# An Improved Mean-Variance Optimization for Nonconvex Economic Dispatch Problems 

Min Jeong Kim*, Hyoung-Yong Song*, Jong-Bae Park*, Jae-Hyung Roh ${ }^{\dagger}$ Sang Un Lee**, and Sung-Yong Son***


#### Abstract

This paper presents an efficient approach for solving economic dispatch (ED) problems with nonconvex cost functions using a 'Mean-Variance Optimization (MVO)' algorithm with KuhnTucker condition and swap process. The aim of the ED problem, one of the most important activities in power system operation and planning, is to determine the optimal combination of power outputs of all generating units so as to meet the required load demand at minimum operating cost while satisfying system equality and inequality constraints. This paper applies Kuhn-Tucker condition and swap process to a MVO algorithm to improve a global minimum searching capability. The proposed MVO is applied to three different nonconvex ED problems with valve-point effects, prohibited operating zones, transmission network losses, and multi-fuels with valve-point effects. Additionally, it is applied to the large-scale power system of Korea. The results are compared with those of the state-of-the-art methods as well.


Keywords: Nonconvex optimization, Economic dispatch, Mean-variance optimization, Kuhn-Tucker condition

## 1. Introduction

Many power system optimization problems including economic dispatch (ED) have nonconvex characteristics with heavy equality and inequality constraints [1]. The aim of the ED problem is to determine the optimal combination of power outputs of all generating units so as to meet the required load demand at minimum operating cost while satisfying system equality and inequality constraints. In ED problems, the cost function of each generator has been approximately expressed by a quadratic function and is solved using mathematical programming [2]. However, each generator has nonconvex input-output characteristics due to prohibited operating zones, valve-point loadings, multi-fuel effects, etc. Therefore it is difficult to solve ED by using the traditional mathematical methods because ED problems should be represented as a nonconvex optimization problem with constraints. Dynamic programming [3] can treat such types of problems, but it suffers from the curse of dimensionality. Over the past decade, many salient methods have been developed to solve these problems, such as the hierarchical numerical method [4], genetic algorithm (GA) [5-7], evolutionary programming [8-10],

[^0]Tabu search [11], neural network approaches [12, 13], differential evolution [14], particle swarm optimization [15-18], hybrid artificial intelligence (AI) method [19], improved PSO [20] and Immune-PSO(IPSO) [21].

MVO suggested by Erlich, Venayagamoorthy and Worawat is one of the stochastic optimization algorithms which is simple to implement. It falls into the category of the so-called "population-based stochastic optimization technique". Its concepts share some similarities and differences from other known stochastic algorithms. Like other algorithms such as differential evolution, genetic algorithm, and particle swarm optimization, it borrows ideas of selection, mutation and crossover from evolutionary computation algorithms. The main distinct feature of the MVO algorithms is a strategic transformation of mutated genes of the offspring based on the mean-variance of the $n$ best population [22].

This paper proposes an improved MVO algorithm with Kuhn-Tucker condition and swap process (KMVO) for the nonconvex ED problems with heavy constraints. KuhnTucker condition is a mathematical judgment for heuristic method that is experience-based techniques for problem solving. Swap process is a method that changes the generator outputs in the direction of reducing total generation cost for searching local minimum. These two methods are applied to MVO algorithm so that optimal solution can be obtained. The suggested KMVO is applied to three different nonconvex ED problems and the largescale power system of Korea. The solutions are compared with those of existing AI methods.

## 2. Formulation of Economic Dispatch Problem

### 2.1 Objective function

The objective of an ED problem is to determine the optimal generation outputs that minimize the total fuel cost while satisfying equality and inequality constraints. The total fuel cost for all the generators in the system and simplified cost function of each generating unit can be represented as ( 1,2 ), respectively:

$$
\begin{gather*}
F_{T}=\sum_{i=1}^{n} F_{i}\left(P_{i}\right)  \tag{1}\\
F_{i}\left(P_{i}\right)=a_{i}+b_{i} P_{i}+c_{i} P_{i}^{2} \tag{2}
\end{gather*}
$$

where

$$
\begin{array}{ll}
F_{T} & \text { total fuel cost, } \\
F_{i} & \text { cost function of generator } i, \\
a_{i}, b_{i}, c_{i} & \text { cost coefficients of generator } i, \\
P_{i} & \text { power output of generator } i, \\
n & \text { number of generators. }
\end{array}
$$

1) ED Problem Considering Valve-Point Effects: The generating units with multi-valve steam turbines exhibit a greater variation in the fuel cost function. Because of the valve point effects, a cost function contains higher order of nonlinearity. Therefore, the cost function (2) is described as the superposition of sinusoidal functions and quadratic functions in (3):

$$
\begin{equation*}
F_{i}\left(P_{i}\right)=a_{i}+b_{i} P_{i}+c_{i} P_{i}^{2}+\left|e_{i} \times \sin \left(f_{i} \times\left(P_{i, \text { min }}-P_{i}\right)\right)\right| \tag{3}
\end{equation*}
$$

where $e_{i}$ and $f_{i}$ are the cost coefficients of generator $i$ reflecting valve-point effects [10].
2) ED Problem Considering Multi-Fuels with ValvePoint Effects: Since the dispatching units can be supplied with multi-fuel sources, each unit can be represented with several piecewise quadratic functions reflecting the effects of different fuel types. In general, the objective function is expressed as the piecewise quadratic functions to represent the input-output curve of a generator with multiple fuels [4] and described as in (4):

$$
F_{i}\left(P_{i}\right)=\left\{\begin{array}{ccc}
a_{i l}+b_{i l} P_{i}+c_{i l} P_{i}^{2}, & \text { fuel } 1, & P_{i, \text { min }} \leq P_{i} \leq P_{i l}  \tag{4}\\
a_{i 2}+b_{i 2} P_{i}+c_{i 2} P_{i}^{2}, & \text { fuel } 2, & P_{i l} \leq P_{i} \leq P_{i 2} \\
\vdots & & \vdots \\
a_{i k}+b_{i k} P_{i}+c_{i k} P_{i}^{2}, & \text { fuel } k, & P_{i k-l} \leq P_{i} \leq P_{i, \max }
\end{array}\right.
$$

where $a_{i k}, b_{i k}, c_{i k}$ are the cost coefficients of generator $i$ for fuel type $k$. To obtain an accurate and practical ED solution, the fuel cost function should be considered with both multi-fuels and valve-point effects simultaneously [7].

Thus, the fuel cost function (3) should be combined with (4), and can be represented as follows:

$$
F_{i}\left(P_{i}\right)=\left\{\begin{array}{ccc}
F_{i l}\left(P_{i}\right), & \text { fuel } 1, & P_{i, \text { min }} \leq P_{i} \leq P_{i l}  \tag{5}\\
F_{i 2}\left(P_{i}\right), & \text { fuel } 2, & P_{i l} \leq P_{i} \leq P_{i 2} \\
\vdots & & \vdots \\
F_{i k}\left(P_{i}\right), & \text { fuel } k, & P_{i k-1} \leq P_{i} \leq P_{i, \max }
\end{array}\right.
$$

where

$$
\begin{equation*}
F_{i k}\left(P_{i}\right)=a_{i k}+b_{i k} P_{i}+c_{i k} P_{i}^{2}+\left|e_{i k} \times \sin \left(f_{i k} \times\left(P_{i k, \text { min }}-P_{i}\right)\right)\right| \tag{6}
\end{equation*}
$$

and $e_{i k}$ and $f_{i k}$ are the cost coefficients of generator $i$ reflecting valve-point effects for fuel type $k$, and $P_{i k \text {, min }}$ is the minimum output of generator $i$ using fuel type $k$.

### 2.2 Equality and inequality constraints

1) Active Power Balance Equation: The ED problem has an equality constraint that the total generation output of all units must be same as total load demand plus the total transmission network losses

$$
\begin{equation*}
\sum_{i=1}^{n} P_{i}=P_{\text {load }}+P_{\text {loss }} \tag{7}
\end{equation*}
$$

where $P_{\text {load }}$ is the total system load. The total transmission network loss, $P_{\text {loss }}$, is a function of the unit power outputs that can be represented using $B$ coefficients [2] as in (8):

$$
\begin{equation*}
P_{\text {loss }}=\sum_{i=1}^{n} \sum_{j=1}^{n} P_{i} B_{i j} P_{j}+\sum_{i=1}^{n} B_{0 i} P_{i}+B_{00} \tag{8}
\end{equation*}
$$

2) Minimum and Maximum Power Limits: Power output of each generator should be within its minimum and maximum limits. The corresponding inequality constraint for each generator is;

$$
\begin{equation*}
P_{i, \text { min }} \leq P_{i} \leq P_{i, \text { max }} \tag{9}
\end{equation*}
$$

where $P_{i, \text { min }}$ and $P_{i, \text { max }}$ are the minimum and maximum output of generator $i$, respectively.
3) ED Problem Considering Prohibited Operating Zones:

In real power system operation, the entire operating range of a generating unit is not always available for load allocation due to physical operation limitations. Units may have prohibited operating zones due to robustness in the shaft bearing caused by the operation of steam values or to faults in machines themselves or associated auxiliaries. Such faults may lead to instability in certain ranges of generator power output [6]. Therefore, for units with
prohibited operating zones, there are additional constraints on the unit operating range as in (10):

$$
\begin{align*}
& P_{i} \in\left\{\begin{array}{l}
P_{i, \min } \leq P_{i} \leq P_{i, 1}^{l} \\
P_{i, k-1}^{u} \leq P_{i} \leq P_{i, k}^{l}, \quad k=2,3, \cdots, p z_{i} \\
P_{i, p z_{i}}^{u} \leq P_{i} \leq P_{i, \max }
\end{array}\right. \\
& i=1,2, \cdots, n_{P Z} \tag{10}
\end{align*}
$$

where $P_{i, k}^{l}$ and $P_{i, k}^{u}$ are, respectively, the lower and upper bounds of prohibited operating zone of unit $i$. Here, $p z_{i}$ is the number of prohibited zones of unit $i$ and $n_{P Z}$ is the number of units which have prohibited operating zones.

