Transceiver Design Using Local Channel State Information at Relays for A Multi-Relay Multi-User MIMO Network

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Abstract

In this paper, we propose an iterative transceiver design in a multi-relay multi-user multiple-input multiple-output (MIMO) system. The design criterion is to minimize sum mean squared error (SMSE) under relay sum power constraint (RSPC) where only local channel state information (CSI)s are available at relays. Local CSI at a relay is defined as the CSI of the channel between BS and the relay in the 1st hop link, and the CSI of the channel between the relay and all users in the 2nd hop link. Exploiting BS transmitter structure which is concatenated with block diagonalization (BD) precoder, each relay's precoder can be determined using local CSI at the relay. The proposed scheme is based on sequential iteration of two stages; stage 1 determines BS transmitter and relay precoders jointly with SMSE duality, and stage 2 determines user receivers. We verify that the proposed scheme outperforms simple amplify-and-forward (SAF), minimum mean squared error (MMSE) relay, and an existing good scheme of [13] in terms of both SMSE and sum-rate performances.

Keywords: MIMO, relay, MSE duality, precoding, transceiver optimization

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1. Introduction

Mutiple-input multiple-output (MIMO) and relay have been dealt with serviceable technologies which greatly contribute to the beyond fourth generation (4G) wireless system. In [1], various relay technologies embodied in long term evolution – advanced (LTE-A) standard are described, and some open research issues which should be resolved in order for relay technology to be practically incorporated into the 4G standard are presented. In a realistic network environment which suffers from significant path loss and shadowing, relays are essential to overcome severe signal power attenuation. Amplify-and-forward (AF) and decode-and-forward (DF) are conventional relaying protocols [2], which are also known as non-regenerative and regenerative relaying, respectively. In a practical implementation perspective, AF relay has been a preferred protocol for its simplicity since DF relay requires processing delay for full decoding. As a solution to improve the QoS of multiple end-users, the deployment of multiple AF relays with multiple antennas can be considered. If all users are also assumed to equip multiple antennas in this case, the property of this all-MIMO network can be efficiently exploited to enhance the end-users' performances. As a simple example, we can also consider that one-to-one coupling between relays and users, which means that each user is served by a certain relay. Similarly, we can imagine that one-to-many coupling between relays and users. In these cases, BS transmits each user's desirable data streams to the user's corresponding relay in the 1^{st} hop channel. Each relay processes the received signal and transmits to their serving users in the 2^{nd} hop channel. Since users experience interference from the relays serving other user groups, interference management among relays should be considered to use multiple antennas of relays and users. According to the relay deployment scenarios above, we should design the linear processing matrices at BS, relay, and users in order to maximize the performance of the network in a given operation environment. To avoid confusion of terminology, we denote the linear processing matrices at BS, each relay, and each user, as transmitter, precoder, and receiver, respectively, for the rest of this paper. Thus, transceiver design for MRMU MIMO network means to determine BS transmitter, relay precoders and user receivers.

There have been several researches dealing with multi-relay multi-user (MRMU) network. Zhao et al. [3] proposed cooperative relaying transmission schemes for the network in which there are two users and two decode-and-forward (DF) relays with single BS, and multiple antennas are equipped to relays and users. Authors optimized the 1^{st} hop link and the 2^{nd} hop link separately rather than jointly. Well known singular value decomposition (SVD) transmission scheme [4] is used for the 1^{st} hop channel and two relays cooperatively choose their beamforming vectors so that each user's desired data can be combined with maximum ratio combining (MRC) [4] for the 2^{nd} hop channel. However, they cannot be easily extended to the network setting of arbitrary numbers of relays and users. Long et al. [5] proposed an energy efficient relaying scheme MRMU network, where multiple antennas are employed to BS while single antenna is employed to each relay and each user in the network. Proposed design of [5] chooses the coefficient of each relay to mitigate inter-user interference at users and minimize transmit power of relays. This cannot be directly applied to the network with multiple antenna relay. In addition, authors of [5] assumed BS transmitter to be zero-forcing (ZF) precoder to make the problem more tractable, which is not certainly an optimal way. Talebi el al. [6] proposed a relaying scheme for MRMU network where BS and each relay employ multiple antennas and each user employs single antenna. It was designed to increase sum rate with the assumption of DF relaying. The schemes in [6] assumed that BS transmitter is fixed to ZF-dirty paper coding (DPC) precoder [7] and each user employs single antenna. After all, even if there have been several researches which investigated transceiver design for MRMU network, relays or users are assumed to have single antenna and BS transmitter is fixed to ZF or ZF-DPC precoder for simplicity of analysis. If multiple antennas are equipped to all nodes and BS transmitter is not fixed to predefined precoder, the transceiver design problem becomes much more involved. Many properties of ultimate sum mean squared error (SMSE) and sum rate of MRMU MIMO networks have yet to be investigated. It should be also noted that there are still many research problems to be studied for regarding the general scope of the MRMU MIMO investigation.

[8], [9] and [10] addressed precoder designs in a multipoint-to-multipoint network with multiple relays. Phan et al. [8] considered multi-relay assisted multi pair network in which all nodes equip single antenna and investigated on the determination of relay coefficients (relay amplification factor), modeling coefficients of multiple relay as a beamforming vector. They presented three kinds of optimization problem such as beamforming with minimum relay power, minimax optimization of individual beamforming power, and beamforming optimization with orthogonal source transmission. Non-smooth optimization algorithms were proposed as a method to solve three optimization problems. Oyman and Paulraj introduced matched filter (MF), ZF, and linear minimum mean squared error (MMSE) relaying and provided performance verification of those schemes in terms of per stream signal to interference ratio (SIR) distribution in [9]. Chalise and Vandendorpe [10] proposed a relay precoder design to satisfy each user's target SINR. Focuses of [9][10] were limited to the optimization of relay precoders without considering the source precoder structure. Moreover, each source and destination is paired one to one. Thus our MRMU MIMO network is not special case of multipoint-to-multipoint network since one BS serves multiple users.

Most transceiver design in relay problems require all channel state information (CSI) in the system with the assumption on centralized processing. Xing et al. [11] provided a unified linear MMSE transceiver design framework based on quadratic matrix programming (QMP) for various wireless networks, which include multi source, relay and multi user MIMO network. In this paper, we are more interested in relay precoder design with local CSIs at a relay to make it more practical in a relay network. We proposed a transceiver design to minimize user SMSE under relay sum power constraint (RSPC) where only local CSIs are available at relays. Local CSI at a relay is defined as the CSI of the channel between BS and the relay in the 1st hop link, and the CSI of the channel between the relay and all users in the 2nd hop link. **Fig. 1** illustrates the local channel of relays. In order to make it possible to design relay precoder with local CSIs at relays, we employ BS transmitter structure composed of two concatenated matrices, of which the former part is fixed to block-diagonalization (BD) precoder [12]. The latter part of the BS transmitter is jointly optimized with relay precoders. Thus, the proposed scheme has advantage over the schemes in [5][6] which simply fix the total BS transmitter to be ZF based precoder. The proposed scheme is based on sequential iteration of two stages. In stage 1, the latter part of BS transmitter and relay precoders are jointly determined by applying SMSE duality for the 2nd hop channel with fixed user receivers. In stage 2, user receivers are determined to MMSE receivers with fixed BS transmitter and relay precoders. Algorithm repeats iteratively until it converges. We numerically verify that the proposed scheme outperforms simple AF (SAF), MMSE relaying [9], and BD-MMSE [13] in terms of both SMSE and sum rate performances. The proposed algorithm considered in this paper is yet to be practical. However, it is expected that this work gives a foundation to examine practical design in the MRMU MIMO network by introducing useful tools such as

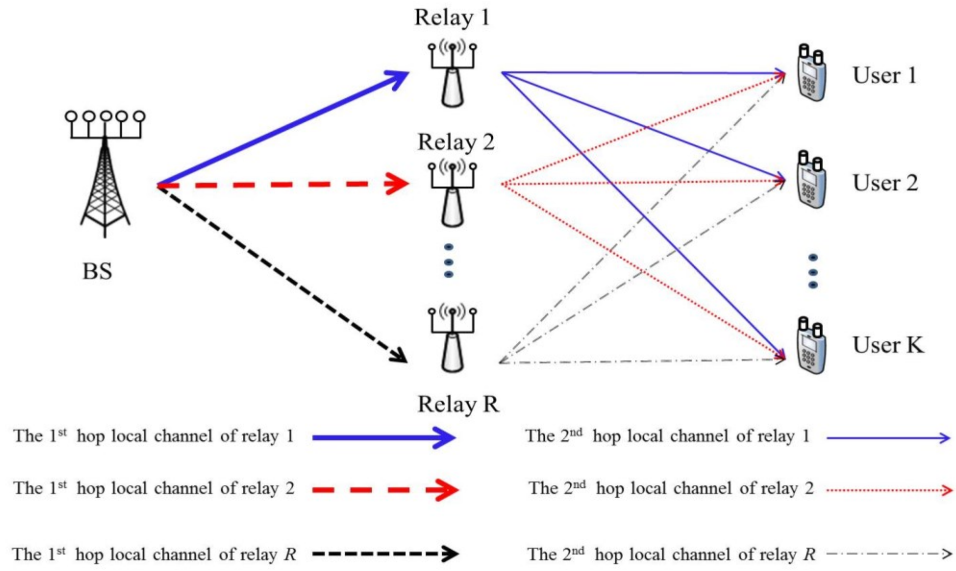

Fig. 1. Multiple relay multi-user MIMO system

BD precoder and SMSE duality and showing how to approach the transceiver design problem. To make the problem more practical, per antenna power constraint (PAPC) or imperfect CSI may be additionally considered, which is left for future research.

The following notations are introduced for the rest of the paper. We use $[\cdot]^H$ and $\text{blkdiag}[\cdot]$ for the transpose, the Hermitian, and the block diagonal matrix composed of $[\cdot]$, respectively. $\text{E}[\cdot]$, $\|\cdot\|_F$, and $\text{tr}[\cdot]$ denote the expectation, the Frobenius norm, and the trace operator, respectively. Boldface uppercase and lowercase fonts denote matrices and vectors, respectively. Finally, $\mathbf{I}_X$ denotes the identity matrix of rank X.

The remainder of this paper is organized as follows. The mathematical expression of the considered system model is specified in section II. In section III, we explain theoretical foundation of transceiver design with only local CSI and provide the structure of BS transmitter. Specific transceiver design procedure is described in section IV. Numerical results are provided in section V. Finally, we make conclusions in section VI.

