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ON SPACES OF WEAK* TO WEAK CONTINUOUS COMPACT OPERATORS

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Abstract. This paper is concerned with the space $\mathcal{K}_{w^*}(X^*, Y)$ of weak* to weak continuous compact operators from the dual space X^* of a Banach space X to a Banach space Y. We show that if X^* or Y^* has the Radon-Nikodým property, $\mathcal{C}$ is a convex subset of $\mathcal{K}_{w^*}(X^*, Y)$ with $0 \in \mathcal{C}$ and T is a bounded linear operator from X^* into Y, then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if $T \in \overline{\{S \in \mathcal{C} : \|S\| \leq \|T\|\}}^{\tau_c}$, where τ_c is the topology of uniform convergence on each compact subset of X, moreover, if $T \in \mathcal{K}_{w^*}(X^*, Y)$, here $\mathcal{C}$ need not to contain 0, then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if $T \in \overline{\mathcal{C}}$ in the topology of the operator norm. Some properties of $\mathcal{K}_{w^*}(X^*, Y)$ are presented.

1. Introduction and the main result

Representations of dual spaces of operator spaces provide a useful tool to study approximation properties of operators. Grothendieck [8] established a representation of the dual space of $\mathcal{L}(X, Y)$, the space of bounded linear operators between Banach spaces X and Y, when endowed with the topology τ_c of uniform convergence on each compact subset of X and the representation was applied to study the approximation property. A Banach space X is said to have the *approximation property* (AP) if the identity operator $id_X \in \overline{\mathcal{F}(X, X)}^{\tau_c}$, where $\mathcal{F}(X, X)$ is the space of finite rank operators on X, and we say that X has the *metric approximation property* (MAP) if $id_X \in \overline{\{T \in \mathcal{F}(X, X) : \|T\| \leq 1\}}^{\tau_c}$. The AP is formally weaker than the MAP, in fact Figiel and Johnson [6] showed that the AP is strictly weaker than the MAP, more precisely, they constructed a separable Banach space having the AP but failing to have the MAP. Grothendieck [8] applied the representation of the dual space of $(\mathcal{L}(X, Y), \tau_c)$ to show that for separable dual spaces, the AP and MAP are equivalent. But it is a long-standing famous problem whether

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the AP and MAP are equivalent for general dual spaces (cf. [2, Problem 3.8]). The main purpose of the paper is to establish an approximation theorem in $\mathcal{K}_{w^*}(X^*, Y)$, the space of weak* to weak continuous compact operators from X^* to Y, using the bidual space of $\mathcal{K}_{w^*}(X^*, Y)$ endowed with the topology of the operator norm. The main result is originated from the following result of Godefroy and Saphar [7].

Theorem 1.1 ([7, Theorem 1.5])**.** *Suppose that X^* or Y^{**} has the Radon-Nikodým property. Let $\mathcal{C}$ be a convex subset of $\mathcal{K}_{w^*}(X^*, Y^*)$ and let $T \in \mathcal{L}(X^*, Y^*)$. Then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if for every $\varepsilon > 0$,*

$$T \in \overline{\{S \in \mathcal{C} : \|S\| < \|T\| + \varepsilon\}}^{\tau_c}.$$

Note that the above mentioned passage from the AP to the MAP for separable dual spaces easily follows from Theorem 1.1. Recently, Choi and Kim [3] used a representation of the dual space of $\mathcal{K}_{w^*}(X^*, Y)$, endowed with the topology of the operator norm, to obtain the following.

Theorem 1.2 ([3, Theorem 2.3])**.** *Suppose that X^* or Y^* has the Radon-Nikodým property. Let $\mathcal{Y}$ be a subspace of $\mathcal{K}_{w^*}(X^*, Y)$ and let $T \in \mathcal{L}(X^*, Y)$. Then $T \in \overline{\mathcal{Y}}^{\tau_c}$ if and only if $T \in \overline{\{S \in \mathcal{Y} : \|S\| \le \|T\|\}}^{\tau_c}$.*

In this paper, we adjust arguments of Feder, Godefroy and Saphar ([5, Theorem 1], [7, Theorem 1.5]) to extend Theorem 1.1:

Theorem 1.3. *Suppose that X^* or Y^* has the Radon-Nikodým property. Let $\mathcal{C}$ be a convex subset of $\mathcal{K}_{w^*}(X^*, Y)$ and let $T \in \mathcal{L}(X^*, Y)$. Then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if for every $\varepsilon > 0$, $T \in \overline{\{S \in \mathcal{C} : \|S\| < \|T\| + \varepsilon\}}^{\tau_c}$.*

The following corollary extends Theorem 1.2.

Corollary 1.4. *Suppose that X^* or Y^* has the Radon-Nikodým property. Let $\mathcal{C}$ be a convex subset of $\mathcal{K}_{w^*}(X^*, Y)$ with $0 \in \mathcal{C}$ and let $T \in \mathcal{L}(X^*, Y)$. Then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if $T \in \overline{\{S \in \mathcal{C} : \|S\| \le \|T\|\}}^{\tau_c}$.*

Proof. Suppose $T \in \overline{\mathcal{C}}^{\tau_c}$. Let K be a compact subset of X^* and let $\varepsilon > 0$. Choose $\delta > 0$ so that $(\delta/(\|T\|+\delta)) \sup_{x^* \in K} \|Tx^*\| < \varepsilon/2$. Then by Theorem 1.3 there exists an $S \in \{S \in \mathcal{C} : \|S\| < \|T\| + \delta\}$ such that $\sup_{x^* \in K} \|Sx^* - Tx^*\| < \varepsilon/2$. Consider $(\|T\|/(\|T\| + \delta))S \in \mathcal{C}$ with $\|(\|T\|/(\|T\| + \delta))S\| < \|T\|$. Then

$$\begin{aligned} &\sup_{x^* \in K} \left\| \frac{\|T\|}{\|T\| + \delta} Sx^* - Tx^* \right\| \\ &\le \frac{\|T\|}{\|T\| + \delta} \sup_{x^* \in K} \|Sx^* - Tx^*\| + \frac{\delta}{\|T\| + \delta} \sup_{x^* \in K} \|Tx^*\| < \varepsilon. \end{aligned}$$

Hence $T \in \overline{\{S \in \mathcal{C} : \|S\| \le \|T\|\}}^{\tau_c}$. □

We end the paper by a section collecting some results concerning the space $\mathcal{K}_{w^*}(X^*, Y)$. First we give a simple characterization of elements in $\mathcal{K}_{w^*}(X^*, Y)$

[Proposition 3.1]. Then we describe $\mathcal{K}_{w^*}(X^*, Y)^*$ in general and look at the particular case when X^* is separable. We end the section by showing how one can simplify the proof of a factorization result for $\mathcal{K}_{w^*}(X^*, Y)$ from [1] and [14]. We use standard Banach space notation as can be found e.g. in [13].