## 3. Overview of Mean-Variance Optimization

A 'Mean-Variance Optimization' (MVO) algorithm is a new stochastic optimization algorithm referred to by Erlich, Venayagamoorthy and Worawat. MVO falls into the category of the so-called "population-based stochastic optimization technique" [22]. The MVO algorithm utilizes a single parent-offspring pair concept using the information of performance of the mean $\bar{x}_{i}$ and variance, $v_{i}$ of $n$ generators.

$$
\begin{gather*}
\bar{x}_{i}=\frac{1}{n} \sum_{j=1}^{n} x_{i}(j)  \tag{11}\\
v_{i}=\frac{1}{n} \sum_{j=1}^{n}\left(x_{i}(j)-\bar{x}_{i}\right)^{2} \quad i=1,2, \cdots, k \tag{12}
\end{gather*}
$$

where $n$ is population size, $k$ is the numbers of problem variables. The range of variables initialized within the allowed limits is $[0,1]$. After computing mean and variance, the fitness of each population is evaluated. For fitness evaluation, however, de-normalization is carried out in every single iteration to calculate fitness using the actual values. The individual with the best fitness, $f_{\text {best }}$ and its corresponding optimization values, $x_{\text {best }}$ are 'parent' of the population for that iteration. And this 'parent' is used for creation of offspring which involves three common evolutionary computation algorithms' operations - selection, mutation and crossover.
$m$ of $k$ variables of the optimization problem are selected for mutation operation in accordance with the following mutation strategies:
A) Random selection
B) Neighbor group selection
a) Moving the group forward in multiple steps
b) Moving the group forward in single steps
C) Sequential selection of the first variable and the selection of the rest randomly (or) and random selection for the rest
Mutation procedure has the transformation and the
corresponding function, which are the key features of the MVO algorithm. A transformation function, $h$, is based on the mean and shape factor, $s_{i}$ as in (13).
$m$ variables selected from $k$ variables are transformed using transformation function

$$
\begin{equation*}
h\left(\bar{x}_{i}, s_{i 1}, s_{i 2}, u_{i}\right)=\bar{x}_{i} \cdot\left(1-e^{-u_{i} \cdot s_{i 1}}\right)+\left(1-\bar{x}_{i}\right) \cdot e^{-\left(1-u_{i}\right) \cdot s_{i 2}} \tag{13}
\end{equation*}
$$

where

$$
\begin{gather*}
x_{i}=h_{x}+\left(1-h_{1}+h_{0}\right) \cdot x_{i}^{\prime}-h_{0}  \tag{14}\\
s_{i}=-\ln \left(v_{i}\right) \cdot f_{S} \tag{15}
\end{gather*}
$$

The scaling factor $f_{s}$ is constant between 0.9 and 10 that allows for controlling the search process.

$$
h_{x}=h\left(u_{i}=x_{i}^{\prime}\right), h_{0}=h\left(u_{i}=0\right), h_{1}=h\left(u_{i}=1\right) \text { and } x_{i}^{\prime}
$$ is a uniform distribution in the range of $[0,1]$.

This paper sets two shape factors, $s_{i 1}$ and $s_{i 2}$ to be equal. The values of remaining un-mutated $(k-m)$ variables are clones of the parent. That is, the offspring is created by combining the 'parent' and $m$ mutated variables [22].

## 4. Improved MVO Algorithm with Kuhn-Tucker Condition and Swap Process

### 4.1 Application of Kuhn-Tucker condition

Economic dispatch problem is a constrained optimization that may be solved using advanced calculus methods that involve the Lagrange function. This is that the constraint function is added to the objective function after the constraint function has been multiplied by an undetermined multiplier as (16):

$$
\begin{equation*}
\ell=F_{T}+\lambda\left(P_{\text {load }}+P_{\text {loss }}-\sum_{i=l}^{n} P_{i}\right) \tag{16}
\end{equation*}
$$

To attain the optimal value for the objective function, the first derivative of the Lagrange function with respect to each of the independent variables are taken and set to be zero. The derivative of the Lagrange function with respect to the undetermined multiplier returns the constraint equation [2]. In the power system, generator has not only an equality constraint like active power balance equation but also an inequality constraint such as minimum and maximum power limit. Lagrange condition includes KuhnTucker condition to consider the inequality constraints as additional term. Then the necessary conditions for optimal dispatch become:

$$
\begin{equation*}
\frac{d F_{i}}{d P_{i}}=\lambda \quad \text { for } \quad P_{i, \min }<P_{i}<P_{i, \max } \tag{17}
\end{equation*}
$$

$$
\begin{array}{ll}
\frac{d F_{i}}{d P_{i}} \leq \lambda \quad \text { for } \quad P_{i}=P_{i, \max } \\
\frac{d F_{i}}{d P_{i}} \geq \lambda \quad \text { for } \quad P_{i}=P_{i, \min } \tag{19}
\end{array}
$$

If a generator's output is outside the limits of the generator, the output of the generator is fixed at that limit and the generator is no longer a participating one in the optimization of dispatch problem. Therefore, if some generators show same output limits - either $P_{\min }$ or $P_{\max }$ through past 5 iterations, their outputs are fixed at their limits. That is, new economic dispatch problem is performed only with the remaining generators. It is easier to solve the new economic dispatch problem since the variables are reduced.

The detailed Kuhn-Tucker process is summarized in the following steps:

Step 1) Implement algorithm more than 5 iterations
Step 2) If $m$ of $n$ generators are fixed at same limit through 5 iterations, their outputs are fixed as limits
Step 3) New ED is implemented without $m$ generators in step 2
Step 4) Go to step 1 until the stopping criteria of algorithm is satisfied

### 4.2 Application of swap process

Swap process can be applied in following situation: The biggest cost reduction when output is reduced by $\alpha$ MW is compared with the smallest cost increase when output is increased by $\alpha$ MW. The $i$ th generator output which has minimum cost increase is reduced by $\alpha$ MW and the $k$ th generator output which has maximum cost reduction is increased by $\alpha$ MW. The total generating cost is reduced through this process. The process is terminated when the minimum cost increase is bigger than the maximum cost reduction or swap process occurs in the same generator.

For searching local minimum, this process is applied to MVO algorithm. The detailed Swap process is summarized as in the following steps.

Step 1) Calculate the cost of $n$ generator outputs, $\operatorname{cost}_{\mathrm{p}}$ when $P=\left[P_{1}, P_{2}, \cdots, P_{n}\right]$
Step 2) Calculate cost of $(P-\alpha)$ MW and $(P+\alpha)$ MW for all generators, $\operatorname{cost}_{\text {minus }}$ and $\operatorname{cost}_{\text {plus }}$, respectively
Step 3) Select two generators which maximize [ $\operatorname{cost}_{p}$ $\operatorname{cost}_{\text {minus }}$ ] and minimize [ $\operatorname{cost}_{\text {plus }}-\operatorname{cost}_{\mathrm{p}}$ ]
Step 4) Modify each generator's output selected in step 3
Step 5) Go to step 1 until the stopping criteria is satisfied
In this paper, $5,4,3,2,1,0.1$ and 0.01 are used
successively as the value of $\alpha$ in swap process.

## 5. Implementation of Improved MVO Algorithm for Economic Dispatch Problems

Since improved MVO algorithm uses normalized values, normalization and de-normalization are carried out in every single iteration during optimization. For obtaining mean and variance, the population size $n$ should be the minimum of the two.

Step 1) Initialize the variables while satisfying the constraints and normalize optimization variables
Step 2) Evaluate fitness using de-normalized variables and store $f_{\text {best }}$ and $x_{\text {best }}$.
Step 3) Calculate means and variances of the normalized variables using (11) and (12).
Step 4) Create offspring through selection, mutation and crossover processes.
Step 5) Apply Kuhn-Tucker condition to best fitness
Step 6) Go to Step 2 until the stopping criteria is satisfied.
Step 7) If stopping criteria is satisfied, implement swap process to get $x_{\text {best }}$.

In the following, the detailed implementation strategies of the proposed method are described.

## 1) Normalization and de-normalization of optimization variables

After initializing variables satisfying the minimum and maximum limits, all variables are normalized as follows:

$$
\begin{equation*}
P_{i, n o r}=\left(P_{i}-P_{i, \min }\right) /\left(P_{i, \max }-P_{i, \min }\right) \tag{20}
\end{equation*}
$$

For evaluating the fitness of objective function, denormalization is carried out using (21).

$$
\begin{equation*}
P_{i}=P_{i, \text { nor }}\left(P_{i, \max }-P_{i, \min }\right)+P_{i, \min } \tag{21}
\end{equation*}
$$

## 2) Stopping Criteria

The improved MVO algorithm is terminated if the iteration reaches a predefined maximum iteration.