2. System Model

A downlink network which consists of a single BS, R relays and K users is considered as in **Fig. 1**. All the nodes in the system equip multiple antennas and the number of antennas of BS, each relay, and each user is denoted as N_S, N_R, and N_U, respectively. The number of the k^{th} user's desired data streams is denoted by L_k and BS transmits $L = \sum_{k=1}^{K} L_k \le N_S$ data streams. The k^{th} user's data $\mathbf{x}_k \in \mathbf{C}^{L_k \times 1}$ is transmitted to a relay after being precoded by $\mathbf{T}_{\text{BS},k} \in \mathbf{C}^{N_S \times L_k}$ at BS. The received signal at the r^{th} relay is written by

$$\mathbf{y}_r = \mathbf{G}_r \mathbf{T}_{\text{BS}} \mathbf{x} + \mathbf{n}_{1,r} \tag{1}$$

where $\mathbf{G}_r \in \mathbf{C}^{N_R \times N_S}$ denotes the 1st hop channel matrix from BS to the r^{th} relay, and $\mathbf{n}_{1,r}$ denotes noise vector at the r^{th} relay. Two stacked matrix $\mathbf{x} = \left[\mathbf{x}_1^H \quad \mathbf{x}_2^H \quad \cdots \right. \quad \in \mathbf{C}^{L\times 1}$ and $\mathbf{T}_{\text{BS}} = \left[\mathbf{T}_{\text{BS},1} \quad \mathbf{T}_{\text{BS},2} \quad \cdots \quad \right]_k \in \mathbf{C}^{N_S \times L}$ are introduced. The received signal at all relays can be written as

$$\mathbf{y} = \mathbf{G}\mathbf{T}_{\text{BS}}\mathbf{x} + \mathbf{n}_1 \tag{2}$$

where $\mathbf{n}_1 = \left[\mathbf{n}_{1,1}^H \quad \mathbf{n}_{1,2}^H \quad \cdots \right. \quad ^H \in \mathbf{C}^{N_R R \times 1}$ and $\mathbf{G} = \left[\mathbf{G}_1^H \quad \mathbf{G}_2^H \quad \cdots \right. \quad ^H \in \mathbf{C}^{N_R R \times N_S}$.
The r^{th} relay weights $\mathbf{y}_r$ by $\mathbf{W}_r \in \mathbf{C}^{N_R \times N_R}$ and transmits it to users. Stacked precoder matrix for all relays can be written as $\mathbf{W} = \text{blkdiag}\left[\mathbf{W}_1 \quad \mathbf{W}_2 \quad \cdots \right. \quad \in \mathbf{C}^{N_R R \times N_R R}$. The 2nd hop MIMO channel between the r^{th} relay and the k^{th} user is denoted by $\mathbf{H}_{kr} \in \mathbf{C}^{N_U \times N_R}$. For notational convenience, we define three kinds of stacked matrices, relay-wide, user-wide, and system-wide 2nd hop channel matrices.

$$\begin{aligned} \mathbf{H}_r &= \left[\mathbf{H}_{1r}^H \quad \mathbf{H}_{2r}^H \quad \cdots \right. \quad ^H \in \mathbf{C}^{N_U K \times N_R} \\ \hat{\mathbf{H}}_k &= \left[\mathbf{H}_{k1} \quad \mathbf{H}_{k2} \quad \cdots \right. \quad \in \mathbf{C}^{N_U \times N_R R} \\ \mathbf{H} &= \left[\mathbf{H}_1 \quad \mathbf{H}_2 \quad \cdots \right. \quad \in \mathbf{C}^{N_U K \times N_R R} \end{aligned} \tag{3}$$

The k^{th} user's received signal $\mathbf{z}_k \in \mathbf{C}^{N_R \times N_S}$ is given by

$$\mathbf{z}_k = \hat{\mathbf{H}}_k \mathbf{W}\left(\mathbf{G}\mathbf{T}\mathbf{x} + \mathbf{n}_1\right) + \mathbf{n}_{2,k} \tag{4}$$

where $\mathbf{n}_{2,k}$ is additive noise at the receiver of the k^{th} user. With the stacked matrices $\mathbf{z} = \left[\mathbf{z}_1^H \quad \mathbf{z}_2^H \quad \cdots \right. \quad \in \mathbf{C}^{N_U K \times 1}$, and $\mathbf{n}_2 = \left[\mathbf{n}_{2,1}^H \quad \mathbf{n}_{2,2}^H \quad \cdots \right. \quad ^H \in \mathbf{C}^{N_U K \times 1}$, the received signal for all users can be written by

$$\mathbf{z} = \mathbf{H}\mathbf{W}\left(\mathbf{G}\mathbf{T}\mathbf{x} + \mathbf{n}_1\right) + \mathbf{n}_2 \tag{5}$$

Finally, $\mathbf{z}_k$ is equalized by linear filter $\mathbf{R}_k \in \mathbf{C}^{L_k \times N_U}$ in order to estimate its desired data $\hat{\mathbf{x}}_k$, i.e. $\hat{\mathbf{x}}_k = \mathbf{R}_k \mathbf{z}_k$. The stacked receiver matrix is given by $\mathbf{R} = \text{blkdiag}\left[\mathbf{R}_1 \quad \mathbf{R}_2 \quad \cdots \right.$ $\in \mathbf{C}^{L \times N_R R}$. The error covariance matrix of the system can be represented as follows.

$$\begin{aligned} \mathbf{E} &= \text{E}\left[\left(\mathbf{x} - \hat{\mathbf{x}}\right)\left(\mathbf{x} - \hat{\mathbf{x}}\right)^H\right] \\ &= \left(\mathbf{RHWGT} - \mathbf{I}_L\right)\left(\mathbf{RHWGT} - \mathbf{I}_L\right)^H + \sigma_1^2 \mathbf{RHWW}^H\mathbf{H}^H\mathbf{R}^H + \sigma_2^2 \mathbf{RR}^H \end{aligned} \tag{6}$$

where $\hat{\mathbf{x}} = \left[\hat{\mathbf{x}}_1^H \quad \hat{\mathbf{x}}_2^H \quad \cdots \right. \quad \in \mathbf{C}^{L\times 1}$. $\mathbf{x}$ is assumed to be uncorrelated Gaussian random vector which has zero mean and identity auto-covariance matrix. Both $\mathbf{n}_1$ and $\mathbf{n}_2$ are also assumed to be uncorrelated Gaussian random vector with zero mean and $\text{E}\left[\mathbf{n}_1\mathbf{n}_1^H\right] = \sigma_1^2 \mathbf{I}_{N_R R}$ and $\text{E}\left[\mathbf{n}_2\mathbf{n}_2^H\right] = \sigma_2^2 \mathbf{I}_{N_U K}$.

Our proposing scheme is applicable for the network configuration in which the number of users is larger than that of relays. In this case, multiple users are classified into each relay's serving group, and then one relay may serve multiple users. If we set the index of relay which serves the k^{th} user to $r(k)$, then BS transmits the k^{th} user's desirable data to the $r(k)$-th relay. Other relays generate interference to the k^{th} user. For mathematical convenience in the derivation of algorithm, we assume that the number of relays and users are fixed to be the same

$(R=K)$. It is also assumed that $r(k)=k$, which means that the k^{th} user is served by the relay which has the same index k. For the feasibility of the scenario, we assume that $L_k \le \min\{N_U, N_R\}$ for all k.

3. The Structure of BS Transmitter

To solve joint BS transmitter and relay precoder design problem for SMSE minimization, we can formulate the following optimization problem.

$$\begin{aligned} &\min_{\mathbf{T},\, \mathbf{W}_r, \forall r,\, \mathbf{R}_k, \forall k} \operatorname{tr}[\mathbf{E}] \\ &\operatorname{tr}\left[\mathbf{T}\mathbf{T}^H\right] \le P_{\text{BS}} \\ &\operatorname{tr}\left[\mathbf{W}\left(\mathbf{G}\mathbf{T}\mathbf{T}^H\mathbf{G}^H + \sigma_1^2 \mathbf{I}_{N_R R}\right)\mathbf{W}^H\right] \le P_{\text{relay}}^{\text{total}} \end{aligned} \tag{7}$$

where P_{BS} denotes the maximum power of a BS, $P_{\text{relay}}^{\text{total}}$ denotes maximum total power of all relays. The determination of the r^{th} relay precoder $\mathbf{W}_r$ generally requires CSI of the 2[nd] hop channel between the other relays and all users. It results in increased signaling overhead and making practical implementation be formidable. In order to understand why the other relays' CSI is needed, we rewrite the objective function of (7) as follows.

$$\begin{aligned} \operatorname{tr}[\mathbf{E}] =& \sum_{k=1}^{K}\sum_{r=1}^{R} \operatorname{tr}\left[\mathbf{R}_k \mathbf{H}_{kr} \mathbf{W}_r \left(\mathbf{G}_r \mathbf{T}_{\text{BS}} \mathbf{T}_{\text{BS}}^H \mathbf{G}_r^H + \sigma_1^2 \mathbf{I}_{N_R}\right) \mathbf{W}_r^H \mathbf{H}_{kr}^H \mathbf{R}_k^H\right] \\ &+ \sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{l=1, l\ne r}^{R} \operatorname{tr}\left[\mathbf{R}_k \mathbf{H}_{kr} \mathbf{W}_r \mathbf{G}_r \mathbf{T}_{\text{BS}} \mathbf{T}_{\text{BS}}^H \mathbf{G}_l^H \mathbf{W}_l^H \mathbf{H}_{kl}^H \mathbf{R}_k^H\right] \\ &- \sum_{k=1}^{K}\sum_{r=1}^{R} \operatorname{tr}\left[\mathbf{R}_k \mathbf{H}_{kr} \mathbf{W}_r \mathbf{G}_r \mathbf{T}_{\text{BS}}\right] - \sum_{k=1}^{K}\sum_{r=1}^{R} \operatorname{tr}\left[\mathbf{T}_{\text{BS}}^H \mathbf{G}_r^H \mathbf{W}_r^H \mathbf{H}_{kr}^H \mathbf{R}_k^H\right] \\ &+ \sigma_2^2 \sum_{k=1}^{K} \operatorname{tr}\left[\mathbf{R}_k \mathbf{R}_k^H\right] + L \end{aligned} \tag{8}$$

Observe that $\mathbf{W}_r$ and $\mathbf{G}_l^H \mathbf{W}_l^H \mathbf{H}_{kl}^H$, $\forall l \ne r$ are multiplied in the 2[nd] term in the right hand side of (8). It means that they jointly affect SMSE. Thus the determination of $\mathbf{W}_r$ requires $\mathbf{G}_l^H \mathbf{W}_l^H \mathbf{H}_{kl}^H$, $\forall l \ne r$.