2. A representation of the bidual space of $\mathcal{K}_{w^*}(X^*, Y)$ and a proof of Theorem 1.3

Godefroy and Saphar [7, Proposition 1.1] established a representation of $\mathcal{K}(X, Y)^{**}$ under the assumption that X^{**} or Y^* has the RNP. In this section, we adopt the factorization argument of Feder, Godefroy and Saphar [5, 7] to represent $\mathcal{K}_{w^*}(X^*, Y)^{**}$ under the assumption that X^* or Y^* has the RNP, and then the representation will be a main tool of the proof of Theorem 1.3.

For Banach spaces Z and W we denote the projective and injective tensor product by $Z \otimes_\pi W$ and $Z \otimes_\varepsilon W$, respectively (cf. see [15, Chapters 2 and 3]). Recall that $\mathcal{L}(Z, W^*)$ is isometrically isomorphic to $(Z \otimes_\pi W)^*$ and that for a net (T_α) in $\mathcal{L}(Z, W^*)$ and $T \in \mathcal{L}(Z, W^*)$

$$T_\alpha \xrightarrow{w^*} T \ \text{ if and only if } \ \sum_n (T_\alpha z_n)(w_n) \longrightarrow \sum_n (T z_n)(w_n)$$

for every (z_n) in Z and (w_n) in W with $\sum_n \|z_n\|\|w_n\| < \infty$ (see [15, p. 24]).

We now have:

Theorem 2.1. *Suppose that X^* or Y^* has the Radon-Nikodým property. Then there exists a w^* to w^* homeomorphic linear isometry Φ from $\overline{\mathcal{K}^*_{w^*}(X^*, Y)}^{w^*}$ (in $(\mathcal{L}(Y^*, X^{**}), w^*)$) onto $\mathcal{K}_{w^*}(X^*, Y)^{**}$ such that*

$$\Phi(\mathcal{K}^*_{w^*}(X^*, Y)) = j(\mathcal{K}_{w^*}(X^*, Y)),$$

where $\mathcal{K}^*_{w^*}(X^*, Y) = \{T^* : T \in \mathcal{K}_{w^*}(X^*, Y)\}$ and $j : \mathcal{K}_{w^*}(X^*, Y) \to \mathcal{K}_{w^*}(X^*, Y)^{**}$ is the natural isometry.

Proof. Suppose that X^* has the Radon-Nikodým property. We define the map $V : Y^* \otimes_\pi X^* \to \mathcal{K}_{w^*}(X^*, Y)^*$ by

$$Vv(T) = \sum_n y_n^*(T x_n^*)$$

for $v = \sum_n y_n^* \otimes x_n^* \in Y^* \otimes_\pi X^*$. Then V is well defined, linear and $\|V\| \leq 1$. First we use the proof of [5, Theorem 1] to show that V is a quotient map and so V^* is an isometry. Let the map $i : Y \to l^\infty(B_{Y^*})$ be defined by $i(y)(y^*) = y^*(y)$ for every $y^* \in B_{Y^*}$. Then i is an isometry and so the map $J_1 : \mathcal{K}_{w^*}(X^*, Y) \to \mathcal{K}_{w^*}(X^*, l^\infty(B_{Y^*}))$ defined by $J_1(T) = iT$ is an isometry. Since $l^\infty(B_{Y^*})$ has the approximation property, $\mathcal{K}_{w^*}(X^*, l^\infty(B_{Y^*}))$ is isometrically isomorphic to $X \otimes_\varepsilon l^\infty(B_{Y^*})$ by the isometry J_2.

$$\mathcal{K}_{w^*}(X^*, Y) \xrightarrow{J_1} \mathcal{K}_{w^*}(X^*, l^\infty(B_{Y^*})) \xrightarrow{J_2} X \otimes_\varepsilon l^\infty(B_{Y^*}).$$

Since $l^\infty(B_{Y^*})^*$ has the approximation property and X^* has the Radon-Nikodým property, $l^\infty(B_{Y^*})^* \otimes_\pi X^*$ is isometrically isomorphic to $(X \otimes_\varepsilon l^\infty(B_{Y^*}))^*$ by the isometry J_3 (see [15, Theorem 5.33]).

$$l^\infty(B_{Y^*})^* \otimes_\pi X^* \xrightarrow{J_3} (X \otimes_\varepsilon l^\infty(B_{Y^*}))^* \overset{(J_2J_1)^*}{\longrightarrow} \mathcal{K}_{w^*}(X^*, Y)^*.$$

Let $J = (J_2J_1)^*J_3$. We show that the following diagram is commutative:

$$\begin{array}{ccc} l^\infty(B_{Y^*})^* \otimes_\pi X^* & \xrightarrow{\quad i^* \otimes id_{X^*} \quad} & Y^* \otimes_\pi X^* \\ & {\scriptstyle J} \searrow \qquad \swarrow {\scriptstyle V} & \\ & \mathcal{K}_{w^*}(X^*, Y)^* & \end{array}$$

