Primary user localization using Bayesian compressive sensing and path-loss exponent estimation for cognitive radio networks

Hoang Anh and Insoo Koo
School of Electrical Engineering, University of Ulsan
680-749 San 29, Mugeo 2-dong, Ulsan, Republic of Korea
E-mail: hoanganh36bk@gmail.com and iskoo@ulsan.ac.kr
*Corresponding author: Insoo Koo

Received July 8, 2013; revised September 14, 2013; accepted September 21, 2013; published October 29, 2013

Abstract

In cognitive radio networks, acquiring the position information of the primary user is critical to the communication of the secondary user. Localization of primary users can help improve the efficiency with which the spectrum is reused, because the information can be used to avoid harmful interference to the network while simultaneity is exploited to improve the spectrum utilization. Despite its inherent inaccuracy, received signal strength based on range has been used as the standard tool for distance measurements in the location detection process. Most previous works have employed the path-loss propagation model with a fixed value of the path loss exponent. However, in actual environments, the path loss exponent for each channel is different. Moreover, due to the complexity of the radio channel, when the number of channel increases, a larger number of RSS measurements are needed, and this results in additional energy consumption. In this paper, to overcome this problem, we propose using the Bayesian compressive sensing method with a calibrated path loss exponent to improve the performance of the PU localization method.

Keywords: Cognitive radio, Bayesian compressive sensing, path loss exponent, angle of arrival method, received signal strength.

This work was supported by the KRF funded by the MEST (NRF-2012R1A1A2038831 and NRF-2013R1A2A2A05004535)

http://dx.doi.org/10.3837/tiis.2013.10.001

1. Introduction

A principal of cognitive radio technology [1] is that the secondary users (SUs) can use the idle spectrum to communicate without creating harmful interference to primary users (PUs), which is required because PUs are authorized with the priority for using the licensed band. The advantages of cognitive radio networks (CRNs) are higher utilization of the spectrum and lower cost, which leads to CRNs' having many practical applications, such as tracking PUs and assisting the communication of SUs when spectrum is a scarce resource. In a CRN that estimates the PU location, several distributed SUs estimate or compute a relevant quantity, like energy or spectrum-sensing decision, and report it to the cluster heads (CHs). Then, the CHs make a final decision about the presence or absence of a PU and estimate its location if present.

According to the information available, the localization system can be categorized into range-free and range-based algorithms [2]-[5]. In the range-free algorithm, the SUs make an independent decision on the presence or absence of the PU, without location information, and only transmit this one bit of information to the CH. The range-based algorithm includes the angle of arrival (AOA), direction of arrival (DOA), and received signal strength (RSS), which rely on having enough location information estimated by smart antenna techniques.

In this paper, we propose a localization scheme using the communication between each group of SUs to calibrate the path loss exponent. After the sensing phase, this proposed localization scheme considers values of the path loss exponent for each pair of PU and SU to find the best-fit value of the path loss exponent that can be obtained at the PU nodes, so that the PU's location precision is improved.

The SU periodically sends a signal to other SUs, and it can calculate the angle of arrival from its neighbor SU and then further calculate the path loss exponent for that direction. After that, by using the temporary location of the PU, we can find the calibrated path loss exponent between each link of PU and SU. Finally, the Bayesian compressive sensing (Bayesian CS) method is used to estimate the precise location of the PU, in the localization phase. Note that the deployment area is divided into disjoint regions based on the communication range between SUs.

The rest of the paper is constructed as follows. The general system model and background about the RSS-based method are presented in Section II. Section III describes the method to calibrate the path loss exponent in the CRN environment by applying the AOA model in each calibration region of the SUs. We also present the proposed Bayesian-CS based localization method using the calibrated path loss exponent to demonstrate the role of PU localization in interference. The simulation results are described in Section IV. Finally, conclusions are drawn in Section V.

2. System Model

In this paper, we assume that the cognitive radio network consists of SUs and PUs. **Fig.1** shows the general system model with the PU at (x_i, y_i), the cluster head (CH), and several SUs randomly distributed at (x_j, y_j), where j equals from *1* to N_{SU}. Assume that the SUs' locations are known and the location of PUs, CHs, and SUs are stationary during the

localization process. We assume that different PUs use disjoint frequency bands, so the localization of one PU is the focus because different PUs can be localized, respectively, in the same way.

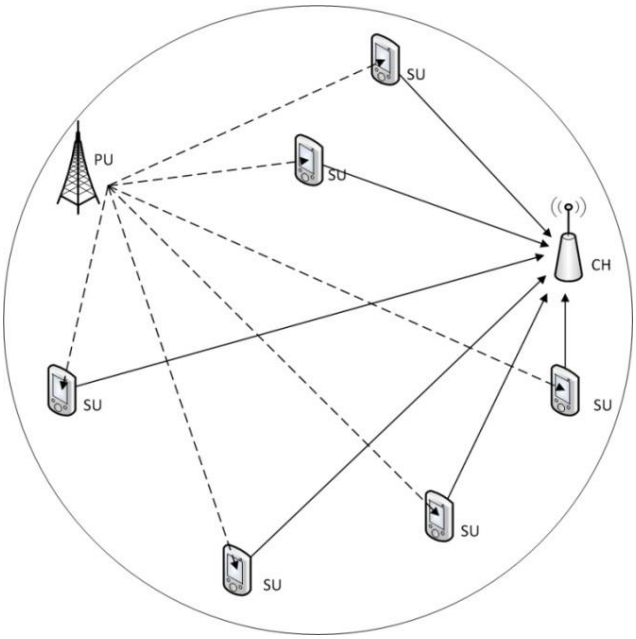

Fig. 1. System model

The RSS of the receive node can often be modeled by the following log normal shadowing path loss model [6]:

$$PL(d)(dB) = PL(d_0)(dB) + 10\gamma \log_{10}\left(\frac{d}{d_0}\right) + \chi(dB) \tag{3}$$

where $PL(d)[dB] = Pt[dBm] - \Pr[dBm]$ is the path loss at distance d from the primary transmitter. $PL(d_o)$ is the path loss at a standard distance d_0, $PL(d_o)$ is a fixed quantity and can be found using the free space model with d_0 set to 1 m in the small grid and 10 m in the large grid ($\geq 1000m$) and $\gamma = 2$. χ is a random variable with a zero-mean Gaussian distribution and variance σ_n^2 on a dB scale. γ is the path loss exponent, which differs by environment. The following Table.1 shows path loss exponents obtained in various radio environments [6].