We introduce BD precoder [12] to eliminate the 2[nd] term in the right hand side of (8). Originally, BD precoder is a well known precoder which block-diagonalizes downlink channel in the MIMO BC network, so that it can make inter-user interference be perfectly eliminated. Let us consider a downlink multi-user MIMO network which consists of a BS and K users. The number of BS antenna and the number of antenna of the k^{th} user is denoted as n_T and n_{R_k}, respectively, and $n_R = \sum_{k=1}^{K} n_{R_k}$. Let $\mathbf{M}_k$ and $\mathbf{H}_k$ be the transmit matrix which precodes the k^{th} user's data streams and the channel matrix from BS to the k^{th} user, respectively. We denote the other channels, which means that the channel matrix from BS to other users, as $\tilde{\mathrm{I}}$, i. e. $\tilde{\mathrm{I}}$ $= \left[\mathbf{H}_1^T, \cdots \right.$ $\cdots$ $\in \mathbf{C}^{\left(n_R - n_{R_k}\right) \times n_T}$. $\mathbf{M}_k$ projects the k^{th} user's data streams into the null space of $\tilde{\mathrm{I}}$. When $\tilde{\imath}$ is a rank of $\tilde{\mathrm{I}}$, the SVD of $\tilde{\mathrm{I}}$ can be written as $\tilde{\mathrm{I}}$, where $\tilde{\text{\textbackslash}}$ means the first $\tilde{\imath}$ right singular vectors, and [illegible] means the last $(n_T - $ [illegible] singular vectors. Since [illegible] forms an orthogonal basis for the null space of [illegible], we set $\mathbf{M}_k$ to be [illegible]. By $\mathbf{M}_k$, the k^{th} user's desired data streams is nullified at the other users. Consequently, there is no user which experiences other user interferences. We call the column-stacked matrix $\mathbf{M} = [\mathbf{M}_1 \quad \mathbf{M}_2 \quad \cdots$ [illegible] as BD precoder. BD precoder effectively transforms downlink channel to block diagonal matrix.

We apply this BD precoder to block-diagonalize the 1st hop channel of our network. For the feasibility of BD precoder at BS, we assume that $N_S > N_R(K-1)$ and $L_k \le N_S - N_R(K-1)$ for all k [12]. BS transmitter is assumed to have following structure.

$$\mathbf{T}_{\text{BS}} = \mathbf{T}_{\text{BD}}\mathbf{T} \tag{9}$$

where $\mathbf{T}_{\text{BD}} = [\mathbf{T}_{\text{BD},1} \quad \mathbf{T}_{\text{BD},2} \quad \cdots$ [illegible] $] \in \mathbf{C}^{N_S \times \bar{L}}$ is the BD precoder (the former part) and $\mathbf{T} \in \mathbf{C}^{\bar{L} \times L}$ is an additional precoder (the latter part). $\mathbf{T}_{\text{BD},k} \in \mathbf{C}^{N_S \times \bar{L}_k}$ makes the k^{th} relay receive the k^{th} user's data streams without suffering interference from the other users' data streams. Note that the k^{th} user is served by the k^{th} relay as explained in the previous section. $\bar{L}_k$ denotes the rank of the $\mathbf{G}\mathbf{T}_{\text{BD},k}$ which is the effective 1st hop channel for the k^{th} user's data streams, and $\bar{L} = \sum_{k=1}^{K} \bar{L}_k$. If $N_S \ge N_R K$, then $\bar{L}_k = N_R$. Otherwise, $\bar{L}_k = L_k$. Thus it always holds that $\bar{L}_k \ge L_k$. The 1st hop channel after being precoded by $\mathbf{T}_{\text{BD}}$ is block-diagonalized. It is denoted by $\mathbf{G}_{\text{eff}} = \mathbf{G}\mathbf{T}_{\text{BD}}$, and the k^{th} block diagonal element of $\mathbf{G}_{\text{eff}}$ is denoted by $\mathbf{G}_{\text{eff},k} \in \mathbf{C}^{N_R \times \bar{L}_k}$. We also assume that $\mathbf{T}$ has block diagonal structure, where the k^{th} block diagonal element $\mathbf{T}_k \in \mathbf{C}^{\bar{L}_k \times L_k}$ precodes the k^{th} user's data streams. The determination of $\mathbf{T}_k$ is discussed in the next section. Inserting $\mathbf{T}_{\text{BS}}$ in (9) into (8), we obtain the following.

$$\begin{aligned}\text{tr}[\mathbf{E}] = &\sum_{k=1}^{K}\sum_{r=1}^{R} \text{tr}\left[\mathbf{R}_k \mathbf{H}_{kr} \mathbf{W}_r \left(\mathbf{G}_{\text{eff},r}\mathbf{T}_r\mathbf{T}_r^H\mathbf{G}_{\text{eff},r}^H + \sigma_1^2 \mathbf{I}_{N_R}\right)\mathbf{W}_r^H \mathbf{H}_{kr}^H \mathbf{R}_k^H\right] \\ &- \sum_{k=1}^{K}\sum_{r=1}^{R} \text{tr}\left[\mathbf{R}_k \mathbf{H}_{kr} \mathbf{W}_r \mathbf{G}_{\text{eff},r}\mathbf{T}_r\right] - \sum_{k=1}^{K}\sum_{r=1}^{R} \text{tr}\left[\mathbf{T}_r^H \mathbf{G}_{\text{eff},r}^H \mathbf{W}_r^H \mathbf{H}_{kr}^H \mathbf{R}_k^H\right] \\ &+ \sigma_2^2 \sum_{k=1}^{K} \text{tr}\left[\mathbf{R}_k \mathbf{R}_k^H\right] + L\end{aligned} \tag{10}$$

The detailed derivation is presented in the Appendix. Observe that the 2nd term in the right hand side of (8) is perfectly eliminated by the BD precoder in (10).

Now, we can rewrite BS power constraint in (7) using the following equation.

$$\text{tr}\left[\mathbf{T}_{\text{BS}}\mathbf{T}_{\text{BS}}^H\right] = \sum_{k=1}^{K} \text{tr}\left[\mathbf{T}_k \mathbf{T}_k^H\right] \tag{11}$$

The detailed derivation is explained in the Appendix. Problem (7) can be written as follows.

$$\begin{aligned}&\min_{\mathbf{T}_k,\, \mathbf{W}_k,\, \mathbf{R}_k,\, \forall k} \text{tr}[\mathbf{E}] \\ &\sum_{k=1}^{K} \text{tr}\left[\mathbf{T}_k \mathbf{T}_k^H\right] \le P_{\text{BS}} \\ &\sum_{k=1}^{K} \text{tr}\left[\mathbf{W}_k \left(\mathbf{G}_{\text{eff},k}\mathbf{T}_k\mathbf{T}_k^H\mathbf{G}_{\text{eff},k}^H + \sigma_1^2 \mathbf{I}_{N_R R}\right)\mathbf{W}_k^H\right] \le P_{\text{relay}}^{\text{total}}\end{aligned} \tag{12}$$

To avoid notational confusion, we clarify the subscripts in this paper. We used subscript r to indicate relay index, and k to indicate user index until so far. In the standpoint of the r^{th} relay, l was used to indicate the index of the other relay except for the r^{th} relay itself. As we assumed that $R = K$ and the k^{th} user is served by the relay of the same index k, we use k to indicate both relay and user indices from now on this paper. (12) is still non-convex problem over $\mathbf{T}_k$, $\mathbf{W}_k$, and $\mathbf{R}_k$ for all k and thus it is hard to derive jointly suboptimal solution. We investigate in-depth on the solution (12) in the following section.

4. BS Transmitter and Relay Precoder Optimization

We try to derive suboptimal solution based on sequential iterative algorithm. Sequential iteration based algorithm is composed of two stages. In the 1st stage, $\mathbf{T}$ and $\mathbf{W}$ are determined by fixing $\mathbf{R}$. Then $\mathbf{R}$ is determined with resultant fixed $\mathbf{T}$ and $\mathbf{W}$ in the 2nd stage. The iteration repeats untils SMSE converges.

4.1 The Structure of a Relay Precoder

The k^{th} relay precoder can be decomposed into two signal processing matrices as follows.

$$\mathbf{W}_k = \mathbf{F}_k \mathbf{B}_k \tag{13}$$

where $\mathbf{B}_k$ and $\mathbf{F}_k$ respectively denote receive matrix and transmit matrix of the k^{th} relay. Overall receive matrix and transmit matrix of all relays are expressed as $\mathbf{B} = \text{blkdiag}\left[\mathbf{B}_1 \quad \mathbf{B}_2 \quad \cdots \right. \in \mathbf{C}^{L \times N_R K}$ and $\mathbf{F} = \text{blkdiag}\left[\mathbf{F}_1 \quad \mathbf{F}_2 \quad \cdots \right. \in \mathbf{C}^{N_R K \times L}$, respectively. $\mathbf{B}_k$ equalizes the k^{th} user's data streams and is determined to be a MMSE filter as follows.

$$\mathbf{B}_k = \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H \left(\mathbf{G}_{\text{eff},k} \mathbf{T}_k \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H + \sigma_1^2 \mathbf{I}_R \right)^{-1}, \ \forall k \tag{14}$$

After substituting $\mathbf{B}_k$ into (10), some mathematical manipulation yields,

$$\begin{aligned} \text{tr}\left[\mathbf{E}\right] = & \sum_{k=1}^{K} \text{tr}\left[\left(\mathbf{I}_{L_k} + \sigma_1^{-2} \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H \mathbf{G}_{\text{eff},k} \mathbf{T}_k \right)^{-1} \right] \\ & + \sum_{k=1}^{K} \text{tr}\left[\left(\mathbf{R}_k \mathbf{H}_{kk} \mathbf{F}_k - \mathbf{I}_{L_k} \right) \mathbf{Q}_k \left(\mathbf{F}_k^H \mathbf{H}_{kk}^H \mathbf{R}_k^H - \mathbf{I}_{L_k} \right) \right] \\ & + \sum_{k=1}^{K} \sum_{l=1, l \neq k}^{K} \text{tr}\left[\mathbf{R}_k \mathbf{H}_{kl} \mathbf{F}_l \mathbf{Q}_l \mathbf{F}_l^H \mathbf{H}_{kl}^H \mathbf{R}_k^H \right] + \sigma_2^2 \sum_{k=1}^{K} \text{tr}\left[\mathbf{R}_k \mathbf{R}_k^H \right] \end{aligned} \tag{15}$$

where $\mathbf{Q}_k = \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H \mathbf{A}_k^{-1} \mathbf{G}_{\text{eff},k} \mathbf{T}_k$ and $\mathbf{A}_k = \mathbf{G}_{\text{eff},k} \mathbf{T}_k \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H + \sigma_1^2 \mathbf{I}_{N_R}$. (See the Appendix for the detailed derivation.)