Let $\mu \in l^\infty(B_{Y^*})^*$, $x^* \in X^*$ and $T \in \mathcal{K}_{w^*}(X^*, Y)$. Then

$$\begin{aligned} J(\mu \otimes x^*)(T) &= (J_2J_1)^*J_3(\mu \otimes x^*)(T) \\ &= J_3(\mu \otimes x^*)(J_2J_1(T)) \\ &= J_3(\mu \otimes x^*)(J_2(iT)) \\ &= \mu(iTx^*) \\ &= i^*(\mu)(Tx^*) \\ &= V(i^*(\mu) \otimes x^*)(T) \\ &= V(i^* \otimes id_{X^*})(\mu \otimes x^*)(T). \end{aligned}$$

It follows that the diagram is commutative. Now let $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$. Since J_2J_1 is an isometry, we see that there exists a $u \in l^\infty(B_{Y^*})^* \otimes_\pi X^*$ so that $J(u) = \varphi$ and $\|u\|_\pi = \|\varphi\|$. Let $v = i^* \otimes id_{X^*}(u)$. Then by the above diagram $\varphi = V(v)$ and we have

$$\|\varphi\| \le \|V\|\|i^* \otimes id_{X^*}(u)\|_\pi \le \|i^*\|\|id_{X^*}\|\|u\|_\pi \le \|u\|_\pi = \|\varphi\|.$$

Thus $\|\varphi\| = \|v\|_\pi$ and so V is a quotient map.

Now we use the proof of [7, Proposition 1.1]. Let the map $W : \mathcal{K}_{w^*}(X^*, Y) \to \mathcal{L}(Y^*, X^{**})$ be defined by $W(T) = T^*$, let $i_1 : \mathcal{L}(Y^*, X^{**}) \to (Y^* \otimes_\pi X^*)^*$ be the isometry and let $i_2 : Y^* \otimes_\pi X^* \to (Y^* \otimes_\pi X^*)^{**}$ be the natural isometry.

$$\begin{array}{ccc} Y^* \otimes_\pi X^* & \xrightarrow{\ V\ } & \mathcal{K}_{w^*}(X^*, Y)^* \\ {\scriptstyle i_2}\downarrow & & \uparrow{\scriptstyle W^*} \\ (Y^* \otimes_\pi X^*)^{**} & \xrightarrow[\ i_1^*\ ]{} & \mathcal{L}(Y^*, X^{**})^* \end{array}$$

Then for every $v = \sum_n y_n^* \otimes x_n^* \in Y^* \otimes_\pi X^*$ and $T \in \mathcal{K}_{w^*}(X^*, Y)$,

$$\begin{aligned} W^*i_1^*i_2(v)(T) &= i_2(v)i_1W(T) = i_1W(T)(v) \\ &= i_1(T^*)(v) = \sum_n (T^*y_n^*)(x_n^*) = \sum_n y_n^*(Tx_n^*) = (Vv)(T). \end{aligned}$$

Thus $W^*i_1^*i_2 = V$. Now consider the following diagram:

$$\begin{array}{ccc} \mathcal{K}_{w^*}(X^*,Y) & \xrightarrow{W} & \mathcal{L}(Y^*,X^{**}) \\ {\scriptstyle j}\downarrow & & \uparrow{\scriptstyle i_1^{-1}} \\ \mathcal{K}_{w^*}(X^*,Y)^{**} & \xrightarrow[V^*]{} & (Y^*\otimes_\pi X^*)^* \end{array}$$

Let $i_3 : \mathcal{L}(Y^*, X^{**}) \to \mathcal{L}(Y^*, X^{**})^{**}$ be the natural isometry. Then for every $T \in \mathcal{K}_{w^*}(X^*, Y)$ and $v \in Y^* \otimes_\pi X^*$,

$$\begin{aligned} i_1(i_1^{-1}V^*j(T))(v) &= (W^*i_1^*i_2)^*j(T)(v) \\ &= (i_1^*i_2)^*W^{**}j(T)(v) \\ &= W^{**}j(T)i_1^*i_2(v) \\ &= i_1^*i_2(v)i_3^{-1}W^{**}j(T) \\ &= i_1^*i_2(v)(W(T)) \\ &= i_2(v)i_1(W(T)) \\ &= i_1(W(T))(v). \end{aligned}$$

Thus the above diagram is commutative and so $i_1^{-1}V^*j(T) = T^*$ for every $T \in \mathcal{K}_{w^*}(X^*, Y)$. Recall that, if the range of an adjoint operator is norm closed, then the range is w^* closed. Thus we have

$$i_1^{-1}V^*(\mathcal{K}_{w^*}(X^*,Y)^{**}) = \overline{i_1^{-1}V^*(j(\mathcal{K}_{w^*}(X^*,Y)))}^{w^*} = \overline{\mathcal{K}^*_{w^*}(X^*,Y)}^{w^*}.$$

We have shown that $i_1^{-1}V^* : \mathcal{K}_{w^*}(X^*, Y)^{**} \to \overline{\mathcal{K}^*_{w^*}(X^*,Y)}^{w^*}$ is a surjective linear isometry. Put

$$\Phi = (i_1^{-1}V^*)^{-1} : \overline{\mathcal{K}^*_{w^*}(X^*,Y)}^{w^*} \to \mathcal{K}_{w^*}(X^*,Y)^{**}.$$

Note that if an adjoint operator is an isomorphism, then the inverse of this adjoint operator is w^* to w^* continuous on its range. Hence Φ is a w^* to w^* homeomorphic linear isometry and for every $T \in \mathcal{K}_{w^*}(X^*, Y)$

$$\Phi(T^*) = (i_1^{-1}V^*)^{-1}(T^*) = (i_1^{-1}V^*)^{-1}i_1^{-1}V^*j(T) = j(T).$$

This completes the proof for the case that X^* has the Radon-Nikodým property.