Table 1. Some typical values of path loss exponent

Environment	Path loss exponent (γ)	σ_n (dB)
Free space	2	4 to 12
Shadowed urban area Cellular radio	3 to 5	9.6
Urban area cellular radio	2.7 to 3.5	8.7
In building Line of sight	1.6 to 1.8	5.2
Blocked by building	4 to 6	6.8
Blocked by factories	2 to 3	5.8

3. The Proposed Primary User Localization Scheme

In this section, we investigate a calibration method for updating the path loss exponent that can improve the accuracy of the proposed localization algorithm. Because the path loss model depends on the power law for the path loss exponent, a small error in the path loss exponent will produce a significant error in distance measuring. In CRNs [7]-[8], we assume that the position information of all SUs are known. In this section, we use the AOA method as a calibration method in order to find the best temporal and spatial match for the path loss exponent around the SUs, instead of using an average value for the path loss exponent (Table.1), which may result in considerable error in measuring the distance during the localization process. **Fig. 2** shows overall flow chart of the proposed localization scheme for CRNs, which is consisted of four phases: initiate phase, sensing phase, path-loss calibration phase and primary user localization phase. In next sub-sections, we will describe each phase in more details.

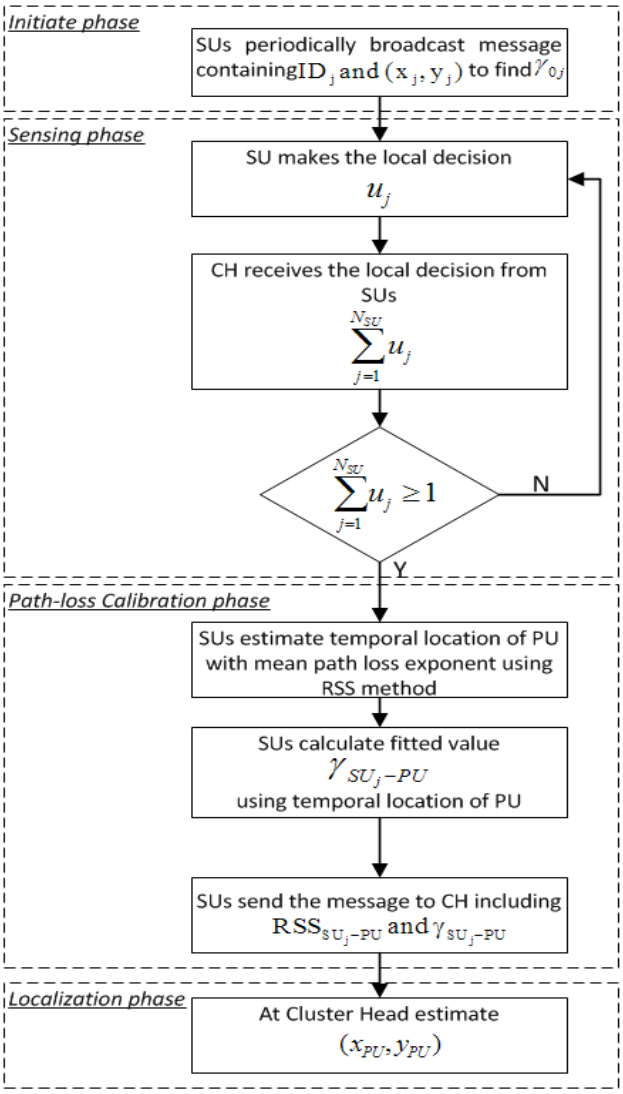

Fig. 2. Overall flow of the proposed localization scheme

3.1 Initiate Phase

SUs periodically broadcast messages containing their IDs and positions to all nodes in their transmission range; here, "broadcast" means on-hop broadcasting. That is, the message of a SU will be sent to other SUs within its transmission range. When an SU receives the messages, it records the RSS values in order to calculate the path loss of the power value, and it extracts the locations. Then, using the path loss value and the distance between any pair of SUs, the path loss exponent for that link can be found. In the above flow chart, $ID_j,(x_j,y_j)$ presents the ID and position of the *j*-th SU, and γ_{0j} presents the path loss exponent between the path of senders SU_0 and receiver SU_j.

We assume that A is sender SU and B is receiver SU, and their location are (x_1,y_1) and (x_2,y_2), respectively. The angle relationship between A and B is determined by:

$$\widehat{BA} = \tan^{-1}\frac{y_2-y_1}{x_2-x_1} \tag{2}$$

Generally, the angle of arrival is in the range $(0:2\pi)$. The direction formed by the sender SU and receiver SU nodes can be obtained by:

$$\beta(A,B) = \begin{cases} \tan^{-1}\dfrac{y_2-y_1}{x_2-x_1} \ (\mathrm{mod}\,2\pi), & x_2 \geq x_1 \\ \pi + \tan^{-1}\dfrac{y_2-y_1}{x_2-x_1}, & x_2 < x_1 \end{cases} \tag{3}$$

Similarly, while an SU periodically broadcasts a message containing its ID and position, the directions from other SUs help the central SU to divide its surrounding space into sectors, where the central SU is the SU that calculates the path loss exponents of the links; any SU can play this role during calibration of the path loss exponent.

For an example, let's assume that SU_0 is the central SU and is within the transmission ranges of SU_1 to SU_4. Then, SU_0 receives the messages and can calculate the angle of arrival from its neighbor SUs by using (3) and, further, can calculate the path loss exponent values between itself and its neighbor SUs as like **Fig. 3**.

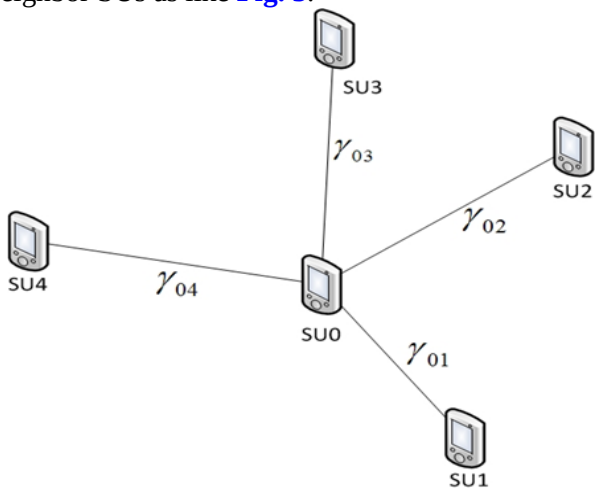

Fig. 3. Example for calculating the angles and path loss exponents between the central and other SUs

3.2 Sensing Phase

In order to protect the PUs from harmful interference, the SUs have to sense the spectrum prior to their transmissions to make sure that it is available at that time instance. In this paper, we assume that the SUs employ the widely-used energy detection method to detect the PU's signal, due to the method's simplicity, speed, and excellent capability [10]-[11]. The principle of energy detection is based on the difference in energy between the PU transmission signal and the noise. The observed energy of the *j*-th SU is given as:

$$y_j = \sum_{k=1}^{N} \left| x_j(k) \right|^2 \tag{4}$$

where $x_j(k)$ is the *k*-th sample of the received signal at the *j*-th SU, and N is the number of samples, $N = 2TW$. T and W are the detection time and the signal bandwidth, respectively.