The physical meaning of $\mathbf{Q}_k$ is the auto-correlation matrix of the k^{th} user's desirable signal at the k^{th} relay. The correlation matrices of all the users' data are originally identity matrices at BS. However, the recovered signal by (14) at the relay is contaminated compared to the original $\mathbf{x}$ since inter-stream interference for each relay is residual even if inter-relay interference is perfectly eliminated. Therefore the correlation matrix of the recovered signal is not identity matrix any more. By matrix inversion lemma, the following equation is hold.

$$\left(\mathbf{I}_{L_k} + \sigma_1^{-2} \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H \mathbf{G}_{\text{eff},k} \mathbf{T}_k \right)^{-1} = \mathbf{I}_{L_k} - \mathbf{T}_k^H \mathbf{G}_{\text{eff},k}^H \mathbf{A}_k^{-1} \mathbf{G}_{\text{eff},k} \mathbf{T}_k = \mathbf{I}_{L_k} - \mathbf{Q}_k \tag{16}$$

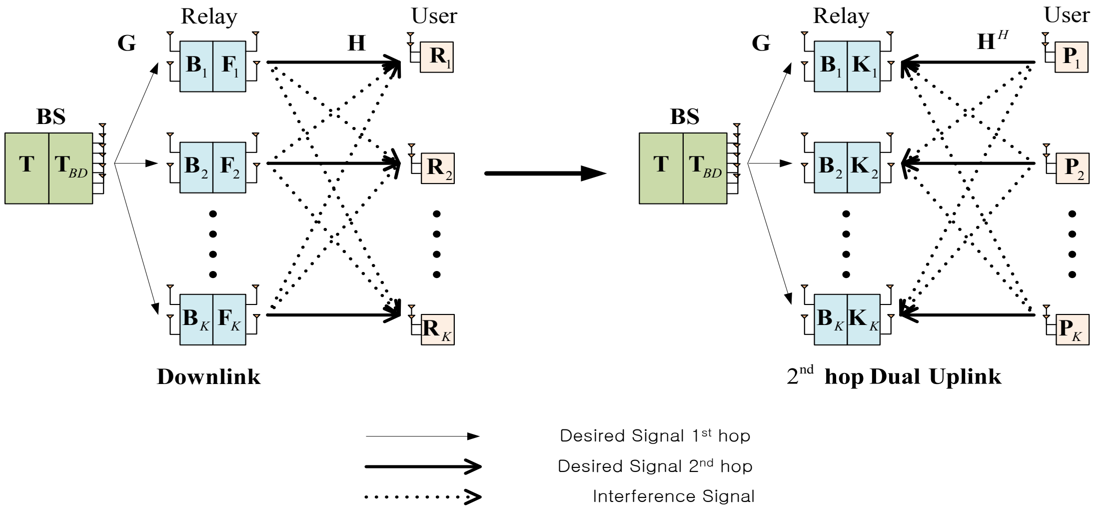

Fig. 2. Duality for the 2nd hop link

Thus the 1st term $\sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{I}_{L_k}+\sigma_1^{-2}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right)^{-1}\right]$ in (15) implies SMSE at all relays. We denote it by $\mathrm{tr}\left[\mathbf{E}_1\right]$. The latter part in (15) is denoted by $\mathrm{tr}\left[\mathbf{E}_2\right]$.

$$\mathrm{tr}\left[\mathbf{E}_1\right]=\sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{I}_{L_k}+\sigma_1^{-2}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right)^{-1}\right] \tag{17}$$

$$\begin{aligned}\mathrm{tr}\left[\mathbf{E}_2\right]=&\sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{R}_k\mathbf{H}_{kk}\mathbf{F}_k-\mathbf{I}_{L_k}\right)\mathbf{Q}_k\left(\mathbf{F}_k^H\mathbf{H}_{kk}^H\mathbf{R}_k^H-\mathbf{I}_{L_k}\right)\right]\\&+\sum_{k=1}^{K}\sum_{l=1,l\neq k}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kl}\mathbf{F}_l\mathbf{Q}_l\mathbf{F}_l^H\mathbf{H}_{kl}^H\mathbf{R}_k^H\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right]\end{aligned} \tag{18}$$

With (14) and $\mathbf{Q}_k$, optimization problem (12) is rewritten as follows.

$$\begin{aligned}&\min_{\mathbf{T}_k,\,\mathbf{F}_k,\,\mathbf{R}_k,\,\forall k}\ \mathrm{tr}\left[\mathbf{E}_1\right]+\mathrm{tr}\left[\mathbf{E}_2\right]\\&\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{T}_k\mathbf{T}_k^H\right]\le P_{\mathrm{BS}}\\&\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{F}_k\mathbf{Q}_k\mathbf{F}_k^H\right]\le P_{\mathrm{relay}}^{\mathrm{total}}\end{aligned} \tag{19}$$

4.2 SMSE Duality for the 2nd Hop Channel

Dual uplink network is illustrated in **Fig. 2**. SMSE duality is used for the 2nd hop channel in the 1st stage to jointly determine $\mathbf{T}$ and $\mathbf{F}$ in a smarter way. If SMSE duality would not be exploited, $\mathbf{T}$ and $\mathbf{F}$ should be determined in an iterative way. That is, updating $\mathbf{F}$ with fixed $\mathbf{T}$ and updating $\mathbf{T}$ with fixed $\mathbf{F}$, which results in the requirement of additional iteration stage. SMSE duality refers to the property that the SMSE in the downlink can be also achieved in the dual uplink, and vice versa while total transmit power of relays in downlink is kept same as the total transmit power of users in the dual uplink. For the given downlink network, we can imagine a network in which the communication direction is reversed. We call this network having the communication link with reversed direction as dual uplink network. While the original downlink network is switched into the dual uplink network, the role of transmitter and receiver is changed each other and the channel is flipped. If we assume that the channel of the original downlink network is denoted by $\mathbf{H}$, the flipped channel is accordingly denoted by $\mathbf{H}^H$. SMSE duality can be applied for various types of networks which include MIMO relay aided network. In this paper, we apply SMSE duality only for the 2nd hop channel as in **Fig. 2**. In the original downlink network, each relay acts as a receiver for the 1st hop channel, and a transmitter for the 2nd hop channel. While on the other hand in the dual uplink channel, the roles of relays and users are changed in the 2nd hop channel.

In the dual uplink network, we denote the user's transmitter as $\mathbf{P}_k$ and the k^{th} relay's receiver as $\mathbf{K}_k$. We also use two stacked matrices $\mathbf{P}$ = blkdiag $[\mathbf{P}_1 \quad \mathbf{P}_2 \quad \cdots$ $\in \mathbf{C}^{N_U K \times L}$ and $\mathbf{K}$ = blkdiag$[\mathbf{K}_1 \quad \mathbf{K}_2 \quad \cdots$ $\in \mathbf{C}^{L \times N_R R}$. Noise at the relay is assumed to have zero mean and σ_2^2 variance. The covariance matrix for SMSE in the dual uplink is defined as follows.

$$\mathbf{E}_2^{\text{ul}} = \sigma_2^2 \mathbf{K}\mathbf{K}^H + \left(\mathbf{K}\mathbf{H}^H\mathbf{P} - \mathbf{I}_L\right)\left(\mathbf{P}^H\mathbf{H}\mathbf{K}^H - \mathbf{I}_L\right) \tag{20}$$

where we assume that the transmit signals from users have zero mean and unit variance. SMSE duality provides the rule on how to determine $\mathbf{K}$ and $\mathbf{P}$ from given $\mathbf{F}$ and $\mathbf{R}$, respectively, with constraint on $\text{tr}\left[\mathbf{E}_2\right] = \text{tr}\left[\mathbf{Q}\mathbf{E}_2^{\text{ul}}\right]$, where $\mathbf{Q}$ = blkdiag$[\mathbf{Q}_1 \quad \mathbf{Q}_2 \quad \cdots$ $\in \mathbf{C}^{L \times L}$. Following proposition explains the SMSE duality for the 2nd hop channel and reveals the relationship among $\mathbf{F}_k$, $\mathbf{K}_k$, $\mathbf{R}_k$, and $\mathbf{P}_k$.

Proposition 1*: Let us set* $\mathbf{F}_k$, $\mathbf{K}_k$, $\mathbf{R}_k$, $\mathbf{P}_k$, and α to be

$$\mathbf{F}_k = \sqrt{\alpha}\mathbf{K}_k^H,\ \mathbf{R}_k^H = \frac{\mathbf{P}_k}{\sqrt{\alpha}},\ \forall k,\ \alpha = \frac{\sum_{k=1}^{K} \text{tr}\left[\mathbf{P}_k^H \mathbf{P}_k\right]}{\sum_{k=1}^{K} \text{tr}\left[\mathbf{K}_k^H \mathbf{Q}_k \mathbf{K}_k\right]} \tag{21}$$

Then, two equalities hold.

$$\text{tr}\left[\mathbf{E}_2\right] = \text{tr}\left[\mathbf{Q}\mathbf{E}_2^{\text{ul}}\right] \tag{22}$$

$$\sum_{k=1}^{K} \text{tr}\left[\mathbf{F}_k \mathbf{Q}_k \mathbf{F}_k^H\right] = \sum_{k=1}^{K} \text{tr}\left[\mathbf{P}_k \mathbf{P}_k^H\right] \tag{23}$$

Proof: (22) is proved straightforwardly by mathematical manipulation. (The detailed derivation is presented in the Appendix.) (23) is also derived directly from (21). These complete the proof of the proposition. ■

(23) implies that total transmit power in the dual uplink is kept same as one in the downlink. In summary, SMSE duality means that SMSE in the dual uplink can be kept same as SMSE in the downlink, with constraint that sum power consumption at users in the dual uplink shall be the same as sum power consumption at relays in the downlink.

Using the transformation in the proposition 1, optimization problem (19) can be reformulated as

$$\begin{aligned} &\min_{\mathbf{T}_k,\, \mathbf{K}_k,\, \mathbf{R}_k,\, \forall k} \text{tr}\left[\mathbf{E}_1\right] + \text{tr}\left[\mathbf{Q}\mathbf{E}_2^{\text{ul}}\right] \\ &\sum_{k=1}^{K} \text{tr}\left[\mathbf{T}_k \mathbf{T}_k^H\right] \le P_{\text{BS}} \\ &\sum_{k=1}^{K} \text{tr}\left[\mathbf{P}_k \mathbf{P}_k^H\right] \le P_{\text{relay}}^{\text{total}} \end{aligned} \tag{24}$$

Observe that the variables are changed from $\mathbf{T}_k$, $\mathbf{F}_k$, and $\mathbf{R}_k$ to $\mathbf{T}_k$, $\mathbf{K}_k$, and $\mathbf{P}_k$. We provide a method to solve $\mathbf{T}_k$, $\mathbf{K}_k$, and $\mathbf{P}_k$, in the next subsection.

4.3 Determination of BS Transmitter and Relay Precoders

In the 1st stage, $\mathbf{R}$ is assumed to be fixed. Consequently, $\mathbf{P}$ in the dual uplink is assumed to be fixed in accordance with $\mathbf{R}$. In a mathematical expression, $\mathrm{tr}\left[\mathbf{Q}\mathbf{E}_2^{\mathrm{ul}}\right]$ is a function of $\mathbf{K}_k$ for all k. The k^{th} relay filters its received signal from users by $\mathbf{K}_k$ and recovers the signal of the k^{th} user. Then, $\mathbf{K}_k$ is determined to a following MMSE filter.

$$\mathbf{K}_k^* = \mathbf{P}_k^H \mathbf{H}_{kk} \left(\hat{\mathbf{H}}_k^H \mathbf{P}\mathbf{P}^H \hat{\mathbf{H}}_k + \sigma_2^2 \mathbf{I}_{N_R}\right)^{-1}, \ \forall k \tag{25}$$

Note that $\mathbf{K}_k$ is determined using ony the local CSI of the k^{th} relay, $\hat{\mathbf{H}}_k$. $\mathbf{F}_k$ for all k is calculated by conversion from uplink to downlink in an analogous way to (21).