Now suppose that Y^* has the Radon-Nikodým property. Define the map $\psi : \mathcal{L}(Y^*, X^{**}) \to \mathcal{L}(X^*, Y^{**})$ by $\psi(T) = T^*j_{X^*}$. Then it is easy to check that ψ is a surjective linear isometry with the inverse $\psi^{-1}(R) = R^*j_{Y^*}$. Let (T_α) be a net in $\mathcal{L}(Y^*, X^{**})$ and $T \in \mathcal{L}(Y^*, X^{**})$ with $T_\alpha \xrightarrow{w^*} T$. Let $v = \sum_n x_n^* \otimes y_n^* \in X^* \otimes_\pi Y^*$. Since $\sum_n y_n^* \otimes x_n^* \in Y^* \otimes_\pi X^*$,

$$\sum_n (T_\alpha y_n^*)(x_n^*) \longrightarrow \sum_n (Ty_n^*)(x_n^*).$$

Thus

$$\begin{aligned}\psi(T_\alpha)(v) &= \sum_n (T_\alpha^* j_{X^*} x_n^*)(y_n^*) = \sum_n j_{X^*}(x_n^*)(T_\alpha y_n^*)\\ &= \sum_n (T_\alpha y_n^*)(x_n^*) \longrightarrow \sum_n (T y_n^*)(x_n^*) = \psi(T)(v).\end{aligned}$$

Hence ψ is w^* to w^* continuous and, similarly, so is ψ^{-1}. Let $S \in \mathcal{K}_{w^*}(X^*, Y)$ and let $x^* \in X^*$ and $y^* \in Y^*$. Then $S^*(y^*) = j_X(x)$ for some $x \in X$ and so we have

$$\begin{aligned}\psi(S^*)(x^*)(y^*) &= S^{**} j_{X^*}(x^*)(y^*) = S^*(y^*)(x^*) = j_X(x)(x^*)\\ &= x^*(x) = x^*(j_X^{-1} S^*(y^*)) = (j_X^{-1} S^*)^*(x^*)(y^*).\end{aligned}$$

Thus $\psi(S^*) = (j_X^{-1} S^*)^* \in \mathcal{K}_{w^*}^*(Y^*, X)$. Similarly, for every $U \in \mathcal{K}_{w^*}(Y^*, X)$ $\psi^{-1}(U^*) = (j_Y^{-1} U^*)^* \in \mathcal{K}_{w^*}^*(X^*, Y)$. Therefore $\psi(\mathcal{K}_{w^*}^*(X^*, Y)) = \mathcal{K}_{w^*}^*(Y^*, X)$ and so

$$\psi(\overline{\mathcal{K}_{w^*}^*(X^*, Y)}^{w^*}) = \overline{\psi(\mathcal{K}_{w^*}^*(X^*, Y))}^{w^*} = \overline{\mathcal{K}_{w^*}^*(Y^*, X)}^{w^*}.$$

Since Y^* has the Radon-Nikodým property, we can find the map

$$\Psi : \overline{\mathcal{K}_{w^*}^*(Y^*, X)}^{w^*} \to \mathcal{K}_{w^*}(Y^*, X)^{**}$$

in the first case. Define the map $\phi : \mathcal{K}_{w^*}(Y^*, X) \to \mathcal{K}_{w^*}(X^*, Y)$ by $\phi(T) = j_Y^{-1} T^*$. Then we see that ϕ is a surjective linear isometry. Then ϕ^{**} is a w^* to w^* homeomorphic isometry from $\mathcal{K}_{w^*}(Y^*, X)^{**}$ onto $\mathcal{K}_{w^*}(X^*, Y)^{**}$. Put

$$\Phi = \phi^{**}\Psi\psi : \overline{\mathcal{K}_{w^*}^*(X^*, Y)}^{w^*} \longrightarrow \mathcal{K}_{w^*}(X^*, Y)^{**}.$$

Then Φ is a w^* to w^* homeomorphic and surjective linear isometry, and

$$\begin{aligned}\Phi(\mathcal{K}_{w^*}^*(X^*, Y)) &= \phi^{**}\Psi(\mathcal{K}_{w^*}^*(Y^*, X))\\ &= \phi^{**}(j(\mathcal{K}_{w^*}(Y^*, X))) = j(\mathcal{K}_{w^*}(X^*, Y)).\end{aligned}$$

□

Remark 2.2. Suppose that X^{**} or Y^* has the Radon-Nikodým property. Let $i : \mathcal{K}(X, Y) \to \mathcal{K}_{w^*}(Y^*, X^*)$ be the surjective linear isometry defined by $i(T) = T^*$. Then $i^{**} : \mathcal{K}(X, Y)^{**} \to \mathcal{K}_{w^*}(Y^*, X^*)^{**}$ is a w^* to w^* homeomorphic and surjective isometry. We can find the map $\Phi : \overline{\mathcal{K}_{w^*}^*(Y^*, X^*)}^{w^*} \to \mathcal{K}_{w^*}(Y^*, X^*)^{**}$ in Theorem 2.1. Here note that $\mathcal{K}_{w^*}^*(Y^*, X^*) = \{T^{**} : T \in \mathcal{K}(X, Y)\}$. Hence $\Phi^{-1} i^{**} : \mathcal{K}(X, Y)^{**} \to \overline{\mathcal{K}_{w^*}^*(Y^*, X^*)}^{w^*}$ is a w^* to w^* homeomorphic isometry and $\Phi^{-1} i^{**}(j(\mathcal{K}(X, Y))) = \mathcal{K}_{w^*}^*(Y^*, X^*)$. Consequently Theorem 2.1 extends [7, Proposition 1.1].

To show Theorem 1.3 we need the following simple but useful lemma which is contained in the proof of [7, Theorem 1.5]. For the sake of completeness we provide the concrete proof.

Lemma 2.3. *Let C be a convex subset of a Banach space B and let $x^{**} \in B^{**}$. If $x^{**} \in \overline{j_B(C)}^{w^*}$ in B^{**}, then for every $\varepsilon > 0$,*

$$x^{**} \in \overline{\{j_B(x) \in j_B(C) : \|x\| < \|x^{**}\| + \varepsilon\}}^{w^*}.$$

Moreover, if $x^{**} \in j_B(B)$ and $x^{**} \in \overline{j_B(C)}^{w^*}$, then $x^{**} \in \overline{j_B(C)}$ in the topology of the norm.