According to the status of the PU, the received signal at the *j*-th SU is given by:

$$x_j(t) = \begin{cases} \mathrm{n}(t), & H_0 \\ h_j s(t) + \mathrm{n}(t), & H_1 \end{cases} \tag{5}$$

where H_0 and H_1 represent the absence and presence of a PU, respectively, $s(t)$ denotes the signal transmitted by the PU, h_j is the channel gain between the *j*-th SU in the cluster and the PU, and $\mathrm{n}(t)$ denotes the additive noise at the SU.

Suppose that the noise in each sample is a Gaussian random variable with a mean of zero and variance σ_n^2. Then, if the PU signal is absent, the sum of the squares of N Gaussian random variables y_j / σ_n^2 follows a central chi-square distribution with N degrees of freedom and a non-centrality parameter $N\eta_j$:

$$\frac{y_j}{\sigma_n^2} \sim \begin{cases} \chi_N^2, & H_0 \\ \chi_N^2(N\eta), & H_1 \end{cases} \tag{6}$$

where $\eta_j = \dfrac{E_s |h_j|^2}{N\sigma_n^2}$ is the SNR of the PU signal at the SU, and the quantity $E_s = \sum_{k=1}^{N} \left| s(k) \right|^2$ represents the transmitted signal energy over a sequence of N samples during each detection interval.

A local decision of the SU can be made as follows:

$$u_j = \begin{cases} H_1 : y_j \geq \lambda \\ H_0, otherwise \end{cases} \tag{7}$$

where λ is the decision threshold of the SU.

With a given false alarm probability P_f, the threshold λ is decided by:

$$\begin{aligned} P_f &= \Pr\{u_j \geq \lambda \mid H_0\} \\ &= Q\left(\frac{\lambda - N\sigma_n^2}{\sqrt{2N\sigma_n^4}}\right) \end{aligned} \tag{8}$$

Then, the detection probability of the *k*-th SU is calculated as follows:

$$\begin{aligned} P_d^j &= \Pr\{u_j \geq \lambda \mid H_1\} \\ &= Q\left(\frac{\lambda - N(1+\eta)\sigma_n^2}{\sqrt{2N(1+2\eta)\sigma_n^4}}\right) \end{aligned} \tag{9}$$

where $Q(.)$ is the tail probability of the standard normal distribution (also called the complementary cumulative distribution function), $Q(x) = \frac{1}{2\pi}\int_x^{+\infty} e^{-\frac{t^2}{2}} dt$.

Assume that the channel to CH is perfect. The local decisions are collected at the cluster head, and then a final decision is made. A final decision of the CH using the OR rule [12] can be made as follows:

$$u_c = \begin{cases} H_1 & \text{when } \sum_{j=1}^{N_{SU}} u_j \geq 1 \\ H_0 & \text{otherwise} \end{cases} \tag{10}$$

where N_{SU} is the number of SUs in the cluster.

3.3 Path-loss Calibration Phase

In the case that a PU is present, its temporal location can be calculated using the RSS method with at least 3 SUs and mean path-loss exponent. However, the temporal location is imprecise, since the mean, rather than the calibrated, path-loss exponent is used to define the distances between the PU and its neighboring SUs. After obtaining the temporal location, the SU receives the signal from the PU in order to improve the precision of the location. One of the SUs, called the central SU, checks the angle of arrival by using the temporal location of the PU. Then, the path loss exponent for the link between the PU and the central SU is used, along with the distances between all pairs of SUs, to find the best-fit value of the path-loss exponent [12]. **Fig. 4** shows the concept of this path-loss calibration:

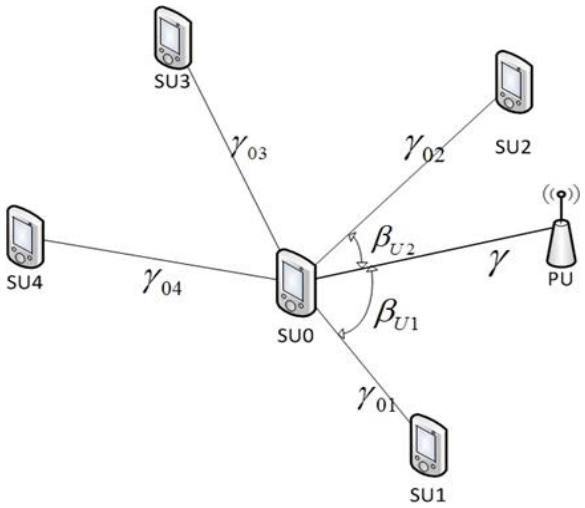

Fig. 4. Example for the path-loss calibration

In the **Fig.4**, the PU is in the sector formed by SU_0, SU_1, and SU_2. The SU_0 checks the angle of arrival based on the temporal location of the PU. To obtain the calibrated value of the path loss exponent over the link between PU and SU_0, the information of distance and path loss over links $SU_0 - SU_1$ and $SU_0 - SU_2$ are used. The best value of the path loss exponent for the link between PU and SU_0 is calculated by:

$$\gamma = \frac{PL_{01}D_{01}\beta_{U1} + PL_{02}D_{02}\beta_{U2}}{D_{01}^2\beta_{U1} + D_{02}^2\beta_{U1}} \tag{11}$$

where PL_{01} and PL_{02} are path loss values and D_{01} and D_{02} are log-normal distances over the links $SU_0 - SU_1$ and $SU_0 - SU_2$, respectively. The angles of arrival between PU and SU_1 and between PU and SU_2 can be calculated respectively by:

$$\beta_{U1} = \begin{cases} \tan^{-1}\dfrac{y_{SU1} - y_{PU}}{x_{SU1} - x_{PU}} \pmod{2\pi}, & x_{SU1} \geq x_{PU} \\ \pi + \tan^{-1}\dfrac{y_{SU1} - y_{PU}}{x_{SU1} - x_{PU}}, & x_{SU1} < x_{PU} \end{cases} \tag{12}$$

and

$$\beta_{U2} = \begin{cases} \tan^{-1}\dfrac{y_{SU2} - y_{PU}}{x_{SU2} - x_{PU}} \pmod{2\pi}, & x_{SU2} \geq x_{PU} \\ \pi + \tan^{-1}\dfrac{y_{SU2} - y_{PU}}{x_{SU2} - x_{PU}}, & x_{SU2} < x_{PU} \end{cases} \tag{13}$$

3.4 Primary User Localization Phase

In this section, we will show how localization in CRN could be viewed as a CS problem and formulated in terms of CS equations. We propose the RSS-based localization scheme

composed of two phases: a training process, which is performed to collect the RSS measurement samples on a grid of SUs, and the reconstructing process, which reduces the area of interest to a smaller number of RSS measurements that then use Bayesian compressive sensing [14]-[16] to estimate the actual location of the PU. **Fig.5** shows the proposed Bayesian CS-based localization system.