Now in order to solve $\mathbf{T}_k$ for all k, the variables of optimization problem (24) are unified to $\mathbf{T}_k$ for all k. Inserting $\mathbf{K}_k$ for all k in (25) into $\mathrm{tr}\left[\mathbf{Q}\mathbf{E}_2^{\mathrm{ul}}\right]$, we obtain the following (See the Appendix for the detailed derivation.)

$$\mathrm{tr}\left[\mathbf{Q}\mathbf{E}_2^{\mathrm{ul}}\right] = \sum_{k=1}^{K} \mathrm{tr}\left[\mathbf{Q}_k \left\{\mathbf{I}_{L_k} + \mathbf{P}_k^H \mathbf{H}_{kk} \left(\sum_{l=1, l\neq k}^{K} \mathbf{H}_{lk}^H \mathbf{P}_l \mathbf{P}_l^H \mathbf{H}_{lk} + \sigma_2^2 \mathbf{I}_{N_R}\right)^{-1} \mathbf{H}_{kk}^H \mathbf{P}_k\right\}\right] \tag{26}$$

In order to clarify the problem, we introduce following several substitutions.

$$\begin{aligned}
f_{1,k}\left(\mathbf{T}_k\right) &= \mathrm{tr}\left[\left(\mathbf{I}_{L_k} + \sigma_1^{-2} \mathbf{T}_k^H \mathbf{G}_{\mathrm{eff},k}^H \mathbf{G}_{\mathrm{eff},k} \mathbf{T}_k\right)^{-1}\right], \ \forall k \\
f_{2,k}\left(\mathbf{T}_k\right) &= \mathrm{tr}\left[\mathbf{Q}_k \left\{\mathbf{I}_{L_k} + \mathbf{P}_k^H \mathbf{H}_{kk} \left(\sum_{l=1, l\neq k}^{K} \mathbf{H}_{lk}^H \mathbf{P}_l \mathbf{P}_l^H \mathbf{H}_{lk} + \sigma_2^2 \mathbf{I}_{N_R}\right)^{-1} \mathbf{H}_{kk}^H \mathbf{P}_k\right\}\right], \ \forall k \\
f_{0,k}\left(\mathbf{T}_k\right) &= f_{1,k}\left(\mathbf{T}_k\right) + f_{2,k}\left(\mathbf{T}_k\right), \ \forall k
\end{aligned} \tag{27}$$

Using (27), the objective function of (24) is simplified as $\mathrm{tr}\left[\mathbf{E}_1\right] + \mathrm{tr}\left[\mathbf{Q}\mathbf{E}_2^{\mathrm{ul}}\right] = \sum_{k=1}^{K} f_{0,k}\left(\mathbf{T}_k\right)$.

Since every $f_{0,k}\left(\mathbf{T}_k\right)$ does not depend on $\mathbf{T}_l$, $\forall l \neq k$, minimizing $\sum_{k=1}^{K} f_{0,k}\left(\mathbf{T}_k\right)$ is decomposed into K individual subproblems which minimize individual $f_{0,k}\left(\mathbf{T}_k\right)$. Next proposition provides the optimal structure for minimizing $f_{0,k}\left(\mathbf{T}_k\right)$.

***Proposition 2**: The optimal structure for minimizing $f_{0,k}\left(\mathbf{T}_k\right)$ is given by,*

$$\mathbf{T}_k^* = \mathbf{U}_{1,k} \begin{pmatrix} \mathbf{\Lambda}_k^{1/2} \\ \mathbf{0} \end{pmatrix} \mathbf{U}_{2,k}^H, \ \forall k \tag{28}$$

where $\mathbf{\Lambda}_k$ is a diagonal matrix of size L_k, and $\mathbf{U}_{1,k} \in \mathbf{C}^{\bar{L}_k \times \bar{L}_k}$ and $\mathbf{U}_{2,k} \in \mathbf{C}^{L_k \times L_k}$ are unitary matrices which are generated from following eigenvalue decompositions.

$$\begin{aligned}
\sigma_1^{-2} \mathbf{G}_{\mathrm{eff},k}^H \mathbf{G}_{\mathrm{eff},k} &= \mathbf{U}_{1,k} \mathbf{D}_{1,k} \mathbf{U}_{1,k}^H \\
\mathbf{D}_{1,k} &= \mathrm{diag}\Big(\lambda_{1,k,1}, \cdots
\end{aligned} \tag{29}$$

$$\begin{aligned}
\mathbf{P}_k^H \mathbf{H}_{kk} \left(\sum_{l=1, l\neq k}^{K} \mathbf{H}_{lk}^H \mathbf{P}_l \mathbf{P}_l^H \mathbf{H}_{lk} + \sigma_2^2 \mathbf{I}_{N_R}\right)^{-1} \mathbf{H}_{kk}^H \mathbf{P}_k &= \mathbf{U}_{2,k} \mathbf{D}_{2,k} \mathbf{U}_{2,k}^H \\
\mathbf{D}_{2,k} &= \mathrm{diag}\Big(\lambda_{2,k,1}, \cdots
\end{aligned} \tag{30}$$

where $\left(\lambda_{1,k,1},\cdots\right.$ *and* $\left(\lambda_{2,k,1},\cdots\right.$ *are arranged up in a descending order.*

Proof: Using (29), $f_{1,k}\left(\mathbf{T}_k\right)$ is written as follows.

$$f_{1,k}\left(\mathbf{T}_k\right)=\operatorname{tr}\left[\left(\mathbf{I}_{L_k}+\mathbf{T}_k^H\mathbf{U}_{1,k}\mathbf{D}_{1,k}\mathbf{U}_{1,k}^H\mathbf{T}_k\right)^{-1}\right],\ \forall k \tag{31}$$

To make $f_{1,k}\left(\mathbf{T}_k\right)$ be minimized, we can consider following structure without loss of generality [14].

$$\bar{\mathbf{T}}_k=\mathbf{U}_{1,k}\begin{pmatrix}\mathbf{\Lambda}_k^{1/2}\\ \mathbf{0}\end{pmatrix},\ \forall k \tag{32}$$

Note that $\bar{L}_k \geq L_k$ for all k as mentioned in the section III. Next, we rewrite $f_{2,k}\left(\mathbf{T}_k\right)$ as follows.

$$\begin{aligned}f_{2,k}\left(\mathbf{T}_k\right)&=\operatorname{tr}\left[\mathbf{Q}_k\left\{\mathbf{I}_{L_k}+\mathbf{P}_k^H\mathbf{H}_{kk}\left(\sum_{l\neq k,l=1}^{K}\mathbf{H}_{lk}^H\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_{lk}+\sigma_2^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{H}_{kk}^H\mathbf{P}_k\right\}^{-1}\right]\\&=\operatorname{tr}\left[\left(\sigma_1^{-2}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k+\mathbf{I}_{L_k}\right)^{-1}\sigma_1^{-2}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\left(\mathbf{I}_{L_k}+\mathbf{U}_{2,k}\mathbf{D}_{2,k}\mathbf{U}_{2,k}^H\right)^{-1}\right]\\&=\operatorname{tr}\left[\left(\mathbf{X}_k+\mathbf{I}_{L_k}\right)^{-1}\mathbf{X}_k\left(\mathbf{D}_{2,k}+\mathbf{I}_{L_k}\right)^{-1}\right],\ \forall k\end{aligned} \tag{33}$$

where $\mathbf{X}_k=\sigma_1^{-2}\mathbf{U}_{2,k}^H\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\mathbf{U}_{2,k}$. $\mathbf{T}_k$ which minimizes $f_{2,k}\left(\mathbf{T}_k\right)$ is known to have following structure [14].

$$\mathbf{T}_k^*=\mathbf{U}_{1,k}\begin{pmatrix}\mathbf{\Lambda}_k^{1/2}\\ \mathbf{0}\end{pmatrix}\mathbf{U}_{2,k}^H,\ \forall k \tag{28}$$

Since $f_{1,k}\left(\mathbf{T}_k^*\right)=f_{1,k}\left(\bar{\mathbf{T}}_k\right)$, $\mathbf{T}_k^*$ minimizes both $f_{1,k}\left(\mathbf{T}_k\right)$ and $f_{2,k}\left(\mathbf{T}_k\right)$, and hence minimize $f_{0,k}\left(\mathbf{T}_k\right)$. This completes the proof. ■

It is left to calculate $\mathbf{\Lambda}_k$. Inserting $\mathbf{T}_k$ in (28) into $\sum_{k=1}^{K}f_{0,k}\left(\mathbf{T}_k\right)$ yields,

$$\operatorname{tr}\left[\mathbf{E}_1+\mathbf{Q}\mathbf{E}_2^{\mathrm{ul}}\right]=\sum_{k=1}^{K}\sum_{l_k=1}^{L_k}\frac{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}\left(\lambda_{2,k,l_k}+1\right)^{-1}}{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}} \tag{34}$$

where Λ_{k,l_k} is the l_k -th diagonal element of $\mathbf{\Lambda}_k$. (See the Appendix for the detailed derivation.) Finally a reformulated optimization problem is written as,

$$\begin{aligned}&\min_{\Lambda_{k,l_k}}\sum_{k=1}^{K}\sum_{l_k=1}^{L_k}\frac{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}\left(\lambda_{2,k,l_k}+1\right)^{-1}}{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}}\\&\text{s. t. }\sum_{k=1}^{K}\sum_{l_k=1}^{L_k}\Lambda_{k,l_k}\leq P_{\mathrm{BS}},\ \Lambda_{k,l_k}\geq 0\end{aligned} \tag{35}$$

(35) is convex optimization problem over Λ_{k,l_k} and easily solved by Karush-Kuhn-Tucker (KKT) Theorem [15] or other well-known solvers. After $\mathbf{T}_k$ is determined, $\mathbf{B}_k$ is readily calculated by (14) and $\mathbf{W}_k$ is spontaneously yielded by (13).

In the 2nd stage, $\mathbf{R}_k$ for all k is determined to following MMSE filter with fixed $\mathbf{T}$ and $\mathbf{W}$.