Proof. Let $\varepsilon > 0$ and let U be a convex w^* closed neighborhood of x^{**}. Then $U \cap j_B(C)$ is not empty. Define the map $\psi : B^{**} \oplus B^{**} \to B^{**}$ by $\psi(x_1^{**}, x_2^{**}) = x_1^{**} - x_2^{**}$. Then ψ is clearly linear and w^* to w^* continuous. Put $V = U \cap j_B(C)$ and $W = \{j_B(x) \in j_B(B) : \|x\| < \|x^{**}\| + \varepsilon/2\}$. Note that $x^{**} \in \overline{V}^{w^*}$ and $x^{**} \in \overline{W}^{w^*}$ by Goldstine's theorem. Thus

$$0 = x^{**} - x^{**} = \psi(x^{**}, x^{**}) \in \psi(\overline{V}^{w^*} \times \overline{W}^{w^*}) = \psi(\overline{V \times W}^{w^*}) \subset \overline{\psi(V \times W)}^{w^*}.$$

Thus there exists a net $(j_B(x_\alpha), j_B(y_\alpha))$ in $V \times W$ so that $j_B(x_\alpha) - j_B(y_\alpha) \xrightarrow{w^*} 0$ in B^{**} and so $x_\alpha - y_\alpha \xrightarrow{w} 0$ in B. Define the map $\widetilde{\psi} : B \oplus B \to B$ by $\widetilde{\psi}(x_1, x_2) = x_1 - x_2$. Then $\widetilde{\psi}(x_\alpha, y_\alpha) \xrightarrow{w} 0$ in B and so $0 \in \overline{\widetilde{\psi}(j_B^{-1}(V) \times j_B^{-1}(W))}^{w} = \overline{\widetilde{\psi}(j_B^{-1}(V) \times j_B^{-1}(W))}$ in the topology of the norm because $\widetilde{\psi}(j_B^{-1}(V) \times j_B^{-1}(W))$ is a convex set in B. Thus there exist $j_B(x_1) \in V$ and $j_B(x_2) \in W$ so that $\|x_1 - x_2\| < \varepsilon/2$. Then $\|x_1\| \le \|x_2\| + \|x_1 - x_2\| < \|x^{**}\| + \varepsilon$. We have shown that $j_B(x_1) \in U \cap \{j_B(x) \in j_B(C) : \|x\| < \|x^{**}\| + \varepsilon\}$. Hence $x^{**} \in \overline{\{j_B(x) \in j_B(C) : \|x\| < \|x^{**}\| + \varepsilon\}}^{w^*}$.

The remaining part follows from convexity of C and that j_B is w to w^* homeomorphic from B onto $j_B(B)$. □

Grothendieck [8] obtained that the dual space $(\mathcal{L}(X,Y), \tau_c)^*$ consists of all functionals f of the form $f(T) = \sum_n y_n^*(Tx_n)$, where (x_n) in X, (y_n^*) in Y^*, and $\sum_n \|x_n\|\|y_n^*\| < \infty$. The *summable weak operator topology* (*swo*) on $\mathcal{L}(X,Y)$ is the topology induced by $(\mathcal{L}(X,Y), \tau_c)^*$ (see [4]). Then, for a net (T_α) in $\mathcal{L}(X,Y)$ and $T \in \mathcal{L}(X,Y)$, $T_\alpha \xrightarrow{swo} T$ if and only if $\sum_n y_n^*(T_\alpha x_n) \longrightarrow \sum_n y_n^*(Tx_n)$ for every (x_n) in X and (y_n^*) in Y^* with $\sum_n \|x_n\|\|y_n^*\| < \infty$, and $\overline{\mathcal{C}}^{\tau_c} = \overline{\mathcal{C}}^{swo}$ for every convex subset $\mathcal{C}$ of $\mathcal{L}(X,Y)$ (cf. see [4, Proposition 3.6]). We are now ready to prove Theorem 1.3.

Proof of Theorem 1.3. Suppose $T \in \overline{\mathcal{C}}^{\tau_c}$ and let $\varepsilon > 0$. By the above note there exists a net (T_α) in $\mathcal{C}$ such that

$$\sum_n (T_\alpha^* y_n^*)(x_n^*) = \sum_n y_n^*(T_\alpha x_n^*) \longrightarrow \sum_n y_n^*(Tx_n^*) = \sum_n (T^* y_n^*)(x_n^*)$$

for every (x_n^*) in X^* and (y_n^*) in Y^* with $\sum_n \|x_n^*\|\|y_n^*\| < \infty$. Thus $T^* \in \overline{\{S^* : S \in \mathcal{C}\}}^{w^*}$ in $\mathcal{L}(Y^*, X^{**})$. Let $\Phi : \overline{\mathcal{K}_{w^*}^*(X^*, Y)}^{w^*} \to \mathcal{K}_{w^*}(X^*, Y)^{**}$ be the map in Theorem 2.1. Then $\Phi(T^*) \in \overline{\Phi(\{S^* : S \in \mathcal{C}\})}^{w^*}$ in $\mathcal{K}_{w^*}(X^*, Y)^{**}$ and $\Phi(\{S^* : S \in \mathcal{C}\}) \subset j(\mathcal{K}_{w^*}(X^*, Y))$. Now by Lemma 2.3,

$$\Phi(T^*) \in \overline{\{\Phi(S^*) \in \Phi(\{S^* : S \in \mathcal{C}\}) : \|S\| < \|\Phi(T^*)\| + \varepsilon\}}^{w^*}.$$

Thus there exists a net (S_β) in $\mathcal{C}$ so that $\Phi(S_\beta^*) \xrightarrow{w^*} \Phi(T^*)$ and $\|S_\beta\| < \|T\| + \varepsilon$ for every β. Then $S_\beta^* \xrightarrow{w^*} T^*$ in $\mathcal{L}(Y^*, X^{**})$, which is equivalent to $S_\beta \xrightarrow{swo} T$ in $\mathcal{L}(X^*, Y)$. Hence, by the above note,

$$T \in \overline{\{S \in \mathcal{C} : \|S\| < \|T\| + \varepsilon\}}^{swo} = \overline{\{S \in \mathcal{C} : \|S\| < \|T\| + \varepsilon\}}^{\tau_c}. \qquad \square$$