## 6. Numerical Tests

The proposed KMVO approach is applied to four different power systems; (i) 40-unit system with valvepoint effects, (ii) 15 -unit system with prohibited operating zones and transmission network losses, (iii) 10-unit system considering multiple fuels with valve-point effects, and (iv) 140 -unit Korean power system with valve-point effects and prohibited operating zones. For each case, 100 independent
trials are conducted to compare the solution quality.
Before implementing the proposed algorithm, some KMVO parameters must be determined in advance. The population size $N P$ and maximum iteration number iter $_{\text {max }}$ are set as 40 and 10,000 , respectively. Since the performance of MVO algorithm depends on the parameters such as shape factor, scaling factor and selection strategies, it is important to determine suitable values of these parameters. To determine the shape factor, the case in which $s_{i}$ is variable according to variance, that is, $f_{s}$ is constant, is compared with other case in which $s_{i}$ is constant. For each case, 100 independent tests are executed for system 1 . In the case in which $f_{s}$ is varied from 1 to 10 with the step size of 1 , there is no $f_{s}$ value that makes minimum cost and average cost coincide. The other case in which $s_{i}$ is varied from 0 to 50 with step size of 5 shows that the optimal value for $s_{i}$ is 30 . Therefore $s_{i}$ is fixed as 30 for four test simulations. As selection strategies, all strategies are applied to each case and the best value is used as a result. After all simulations, it is observed that the mutation strategy C is superior to other strategies.

## 1) Test System 1: System with Valve-point Effects

The test system consists of 40 generating units and the input data are described in [10]. The total demand is set as 10,500MW.


Fig. 1. Convergence characteristics of the shape factor for Test system 1

Table 1. Determination of shape factor for test system 1

| Case | $s_{i}$ | Minimum <br> Cost $(\$)$ | Average <br> Cost $(\$)$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | $121,668.1040$ | $122,114.2391$ |
| 2 | 5 | $121,539.4381$ | $121,898.9609$ |
| 3 | 10 | $121,502.3348$ | $121,848.1305$ |
| 4 | 15 | $121,521.8453$ | $121,918.7053$ |
| 5 | 20 | $121,534.8039$ | $121,889.0578$ |
| 6 | 25 | $121,531.7715$ | $121,890.3810$ |
| 7 | $\mathbf{3 0}$ | $\mathbf{1 2 1 , 4 8 7 . 3 6 0 0}$ | $\mathbf{1 2 1 , 8 4 5 . 6 7 0 0}$ |
| 8 | 35 | $121,512.9815$ | $121,876.6371$ |
| 9 | 40 | $121,551.4565$ | $121,866.3985$ |
| 10 | 45 | $121,515.6051$ | $121,857.4572$ |
| 11 | 50 | $121,515.4220$ | $121,859.9668$ |

In Table 2, the results of the proposed KMVO algorithm are compared with those of evolutionary programming (EP) [10], MPSO [16], PSO-SQP [17], DEC-SQP [14], NPSO [18], NPSO-LRS [18], CTPSO [20], CSPSO [20],

Table 2. Comparison of the result of each method for test system 1

| Methods | Minimum Cost (\$) | Average Cost (\$) |
| :---: | :---: | :---: |
| EP [10] | $122,624.3500$ | $123,382.0000$ |
| MPSO [16] | $122,252.2650$ | N/A |
| PSO-SQP [17] | $122,094.6700$ | $122,245.2500$ |
| DEC-SQP [14] | $121,741.9793$ | $122,295.1278$ |
| NPSO [18] | $121,704.7391$ | $122,221.3697$ |
| NPSO-LRS [18] | $121,664.4308$ | $122,209.3185$ |
| CTPSO[20] | $121,703.6056$ | $121,953.3959$ |
| CSPSO[20] | $121,444.9581$ | $121,954.0564$ |
| COPSO[20] | $121,420.8975$ | $121,508.9769$ |
| CCPSO[20] | $121,412.5483$ | $121,454.3269$ |
| KMVO | $121,412.5363$ | $121,437.8247$ |

Table 3. Generation output of each generator and the corresponding cost in 40 -unit test system 1 of NPSO-LRS, CCPSO, and KMVO

| Unit | NPSO-LRS[18] | CCPSO[20] | KMVO |
| :---: | :---: | :---: | :---: |
| 1 | 113.9761 | 110.7998 | 110.7998 |
| 2 | 113.9986 | 110.7999 | 110.7998 |
| 3 | 97.4241 | 97.3999 | 97.3999 |
| 4 | 179.7327 | 179.7331 | 179.7331 |
| 5 | 89.6511 | 87.7999 | 87.7999 |
| 6 | 105.4044 | 140.0000 | 140.0000 |
| 7 | 259.7502 | 259.5997 | 259.5997 |
| 8 | 288.4534 | 284.5997 | 284.5997 |
| 9 | 284.6460 | 284.5997 | 284.5997 |
| 10 | 204.8120 | 130.0000 | 130.0000 |
| 11 | 168.8311 | 94.0000 | 94.0000 |
| 12 | 94.0000 | 94.0000 | 94.0000 |
| 13 | 214.7663 | 214.7598 | 214.7598 |
| 14 | 394.2852 | 394.2794 | 394.2794 |
| 15 | 304.5187 | 394.2794 | 394.2794 |
| 16 | 394.2811 | 394.2794 | 394.2794 |
| 17 | 489.2807 | 489.2794 | 489.2794 |
| 18 | 489.2832 | 489.2794 | 489.2794 |
| 19 | 511.2845 | 511.2794 | 511.2794 |
| 20 | 511.3049 | 511.2794 | 511.2794 |
| 21 | 523.2916 | 523.2794 | 523.2794 |
| 22 | 523.2853 | 523.2794 | 523.2794 |
| 23 | 523.2797 | 523.2794 | 523.2794 |
| 24 | 523.2994 | 523.2794 | 523.2794 |
| 25 | 523.2865 | 523.2794 | 523.2794 |
| 26 | 523.2936 | 523.2794 | 523.2794 |
| 27 | 10.0000 | 10.0000 | 10.0000 |
| 28 | 10.0001 | 10.0000 | 10.0000 |
| 29 | 10.0000 | 10.0000 | 10.0000 |
| 30 | 89.0139 | 87.8000 | 87.7999 |
| 31 | 190.0000 | 190.0000 | 190.0000 |
| 32 | 190.0000 | 190.0000 | 190.0000 |
| 33 | 190.0000 | 190.0000 | 190.0000 |
| 34 | 199.9998 | 164.7998 | 164.7998 |
| 35 | 165.1397 | 194.3976 | 194.3977 |
| 36 | 172.0275 | 200.0000 | 200.0000 |
| 37 | 110.0000 | 110.0000 | 110.0000 |
| 38 | 110.0000 | 110.0000 | 110.0000 |
| 39 | 93.0962 | 110.0000 | 110.0000 |
| 40 | 511.2996 | 511.2794 | 511.2794 |
| TP | 10500 | 10500 | 10500 |
| TC | 121664.4308 | 121412.5483 | 121412.5363 |

* TP: total power [MW], TC: total generation cost [\$].

COPSO [20], and CCPSO[20]. The result of KMVO is similar to the best solution previously found by CCPSO and has shown the superiority to other algorithms, so it is almost the global solution.

## 2) Test System 2: System with Prohibited Operating Zones, Ramp Rate Limits, and Transmission Network Losses

Experiments are performed on the 15 -unit power system, which considers the prohibited operating zones and transmission network losses. Units $2,5,6$, and 12 have two or three prohibited operating zones. The system supplies a load of $2,630 \mathrm{MW}$. The input data and $B$ coefficients for transmission network losses are provided in [15].

Table 4. Comparison of the results of each method for test system 2

| Unit | GA [15] | PSO [15] | CCPSO[20] | KMVO |
| :--- | :---: | :---: | :--- | :--- |
| 1 | 415.3108 | 439.1162 | 455.0000 | 454.9993 |
| 2 | 359.7206 | 407.9727 | 380.0000 | 454.9676 |
| 3 | 104.4250 | 119.6324 | 130.0000 | 129.9999 |
| 4 | 74.9853 | 129.9925 | 130.0000 | 130.0000 |
| 5 | 380.2844 | 151.0681 | 170.0000 | 235.1029 |
| 6 | 426.7902 | 459.9978 | 460.0000 | 460.0000 |
| 7 | 341.3164 | 425.5601 | 430.0000 | 465.0000 |
| 8 | 124.7867 | 98.5699 | 71.7526 | 60.0000 |
| 9 | 133.1445 | 113.4936 | 58.9090 | 25.0000 |
| 10 | 89.2567 | 101.1142 | 160.0000 | 27.6110 |
| 11 | 60.0572 | 33.9116 | 80.0000 | 79.8528 |
| 12 | 49.9998 | 79.9583 | 80.0000 | 79.9863 |
| 13 | 38.7713 | 25.0042 | 25.0000 | 25.0000 |
| 14 | 41.9425 | 41.4140 | 15.0000 | 15.0036 |
| 15 | 22.6445 | 35.6140 | 15.0000 | 15.0000 |
| TP | 2668.4 | 2662.4 | 2660.6616 | 2657.5233 |
| $P_{\text {loss }}$ | 38.2782 | 32.4306 | 30.6616 | 27.5233 |
| TC | 33,113 | 32,858 | 32,704 | 32,555 |

In Table 4, the best result of KMOV is compared with that of GA [15], PSO [15] and CCPSO [20]. As shown in table 4, KMVO provides the better solution than other solutions.