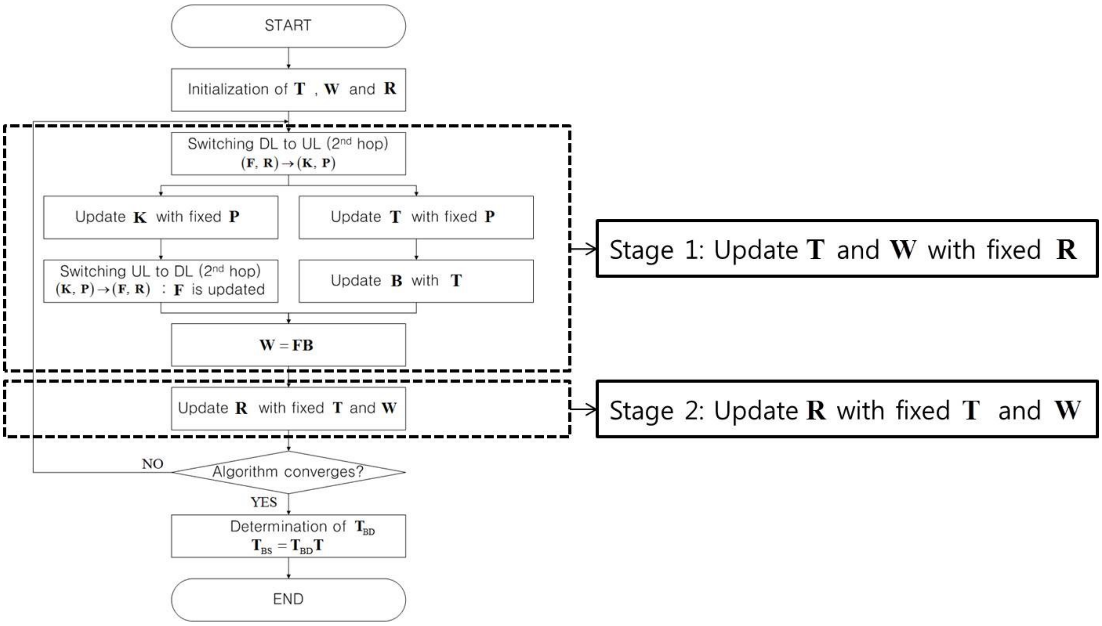

Fig. 3. Flow chart of transceiver design with local CSI at relays

$$\mathbf{R}_k = \mathbf{T}_k^H \mathbf{G}_{\mathrm{eff},k}^H \mathbf{W}_k^H \mathbf{H}_{kk}^H \left[\hat{\mathbf{H}}_k \mathbf{W} \left(\mathbf{G}_{\mathrm{eff}} \mathbf{T} \mathbf{T}^H \mathbf{G}_{\mathrm{eff}}^H + \sigma_1^2 \mathbf{I}_{N_R R} \right) \mathbf{W}^H \hat{\mathbf{H}}_k^H + \sigma_2^2 \mathbf{I}_{N_U} \right]^{-1}, \ \forall k \qquad (36)$$

In summary, $\mathbf{T}$ and $\mathbf{W}$ are updated with given fixed $\mathbf{R}$ in the 1st stage. Subsequently with given fixed $\mathbf{T}$ and $\mathbf{W}$, $\mathbf{R}$ is updated in the 2nd stage. If $\mathbf{R}$ is fixed in the 1st stage, then $\mathbf{P}$ is also fixed due to the connection between them (21). From fixed $\mathbf{P}$, $\mathbf{K}_k$ for all k is updated by (25), while simultaneously $\mathbf{T}_k$ for all k is updated by (28). Using the connection between $\mathbf{K}_k$ and $\mathbf{F}_k$ (21), $\mathbf{F}_k$ is readily updated. Simultaneously, $\mathbf{B}_k$ is updated depending on $\mathbf{T}_k$ by (14). Since $\mathbf{W}_k = \mathbf{F}_k \mathbf{B}_k$ (13), determination of $\mathbf{F}_k$ and $\mathbf{B}_k$ for all k readily yields $\mathbf{W}$. In the next stage, $\mathbf{R}_k$ for all k is updated to (37) with resultant fixed $\mathbf{T}$ and $\mathbf{W}$. The 1st and the 2nd stages repeat iteratively until SMSE converges. The whole procedure is summarized in **Fig. 3**. We call this proposed algorithm as joint BS and distributed multi-relay (JBDMR) for the rest of this paper. Matrix updates at each stage yield a non-increasing SMSE value that is lower bounded by zero. Thus the SMSE is guaranteed to converge through some number of iterations. However, the convergence to optimal point is not guaranteed since the primal problem is non-convex.

5. Numerical Results

We analyze the SMSE and sum rate performance of JBDMRs with other relaying schemes in this section. The unit of sum rate is bit per second per hertz (bps/Hz). $P_{\mathrm{relay}}^{\mathrm{total}}$ is set to be equal to P_{BS} and SNR in following figures is defined as $P_{\mathrm{BS}} / \sigma_1^2$ and $\sigma_1^2 = \sigma_2^2 = 1$. Network configuration is denoted by $\left(R, K, N_S, N_R, N_U\right)$. L_k is set to be one for all k, which can achieve the best sum rate of all relaying schemes in each network configuration. Both the 1st hop and the 2nd hop channels experience uncorrelated Rayleigh block fading with unit variance.

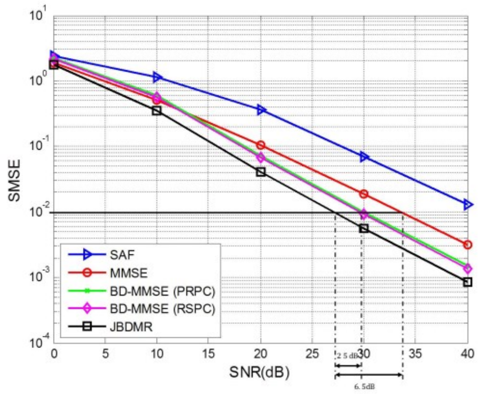

Fig. 4. SMSE comparison of various relaying schemes for $(3,3,9,3,3)$ network

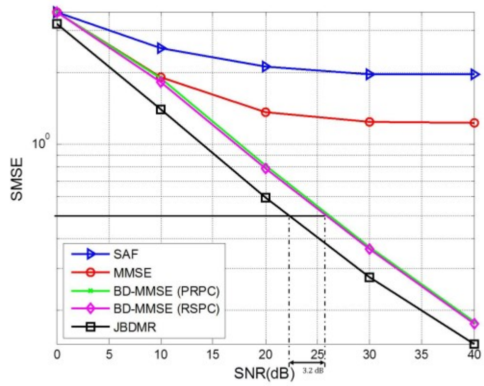

Fig. 5. SMSE comparison of various relaying schemes for $(4,4,8,2,2)$ network

For both schemes, $\mathbf{T}_{\text{BS}}$ and $\mathbf{W}$ are initialized as

$$\mathbf{T}_{\text{BS}} = \sqrt{\frac{P_{\text{BS}}}{L}}\mathbf{I}_{N_S}(1:L),\ \mathbf{W}_k = \sqrt{\frac{P_{\text{relay}}}{\text{tr}\left[\mathbf{G}_k\mathbf{T}\mathbf{T}^H\mathbf{G}_k^H + \sigma_1^2\mathbf{I}_{N_R}\right]}}\mathbf{I}_{N_R},\ \forall k \quad (37)$$

where $\mathbf{I}_X(1:L)$ denotes the 1^{st} L columns of $\mathbf{I}_X$. JBDMR terminates if the SMSE difference between each iteration becomes smaller than a predefined accuracy ε which is set to be 10^{-6}. All users are assumed to be simultaneously served by a two hop relay aided BS simultaneously.

SAF relaying and MMSE relaying are introduced for comparison, which are the most referred schemes in many literatures of relay aided network. For SAF relaying, $\mathbf{T}_{\text{BS}}$ and $\mathbf{W}_k$ follow (37) and $\mathbf{R}_k$ follows below.

$$\mathbf{R}_k = \mathbf{T}_{\text{BS},k}^H\mathbf{G}^H\mathbf{W}^H\hat{\mathbf{H}}_k^H\left[\hat{\mathbf{H}}_k\mathbf{W}\left(\mathbf{G}\mathbf{T}_{\text{BS}}\mathbf{T}_{\text{BS}}^H\mathbf{G}^H + \mathbf{I}_{N_R R}\right)\mathbf{W}^H\hat{\mathbf{H}}_k^H + \mathbf{I}_{N_U}\right]^{-1},\ \forall k \quad (38)$$

For the MMSE relaying, a well-defined form of relay precoders from [9] is used. We also consider a more refined MMSE scheme, BD-MMSE [12]. Originally BD-MMSE operates under PRPC. However, performance of BD-MMSE modified for RSPC is also evaluated for fair comparison.

5.1 Comparisons of SMSE

Fig. 4 compares SMSE performance of a $(3,3,9,3,3)$ network. BD-MMSE shows approximately the same SMSE for both RSPC and PRPC cases. JBDMR is shown to provide the best SMSE performance among all compared relaying schemes in all SNR regions. JBDMR achieves about 6.5 dB and 2.5 dB gain over MMSE relaying and BD-MMSE (RSPC), respectively, at the SMSE of 10^{-2}. **Fig. 5** illustrates SMSE comparison in a $(4,4,8,2,2)$ network. JBDMR shows 3.2 dB gain over BD-MMSE (RSPC) at an SMSE of 0.5×10^{-1}. In this network configuration, SMSE of MMSE relaying is significantly degraded in all SNR regions, while SMSE of BD-MMSE and JBDMR keep decreasing with increasing SNR. In the cases of BD-MMSE and JBDMR, BD precoder at the transmitter makes inter-relay interference be free, while SAF and MMSE relaying suffer severe inter-relay interference in interference limited region. Using SAF and MMSE relaying, residual degree of freedom at

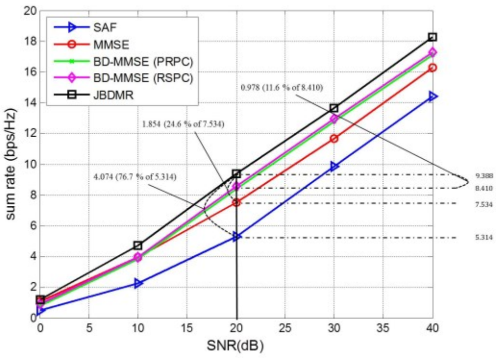

Fig. 6. Sum rate comparison of various relaying schemes for $(3,3,9,3,3)$ network

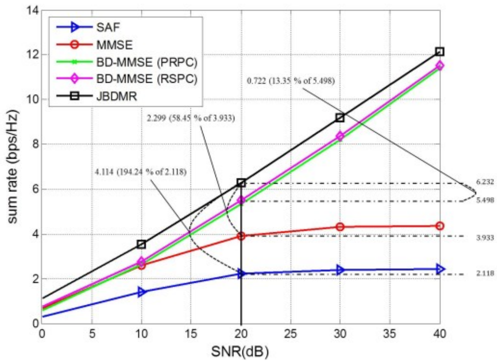

Fig. 7. Sum rate comparison of various relaying schemes for $(4,4,8,2,2)$ network

each relay is sufficient to mitigate interference in the $(3,3,9,3,3)$ network, while it is insufficient in the $(4,4,8,2,2)$ network. Moreover, due to the additional optimization of $\mathbf{T}$ (the latter part of $\mathbf{T}_{BS}$), JBDMR outperforms BD-MMSE. Thus JBDMR significantly improves SMSE performance for all considered typical network configuration and SNRs.