Corollary 2.4. *Suppose that X^* or Y^* has the Radon-Nikodým property. Let $\mathcal{C}$ be a convex subset of $\mathcal{K}_{w^*}(X^*, Y)$ and let $T \in \mathcal{K}_{w^*}(X^*, Y)$. Then $T \in \overline{\mathcal{C}}^{\tau_c}$ if and only if $T \in \overline{\mathcal{C}}$ in the topology of the operator norm.*

Proof. If $T \in \overline{\mathcal{C}}^{\tau}$, then by the proof of Theorem 1.3, $\Phi(T^*) \in \overline{\Phi(\{S^* : S \in \mathcal{C}\})}^{w^*}$ in $\mathcal{K}_{w^*}(X^*, Y)^{**}$, $\Phi(\{S^* : S \in \mathcal{C}\}) \subset j(\mathcal{K}_{w^*}(X^*, Y))$ and $\Phi(T^*) \in j(\mathcal{K}_{w^*}(X^*, Y))$. By Lemma 2.3 $\Phi(T^*) \in \overline{\Phi(\{S^* : S \in \mathcal{C}\})}$. Hence $T \in \overline{\mathcal{C}}$. $\square$

3. Some properties of $\mathcal{K}_{w^*}(X^*, Y)$

The operator $T = \sum_n x_n \otimes y_n$ with $\sum_n \|x_n\|\|y_n\| < \infty$ from X^* to Y is a simple example of a w^* to w continuous compact operator because the operator is a limit of w^* to w continuous finite rank operators and the space $\mathcal{K}_{w^*}(X^*, Y)$ is closed in the topology of the operator norm. A Banach space X is reflexive if and only if the space $\mathcal{K}(X^*, Y)$ of compact operators and $\mathcal{K}_{w^*}(X^*, Y)$ are the same. Indeed, if X is nonreflexive, then there exists an $x_0^{**} \in X^{**}$ so that x_0^{**} is not a w^* continuous linear functional. Then the operator $x_0^{**}(\cdot)y \in \mathcal{K}(X^*, Y)$ for every $y \in Y$ but $x_0^{**}(\cdot)y \notin \mathcal{K}_{w^*}(X^*, Y)$. Also $\mathcal{K}(X, Y)$ is isometrically isomorphic to $\mathcal{K}_{w^*}(X^{**}, Y)$ by the map $T \leftrightarrow j_Y^{-1}T^{**}$.

The bw^* topology is strictly stronger than the w^* topology (cf. see [13, Corollary 2.7.7]). But for $T \in \mathcal{L}(X^*, Y)$, T is w^* to w continuous if and only if T is bw^* to w continuous. Indeed, if T is bw^* to w continuous, then for every net (x_α^*) in X^* and $x^* \in X^*$ with $x_\alpha^* \xrightarrow{bw^*} x^*$

$$(T^*y^*)x_\alpha^* = y^*(Tx_\alpha^*) \longrightarrow y^*(Tx^*) = (T^*y^*)x^*$$

for every $y^* \in Y^*$, which shows $T^*y^* \in (X^*, bw^*)^*$. Since $(X^*, bw^*)^* = (X^*, w^*)^*$ (see [13, Theorem 2.7.8]), $T^*(Y^*) \subset j_X(X)$. Hence T is w^* to w continuous because T is w^* to w continuous if and only if $T^*(Y^*) \subset j_X(X)$.

We now establish some criteria of w^* to w continuous compact operators.

Proposition 3.1. *For $T \in \mathcal{L}(X^*, Y)$ the following assertions are equivalent.*

(a) *T is bw^* to norm continuous.*
(b) *T is w^* to w continuous compact.*
(c) *T is bw^* to w continuous compact.*
(d) *$Tx_\alpha^* \xrightarrow{norm} Tx^*$ whenever $x_\alpha^* \xrightarrow{w^*} x^*$ in B_{X^*}.*

Proof. From the above note we only need to show (a)⇒(c)⇒(d)⇒(a).

(a)⇒(c) Let (x^*_α) be a net in B_{X^*}. Then there exists a subnet (x^*_β) of (x^*_α) and $x^* \in B_{X^*}$ so that $x^*_\beta \xrightarrow{bw^*} x^*$ because the bw^* and w^* topology are the same on B_{X^*} (see [13, Theorem 2.7.2]) and B_{X^*} is w^* compact. Thus by the assumption (a)

$$Tx^*_\beta \xrightarrow{norm} Tx^*,$$

which shows that $T(B_{X^*})$ is norm compact in Y. Hence T is bw^* to w continuous compact.

(c)⇒(d) Let (x^*_α) be a net in B_{X^*} and $x^* \in B_{X^*}$ with $x^*_\alpha \xrightarrow{w^*} x^*$. Then $x^*_\alpha \xrightarrow{bw^*} x^*$ and so $Tx^*_\alpha \xrightarrow{w} Tx^*$ by the assumption (c). Since the norm closure $\overline{T(B_{X^*})}$ in Y is norm compact, the norm and w topology are the same on $\overline{T(B_{X^*})}$. Hence

$$Tx^*_\alpha \xrightarrow{norm} Tx^*.$$

(d)⇒(a) If $Tx^*_\alpha \xrightarrow{norm} Tx^*$ whenever $x^*_\alpha \xrightarrow{w^*} x^*$ in B_{X^*}, then $Tx^*_\alpha \xrightarrow{norm} Tx^*$ whenever $t > 0$ and $x^*_\alpha \xrightarrow{w^*} x^*$ in tB_{X^*}. Therefore T is w^* to norm continuous with respect to the relative w^* topology of tB_{X^*} whenever $t > 0$. Let V be a norm open set in Y. Then for every $t > 0$, $T^{-1}(V) \cap tB_{X^*}$ is a relatively w^* open set in tB_{X^*}. By [13, Corollary 2.7.4] $T^{-1}(V)$ is a bw^* open set in X^*. Hence T is bw^* to norm continuous. □