## 3) Test System 3: Multi-Fuels with Valve-Point Effect

The test system consists of 10 generating units considering multi-fuels with valve-point effects. The input data and related constraints of the test system are given in [7]. The total system demand is set as $2,700 \mathrm{MW}$.

In Table 5, the generation outputs, fuel types, and corresponding costs of the best solution obtained from the proposed KMVO are compared with those of NPSO-LRS [18], CTPSO [20], CCPSO [20]. For accurate comparison, other results are recalculated using Microsoft Excel 2010. The result shows that KMVO provides best solution
similar to CCPSO [20] while satisfying the system constraints exactly.

Table 5. Comparison of the results of each method for test system 3

| Unit | NPSO-LRS[18] |  | CTPSO[20] |  | CCPSO[20] |  | KMVO |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | GEN | F | GEN | F | GEN | F | GEN |
| 1 | 2 | 223.3352 | 2 | 218.6807 | 2 | 218.5940 | 2 | 218.5940 |
| 2 | 1 | 212.1957 | 1 | 211.4642 | 1 | 211.7117 | 1 | 211.7117 |
| 3 | 1 | 276.2167 | 1 | 280.6545 | 1 | 280.6571 | 1 | 280.6571 |
| 4 | 3 | 239.4187 | 3 | 240.4457 | 3 | 239.6394 | 3 | 239.6394 |
| 5 | 1 | 274.6470 | 1 | 276.4034 | 1 | 279.9346 | 1 | 279.9345 |
| 6 | 3 | 239.7974 | 3 | 240.1769 | 3 | 239.5051 | 3 | 239.6394 |
| 7 | 1 | 285.5388 | 1 | 287.8657 | 1 | 287.7275 | 1 | 287.7275 |
| 8 | 3 | 240.6323 | 3 | 240.5800 | 3 | 239.6394 | 3 | 239.6394 |
| 9 | 3 | 429.2637 | 3 | 428.5886 | 3 | 426.7226 | 3 | 426.5884 |
| 10 | 1 | 278.9541 | 1 | 275.1403 | 1 | 275.8686 | 1 | 275.8686 |
| TP | 2700.0000 |  | 2700.0000 | 2700.0000 |  | 2700.0000 |  |  |
| TC | 624.1273 |  | 623.8593 |  | 623.8273 |  | 623.8267 |  |

## 4) Large-scale Power System of Korea

To investigate the applicability of the proposed KMVO to the large-scale power systems, experiments are conducted on the Korean power system. The system consists of 140 thermal generating units. The input data are given in Table 10 of the Appendix. This system supplies a load of $49,342 \mathrm{MW}$.

Table 6. Convergence results for Korean power system with convex cost functions

| Methods | Minimum <br> $\operatorname{Cost}(\$)$ | Average <br> $\operatorname{Cost}(\$)$ | Maximum <br> $\operatorname{Cost}(\$)$ |
| :---: | :---: | :---: | :---: |
| CTPSO[20] | $1,655,685$ | $1,655,685$ | $1,655,685$ |
| CSPSO[20] | $1,655,685$ | $1,655,685$ | $1,655,685$ |
| COPSO[20] | $1,655,685$ | $1,655,685$ | $1,655,685$ |
| CCPSO[20] | $1,655,685$ | $1,655,685$ | $1,655,685$ |
| KMVO | $1,577,607$ | $1,586,547$ | $1,594,251$ |

Table 7. Generation output of each generator in Korean power system with convex cost function

| Gen | Output | Gen | Output |
| :---: | :---: | :---: | :---: |
| COAL \#01 | 116.3651 | LNG CC \#29 | 157.8965 |
| COAL \#02 | 186.7814 | LNG CC \#30 | 276.0046 |
| COAL \#03 | 184.1960 | LNG CC \#31 | 199.8780 |
| COAL \#04 | 151.1263 | LNG CC \#32 | 257.4908 |
| COAL \#05 | 115.4063 | LNG CC \#33 | 207.4134 |
| COAL \#06 | 170.4321 | LNG CC \#34 | 252.4126 |
| COAL \#07 | 488.5877 | LNG CC \#35 | 204.3870 |
| COAL \#08 | 486.6820 | LNG CC \#36 | 417.5256 |
| COAL \#09 | 496.0000 | LNG CC \#37 | 513.8888 |
| COAL \#10 | 495.9421 | LNG CC \#38 | 531.0000 |
| COAL \#11 | 496.0000 | LNG CC \#39 | 345.2138 |
| COAL \#12 | 495.2605 | LNG CC \#40 | 61.6248 |
| COAL \#13 | 505.9998 | LNG CC \#41 | 115.0000 |
| COAL \#14 | 507.8446 | LNG CC \#42 | 124.1302 |
| COAL \#15 | 504.3031 | LNG CC \#43 | 135.3891 |
| COAL \#16 | 503.2839 | LNG CC \#44 | 207.0000 |
| COAL \#17 | 506.0000 | LNG CC \#45 | 208.2738 |
| COAL \#18 | 506.0000 | LNG CC \#46 | 194.0125 |
| COAL \#19 | 484.7896 | LNG CC \#47 | 251.0230 |
| COAL \#20 | 505.0000 | LNG CC \#48 | 191.5331 |
| COAL \#21 | 505.0000 | LNG CC \#49 | 231.5253 |


| COAL \#22 | 489.5698 | NUCLEAR\#1 | 577.9281 |
| :---: | :---: | :---: | :---: |
| COAL \#23 | 497.3235 | NUCLEAR\#2 | 643.1416 |
| COAL \#24 | 493.7546 | NUCLEAR\#3 | 984.0000 |
| COAL \#25 | 531.9461 | NUCLEAR\#4 | 978.0000 |
| COAL \#26 | 533.3515 | NUCLEAR\#5 | 679.9139 |
| COAL \#27 | 549.0000 | NUCLEAR\#6 | 720.0000 |
| COAL \#28 | 540.8276 | NUCLEAR\#7 | 718.0000 |
| COAL \#29 | 480.6153 | NUCLEAR\#8 | 719.7980 |
| COAL \#30 | 499.6192 | NUCLEAR\#9 | 963.3888 |
| COAL \#31 | 502.9783 | NUCLEAR\#10 | 957.9991 |
| COAL \#32 | 499.7606 | NUCLEAR\#11 | 1001.9413 |
| COAL \#33 | 505.9990 | NUCLEAR\#12 | 1006.0000 |
| COAL \#34 | 505.9995 | NUCLEAR\#13 | 1013.0000 |
| COAL \#35 | 496.6364 | NUCLEAR\#14 | 1020.0000 |
| COAL \#36 | 500.0000 | NUCLEAR\#15 | 953.9991 |
| COAL \#37 | 237.6282 | NUCLEAR\#16 | 951.7433 |
| COAL \#38 | 239.4089 | NUCLEAR\#17 | 1006.0000 |
| COAL \#39 | 773.9897 | NUCLEAR\#18 | 1013.0000 |
| COAL \#40 | 768.7938 | NUCLEAR\#19 | 1021.0000 |
| LNG \#1 | 11.9754 | NUCLEAR\#20 | 1015.0000 |
| LNG \#2 | 5.7723 | OIL \#1 | 94.0000 |
| LNG CC \#01 | 193.2953 | OIL \#2 | 94.0000 |
| LNG CC \#02 | 204.9660 | OIL \#3 | 111.1225 |
| LNG CC \#03 | 201.5581 | OIL \#4 | 244.0655 |
| LNG CC \#04 | 240.1244 | OIL \#5 | 305.3824 |
| LNG CC \#05 | 213.2612 | OIL \#6 | 256.8998 |
| LNG CC \#06 | 214.9590 | OIL \#7 | 117.8872 |
| LNG CC \#07 | 249.3393 | OIL \#8 | 96.1081 |
| LNG CC \#08 | 238.9636 | OIL \#9 | 133.2779 |
| LNG CC \#09 | 183.6487 | OIL \#10 | 179.3312 |
| LNG CC \#10 | 185.6466 | OIL \#11 | 12.7716 |
| LNG CC \#11 | 226.3351 | OIL \#12 | 5.3750 |
| LNG CC \#12 | 196.5095 | OIL \#13 | 15.0001 |
| LNG CC \#13 | 180.0000 | OIL \#14 | 12.2708 |
| LNG CC \#14 | 192.9887 | OIL \#15 | 12.0000 |
| LNG CC \#15 | 108.1703 | OIL \#16 | 11.3371 |
| LNG CC \#16 | 199.0215 | OIL \#17 | 131.3862 |
| LNG CC \#17 | 201.7144 | OIL \#18 | 4.9092 |
| LNG CC \#18 | 224.6738 | OIL \#19 | 18.0074 |
| LNG CC \#19 | 166.3843 | OIL \#20 | 5.6798 |
| LNG CC \#20 | 95.0000 | OIL \#21 | 56.7364 |
| LNG CC \#21 | 218.6087 | OIL \#22 | 5.0000 |
| LNG CC \#22 | 160.0929 | OIL \#23 | 46.2592 |
| LNG CC \#23 | 339.4153 | OIL \#24 | 46.4731 |
| LNG CC \#24 | 396.3759 | OIL \#25 | 41.0000 |
| LNG CC \#25 | 421.6618 | OIL \#26 | 17.0027 |
| LNG CC \#26 | 219.9153 | OIL \#27 | 18.5819 |
| LNG CC \#27 | 144.8121 | OIL \#28 | 7.9656 |
| LNG CC \#28 | 287.0772 | OIL \#29 | 35.2237 |
|  | TP | 49,342 |  |
|  | TC | 1,577,607 |  |

In order to show the applicability of the KMVO to the large-scale power system, it is applied to Korean power system with convex cost function. The result is shown in table 6 and each generator's output and the corresponding total cost of KMOV are provided in table 7.

