ML-Based Estimation Algorithm of Frequency Offset for 2×2 STBC-OFDM Systems

Ming Lei, Minjian Zhao, Jie Zhong, and Yunlong Cai

In this letter, we propose a novel frequency offset estimation algorithm for space-time block code (STBC) orthogonal frequency division multiplexing systems. The algorithm mainly exploits the specific construction of STBC so that it does not need any additional pilots or sequences in the data field. The estimator is derived on the basis of the maximum likelihood theory. Simulation results show that this method can provide a significant performance improvement in terms of the estimation accuracy of the frequency offset.

Keywords: STBC-OFDM, frequency offset, ML estimation.

I. Introduction

The application of multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems combined with space-time block code (STBC) can offer a diversity gain to improve the performance in wireless fading channels [1]. The MIMO-OFDM system is very sensitive to the carrier frequency offset (FO) since the orthogonality among subcarriers is destroyed in such a case. In a practical system, the residual FO (RFO) inevitably exists after the frequency synchronization. Especially when the frame is long enough, the RFO will accumulate greatly. This significantly affects the bit error rate (BER) of the system. To obtain an appropriate correction, two popular tracking algorithms are proposed to estimate the RFO. One is the cyclic prefix (CP)-aided algorithm, which has been applied in digital video broadcast systems, with lengths of 25%, 12.5%, 6.25%, and 3.125% of an OFDM symbol [2]. The other one is the pilot-aided algorithm, which uses the pilots embedded within the data payload. In an OFDM system such as IEEE 802.11a, four subcarriers are used for this purpose [3]. This method has also been applied both in MIMO-OFDM [4], [5] and STBC-OFDM systems [6], [7].

The FO estimation schemes described above require additional aided data. Our approach in this letter focuses on the use of transmit data for FO estimation. It exploits the data redundancy from STBC in MIMO-OFDM symbols. We derive the maximum likelihood (ML) estimator of the RFO and propose the related estimation algorithm. We give the mean-squared error (MSE) expression for the estimator and analyze the benefits of the algorithm.

The main contributions of this letter are as follows:

- A novel FO estimation algorithm without aided pilots is proposed for STBC-OFDM systems.
- The relationship between two successive signals at receive antennas is established.
- Simulations are performed in comparison with existing algorithms and show a great improvement in terms of the estimation of FO and system performance.

II. Signal Model

A 2×2 STBC-OFDM system is considered in this letter. The STBC signal transmit matrix across the transmit antennas at the conjunctive time instant has the structure as [1]

$$\mathbf{S}=\begin{bmatrix} s_{2n+1}^{(0)} & s_{2n+1}^{(1)\,*} \\ s_{2n+1}^{(1)} & -s_{2n+1}^{(0)\,*} \end{bmatrix},$$

where n=0, 1, 2,…, the row wise of matrix **S** denotes the temporal domain, and the column wise of matrix **S** denotes the

---

Manuscript received Sept. 22, 2011; revised Dec. 29, 2011, accepted Jan. 16, 2012.

The work in this letter was supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. Y1101077.

Ming Lei (phone: +86 0571 8795 3249, lm1029@gmail.com), Minjian Zhao (mjzhao@zju.edu.cn), Jie Zhong (zhongjie@zju.edu.cn), and Yunlong Cai (ylcai@zju.edu.cn) are with the Department of Information Science & Electronic Engineering, Zhejiang University, Zhejiang, China.

http://dx.doi.org/10.4218/etrij.12.0211.0412

 © 2012 ETRI

spatial domain.

The successive OFDM received signal on subcarrier k can be expressed as [8]

$$\boldsymbol{R}_{2n+1,k} = e^{j\theta}\mathbf{H}_k^0 \boldsymbol{s}_{2n+1,k} + \boldsymbol{N}_{2n+1,k},$$
$$\boldsymbol{R}_{2n+2,k} = e^{j\theta}\mathbf{H}_k^1 \boldsymbol{s}_{2n+1,k}^* e^{j\frac{2\pi}{N}\varepsilon(N+N_g)} + \boldsymbol{N}_{2n+2,k},$$

where

$$\mathbf{H}_k^0 = \begin{bmatrix} H_k^{(0,0)} & H_k^{(0,1)} \\ H_k^{(1,0)} & H_k^{(1,1)} \end{bmatrix}, \quad \mathbf{H}_k^1 = \begin{bmatrix} -H_k^{(0,1)} & H_k^{(0,0)} \\ -H_k^{(1,1)} & H_k^{(1,0)} \end{bmatrix},$$

$$\boldsymbol{s}_{t,k} = [s_{t,k}^{(0)} \ s_{t,k}^{(1)}]^T, \quad \boldsymbol{N}_{t,k} = [N_{t,k}^{(0)} \ N_{t,k}^{(1)}]^T,$$

where ε denotes the FO normalized by the subcarrier spacing, θ is the unknown phase offset at time index $2n+1$, N is the number of subcarriers, N_g is the length of CP, $s_{t,k}^{(p)}$ represents the OFDM symbol sent by transmit antenna p on subcarrier k at time index t, $N_{t,k}^{(q)}$ stands for the additive white Gaussian noise observed by receive antenna q, and $H_k^{(q,p)}$ is the channel state information (CSI) on subcarrier k between the p-th transmit antenna and the q-th receive antenna. All the elements in the matrices and vectors are complex-valued.