Now we summarize some results for the space $\mathcal{K}_{w^*}(X^*, Y)$. First, we comment on the dual space of $\mathcal{K}_{w^*}(X^*, Y)$ (see P. Harmand, D. Werner and W. Werner [9, pp. 265, 266]). We say that a linear functional φ on $\mathcal{K}_{w^*}(X^*, Y)$ is an *integral linear functional* if there exists a regular Borel measure μ on $B_{X^*} \times B_{Y^*}$, where B_{X^*} and B_{Y^*} are equipped with the w^* topology, so that

$$\varphi(T) = \int_{B_{X^*} \times B_{Y^*}} y^*(Tx^*) d\mu$$

for all $T \in \mathcal{K}_{w^*}(X^*, Y)$. We denote the space of integral linear functionals on $\mathcal{K}_{w^*}(X^*, Y)$ by $\mathcal{I}_{w^*}$ and define the norm on $\mathcal{I}_{w^*}$ by

$$\|\varphi\|_I = \inf\{\|\mu\| : \mu \text{ represents } \varphi\}.$$

Let $C(B_{X^*} \times B_{Y^*})$ be the Banach space of scalar valued continuous functions on $B_{X^*} \times B_{Y^*}$. Our first application of Proposition 3.1 is for the proof of the following well-known and very useful observation.

Lemma 3.2. *$\mathcal{K}_{w^*}(X^*, Y)$ is isometrically isomorphic to a subspace of $C(B_{X^*} \times B_{Y^*})$.*

Proof. We consider the map $\Lambda : \mathcal{K}_{w^*}(X^*, Y) \to C(B_{X^*} \times B_{Y^*})$ defined by

$$\Lambda(T)(x^*, y^*) = y^*(Tx^*).$$

From Proposition 3.1(d), it is easy to check that $\Lambda(T) \in C(B_{X^*} \times B_{Y^*})$ for all $T \in \mathcal{K}_{w^*}(X^*, Y)$ and Λ is a linear isometry. Hence the conclusion follows. □

We are now ready to represent the dual space of $\mathcal{K}_{w^*}(X^*, Y)$.

Theorem 3.3. *$\mathcal{K}_{w^*}(X^*, Y)^*$ is isometrically isomorphic to $\mathcal{I}_{w^*}$.*

Proof. If $\psi \in \mathcal{K}_{w^*}(X^*, Y)^*$, then by Lemma 3.2, Hahn-Banach extension and Riesz representation theorem, there exists a regular Borel measure μ on $B_{X^*} \times B_{Y^*}$ such that
$$\psi(T) = \int_{B_{X^*}\times B_{Y^*}} y^*(Tx^*)d\mu$$
for all $T \in \mathcal{K}_{w^*}(X^*, Y)$ and $\|\psi\| = \|\mu\|$ and so $\|\psi\| \geq \|\psi\|_I$. Also for every such representation ν of ψ, we see $\|\psi\| \leq \|\nu\|$. Hence $\|\psi\| = \|\psi\|_I$. Since for every $\varphi \in \mathcal{I}_{w^*}$ clearly $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$, the conclusion follows. □

Remark 3.4. Under the assumption that X^* or Y^* has the Radon-Nikodym property, elements of $\mathcal{K}_{w^*}(X^*, Y)^*$ can be represented by a series form, more precisely, for every $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$ and $\varepsilon > 0$ there exist (x_n^*) in X^* and (y_n^*) in Y^* with $\sum_n \|x_n^*\|\|y_n^*\| < \|\varphi\| + \varepsilon$ such that $\varphi(T) = \sum_n y_n^*(Tx_n^*)$ for all $T \in \mathcal{K}_{w^*}(X^*, Y)$. Indeed, if X^* or Y^* has the Radon-Nikodym property, then the map $V : Y^* \otimes_\pi X^* \to \mathcal{K}_{w^*}(X^*, Y)^*$, in the proof of Theorem 2.1, is a quotient map. Thus for every $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$ there exists $v = \sum_n y_n^* \otimes x_n^* \in Y^* \otimes_\pi X^*$ with $\|v\|_\pi = \|\varphi\|$ such that $\varphi(T) = \sum_n y_n^*(Tx_n^*)$ for all $T \in \mathcal{K}_{w^*}(X^*, Y)$. Another proof of this was presented in [3, Theorem 1.2].

We need the following lemma to obtain a more concrete representation of $\mathcal{K}_{w^*}(X^*, Y)^*$ than the one in Remark 3.4 when X^* is separable.

Lemma 3.5 ([12, Lemma 1.e.16])**.** *Let X be a separable Banach space and $\varepsilon > 0$. Then there exists a sequence $(f_i)_{i=1}^\infty$ of functions on B_X so that $x = \sum_{i=1}^\infty f_i(x)$, for every $x \in B_X$, each $f_i(x)$ is of the form $\sum_{j=1}^\infty \chi_{E_{i,j}}(x)x_{i,j}$, where $\{E_{i,j}\}_{j=1}^\infty$ are disjoint Borel subsets of B_X, $\{x_{i,j}\}_{j=1}^\infty \subset B_X$ and*
$$\sum_{i=1}^\infty \|f_i\|_\infty < 1 + \varepsilon \text{ with } \|f_i\|_\infty = \sup_x \|f_i(x)\| = \sup_j \|x_{i,j}\|.$$

We now have:

Corollary 3.6. *Suppose that X^* is separable. If $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$, then for $\varepsilon > 0$ there exist sequences $(x_{i,j}^*)$ in X^* and $(y_{i,j}^*)$ in Y^* with $\sum_{i=1}^\infty \sup_j \|x_{i,j}^*\| < 1 + \varepsilon$ and $\sum_{j=1}^\infty \|y_{i,j}^*\| \leq \|\varphi\|$ for every i so that*
$$\varphi(T) = \sum_{i=1}^\infty \sum_{j=1}^\infty y_{i,j}^*(Tx_{i,j}^*)$$
for every $T \in \mathcal{K}_{w^*}(X^*, Y)$.