5.2 Comparison of Sum Rate

For the same network configurations in the preceding subsection, we compare sum rate performances in **Fig. 6** and **Fig. 7**. It is observed that characteristics of sum rate performance have characteristics similar to those of SMSE in **Fig. 4** and **Fig. 5**. JBDMR always outperform SAF, MMSE relaying and BD-MMSE in sum rate performance in all SNR region. More specifically, JBDMR outperforms SAF and MMSE relaying in sum rate by about 76.7%, 24.6%, and 11.6%, respectively, when SNR = 20 dB in the $(3,3,9,3,3)$ network, which is shown in **Fig. 6**. **Fig. 7** displays sum rate comparison in the $(4,4,8,2,2)$ network. Sum rate of MMSE relaying is significantly degraded when SNR is over 20dB. While on the other hand, sum rate of BD-MMSE and JBDMR continuously increases even in that SNR region. JBDMR provides 194.24%, 58.45%, and 13.35% improved performance over SAF, MMSE relaying and BD-MMSE (RSPC), respectively, for the $(4,4,8,2,2)$ network in **Fig. 7**. These results verify that JBDMR shows superior performance to those of conventional schemes such SAF, MMSE relaying in terms of sum rate for all considered typical network configuration and SNRs.

6. Conclusion

We proposed an iterative transceiver design called JBDMR using local CSI at relays in an MRMU MIMO network. Construction of BS transmitter as the product of BD precoder and individual precoder for each relay made it possible to design the transceiver with local CSI. The numerical results verified that JBDMR outperforms simple SAF, MMSE relaying, and BD-MMSE in terms of both SMSE and sum rate performances. Even though the proposed scheme made a step forward to make MRMU MIMO system more practical, there are still many issues to remain to be resolved. The most realistic power constraint will be PAPC which often makes optimum solution for transceiver design difficult to be determined. Even though we assumed perfect local CSI, it cannot be available in practice due to noise and limited backhaul capacity. In multi-cell environment, inter-cell interference from relays in other cells may degrade performance significantly. These problems will be addressed thoroughly in future research.

Appendix

Derivation of (10)

We rewrite the 1st and the 2nd terms of (8) as follows.

$$\begin{aligned}&\sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kr}\mathbf{W}_r\left(\mathbf{G}_r\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\mathbf{G}_r^H+\sigma_1^2\mathbf{I}_{N_R}\right)\mathbf{W}_r^H\mathbf{H}_{kr}^H\mathbf{R}_k^H\right]\\&+\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{l=1,l\neq r}^{R}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kr}\mathbf{W}_r\left(\mathbf{G}_r\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\mathbf{G}_l^H\right)\mathbf{W}_l^H\mathbf{H}_{kl}^H\mathbf{R}_k^H\right]\\&=\mathrm{tr}\left[\mathbf{RHW}\left(\mathbf{G}\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\mathbf{G}^H+\sigma_1^2\mathbf{I}_{N_RK}\right)\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H\right]\end{aligned}\tag{39}$$

Substituting $\mathbf{T}_{\mathrm{BS}}$ in (9) into (39) and expanding yields

$$\begin{aligned}&\mathrm{tr}\left[\mathbf{RHW}\left(\mathbf{G}\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\mathbf{G}^H+\sigma_1^2\mathbf{I}_{N_RK}\right)\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H\right]\\&=\mathrm{tr}\left[\mathbf{RHW}\left(\mathbf{G}_{\mathrm{eff}}\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\mathbf{G}_{\mathrm{eff}}^H+\sigma_1^2\mathbf{I}_{N_RK}\right)\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H\right]\\&=\sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kr}\mathbf{W}_r\left(\mathbf{G}_{\mathrm{eff},r}\mathbf{T}_r\mathbf{T}_r^H\mathbf{G}_{\mathrm{eff},r}^H+\sigma_1^2\mathbf{I}_{N_R}\right)\mathbf{W}_r^H\mathbf{H}_{kr}^H\mathbf{R}_k^H\right]\end{aligned}\tag{40}$$

Then, (10) is achieved and the derivation is completed.

Derivation of (11)

The order of matrices in trace operator can be circulary changed ($\mathrm{tr}[\mathbf{ABC}]=\mathrm{tr}[\mathbf{BCA}]$). We can reorder the matrices in $\mathrm{tr}\left[\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\right]$ as follows.

$$\mathrm{tr}\left[\mathbf{T}_{\mathrm{BS}}\mathbf{T}_{\mathrm{BS}}^H\right]=\mathrm{tr}\left[\mathbf{T}_{\mathrm{BD}}\mathbf{T}\mathbf{T}^H\mathbf{T}_{\mathrm{BD}}^H\right]=\mathrm{tr}\left[\mathbf{T}_{\mathrm{BD}}^H\mathbf{T}_{\mathrm{BD}}\mathbf{T}\mathbf{T}^H\right]\tag{41}$$

Note that the $\mathbf{T}_{\mathrm{BD}}$ is column-stacked matrix of which the column submatrix $\mathbf{T}_{\mathrm{BD},k}$ forms orthonomal basis. That is, $\mathbf{T}_{\mathrm{BD},k}^H\mathbf{T}_{\mathrm{BD},k}=\mathbf{I}_{\bar{L}_k}$. The detailed structure of $\mathbf{T}_{\mathrm{BD}}^H\mathbf{T}_{\mathrm{BD}}\mathbf{T}\mathbf{T}^H$ is represented as follows.

$$\begin{aligned}&\begin{bmatrix}\mathbf{I}_{N_R} & \mathbf{X}_{12} & \mathbf{X}_{13} & \cdots & \\ \mathbf{X}_{21} & \mathbf{I}_{N_R} & \mathbf{X}_{23} & \cdots & \\ \mathbf{X}_{31} & \mathbf{X}_{32} & \mathbf{I}_{N_R} & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{K1} & \mathbf{X}_{K2} & \mathbf{X}_{K3} & \cdots & \end{bmatrix}\begin{bmatrix}\mathbf{T}_1\mathbf{T}_1^H & \mathbf{0} & \mathbf{0} & \cdots & \\ \mathbf{0} & \mathbf{T}_2\mathbf{T}_2^H & \mathbf{0} & \cdots & \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_3\mathbf{T}_3^H & \cdots & \\ & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \end{bmatrix}\\&=\begin{bmatrix}\mathbf{T}_1\mathbf{T}_1^H & \mathbf{X}_{12}\mathbf{T}_2\mathbf{T}_2^H & \mathbf{X}_{13}\mathbf{T}_3\mathbf{T}_3^H & \cdots & {}_K\mathbf{T}_K^H \\ \mathbf{X}_{21}\mathbf{T}_1\mathbf{T}_1^H & \mathbf{T}_2\mathbf{T}_2^H & \mathbf{X}_{23}\mathbf{T}_3\mathbf{T}_3^H & \cdots & {}_K\mathbf{T}_K^H \\ \mathbf{X}_{31}\mathbf{T}_1\mathbf{T}_1^H & \mathbf{X}_{32}\mathbf{T}_2\mathbf{T}_2^H & \mathbf{T}_3\mathbf{T}_3^H & \cdots & {}_K\mathbf{T}_K^H \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{K1}\mathbf{T}_1\mathbf{T}_1^H & \mathbf{X}_{K2}\mathbf{T}_2\mathbf{T}_2^H & \mathbf{X}_{K3}\mathbf{T}_3\mathbf{T}_3^H & \cdots & \Gamma_K^H\end{bmatrix}\end{aligned}\tag{42}$$

where $\mathbf{X}_{mn}$ denotes submatrix of off-block diagonal elements of $\mathbf{T}_{\mathrm{BD}}^H\mathbf{T}_{\mathrm{BD}}$. Since trace operater does not care off-block diagonal elements, $\mathrm{tr}\left[\mathbf{T}_{\mathrm{BD}}^{H}\mathbf{T}_{\mathrm{BD}}\mathbf{T}\mathbf{T}^{H}\right]=\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{T}_{k}\mathbf{T}_{k}^{H}\right]$ and this completes the derivation of (11).

Derivation of (15)

Inserting $\mathbf{F}_k$ and $\mathbf{R}_k$ into $\mathrm{tr}\left[\mathbf{E}_2\right]$ in (18), we obtain the following.

$$
\begin{aligned}
\mathrm{tr}[\mathbf{E}] &= \sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kr}\mathbf{W}_r\left(\mathbf{G}_{\mathrm{eff},r}\mathbf{T}_r\mathbf{T}_r^H\mathbf{G}_{\mathrm{eff},r}^H+\sigma_1^2\mathbf{I}_{N_R}\right)\mathbf{W}_r^H\mathbf{H}_{kr}^H\mathbf{R}_k^H\right] \\
&\quad -\sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kr}\mathbf{W}_r\mathbf{G}_{\mathrm{eff},r}\mathbf{T}_r\right]-\sum_{k=1}^{K}\sum_{r=1}^{R}\mathrm{tr}\left[\mathbf{T}_r^H\mathbf{G}_{\mathrm{eff},r}^H\mathbf{W}_r^H\mathbf{H}_{kr}^H\mathbf{R}_k^H\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right]+L \\
&= L-\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{A}_k^{-1}\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right]+\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{A}_k^{-1}\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right] \\
&\quad +\mathrm{tr}\left[\mathbf{RHW}\left(\mathbf{G}_{\mathrm{eff}}\mathbf{T}\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H+\sigma_1^2\mathbf{I}_{N_R R}\right)\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H\right] \\
&\quad -\mathrm{tr}\left[\mathbf{RHW}\mathbf{G}_{\mathrm{eff}}\mathbf{T}\right]-\mathrm{tr}\left[\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right] \\
&= L-\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{A}_k^{-1}\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right] \\
&\quad +\mathrm{tr}\left[\left(\mathbf{RHW}-\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H\mathbf{A}^{-1}\right)\mathbf{A}\left(\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H-\mathbf{A}^{-1}\mathbf{G}_{\mathrm{eff}}\mathbf{T}\right)\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right] \\
&\overset{(a)}{=}\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{I}_{L_k}+\sigma_1^{-1}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right]^{-1} \\
&\quad +\mathrm{tr}\left[\left(\mathbf{RHF}\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H\mathbf{A}^{-1}-\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H\mathbf{A}^{-1}\right)\mathbf{A}\left(\mathbf{A}^{-1}\mathbf{G}_{\mathrm{eff}}\mathbf{T}\mathbf{W}^H\mathbf{H}^H\mathbf{R}^H-\mathbf{A}^{-1}\mathbf{G}_{\mathrm{eff}}\mathbf{T}\right)\right] \\
&\quad +\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right] \\
&= \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{I}_{L_k}+\sigma_1^{-1}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right]^{-1}+\mathrm{tr}\left[\left(\mathbf{RHF}-\mathbf{I}_L\right)\mathbf{Q}\left(\mathbf{F}^H\mathbf{H}^H\mathbf{R}^H-\mathbf{I}_L\right)\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right] \\
&= \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{I}_{L_k}+\sigma_1^{-1}\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\right]^{-1} \\
&\quad +\sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{R}_k\mathbf{H}_{kk}\mathbf{F}_k-\mathbf{I}_{L_k}\right)\mathbf{Q}_k\left(\mathbf{F}_k^H\mathbf{H}_{kk}^H\mathbf{R}_k^H-\mathbf{I}_{L_k}\right)\right]+\sum_{k=1}^{K}\sum_{l=1,l\neq k}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kl}\mathbf{F}_l\mathbf{Q}_l\mathbf{F}_l^H\mathbf{H}_{kl}^H\mathbf{R}_k^H\right] \\
&\quad +\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right]
\end{aligned}
\tag{43}
$$

where $\mathbf{T}^H\mathbf{G}_{\mathrm{eff}}^H\mathbf{A}^H$ = $\mathrm{blkdiag}\left[\mathbf{T}_1^H\mathbf{G}_{\mathrm{eff},1}^H\mathbf{A}_1^{-1},\ \mathbf{T}_2^H\mathbf{G}_{\mathrm{eff},2}^H\mathbf{A}_2^{-1},\ \cdots\ \mathbf{G}_{\mathrm{eff},K}^H\mathbf{A}_K^{-1}\right]$, $\mathbf{A}$ = $\mathrm{blkdiag}[\mathbf{A}_1,\ \mathbf{A}_2,\ \cdots$ and (a) follows from the matrix inversion lemma. The derivation leads (15).