Table 8. Convergence results for Korean power system with nonconvex cost functions

| Methods | Minimum <br> Cost $(\$)$ | Average <br> Cost $(\$)$ | Maximum <br> Cost $(\$)$ |
| :---: | :---: | :---: | :---: |
| CTPSO[20] | 1657962.73 | 1657964.06 | 1658002.79 |
| CSPSO[20] | 1657962.73 | 1657962.74 | 1657962.85 |
| COPSO[20] | 1657962.73 | 1657962.73 | 1657962.73 |
| CCPSO[20] | 1657962.73 | 1657962.73 | 1657962.73 |
| KMVO | 1568450.94 | 1590666.49 | 1609134.60 |

Table 9. Generation output of each generator in Korean power system with nonconvex cost function

| Gen | Output | Gen | Output |
| :---: | :---: | :---: | :---: |
| COAL \#01 | 82.5513 | LNG CC \#29 | 157.2506 |
| COAL \#02 | 188.6216 | LNG CC \#30 | 318.6942 |
| COAL \#03 | 189.9989 | LNG CC \#31 | 235.7487 |
| COAL \#04 | 182.3628 | LNG CC \#32 | 281.5549 |
| COAL \#05 | 90.0000 | LNG CC \#33 | 190.5204 |
| COAL \#06 | 187.4329 | LNG CC \#34 | 305.2888 |
| COAL \#07 | 490.0000 | LNG CC \#35 | 272.3050 |
| COAL \#08 | 490.0000 | LNG CC \#36 | 409.9372 |
| COAL \#09 | 489.9706 | LNG CC \#37 | 530.6425 |
| COAL \#10 | 495.9315 | LNG CC \#38 | 531.0000 |
| COAL \#11 | 495.9994 | LNG CC \#39 | 212.6973 |
| COAL \#12 | 496.0000 | LNG CC \#40 | 56.0000 |
| COAL \#13 | 505.9958 | LNG CC \#41 | 115.0000 |
| COAL \#14 | 509.0000 | LNG CC \#42 | 115.0000 |
| COAL \#15 | 506.0000 | LNG CC \#43 | 115.1458 |
| COAL \#16 | 505.0000 | LNG CC \#44 | 207.0000 |
| COAL \#17 | 506.0000 | LNG CC \#45 | 208.0936 |
| COAL \#18 | 506.0000 | LNG CC \#46 | 183.8922 |
| COAL \#19 | 505.0000 | LNG CC \#47 | 188.5782 |
| COAL \#20 | 505.0000 | LNG CC \#48 | 222.7599 |
| COAL \#21 | 505.0000 | LNG CC \#49 | 226.0267 |
| COAL \#22 | 504.9890 | NUCLEAR\#1 | 580.0000 |
| COAL \#23 | 505.0000 | NUCLEAR\#2 | 644.8960 |
| COAL \#24 | 505.0000 | NUCLEAR\#3 | 984.0000 |
| COAL \#25 | 536.9995 | NUCLEAR\#4 | 977.9915 |
| COAL \#26 | 537.0000 | NUCLEAR\#5 | 682.0000 |
| COAL \#27 | 549.0000 | NUCLEAR\#6 | 719.8723 |
| COAL \#28 | 549.0000 | NUCLEAR\#7 | 716.3277 |
| COAL \#29 | 501.0000 | NUCLEAR\#8 | 719.9999 |
| COAL \#30 | 501.0000 | NUCLEAR\#9 | 963.7387 |
| COAL \#31 | 505.6595 | NUCLEAR\#10 | 957.0750 |
| COAL \#32 | 506.0000 | NUCLEAR\#11 | 1007.0000 |
| COAL \#33 | 506.0000 | NUCLEAR\#12 | 1006.0000 |
| COAL \#34 | 506.0000 | NUCLEAR\#13 | 1013.0000 |
| COAL \#35 | 500.0000 | NUCLEAR\#14 | 1019.9919 |
| COAL \#36 | 487.7316 | NUCLEAR\#15 | 954.0000 |
| COAL \#37 | 241.0000 | NUCLEAR\#16 | 952.0000 |
| COAL \#38 | 241.0000 | NUCLEAR\#17 | 1005.9999 |
| COAL \#39 | 774.0000 | NUCLEAR\#18 | 1012.9999 |
| COAL \#40 | 768.5452 | NUCLEAR\#19 | 1020.9987 |
| LNG \#1 | 5.3027 | NUCLEAR\#20 | 1013.3990 |
| LNG \#2 | 3.0000 | OIL \#1 | 95.8236 |
| LNG CC \#01 | 206.6909 | OIL \#2 | 98.1712 |
| LNG CC \#02 | 183.3619 | OIL \#3 | 94.0000 |
| LNG CC \#03 | 215.0000 | OIL \#4 | 261.0264 |
| LNG CC \#04 | 248.6941 | OIL \#5 | 247.9809 |
| LNG CC \#05 | 250.0000 | OIL \#6 | 278.0867 |
| LNG CC \#06 | 241.7546 | OIL \#7 | 95.9819 |
| LNG CC \#07 | 160.0000 | OIL \#8 | 95.0000 |
| LNG CC \#08 | 210.0428 | OIL \#9 | 125.8192 |
| LNG CC \#09 | 208.6296 | OIL \#10 | 175.2850 |
| LNG CC \#10 | 165.0000 | OIL \#11 | 3.2382 |
| LNG CC \#11 | 201.4451 | OIL \#12 | 5.7585 |
| LNG CC \#12 | 165.4062 | OIL \#13 | 16.6915 |
| LNG CC \#13 | 184.5243 | OIL \#14 | 9.2993 |
| LNG CC \#14 | 180.0000 | OIL \#15 | 12.7125 |
| LNG CC \#15 | 166.6488 | OIL \#16 | 10.0000 |
| LNG CC \#16 | 198.0000 | OIL \#17 | 114.8288 |
| LNG CC \#17 | 311.1821 | OIL \#18 | 6.0628 |
| LNG CC \#18 | 198.6260 | OIL \#19 | 5.7295 |
| LNG CC \#19 | 164.9270 | OIL \#20 | 5.0000 |
| LNG CC \#20 | 95.0000 | OIL \#21 | 51.7665 |
| LNG CC \#21 | 181.5410 | OIL \#22 | 5.9033 |
| LNG CC \#22 | 171.8393 | OIL \#23 | 48.5351 |
| LNG CC \#23 | 358.9548 | OIL \#24 | 43.2163 |
| LNG CC \#24 | 347.2441 | OIL \#25 | 41.1533 |
| LNG CC \#25 | 376.8657 | OIL \#26 | 17.0000 |
| LNG CC \#26 | 371.0256 | OIL \#27 | 8.7338 |
| LNG CC \#27 | 140.0580 | OIL \#28 | 11.2042 |
| LNG CC \#28 | 229.6166 | OIL \#29 | 26.3946 |
|  | TP | 49,342 |  |
| TC |  | 1,568,450.94 |  |

Additionally, it is applied to Korean power system with nonconvex cost function. It is assumed that 12 generators have the cost function with valve-point effects and 4 generators are considered the prohibited operating zones. The data are given in Tables 11-12 in Appendix.

The Table 8 shows that results of KMVO are superior to CCPSO [20] and the average and maximum cost of KMVO are even better than the minimum cost of CCPSO. Each generator's outputs and the corresponding total cost are provided in Table 9.

## 7. Conclusions

This paper proposes an improved "Mean-variance optimization" algorithm for solving nonconvex ED problems. The proposed algorithm includes Kuhn-Tucker condition and swap process to improve the performance of MVO algorithm. Kuhn-Tucker condition is applied to MVO algorithm to enhance capability of searching global minimum by reducing the number of generators for economic dispatch problems. Also swap process is used to obtain the optimal solution. The proposed KMVO is successfully applied to three different nonconvex ED problems with valve-point effects, prohibited operating zones, transmission network losses and multi-fuels with valve-point effects. Also it is applied to the large-scale Korean power system with convex and nonconvex cost functions. The KMVO algorithm has found a better solution for the four test systems than other existing solutions. The results show that the proposed KMVO can successfully be applied and used for general nonconvex ED problems with several constraints.