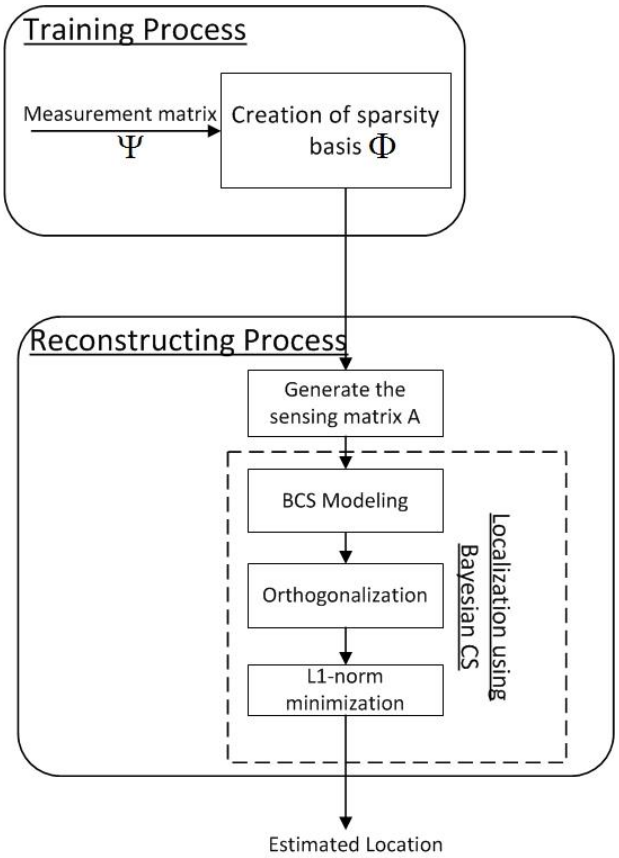

Fig. 5. The proposed Bayesian CS-based localization scheme

3.4.1 Training Process

We consider a case in which the N_{PU} 's are located in an isotropic area, which is divided into a discrete grid with *N* points, but the exact PU locations are unknown. The goal is to determine the locations of PUs accurately, using only a small number of noisy RSS measurements and simple operations. This problem has a sparse nature, that is, $N_{PU} \ll N$. Furthermore, the number of measurements *M* is much smaller than the grid size *N*.

In the localization problem, since the location of each PU is unique within the discrete spatial domain at a certain time, it can be modeled as an ideal sparse vector. Thus, the localization problem can be well formulated as a sparse matrix recovery problem in the discrete spatial domain. To perform localization, at first we need a training process which is

performed by interactions between the CHs and SUs as like **Fig. 6**. Through the training process, basic matrix measurement matrix and radio map will be built.

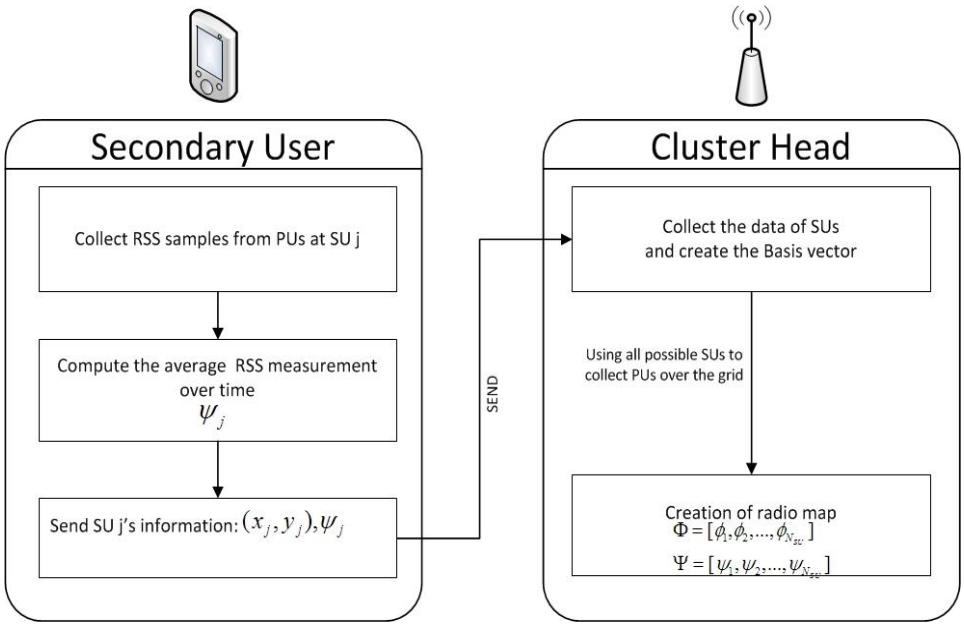

Fig. 6. Cooperation between CHs and SUs during the training process

A. Basis matrix (Ψ)

During the training process, the time samples of RSS measurements are collected at known SU locations. The τ time sample of the RSS that records the RSS value at the *j*th SU from the *i*th PU over the grid for all $1 \le i \le N, 1 \le j \le N$ is denoted as $\psi_{i,j}(\tau), \tau = 1,...,q(q>1)$ where q is the total number of time samples collected.

According to Section.2, the RSS propagation model is expressed as:

$$RSS[dB] = P_t[dB] - PL[dB] - 20\log\left(\frac{4\pi f}{c}\right) + \chi \tag{14}$$

where P_t is the transmission power of the PU, PL is the path loss model with calibration path loss exponent γ mentioned in the previous chapter, and χ is the zero-mean Gaussian distribution.

Therefore, from the above equation, each value of $\psi_{i,j}$ corresponds to the mean value of the RSS signal that SU *j* receives from PU *i* at a specific location:

$$\psi_{i,j} = RSS_{d_{ij}} \tag{15}$$

The RSS value recovered at each possible position point *j* in the grid that SU may occupy can be represented as

$$\begin{aligned} \psi_j &= [\psi_{1,j}, \psi_{2,j}, ..., \psi_{N,j}]^T \\ j &= 1, 2, ..., N \end{aligned} \tag{16}$$

where N is the size of the grid. The average RSS value over the time domain, at SU *j* from PU *i*, can be expressed as $\psi_{i,j} = \frac{1}{q}\sum_{\tau=1}^{q}\psi_{i,j}(\tau)$ with $\tau = 1, ..., q,\ q > 1$.