Proof of (22) in Proposition 3

We prove (22) by direct substitution. Inserting $\mathbf{F}_k=\sqrt{\alpha}\mathbf{K}_k^H$ and $\mathbf{R}_k^H=\frac{\mathbf{P}_k}{\sqrt{\alpha}}$ in (21) into $\mathrm{tr}\left[\mathbf{E}_2\right]$ in (18), we obtain the following.

$$\begin{aligned}
\mathrm{tr}\left[\mathbf{E}_2\right] &= \sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{R}_k\mathbf{H}_{kk}\mathbf{F}_k-\mathbf{I}_{L_k}\right)\mathbf{Q}_k\left(\mathbf{F}_k^H\mathbf{H}_{kk}^H\mathbf{R}_k^H-\mathbf{I}_{L_k}\right)\right] \\
&\quad+\sum_{k=1}^{K}\sum_{l=1,l\neq k}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{H}_{kl}\mathbf{F}_l\mathbf{Q}_l\mathbf{F}_l^H\mathbf{H}_{kl}^H\mathbf{R}_k^H\right]+\sigma_2^2\sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{R}_k\mathbf{R}_k^H\right] \\
&= \mathrm{tr}\left[\left(\mathbf{RHF}-\mathbf{I}_L\right)\mathbf{Q}\left(\mathbf{F}^H\mathbf{H}^H\mathbf{R}^H-\mathbf{I}_L\right)\right]+\sigma_2^2\mathrm{tr}\left[\mathbf{RR}^H\right] \\
&\overset{(b)}{=} \mathrm{tr}\left[\left(\mathbf{P}^H\mathbf{HK}^H-\mathbf{I}_L\right)\mathbf{Q}\left(\mathbf{KH}^H\mathbf{P}-\mathbf{I}_L\right)\right]+\frac{\sigma_2^2\mathrm{tr}\left[\mathbf{P}^H\mathbf{P}\right]}{\alpha} \\
&= \mathrm{tr}\left[\mathbf{Q}\left(\mathbf{KH}^H\mathbf{P}-\mathbf{I}_L\right)\left(\mathbf{P}^H\mathbf{HK}^H-\mathbf{I}_L\right)\right]+\sigma_2^2\mathrm{tr}\left[\mathbf{K}^H\mathbf{QK}\right] \\
&= \mathrm{tr}\left[\mathbf{Q}\left\{\left(\mathbf{KH}^H\mathbf{P}-\mathbf{I}_L\right)\left(\mathbf{P}^H\mathbf{HK}^H-\mathbf{I}_L\right)+\mathbf{KK}^H\right\}\right] \\
&= \mathrm{tr}\left[\mathbf{QE}_2^{\mathrm{ul}}\right]
\end{aligned} \tag{44}$$

where (b) follows from the 3rd equality in (21).

Derivation of (26)

Substituting $\mathbf{K}_k$ in (25) into $\mathrm{tr}\left[\mathbf{QE}_2^{\mathrm{ul}}\right]$ yields,

$$\begin{aligned}
\mathrm{tr}\left[\mathbf{QE}_2^{\mathrm{ul}}\right] &= \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{Q}_k\left\{\sigma_2^2\mathbf{K}_k\mathbf{K}_k^H+\left(\mathbf{K}_k\mathbf{H}_{kk}^H\mathbf{P}_k-\mathbf{I}_{L_k}\right)\left(\mathbf{P}_k^H\mathbf{H}_{kk}\mathbf{K}_k^H-\mathbf{I}_{L_k}\right)+\sum_{l=1,l\neq k}^{K}\mathbf{K}_k\mathbf{H}_{lk}^H\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_{lk}\mathbf{K}_k^H\right\}\right] \\
&= \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{Q}_k\left\{\sigma_2^2\mathbf{K}_k\left(\mathbf{H}_k^H\mathbf{P}\mathbf{P}_k^H\mathbf{H}_k+\sigma_2^2\mathbf{I}_{N_R}\right)\mathbf{K}_k^H-\mathbf{K}_k\mathbf{H}_{kk}^H\mathbf{P}_k-\mathbf{P}_k^H\mathbf{H}_{kk}\mathbf{K}_k^H+\mathbf{I}_{L_k}\right\}\right] \\
&= \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{Q}_k\left\{\mathbf{I}_{L_k}-\mathbf{P}_k^H\mathbf{H}_{kk}\left(\mathbf{H}_{kk}^H\mathbf{P}_k\mathbf{P}_k^H\mathbf{H}_{kk}+\sum_{l=1,l\neq k}^{K}\mathbf{H}_{lk}^H\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_{lk}\right)^{-1}\mathbf{H}_{kk}^H\mathbf{P}_k\right\}\right] \\
&\overset{(c)}{=} \sum_{k=1}^{K}\mathrm{tr}\left[\mathbf{Q}_k\left\{\mathbf{I}_{L_k}+\mathbf{P}_k^H\mathbf{H}_{kk}\left(\sum_{l=1,l\neq k}^{K}\mathbf{H}_{lk}^H\mathbf{P}_l\mathbf{P}_l^H\mathbf{H}_{lk}+\sigma_2^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{H}_{kk}^H\mathbf{P}_k\right\}\right]
\end{aligned} \tag{45}$$

where (c) follows from the matrix inversion lemma. The derivation is completed.

Derivation of (34)

Substituting $\mathbf{T}_k$ in (28) into $\mathrm{tr}\left[\mathbf{E}_1+\mathbf{QE}_2^{\mathrm{ul}}\right]$ yields,

$$\begin{aligned}
\mathrm{tr}\left[\mathbf{E}_1+\mathbf{QE}_2^{\mathrm{ul}}\right] &= \sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{I}_{L_k}+\sigma_1^{-2}\mathbf{T}_k^H\mathbf{U}_{1,k}\mathbf{D}_{1,k}\mathbf{U}_{1,k}^H\mathbf{T}_k\right)^{-1}+\left(\mathbf{X}_k+\mathbf{I}_{L_k}\right)^{-1}\mathbf{X}_k\left(\mathbf{D}_{2,k}+\mathbf{I}_{L_k}\right)^{-1}\right] \\
&= \sum_{k=1}^{K}\mathrm{tr}\left[\left(\mathbf{I}_{L_k}+\mathbf{U}_{2,k}^H\mathbf{\Lambda}_k^{1/2}\mathbf{D}_{1,k}\mathbf{\Lambda}_k^{1/2}\mathbf{U}_{2,k}\right)^{-1}+\left(\mathbf{\Lambda}_k^{1/2}\mathbf{D}_{1,k}\mathbf{\Lambda}_k^{1/2}+\mathbf{I}_{L_k}\right)^{-1}\mathbf{\Lambda}_k^{1/2}\mathbf{D}_{1,k}\mathbf{\Lambda}_k^{1/2}\left(\mathbf{D}_{2,k}+\mathbf{I}_{L_k}\right)^{-1}\right] \\
&= \sum_{k=1}^{K}\sum_{l_k=1}^{L_k}\frac{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}\left(\lambda_{2,k,l_k}+1\right)^{-1}}{1+\lambda_{1,k,l_k}\Lambda_{k,l_k}}
\end{aligned} \tag{46}$$

where $\mathbf{X}_k=\sigma_1^{-2}\mathbf{U}_{2,k}^H\mathbf{T}_k^H\mathbf{G}_{\mathrm{eff},k}^H\mathbf{G}_{\mathrm{eff},k}\mathbf{T}_k\mathbf{U}_{2,k}$. The derivation is completed.

References

[1] K. Loa, C. C. Wu, S. T. Sheu, Y. Yuan, M. Chion, D. Huo, and L. Xu, "IMT-advanced relay standards," *IEEE Communications Magazine*, vol. 48, no. 8, pp. 40-48, 2010. Article (CrossRef Link)

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform.*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004. Article (CrossRef Link)

[3] J. Zhao, M. Kuhn, A. Witteneben, and G. Bauch, "Cooperative transmission schemes for decode-and-forward relaying," in *Proc. of PIMRC '07*. Article (CrossRef Link)

[4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005. Article (CrossRef Link)

[5] H. Long, W. Ziang, Y. Zhang, J. Wang, and W. Wang, "Multi-user precoding and energy-efficient relaying scheme in multi-relay systems," in *Proc. of ICC 2011*. Article (CrossRef Link)

[6] A. Talebi and W. A. Krzymien, "Multiple-antenna multiple-relay system with precoding for multiuser transmission," in *Proc. of IWCMC 2009*. Article (CrossRef Link)

[7] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691-1706, Jul. 2003. Article (CrossRef Link)

[8] A. H. Phan, H. D. Tuan. H. H. Kham and H. H. Nguyen, "Beamforming optimization in multi-user amplify-and-forward wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1510-1520, Apr. 2012. Article (CrossRef Link)

[9] O. Oyman and A. J. Paulraj, "Design and analysis of linear distributed MIMO relaying algorithms," *IEE Proc. Commun.*, vol. 153, no. 4, pp. 565-572, Aug. 2006. Article (CrossRef Link)

[10] B. K. Chalise and L. Vandendorpe, "Optimization of MIMO relays for multipoint-to-multipoint communications: nonrobust and robust designs," *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6355-6368, Oct. 2008. Article (CrossRef Link)

[11] C. Xing, S. Li, Z. Fei, and J. Kuang, "How to understand linear minimum mean-square-error transceiver design for multiple-input-multiple-output systems from quadratic matrix programming," *IET Commun.*, vol. 7, Iss. 12, pp. 1231-1242, Aug. 2013. Article (CrossRef Link)

[12] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 461-471, Feb. 2004. Article (CrossRef Link)

[13] Y.-M. Cho, S. Jang, D. K. Kim, and J. Yang, "Minimum sum-mse design for distributed multi-relay aided multi-user MIMO network," in *Proc. of IEEE MWSCAS.*, Aug. 2011. Article (CrossRef Link)

[14] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint tx-rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381-2401, Sep. 2003. Article (CrossRef Link)

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004. Article (CrossRef Link)

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