## References

[1] K. Y. Lee and M. A. El-Sharkawi (Editors), Modern Heuristic Optimization Techniques with Applications to Power Systems, IEEE Power Engineering Society (02TP160), 2002.
[2] A. J. Wood and B. F. Wollenberg, Power Generation, Operation, and Control. New York: John Wiley \& Sons, Inc., 1984.
[3] Z. X. Liang and J. D. Glover, "A zoom feature for a dynamic programming solution to economic dispatch including transmission losses," IEEE Trans. on Power Systems, Vol. 7. No. 2, pp. 544-550, May 1992
[4] C. E. Lin and G. L. Viviani, "Hierarchical economic dispatch for piece-wise quadratic cost functions," IEEE Trans. Power App. Syst., Vol. PAS-103, No.6, pp. 1170-1175, June 1984.
[5] D. C. Walters and G. B. Sheble, "Genetic algorithm solution of economic dispatch with the valve point loading," IEEE Trans. on Power Systems, Vol. 8, No. 3, pp. 1325-1332, Aug. 1993.
[6] S. O. Orero and M. R. Irving, "Economic dispatch of generators with prohibited operating zones: a genetic algorithm approach," IEE Proc.-Gener. Transm. Distrib., Vol. 143, No. 6, pp. 529-534, Nov. 1996.
[7] C. L. Chiang, "Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels," IEEE Trans. on Power Systems, Vol. 20, No. 4, pp. 1690-1699, Nov. 2005.
[8] H. T. Yang, P. C. Yang, and C. L. Huang, "Evolutionary programming based economic dispatch for units with non-smooth fuel cost functions," IEEE Trans. on Power Systems, Vol. 11, No. 1, pp. 112118, Feb. 1996.
[9] Y. M. Park, J. R. Won and J. B. Park, "A new approach to economic load dispatch based on improved evolutionary programming," Engineering Intelligent Systems for Electrical Engineering and Communications, Vol. 6, No. 2, pp. 103-110, June 1998.
[10] N. Sinha, R. Chakrabarti, and P. K. Chattopadhyay, "Evolutionary programming techniques for economic load dispatch," IEEE Trans. on Evolutionary Computations, Vol. 7, No. 1, pp. 83-94, Feb. 2003.
[11] W. M. Lin, F. S. Cheng, and M. T. Tsay, "An improved Tabu search for economic dispatch with multiple minima," IEEE Trans. on Power Systems, Vol. 17, No. 1, pp. 108-112, Feb. 2002.
[12] J. H. Park, Y. S. Kim, I. K. Eom, and K. Y. Lee, "Economic load dispatch for piecewise quadratic cost function using Hopfield neural network," IEEE Trans. on Power Systems, Vol. 8, No. 3, pp. 1030-1038, August 1993.
[13] K. Y. Lee, A. Sode-Yome, and J. H. Park, "Adaptive Hopfield neural network for economic load dispatch," IEEE Trans. on Power Systems, Vol. 13, No. 2, pp. 519-526, May 1998.
[14] L. S. Coelho and V. C. Mariani, "Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valvepoint effect," IEEE Trans. on Power Systems, Vol. 21, No. 2, May 2006.
[15] Z. L. Gaing, "Particle swarm optimization to solving the economic dispatch considering the generator constraints," IEEE Trans. on Power Systems, Vol. 18, No. 3, pp. 1187-1195, Aug. 2003.
[16] J. B. Park, K. S. Lee, J. R. Shin, and K. Y. Lee, "A particle swarm optimization for economic dispatch with nonsmooth cost functions," IEEE Trans. on Power Systems, Vol. 20, No. 1, pp. 34-42, Feb. 2005.
[17] T. A. A. Victoire and A. E. Jeyakumar, "Hybrid PSO-SQP for economic dispatch with valve-point effect," Electric Power Systems Research, Vol. 71, pp. 51-59, Sep. 2004.
[18] A. I. Selvakumar and K. Thanushkodi, "A new particle swarm optimization solution to nonconvex economic dispatch problems," IEEE Trans. on Power

Systems, Vol. 22, No. 1, pp. 42-51, Feb. 2007.
[19] W. M. Lin, F. S. Cheng, and M. T. Tsay, "Nonconvex economic dispatch by integrated artificial intelligence," IEEE Trans. on Power Systems, Vol. 16, No. 2, pp. 307-311, May 2001.
[20] J. B. Park, Y. W. Jeong, J. R. Shin, and K. Y. Lee, "An improved particle swarm optimization for nonconvex economic dispatch problems," IEEE Trans. on Power Systems, Vol. 25, No. 1, pp. 156-166, Feb. 2010.
[21] H. Y. Song, Y. G. Park, J. H. Roh, J. B. Park, "Immune-PSO for economic dispatch with valve point effect," Advanced Materials Research, Vol. 452-453, pp. 1054-1058, 2012.
[22] Erlich. I, Venayagamoorthy. G. K, Worawat. N, "A mean-variance optimization algorithm," IEEE World Congress on Computational Intelligence, pp. 344-349, July. 2010

## Appendix <br> The characteristics data of generating units for Korean power system are given in table 10-12.

Table 10. Generating unit data of Korean power system

| Gen | a | b | c | $\mathrm{P}_{\text {min }}$ | $\mathrm{P}_{\text {max }}$ | UR | DR | $\mathrm{P}^{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| COAL\#01 | 1220.645 | 61.242 | 0.032888 | 71 | 119 | 30 | 120 | 98.4 |
| COAL\#02 | 1315.118 | 41.095 | 0.008280 | 120 | 189 | 30 | 120 | 134.0 |
| COAL\#03 | 874.288 | 46.310 | 0.003849 | 125 | 190 | 60 | 60 | 141.5 |
| COAL\#04 | 874.288 | 46.310 | 0.003849 | 125 | 190 | 60 | 60 | 183.3 |
| COAL\#05 | 1976.469 | 54.242 | 0.042468 | 90 | 190 | 150 | 150 | 125.0 |
| COAL\#06 | 1338.087 | 61.215 | 0.014992 | 90 | 190 | 150 | 150 | 91.3 |
| COAL\#07 | 1818.299 | 11.791 | 0.007039 | 280 | 490 | 180 | 300 | 401.1 |
| COAL\#08 | 1133.978 | 15.055 | 0.003079 | 280 | 490 | 180 | 300 | 329.5 |
| COAL\#09 | 1320.636 | 13.226 | 0.005063 | 260 | 496 | 300 | 510 | 386.1 |
| COAL\#10 | 1320.636 | 13.226 | 0.005063 | 260 | 496 | 300 | 510 | 427.3 |
| COAL\#11 | 1320.636 | 13.226 | 0.005063 | 260 | 496 | 300 | 510 | 412.2 |
| COAL\#12 | 1106.539 | 14.498 | 0.003552 | 260 | 496 | 300 | 510 | 370.1 |
| COAL\#13 | 1176.504 | 14.651 | 0.003901 | 260 | 506 | 600 | 600 | 301.8 |
| COAL\#14 | 1176.504 | 14.651 | 0.003901 | 260 | 509 | 600 | 600 | 368.0 |
| COAL\#15 | 1176.504 | 14.651 | 0.003901 | 260 | 506 | 600 | 600 | 301.9 |
| COAL\#16 | 1176.504 | 14.651 | 0.003901 | 260 | 505 | 600 | 600 | 476.4 |
| COAL\#17 | 1017.406 | 15.669 | 0.002393 | 260 | 506 | 600 | 600 | 283.1 |
| COAL\#18 | 1017.406 | 15.669 | 0.002393 | 260 | 506 | 600 | 600 | 414.1 |
| COAL\#19 | 1229.131 | 14.656 | 0.003684 | 260 | 505 | 600 | 600 | 328.0 |
| COAL\#20 | 1229.131 | 14.656 | 0.003684 | 260 | 505 | 600 | 600 | 389.4 |
| COAL\#21 | 1229.131 | 14.656 | 0.003684 | 260 | 505 | 600 | 600 | 354.7 |
| COAL\#22 | 1229.131 | 14.656 | 0.003684 | 260 | 505 | 600 | 600 | 262.0 |
| COAL\#23 | 1267.894 | 14.378 | 0.004004 | 260 | 505 | 600 | 600 | 461.5 |
| COAL\#24 | 1229.131 | 14.656 | 0.003684 | 260 | 505 | 600 | 600 | 371.6 |
| COAL\#25 | 975.926 | 16.261 | 0.001619 | 280 | 537 | 300 | 300 | 462.6 |
| COAL\#26 | 1532.093 | 13.362 | 0.005093 | 280 | 537 | 300 | 300 | 379.2 |
| COAL\#27 | 641.989 | 17.203 | 0.000993 | 280 | 549 | 360 | 360 | 530.8 |
| COAL\#28 | 641.989 | 17.203 | 0.000993 | 280 | 549 | 360 | 360 | 391.9 |
| COAL\#29 | 911.533 | 15.274 | 0.002473 | 260 | 501 | 180 | 180 | 480.1 |
| COAL\#30 | 910.533 | 15.212 | 0.002547 | 260 | 501 | 180 | 180 | 319.0 |
| COAL\#31 | 1074.810 | 15.033 | 0.003542 | 260 | 506 | 600 | 600 | 329.5 |
| COAL\#32 | 1074.810 | 15.033 | 0.003542 | 260 | 506 | 600 | 600 | 333.8 |
| COAL\#33 | 1074.810 | 15.033 | 0.003542 | 260 | 506 | 600 | 600 | 390.0 |
| COAL\#34 | 1074.810 | 15.033 | 0.003542 | 260 | 506 | 600 | 600 | 432.0 |
| COAL\#35 | 1278.460 | 13.992 | 0.003132 | 260 | 500 | 660 | 660 | 402.0 |
| COAL\#36 | 861.742 | 15.679 | 0.001323 | 260 | 500 | 900 | 900 | 428.0 |
| COAL\#37 | 408.834 | 16.542 | 0.002950 | 120 | 241 | 180 | 180 | 178.4 |
| COAL\#38 | 408.834 | 16.542 | 0.002950 | 120 | 241 | 180 | 180 | 194.1 |
| COAL\#39 | 1288.815 | 16.518 | 0.000991 | 423 | 774 | 600 | 600 | 474.0 |
| COAL\#40 | 1436.251 | 15.815 | 0.001581 | 423 | 769 | 600 | 600 | 609.8 |
| LNG\#1 | 669.988 | 75.464 | 0.902360 | 3 | 19 | 210 | 210 | 17.8 |
| LNG\#2 | 134.544 | 129.544 | 0.110295 | 3 | 28 | 366 | 366 | 6.9 |
| LNG_CC\#01 | 3427.912 | 56.613 | 0.024493 | 160 | 250 | 702 | 702 | 224.3 |
| LNG_CC\#02 | 3751.772 | 54.451 | 0.029156 | 160 | 250 | 702 | 702 | 210.0 |
| LNG_CC\#03 | 3918.780 | 54.736 | 0.024667 | 160 | 250 | 702 | 702 | 212.0 |
| LNG_CC\#04 | 3379.580 | 58.034 | 0.016517 | 160 | 250 | 702 | 702 | 200.8 |
| LNG_CC\#05 | 3345.296 | 55.981 | 0.026584 | 160 | 250 | 702 | 702 | 220.0 |