The basis matrix $\mathbf{\Psi}$ is represented by

$$\mathbf{\Psi}_{\text{NxN}} = \begin{bmatrix} \psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,N} \\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N,1} & \psi_{N,2} & \cdots & \psi_{N,N} \end{bmatrix} \tag{17}$$

The average of these time samples is computed on the CH. This basis matrix gives a sufficient representation of the spatial RSS properties in the cognitive radio environment.

B. Measurement Matrix ($\mathbf{\Phi}$)

Instead of measuring all the RSS values on the overall grid, only a small number of measurements are collected at several arbitrary grid points in which SUs are located. In the paper, we denote these measurements as measurement matrix $\mathbf{\Phi}$. Each row of $\mathbf{\Phi}$ represents the location of each SU where an element of 1 indicates the grid point in which the SU is located. If you denote ϕ_j as the *j*-th low vector of $\mathbf{\Phi}$ where $j = 1, 2, ..., M$ and M is the number of SUs, then $\phi_j(3)$ indicates that the j^{th} SU node is located at grid 3 when the vector ϕ_j is given as $[0, 0, 1, ..., 0, ..., 0]$. Thus the measurement matrix can be represented by

$$\mathbf{\Phi}_{\text{MxN}} = [\phi_1, \phi_2, ..., \phi_M]^T \tag{18}$$

C. Radio Map

Finally, **Fig. 7** shows the framework for the creation of the radio map. The radio map is the table of $[(x_j, y_j), \psi_j, \Phi_j]$ where (x_j, y_j) represents the coordinates of the j^{th} SU. If no RSS measurements are found for a PU at SU due to the absence of any PU, the corresponding RSS entity in the radio map is set to a small value that implies zero.

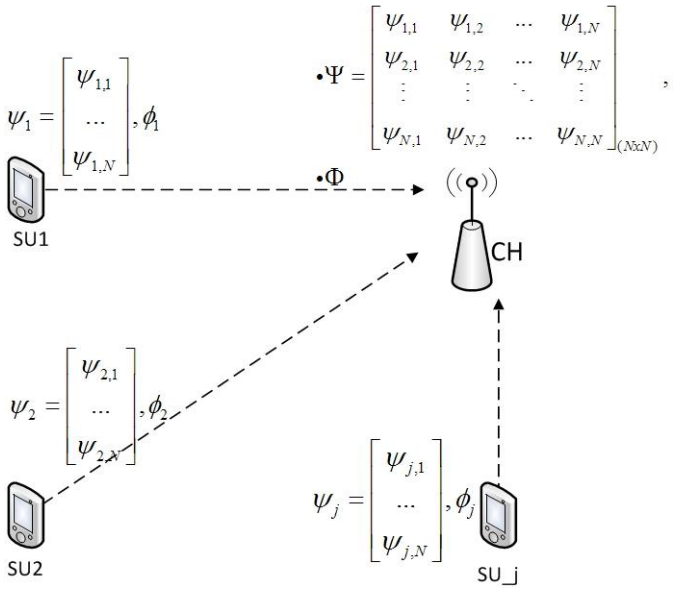

Fig. 7. Example for creating radio map

3.4.2 Reconstructing Process

Without loss of generality, we can assume that one PU's location can be formulated as a 1-sparse vector $\boldsymbol{\theta}_\mathbf{i}$ where $\boldsymbol{\theta}_\mathbf{i}$ is an N×1 vector with all elements equal to zero except $\boldsymbol{\theta}_i(n)=1,\ n=1,...,N$ where n is the index of the PU where the PU is located. For instance, the vector $\boldsymbol{\theta}_\mathbf{i}=[0,1,0,...,0]^T_{(\mathrm{Nx1})}$ indicates that the i^{th} PU node is located at grid 2. Therefore, the locations of the detectable PUs over the grid will be a *K*-sparse matrix and are represented by $\boldsymbol{\Theta}$ such that we have

$$\boldsymbol{\Theta}_{\mathrm{NxN_{PU}}}=[\boldsymbol{\theta}_\mathbf{1},...,\boldsymbol{\theta}_\mathbf{i},...,\boldsymbol{\theta}_\mathbf{N_{PU}}] \tag{19}$$

Fig. 8 shows the cooperation between CHs and SUs during the reconstruction process.

Due to the wide deployment of SUs, the numbers of SUs are generally much greater than that required for localization, which leads to biased estimates. In the paper, we assume the use of *M* possible positions of SUs for localization, $M \le N_{SU}$.

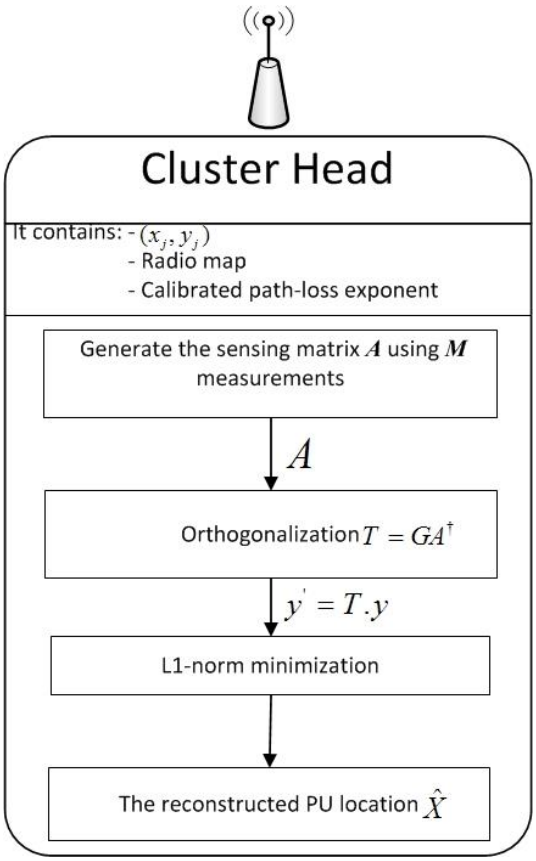

Fig. 8. Cooperation process between CHs and SUs during the reconstructing process

Based on the above equation and notations, the MxN_{PU} matrix $\mathbf{y}$ represents the compressive noisy RSS measurements from N_{PU} PUs at M SUs with each row vector indicating one measurement value. Finally, according to the CS theory, the compressive noisy RSS measurement is obtained by multiplying a random matrix by the original signal, which can be expressed as

$$\begin{aligned} \mathbf{y}_{\mathrm{MxN_{PU}}} &= \mathbf{\Phi}_{\mathrm{MxN}} \mathbf{\Psi}_{\mathrm{NxN}} \mathbf{\Theta}_{\mathrm{NxN_{PU}}} + \boldsymbol{\varepsilon}_{\mathrm{MxN_{PU}}} \\ &= \mathbf{A}_{\mathrm{MxN}} \mathbf{X}_{\mathrm{NxN_{PU}}} + \boldsymbol{\varepsilon}_{\mathrm{MxN_{PU}}} \\ \text{where } \mathbf{A} &= \mathbf{\Phi}\mathbf{\Psi};\ \mathbf{X} = \mathbf{\Theta} \end{aligned} \tag{20}$$

3.4.3 Bayesian Compressed Sensing

Compressed sensing is a technique for acquiring and reconstructing a signal utilizing the prior knowledge that it is sparse or compressible with limited (incomplete) measurements.