Proof. Let $\varphi \in \mathcal{K}_{w^*}(X^*, Y)^*$ and let $\varepsilon > 0$. Then by Theorem 3.3 there exists a regular Borel measure μ on $B_{X^*} \times B_{Y^*}$ with $\|\varphi\| = \|\mu\|$ so that
$$\varphi(T) = \int_{B_{X^*}\times B_{Y^*}} y^*(Tx^*)d\mu$$

for every $T \in \mathcal{K}_{w^*}(X^*, Y)$. Then by Lemma 3.5, for every $T \in \mathcal{K}_{w^*}(X^*, Y)$,

$$\begin{aligned}\varphi(T) &= \int_{B_{X^*}\times B_{Y^*}} y^*T\Big(\sum_{i=1}^{\infty} f_i(x^*)\Big)d\mu \\ &= \sum_{i=1}^{\infty}\int_{B_{X^*}\times B_{Y^*}} y^*T\Big(\sum_{j=1}^{\infty} \chi_{E_{i,j}}(x^*)x^*_{i,j}\Big)d\mu \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\int_{E_{i,j}\times B_{Y^*}} y^*(Tx^*_{i,j})d\mu \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} y^*_{i,j}(Tx^*_{i,j}),\end{aligned}$$

where $y^*_{i,j}$ is the functional on Y defined by $y^*_{i,j}(y) = \int_{E_{i,j}\times B_{Y^*}} y^*(y)d\mu$. Since for every i, j, and $y \in B_Y$ $|y^*_{i,j}(y)| \le \int_{E_{i,j}\times B_{Y^*}} |y^*(y)|d|\mu| \le |\mu|(E_{i,j} \times B_{Y^*})$, $\|y^*_{i,j}\| \le |\mu|(E_{i,j} \times B_{Y^*})$ for every i and j. Hence for every i, we have $\sum_{j=1}^{\infty} \|y^*_{i,j}\| \le \|\mu\| = \|\varphi\|$ and $\sum_{i=1}^{\infty} \sup_j \|x^*_{i,j}\| < 1 + \varepsilon$. □

Next we present a variant of a result of Kalton [10]. Recall the *weak operator topology* (*wo*) on $\mathcal{L}(X, Y)$. For a net (T_α) in $\mathcal{L}(X, Y)$ and $T \in \mathcal{L}(X, Y)$ $T_\alpha \overset{wo}{\longrightarrow} T$ if and only if $y^*(T_\alpha x) \longrightarrow y^*(Tx)$ for every $x \in X$ and $y^* \in Y^*$. The following are the $\mathcal{K}_{w^*}(X^*, Y)$ versions of [10, Theorem 1] and [10, Corollary 3], respectively.

Proposition 3.7. *Let $\mathcal{A}$ be a subset of $\mathcal{K}_{w^*}(X^*, Y)$. Then $\mathcal{A}$ is wo compact if and only if $\mathcal{A}$ is weakly compact.*

Corollary 3.8. *Let (T_n) be a sequence in $\mathcal{K}_{w^*}(X^*, Y)$ and $T \in \mathcal{K}_{w^*}(X^*, Y)$. Then $T_n \overset{wo}{\longrightarrow} T$ if and only if $T_n \overset{weak}{\longrightarrow} T$.*

Finally we consider a factorization of elements in $\mathcal{K}_{w^*}(X^*, Y)$.

Lemma 3.9 ([11, Lemma 1.1 and Theorem 2.2])**.** *If $T \in \mathcal{K}(X, Y)$, then there exist a separable reflexive Banach space Z with $\overline{T(B_X)}/\|T\| \subset B_Z \subset B_Y$, $S \in \mathcal{K}(X, Z)$, and the inclusion map $J \in \mathcal{K}(Z, Y)$ such that $\|J\| = 1$, $T = JS$, and $\|S\| = \|T\|$.*

The following theorem is essentially contained in Aron, Lindström, Ruess, Ryan [1], and Mikkor, Oja [14]. But we use Proposition 3.1 to slightly simplify the existing proof.

Proposition 3.10. *If $T \in \mathcal{K}_{w^*}(X^*, Y)$, then there exist a separable reflexive Banach space Z, $R \in \mathcal{K}_{w^*}(X^*, Z^{**})$ with $\|R\| = \|T\|$, $U \in \mathcal{K}_{w^*}(Z^{**}, Y)$ with $\|U\| = 1$ such that $T = UR$.*

Proof. Let $T \in \mathcal{K}_{w^*}(X^*, Y)$. Then by Lemma 3.9, there exist a separable reflexive Banach space Z with $\overline{T(B_{X^*})}/\|T\| \subset B_Z \subset B_Y$, $S \in \mathcal{K}(X^*, Z)$, and the inclusion map $J \in \mathcal{K}(Z, Y)$ such that $\|J\| = 1$, $T = JS$, and $\|S\| = \|T\|$. Let $R = j_Z S \in \mathcal{K}(X^*, Z^{**})$ and $U = J j_Z^{-1} \in \mathcal{K}(Z^{**}, Y)$. Then $\|R\| = \|T\|$, $\|U\| = 1$, and $T = UR$. If (x^*_α) in B_{X^*} and $x^* \in B_{X^*}$ with $x^*_\alpha \xrightarrow{w^*} x^*$, then by Proposition 3.1(d)

$$Tx^*_\alpha \xrightarrow{\|\cdot\|_Y} Tx^*.$$

Since $(Tx^*_\alpha/\|T\|)$ and $Tx^*/\|T\|$ in $\overline{T(B_{X^*})}/\|T\|$, by [11, Lemma 2.1(ii)]

$$Tx^*_\alpha/\|T\| \xrightarrow{\|\cdot\|_Z} Tx^*/\|T\|.$$

Consequently $Tx^*_\alpha \xrightarrow{\|\cdot\|_Z} Tx^*$ and so $Sx^*_\alpha \xrightarrow{\|\cdot\|_Z} Sx^*$ because $Sx^* = Tx^*$ for all $x^* \in X^*$ (see [11, Theorem 2.2]). Therefore

$$Rx^*_\alpha = j_Z Sx^*_\alpha \xrightarrow{\|\cdot\|_{Z^{**}}} j_Z Sx^* = Rx^*.$$

Hence $R \in \mathcal{K}_{w^*}(X^*, Z^{**})$ by Proposition 3.1(d). Since Z is reflexive, $U \in \mathcal{K}_{w^*}(Z^{**}, Y)$. □

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