| LNG_CC\#06 | 3138.754 | 61.520 | 0.007540 | 160 | 250 | 702 | 702 | 232.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LNG_CC\#07 | 3453.050 | 58.635 | 0.016430 | 160 | 250 | 702 | 702 | 168.0 |
| LNG_CC\#08 | 5119.300 | 44.647 | 0.045934 | 160 | 250 | 702 | 702 | 208.4 |
| LNG_CC\#09 | 1898.415 | 71.584 | 0.000044 | 165 | 504 | 1350 | 1350 | 443.9 |
| LNG_CC\#10 | 1898.415 | 71.584 | 0.000044 | 165 | 504 | 1350 | 1350 | 426.0 |
| LNG_CC\#11 | 1898.415 | 71.584 | 0.000044 | 165 | 504 | 1350 | 1350 | 434.1 |
| LNG_CC\#12 | 1898.415 | 71.584 | 0.000044 | 165 | 504 | 1350 | 1350 | 402.5 |
| LNG_CC\#13 | 2473.390 | 85.120 | 0.002528 | 180 | 471 | 1350 | 1350 | 357.4 |
| LNG_CC\#14 | 2781.705 | 87.682 | 0.000131 | 180 | 561 | 720 | 720 | 423.0 |
| LNG_CC\#15 | 5515.508 | 69.532 | 0.010372 | 103 | 341 | 720 | 720 | 220.0 |
| LNG_CC\#16 | 3478.300 | 78.339 | 0.007627 | 198 | 617 | 2700 | 2700 | 369.4 |
| LNG_CC\#17 | 6240.909 | 58.172 | 0.012464 | 100 | 312 | 1500 | 1500 | 273.5 |
| LNG_CC\#18 | 9960.110 | 46.636 | 0.039441 | 153 | 471 | 1656 | 1656 | 336.0 |
| LNG_CC\#19 | 3671.997 | 76.947 | 0.007278 | 163 | 500 | 2160 | 2160 | 432.0 |
| LNG_CCH20 | 1837.383 | 80.761 | 0.000044 | 95 | 302 | 900 | 900 | 220.0 |
| LNG_CCH21 | 3108.395 | 70.136 | 0.000044 | 160 | 511 | 1200 | 1200 | 410.6 |
| LNG_CCH22 | 3108.395 | 70.136 | 0.000044 | 160 | 511 | 1200 | 1200 | 422.7 |
| LNG_CCH23 | 7095.484 | 49.840 | 0.018827 | 196 | 490 | 1014 | 1014 | 351.0 |
| LNG_CC\#24 | 3392.732 | 65.404 | 0.010852 | 196 | 490 | 1014 | 1014 | 296.0 |
| LNG_CC\#25 | 7095.484 | 49.840 | 0.018827 | 196 | 490 | 1014 | 1014 | 411.1 |
| LNG_CC\#26 | 7095.484 | 49.840 | 0.018827 | 196 | 490 | 1014 | 1014 | 263.2 |
| LNG_CC\#27 | 4288.320 | 66.465 | 0.034560 | 130 | 432 | 1350 | 1350 | 370.3 |
| LNG_CCH28 | 13813.001 | 22.941 | 0.081540 | 130 | 432 | 1350 | 1350 | 418.7 |
| LNG_CCH29 | 4435.493 | 64.314 | 0.023534 | 137 | 455 | 1350 | 1350 | 409.6 |
| LNG_CC\#30 | 9750.750 | 45.017 | 0.035475 | 137 | 455 | 1350 | 1350 | 412.0 |
| LNG_CC\#31 | 1042.366 | 70.644 | 0.000915 | 195 | 541 | 780 | 780 | 423.2 |
| LNG_CC\#32 | 1159.895 | 70.959 | 0.000044 | 175 | 536 | 1650 | 1650 | 428.0 |
| LNG_CC\#3 | 1159.895 | 70.959 | 0.000044 | 175 | 540 | 1650 | 1650 | 436.0 |
| LNG_CC\#34 | 1303.990 | 70.302 | 0.001307 | 175 | 538 | 1650 | 1650 | 428.0 |
| LNG_CC\#35 | 1156.193 | 70.662 | 0.000392 | 175 | 540 | 1650 | 1650 | 425.0 |
| LNG_CC\#36 | 2118.968 | 71.101 | 0.000087 | 330 | 574 | 1620 | 1620 | 497.2 |
| LNG_CC\#37 | 779.519 | 37.854 | 0.000521 | 160 | 531 | 1482 | 1482 | 510.0 |
| LNG_CC\#38 | 829.888 | 37.768 | 0.000498 | 160 | 531 | 1482 | 1482 | 470.0 |
| LNG_CC\#39 | 2333.690 | 67.983 | 0.001046 | 200 | 542 | 1668 | 1668 | 464.1 |
| LNG_CC\#40 | 2028.954 | 77.838 | 0.132050 | 56 | 132 | 120 | 120 | 118.1 |
| LNG_CC\#41 | 4412.017 | 63.671 | 0.096968 | 115 | 245 | 180 | 180 | 141.3 |
| LNG_CC\#42 | 2982.219 | 79.458 | 0.054868 | 115 | 245 | 120 | 180 | 132.0 |
| LNG_CC\#43 | 2982.219 | 79.458 | 0.054868 | 115 | 245 | 120 | 180 | 135.0 |
| LNG_CC\#44 | 3174.939 | 93.966 | 0.014382 | 207 | 307 | 120 | 180 | 252.0 |
| LNG_CC\#45 | 3218.359 | 94.723 | 0.013161 | 207 | 307 | 120 | 180 | 221.0 |
| LNG_CC\#46 | 3723.822 | 66.919 | 0.016033 | 175 | 345 | 318 | 318 | 245.9 |
| LNG_CC\#47 | 3551.405 | 68.185 | 0.013653 | 175 | 345 | 318 | 318 | 247.9 |
| LNG_CC\#48 | 4322.615 | 60.821 | 0.028148 | 175 | 345 | 318 | 318 | 183.6 |
| LNG_CC\#49 | 3493.739 | 68.551 | 0.013470 | 175 | 345 | 318 | 318 | 288.0 |
| NUCLEAR\#01 | 226.799 | 2.842 | 0.000064 | 360 | 580 | 18 | 18 | 557.4 |
| NUCLEAR\#02 | 382.932 | 2.946 | 0.000252 | 415 | 645 | 18 | 18 | 529.5 |
| NUCLEAR\#03 | 156.987 | 3.096 | 0.000022 | 795 | 984 | 36 | 36 | 800.8 |
| NUCLEAR\#04 | 154.484 | 3.040 | 0.000022 | 795 | 978 | 36 | 36 | 801.5 |
| NUCLEAR\#05 | 332.834 | 1.709 | 0.000203 | 578 | 682 | 138 | 204 | 582.7 |
| NUCLEAR\#06 | 326.599 | 1.668 | 0.000198 | 615 | 720 | 144 | 216 | 680.7 |
| NUCLEAR\#07 | 345.306 | 1.789 | 0.000215 | 612 | 718 | 144 | 216 | 670.7 |
| NUCLEAR\#08 | 350.372 | 1.815 | 0.000218 | 612 | 720 | 144 | 216 | 651.7 |
| NUCLEAR\#09 | 370.377 | 2.726 | 0.000193 | 758 | 964 | 48 | 48 | 921.0 |
| NUCLEAR\#10 | 367.067 | 2.732 | 0.000197 | 755 | 958 | 48 | 48 | 916.8 |
| NUCLEAR\#11 | 124.875 | 2.651 | 0.000324 | 750 | 1007 | 36 | 54 | 911.9 |
| NUCLEAR\#12 | 130.785 | 2.798 | 0.000344 | 750 | 1006 | 36 | 54 | 898.0 |
| NUCLEAR\#13 | 878.746 | 1.595 | 0.000690 | 713 | 1013 | 30 | 30 | 905.0 |
| NUCLEAR\#14 | 827.959 | 1.503 | 0.000650 | 718 | 1020 | 30 | 30 | 846.5 |
| NUCLEAR\#15 | 432.007 | 2.425 | 0.000233 | 791 | 954 | 30 | 30 | 850.9 |
| NUCLEAR\#16 | 445.606 | 2.499 | 0.000239 | 786 | 952 | 30 | 30 | 843.7 |
| NUCLEAR\#17 | 467.223 | 2.674 | 0.000261 | 795 | 1006 | 36 | 36 | 841.4 |
| NUCLEAR\#18 | 475.940 | 2.692 | 0.000259 | 795 | 1013 | 36 | 36 | 835.7 |
| NUCLEAR\#19 | 899.462 | 1.633 | 0.000707 | 795 | 1021 | 36 | 36 | 828.8 |
| NUCLEAR\#20 | 1000.367 | 1.816 | 0.000786 | 795 | 1015 | 36 | 36 | 846.0 |
| OIL\#01 | 1269.132 | 89.830 | 0.014355 | 94 | 203 | 120 | 120 | 179.0 |
| OIL\#02 | 1269.132 | 89.830 | 0.014355 | 94 | 203 | 120 | 120 | 120.8 |
| OIL\#03 | 1269.132 | 89.830 | 0.014355 | 94 | 203 | 120 | 120 | 121.0 |
| OIL\#04 | 4965.124 | 64.125 | 0.030266 | 244 | 379 | 480 | 480 | 317.4 |
| OIL\#05 | 4965.124 | 64.125 | 0.030266 | 244 | 379 | 480 | 480 | 318.4 |
| OIL\#06 | 4965.124 | 64.125 | 0.030266 | 244 | 379 | 480 | 480 | 335.8 |
| OIL\#07 | 2243.185 | 76.129 | 0.024027 | 95 | 190 | 240 | 240 | 151.0 |
| OIL\#08 | 2290.381 | 81.805 | 0.001580 | 95 | 189 | 240 | 240 | 129.5 |
| OIL\#09 | 1681.533 | 81.140 | 0.022095 | 116 | 194 | 120 | 120 | 130.0 |
| OIL\#10 | 6743.302 | 46.665 | 0.076810 | 175 | 321 | 180 | 180 | 218.9 |
| OIL\#11 | 394.398 | 78.412 | 0.953443 | 2 | 19 | 90 | 90 | 5.4 |
| OIL\#12 | 1243.165 | 112.088 | 0.000044 | 4 | 59 | 90 | 90 | 45.0 |
| OIL\#13 | 1454.740 | 90.871 | 0.072468 | 15 | 83 | 300 | 300 | 20.0 |
| OIL\#14 | 1011.051 | 97.116 | 0.000448 | 9 | 53 | 162 | 162 | 16.3 |
| OIL\#15 | 909.269 | 83.244 | 0.599112 | 12 | 37 | 114 | 114 | 20.0 |
| OIL\#16 | 689.378 | 95.665 | 0.244706 | 10 | 34 | 120 | 120 | 22.1 |
| OIL\#17 | 1443.792 | 91.202 | 0.000042 | 112 | 373 | 1080 | 1080 | 125.0 |
| OIL\#18 | 535.553 | 104.501 | 0.085145 | 4 | 20 | 60 | 60 | 10.0 |
| OIL\#19 | 617.734 | 83.015 | 0.524718 | 5 | 38 | 66 | 66 | 13.0 |
| OLL\#20 | 90.966 | 127.795 | 0.176515 | 5 | 19 | 12 | 6 | 7.5 |
| OIL\#21 | 974.447 | 77.929 | 0.063414 | 50 | 98 | 300 | 300 | 53.2 |
| OIL\#22 | 263.810 | 92.779 | 2.740485 | 5 | 10 | , | 6 | 6.4 |
| OIL\#23 | 1335.594 | 80.950 | 0.112438 | 42 | 74 | 60 | 60 | 69.1 |
| OIL\#24 | 1033.871 | 89.073 | 0.041529 | 42 | 74 | 60 | 60 | 49.9 |
| OIL\#25 | 1391.325 | 161.288 | 0.000911 | 41 | 105 | 528 | 528 | 91.0 |
| OIL\#26 | 4477.110 | 161.829 | 0.005245 | 17 | 51 | 300 | 300 | 41.0 |
| OIL\#27 | 57.794 | 84.972 | 0.234787 |  | 19 | 18 | 30 | 13.7 |
| OIL\#28 | 57.794 | 84.972 | 0.234787 | 7 | 19 | 18 | 30 | 7.4 |
| OIL\#29 | 1258.437 | 16.087 | 1.111878 | 26 | 40 | 72 | 120 | 28.6 |