When $\mathbf{\Phi}$ satisfies the restricted isometry property (RIP) [17], it requires only

$M \geq (N_{PU} \log(N / N_{PU}))$ to recover $\boldsymbol{\Theta}$ with high probability. The recovery algorithm can be expressed as:

$$\begin{aligned} &\min \| \boldsymbol{\theta}_{\mathbf{i}} \|_1 = \min \sum_j | \boldsymbol{\theta}_{\mathbf{i}}(\mathrm{j}) | \quad \text{for} \quad i = 1, \ldots, \mathrm{N}_{PU} \\ &\text{subject to } \mathbf{y} = \mathbf{AX} \end{aligned} \tag{21}$$

which is called *basic pursuit* [18].

Compared to the theoretical optimization methods [19], Bayesian compressed sensing provides a better solution based on Bayes rule [20]-[21]. In Eqn. (20), the components of $\boldsymbol{\varepsilon}$ can be approximated as zero-mean uncorrelated Gaussian noise with unknown variance σ^2. Therefore, the probability density function (pdf) of $\boldsymbol{\varepsilon}$ is given by:

$$p(\boldsymbol{\varepsilon}) = \prod_{i=1}^{M} N(\varepsilon_i \mid 0, \sigma^2) \tag{22}$$

The observation of noisy compressed measurement data $\mathbf{y}$ is also a random process with conditional distribution, $p(\mathbf{y} \mid \mathbf{X}, \sigma^2)$. Thus the conditional distribution of the observations becomes a Gaussian likelihood model [21] such that we have:

$$p(\mathbf{y} \mid \mathbf{X}, \sigma^2) = (2\pi\sigma^2)^{-M/2} \exp(-\frac{1}{2\sigma^2} \| \mathbf{y} - \mathbf{AX} \|^2) \tag{23}$$

A widely used sparseness representation is the Laplace density function. However, because Laplace density is not conjugate, we adopt the zero-mean Gaussian used earlier in the following function:

$$\begin{aligned} p(\mathbf{X} \mid \boldsymbol{\alpha}) &= \prod_{i=1}^{N} N(X_i \mid 0, \boldsymbol{\alpha}^{-1}) \\ &= (2\pi)^{-2/N} \prod_{n=1}^{N} \alpha_n^{1/2} \exp(-\frac{\alpha_n x_n^2}{2}) \end{aligned} \tag{24}$$

where $\boldsymbol{\alpha}_{\mathrm{Nx1}}$ is the N independent hyperparameter, which is the inverse variance, i.e., the precision of the prior Gaussian distribution. This approach is based on relevance vector machine (RVM) [22] .

Assuming that $\boldsymbol{\alpha}$ is known, using Gaussian likelihood and Bayes' rule, the conditional probability density function for $\mathbf{X}_{\mathrm{NxN_{PU}}}$ can be represented as:

$$p(\mathbf{X}\,|\,\mathbf{y},\boldsymbol{\alpha},\sigma^2)=\frac{p(\mathbf{y}\,|\,\mathbf{X},\sigma^2)\,p(\mathbf{X}\,|\,\boldsymbol{\alpha})}{p(\mathbf{y}\,|\,\boldsymbol{\alpha},\sigma^2)}$$
$$=(2\pi)^{-2/N}\left|\boldsymbol{\Sigma}\right|^{-1/2}\exp(-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu})) \tag{25}$$

which is a Gaussian distribution $N(\boldsymbol{\mu},\boldsymbol{\Sigma})$ with mean and covariance given by

$$\begin{aligned}\boldsymbol{\mu}_{\mathrm{NxN_{NP}}}&=\sigma^{-2}\boldsymbol{\Sigma}\mathbf{A}^T\mathbf{y}\\ \boldsymbol{\Sigma}_{\mathrm{NxN}}&=(\mathbf{B}+\sigma^{-2}\mathbf{A}^T\mathbf{A})^{-1}]\\ \mathbf{B}_{\mathrm{NxN}}&=diag\{\boldsymbol{\alpha}\}\\ &=\{\alpha_1,\alpha_2,...,\alpha_N\}\end{aligned} \tag{26}$$

Therefore, the posterior density function of the observation y is a multivariate Gaussian distribution with mean $E[\mathbf{y}]=\mathbf{A}\boldsymbol{\mu}$ and covariance $Cov[\mathbf{y}]=\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^T$.

3.4.4 Orthogonalization and Signal Recovery

Since $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are generally coherent in the spatial domain, which violates the incoherence requirement for the CS theory, an orthogonalization process is applied so as to $\mathbf{A}_{\mathrm{MxN}}=\boldsymbol{\Phi}\boldsymbol{\Psi}$ can satisfy RIP condition. That is, the measurement vector $\mathbf{y}$ can be preprocessed by an orthogonalization operator $\mathbf{T}$ such that $\mathbf{y}'=\mathbf{T}\mathbf{y}$. The orthogonalization operator $\mathbf{T}$ is defined as

$$\mathbf{T}_{\mathrm{MxM}}=\mathbf{G}\mathbf{A}^{\dagger} \tag{27}$$

where $\mathbf{G}_{\mathrm{MxN}}=orth(\mathbf{A}^T)^T$ is an orthogonal basis for the range of $\mathbf{A}^T$, and $\mathbf{A}^{\dagger}{}_{\mathrm{NxM}}$ is a pseudo-inverse matrix of $\mathbf{A}$ and $\mathbf{A}^{\dagger}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$.

