inference) for two-dimension system.



(a) Training data



(b) Testing data

Fig. 7. Predicting errors (5 clusters, linear inference) for six-dimension system.

4. Conclusion

In this paper, we introduced a fuzzy scatter partition-based fuzzy inference system to analysis the nonlinear characteristics. In order to generate the rules of the system for nonlinear process, the input spaces of the proposed model were divided as the scatter form using FCM clustering algorithm. By this method, we could alleviate the problem of the curse of dimensionality and design the FIS that is compact and simple.

From the results, we were able to design preferred model using a very small number of rules in a high dimensional nonlinear systems. Through the use of a performance index, we were able to achieve a balance between the approximation and generalization abilities of the resulting model. Finally, this approach would find potential application to system modeling in many fields.

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Keon-Jun Park

He received the B.S degree in Electrical & Electronics Engineering from the Wonkwang University, Iksan, South Korea, in 2003, and the M.S degree in Control and Instrumentation Engineering in 2005. He received the Ph.D degree in Electrical Engineering from University of Suwon, Suwon, South Korea, in 2010. He is currently a research professor with Information & Communication Engineering, Wonkwang University, Korea. His research interests include computational intelligence, intelligence modeling and control, and pattern recognition.

Wei Huang

He received the MS degree at school of information Engineering, East ChinaInstitute of Technology, China, in 2006, and Ph.D. degree at State Key Laboratory ofSoftware Engineering, Wuhan University, China, in 2011. He is currently a lecturer in theSchool of Computer and Communication Engineering, Tianjin University of Technology,Tianjin, China. His research interests include evolutionary computation, operations research,fuzzy system, natural language processing, and software reliability.

Yong-Kab Kim

He received the B.S degree in Electronics Engineering from Ajou University and the M.S.E degree in Electrical & Computer Engineering from University of Alabama in Huntsville. He received his Ph.D degree in Electrical & Computer Engineering in North Carolina State University. He is currently in Professor at School of Information and Communication Engineering, Wonkwang University, Korea. His research interests are Remote Sensing for Visible Communication, Optical Fiber Sensing, Intelligent Control, Advanced Computational Intelligence.