Table 11. Unit data with valve-point loading

| Generator | $a$ | $b$ | $c$ | $e$ | $f$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| COAL\#05 | 1976.469 | 54.242 | 0.042468 | 700 | 0.080 |
| COAL\#10 | 1320.636 | 13.226 | 0.005063 | 600 | 0.055 |
| COAL\#15 | 1176.504 | 14.651 | 0.003901 | 800 | 0.060 |
| COAL\#22 | 1229.131 | 14.656 | 0.003684 | 600 | 0.050 |
| COAL\#33 | 1074.810 | 15.033 | 0.003542 | 600 | 0.043 |
| COAL\#40 | 1436.251 | 15.815 | 0.001581 | 600 | 0.043 |
| LNG_CC\#10 | 1898.415 | 71.584 | 0.000044 | 1100 | 0.043 |
| LNG_CC\#28 | 13813.001 | 22.941 | 0.081540 | 1200 | 0.030 |
| LNG_CC\#30 | 9750.750 | 45.017 | 0.035475 | 1000 | 0.050 |
| LNG_CC\#42 | 2982.219 | 79.458 | 0.054868 | 1000 | 0.050 |
| OIL\#08 | 2290.381 | 81.805 | 0.001580 | 600 | 0.070 |
| OIL\#10 | 6743.302 | 46.665 | 0.076810 | 1200 | 0.043 |

Table 12. Prohibit zones of units

| Generator | Zone 1 | Zone 2 | Zone 3 |
| :---: | :---: | :---: | :---: |
| COAL\#08 | $[305,335]$ | $[420,450]$ |  |
| COAL\#32 | $[320,350]$ | $[390,420]$ |  |
| LNG_CC\#32 | $[230,255]$ | $[365,395]$ | $[430,455]$ |
| OIL\#25 | $[50,75]$ | $[85,95]$ | - |



Min Jeong Kim She received B.S degree in electrical engineering from Konkuk University in 2011. Her research interests include power system operation and planning, electricity markets, and economics.


Hyoung-Yong Song He received B.S. and M.S degrees from Konkuk University, Seoul, Korea, in 2007 and 2009, respectively. His research interests include power system operation and planning, electricity markets, and computational intelligence and their application to power system.


Jong-Bae Park He received B.S., M.S., and Ph.D. degrees from Seoul National University in 1987, 1989, and 1998, respectively. For 1989-1998, he was with Korea Electric Power Corporation, and for 1998-2001 he was an Assistant Professor at Anyang University, Korea. For 2006-2008, he was a guest researcher of EPRI, USA. From 2001, he has been with Electrical Engineering Department at Konkuk University as Professor. His major research topics include power system operation, planning, economics, and markets.


Jae-Hyung Roh He received the B.S. degree in Nuclear Engineering from Seoul National University, Korea, in 1993 and the M.S. degree in Electrical Engineering from Hongik University, Korea, in 2002. He received Ph.D. degree in Electrical engineering from Illinois Institute of Technology, Chicago, USA. For 1992-2001, he was with Korea Electric Power Corporation, and for 2001-2010, he was with Korea Power Exchange. Since 2010, he has been with Electrical Engineering Department at Konkuk University, Seoul, as an Assistant Professor. His research interests include power systems restructuring, smart grid and resource planning.


Sang Un Lee He was born in Jecheon city, Korea, on 1963. He received the B. Sc. degree in avionics from the Korea Aerospace University in 1997. He received the M. Sc. and Ph. D. degrees in Computer Science from Gyeongsang National University, Korea, in 1997 and 2001, respectively. He is currently an Associate Professor with the Department of Multimedia Science, Gangneung-Wonju National University, Korea. He is interested in software quality assurance and reliability modeling, software engineering, software project management, neural networks, and algorithm.


Sung-Yong Son He received the B.S. and M.S. degrees from Korea Advanced Institute of Science and Technology (KAIST), Korea in 1999 and 1992, respectively, and Ph.D. degree in Mechanical Engineering from University of Michigan, Ann Arbor, in 2000. From 2000 to 2005, he worked at 4DHomeNet and Icross-technology, respectively. He is an associate professor in the Department of Energy\&IT, Gachon University, Korea. His special fields of interest included Smart Grid architectures.


[^0]:    $\dagger$ CorWresponding Author: Dept. of Electrical Engineering, Konkuk University (jhroh@konkuk.ac.kr)

    * Dept. of Electrical Engineering, Konkuk Univerity, Korea. (envyhoi @konkuk.ac.kr)
    ** Dept. of Multimedia Science, Gangneung-Wonju National Univerity, Korea. (sulee@gwnu.ac.kr)
    *** Dept. of Energy\&IT, Gachon Univerity, Korea. (xtra@gachon.ac.kr) Received: July 9, 2012; Accepted: September 10, 2012