Thus, when $\boldsymbol{\Phi}$ satisfies the restricted isometry property (RIP), it requires only $M\geq(N_{PU}\log(N/N_{PU}))$ for $\mathbf{X}$ to be fully recoverable from $\mathbf{y}'_{\mathrm{MxN_{NP}}}$ with high probability, through the following l_1-minimization program:

$$\begin{aligned}&\hat{\mathbf{X}}=\arg\min_{\theta\in R^N}\|\mathbf{X}\|_1 \quad \text{such that } \mathbf{y}'=\mathbf{G}\mathbf{X}+\boldsymbol{\varepsilon}'\\ &\text{where } \boldsymbol{\varepsilon}'=\mathbf{T}\boldsymbol{\varepsilon}\end{aligned} \tag{28}$$

4. Simulation Results

In this section, to demonstrate the performance of the proposed scheme, we describe two other reconstruction methods: the basis pursuit (BP method) and Bayesian compressive sensing (the Bayesian CS method). We utilize Matlab and the l_1 norm packet provided in [23] to obtain the localization results.

We consider a secondary network including 5 CHs and 50 SUs that are randomly distributed and whose positions are known, and a primary network including 10 PUs randomly distributed in a 100 meter square, whose positions are unknown. For this area, a grid-based structure is considered divided into cells of size 0.5m × 0.5m (N=400). A total of q time samples are collected 10 times during the training process for calculating the received signal strength, and 100 run times are conducted for performance evaluation.

As the CS theory indicates regarding the restricted isometry property (RIP), the minimum number of measurements is required as following:

$$\begin{aligned} M &\geq (N_{PU} \log(N / N_{PU})) \\ &= (10 \log(400/10)) \approx 16 \end{aligned} \tag{29}$$

Since 50 SUs take measurements, restricted isometry property (RIP) is satisfied in the simulation.

In the following simulation, the localization error is measured according to the number of measurements needed, using the compressive sensing approach via the above three recovery programs: the proposed scheme, the Bayesian CS method, and the basis pursuit method. Given a false alarm probability of 0.01, the number of measurements varies from the minimum number of measurements to the number of total SUs. The localization error is defined as the average Euclidean distances between the true positions and the recovered positions of detected PUs as following:

$$P_e = \frac{1}{N_{PU}} \sum_{i=1}^{N_{PU}} \sqrt{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2} \tag{30}$$

As shown in **Fig.9**, the localization errors of with all three considered methods decrease sharply and become very small as the number of measurements increases. To observe the localization errors, here we only consider more than M=16 cases because the minimum number of measurements to satisfy RIP is 16 according to (29). Our proposed scheme has far better performance than the basis pursuit method in term of the accuracy of localization. We

also observe that our proposed scheme has better performance than the Bayesian CS method, which uses only the given path-loss exponent.

Fig.10 shows that the interference rate depends strongly on the accuracy of the localization method. Through simulation, it is verified that the improved accuracy of localization by the proposed method can improve the interference rate significantly as the maximum transmission range of the SU increase from 16m to 25m.

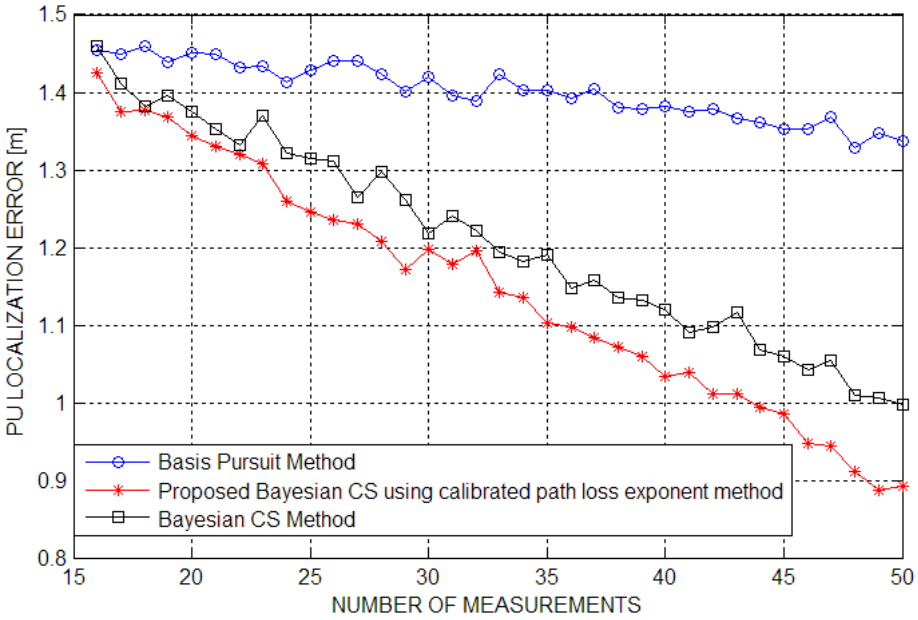

Fig. 9. Localization error according to the number of measurements

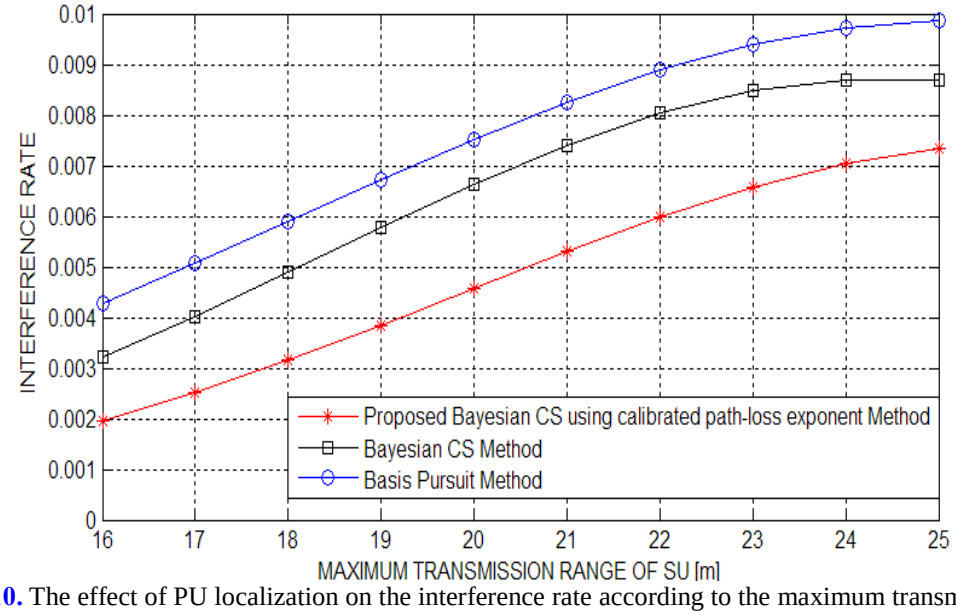

Fig. 10. The effect of PU localization on the interference rate according to the maximum transmission range of SUs

7. Conclusions

In cognitive radio networks (CRNs), acquiring the position information of primary users (PUs) is very important to the communication of secondary users (SUs). Accurate localization of a PU can improve the efficiency with which the spectrum is reused because it can be used to avoid harmful interference in the network while simultaneity is exploited to improve the spectrum utilization. Despite its inherent inaccuracy, received signal strength(RSS) range-based method has been used as the standard tool for distance measurements in the location detection process. As the number of channels increases, a larger number of RSS measurements are required for localization, which results in additional energy consumption. In this paper, we focused our efforts on improving localization accuracy by using Bayesian compressive sensing (CS) with a calibrated path-loss exponent based on nearby SU's information. Compared with other methods, the proposed scheme demonstrates more precise location of PUs with a lower number of noisy measurements.

Acknowledgement

This work was supported by the KRF funded by the MEST (NRF-2012R1A1A2038831 and NRF-2013R1A2A2A05004535)

References

[1] S. Haykin, "Cognitive radio: Brain empowered wireless communications," *IEEE J. Sel. Areas Comm.*, vol. 23, no. 2, pp.Feb.2008. Article (CrossRef Link)

[2] A. Uchiyama, S. Fujii, K. Maeda, T. Umedu, H. Yamaguchi, and T. Hig -ashino, "Ad-hoc localization in urban district," in *Proc. of IEEE INFOCOM*, Anchorage, AK, pp. 2306–2310, 2007.

[3] D. Gong, Z Ma. Y. Li, W. Ches, and Z. Cao, "High geometric range free localization in opportunistic cognitive sensor network," in *Proc. of IEEE International Personal Commun*., pp. 139-143, 2008.

[4] N. Bulusu, J. Heidemann, and D, Estrin, "GPS-less Low Cost Outdoor Localization for Very Small Devices," *IEEE Personal Communications Magazine*, vol. 7 no. 5, Oct., 2000. Article (CrossRef Link)

[5] Tian He, Chengdu Huang, Brian M.Blum, John A.Stankovic, and Tarek Abdelzaher, "Range-Free Localization Schemes for Large Scale Sensor Networks," in *Proc. of ACM MOBICOM*, 2003.

[6] Tapan K.Sarkar, Zhong Ji, Kyungjung Kim, Abdellatif Medourf, and Magdalena Salazar-Palma, "A Survey of Various Propagation Models for Mobile Communication," *IEEE Antennas and Propagation Magazine*, vol. 45 (2003) pp. 51-82. Article (CrossRef Link)

[7] H. Arslan, *Cognitive Radio, Software Defined Radio, and Adaptive Wireless Systems*, Springer, 2007.

[8] N. Radhi, K. Aziz, S. Hamad and H.S. Al-Raweshidy, "Estimate primary user localization using cognitive radio networks," in *Proc. of International Conference on Innovations in Information Technology* , pp. 381-385, 2011.

[9] T. Rappaport, *Wireless Communication: Principles and Practice*, 2nd Edition, Prentice-Hall 2002.

[10] M.A. Abdulsattar and Zahir A. Hussein, "Energy detection technique for spectrum sensing in Cognitive radio: A survey," *International Journal of Computer Networks & Communications (IJCNC)*, Vol.4, No.5, pp. 223-242, 2012. Article (CrossRef Link)

[11] M.A. Hossain, Md. Hossain and Md. Ibrahim Abdullah, "Performance analysis of Cooperative Spectrum sensing in Cognitive Radio," *International Journal of Innovation and Applied Studies*, Vol.1, No.2, pp. 236-245, 2012. Article (CrossRef Link)
[12] T.X Cong and Insoo Koo, "An RSS-based Localization Scheme Using Direction Calibration and Reliability Factor Information for WSN," *KSII Transactions on Internet and Information Systems*, Vol.4, No.1 (Feb. 2010). Article (CrossRef Link)
[13] C. Kocks, S. Wang, G.H. Bruck and P. Jung, "Maximum likelihood localization estimation based on received signal strength," *in Proc. of International Symposium on Applied Sciences in Biomedical and Communication Technologies (ISABEL)*, 2010.
[14] D. Donoho, "Compressive Sensing," *IEEE Trans. Inform. Theory*, 2006, 52(4), pp.1289-1306. Article (CrossRef Link)
[15] E. Candes, J. Romberg and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information" *IEEE Trans. Inform. Theory*, 2006, 52(2), pp.489-509. Article (CrossRef Link)
[16] Wei Dai, O. Milenkovic, "Subspace Pursuit for Compressive Sensing Signal Reconstruction," *IEEE Trans. Inform. Theory*, 2009, 55(5), pp.2230-2249. Article (CrossRef Link)
[17] R. Baraniuk, M. Davenport, R. DeVore and M. Wakin, "A Simple Proof of the Restricted Isometry Property for Random Matrices," *Constructive Approximation*, Vol.28, Issue 3, pp.253-263, 2008. Article (CrossRef Link)
[18] S. S. Chen, D. L. Donoho, and M. A. Saunders, "Atomic decomposition by basis pursuit," SIAM *Journal of Scientific Computing*, 43(1), pp.129-159, 1998.
[19] L. Rebollo-Neira and D. Lowe, "Optimized Orthogonal Matching Pursuit Approach." *IEEE Signal Processing Letters*, 9(4), pp.137-140, 2002. Article (CrossRef Link)
[20] S. Hong, "Direct spectrum sensing from compressed measurements," in *Proc. of Military Communications Conference*, pp. 1187 –1192, Nov. 2010. Article (CrossRef Link)
[21] S. Ji, Y.Xue and L. Carin "Bayesian Compressive Sensing," *IEEE Transaction on Signal Processing*, vol. 56, no. 6, 2008. Article (CrossRef Link)
[22] M.E. Tipping, "Sparse Bayesian Leaning and the Relevance Vector Machine," *Journal of Machine Leaning Research*, pp. 211-244, 2001. Article (CrossRef Link)
[23] http://dsp.rice.edu/cs.

Hoang Anh received his B.E. degree in Electronics & Telecommunications Engineering from the Hanoi University of Technology, Vietnam, in 2008. Recently, He obtained Master degree at the School of Electrical Engineering, University of Ulsan, South Korea. His research interests include localization for cognitive radio and sensor network.

Insoo Koo received the B.E. degree from the Kon-Kuk University, Seoul, Korea, in 1996, and received the M.S. and Ph.D. degrees from the Gwangju Institute of Science and Technology (GIST), Gwangju, Korea, in 1998 and 2002, respectively. From 2002 to 2004, he was with Ultrafast Fiber-Optic Networks (UFON) research center in GIST, as a research professor. For one year from September 2003, he was a visiting scholar at Royal Institute of Science and Technology, Sweden. In 2005, he joined University of Ulsan where he is now full professor. His research interests include next generation wireless communication systems and wireless sensor networks.