We make some assumptions as follows: The FO values at each receive antenna are supposed to be the same [9]; the effect of the inter-carrier interference (ICI) caused by the RFO is small; different receive antennas do not correlate to each other; and the noise power at each antenna is considered to be equivalent.

In the considered system, the symbol at time index 0 is the training symbol, which is used for channel estimation. The remaining symbols in the frame structure are all for the data transmission.

III. Proposed Estimation Algorithm

By stacking the symbols on each subcarrier, we obtain the received vector at both receive antennas:

$$\boldsymbol{R}_{2n+1} = e^{j\theta}\mathbf{H}^0 \boldsymbol{s}_{2n+1} + \boldsymbol{N}_{2n+1}, \tag{1}$$

$$\boldsymbol{R}_{2n+2} = e^{j\theta}\mathbf{H}^1 \boldsymbol{s}_{2n+1}^* e^{j\frac{2\pi}{N}\varepsilon(N+N_g)} + \boldsymbol{N}_{2n+2}. \tag{2}$$

The quantities $\mathbf{H}^i$, $\boldsymbol{s}_t$, and $\boldsymbol{N}_t$ are defined as

$$\mathbf{H}^i = \begin{bmatrix} \mathbf{H}_0^i & 0 & \cdots & 0 \\ 0 & \mathbf{H}_1^i & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{H}_{N-1}^i \end{bmatrix}, \quad i = 0,1,$$

$$\boldsymbol{s}_t = [\boldsymbol{s}_{t,0} \ \boldsymbol{s}_{t,1} \ \cdots \ \boldsymbol{s}_{t,N-1}]^T,$$
$$\boldsymbol{N}_t = [\boldsymbol{N}_{t,0} \ \boldsymbol{N}_{t,1} \ \cdots \ \boldsymbol{N}_{t,N-1}]^T, \quad t = 2n+1, 2n+2,$$

where the dimensions of the matrices $\mathbf{H}^i$, $\boldsymbol{s}_t$, and $\boldsymbol{N}_t$ are $2N\times 2N$, $2N\times 1$, and $2N\times 1$, respectively. The received signal $\boldsymbol{R}_t$ is also a $2N\times 1$ vector.

As we assumed before that in the process of FO estimation, the influence of ICI in the system can be ignored. Therefore, the matrix $\mathbf{H}^i$ is a quasi-diagonal matrix. From (1) and (2), we get the relationship between $\boldsymbol{R}_{2n+1}$ and $\boldsymbol{R}_{2n+2}$ as

$$\boldsymbol{R}_{2n+2} = e^{j(2\theta + \frac{2\pi}{N}\varepsilon(N+N_g))}\mathbf{H}^1((\mathbf{H}^0)^{-1}\boldsymbol{R}_{2n+1})^* - e^{j(2\theta + \frac{2\pi}{N}\varepsilon(N+N_g))}\mathbf{H}^1((\mathbf{H}^0)^{-1}\boldsymbol{N}_{2n+1})^* + \boldsymbol{N}_{2n+2}.$$

Based on the analysis in [10], the ML estimation of the FO, given the observations $\boldsymbol{R}_{2n+1}$ and $\boldsymbol{R}_{2n+2}$, is derived from (1) and (2), which maximizes the conditional joint density function of the observations. That is,

$$\hat{\varepsilon} = \arg\max_{\varepsilon}(f(\boldsymbol{R}_{2n+1}, \boldsymbol{R}_{2n+2} \mid \varepsilon)) = \arg\max_{\varepsilon}(f(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon) f(\boldsymbol{R}_{2n+1} \mid \varepsilon)).$$

According to the definition in (1), ε gives no information about $\boldsymbol{R}_{2n+1}$. Therefore,

$$f(\boldsymbol{R}_{2n+1} \mid \varepsilon) = f(\boldsymbol{R}_{2n+1}).$$

Thus, the ML function can be expressed as

$$\hat{\varepsilon} = \arg\max_{\varepsilon}(f(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon)).$$

Since $\boldsymbol{N}_{2n+1}$ and $\boldsymbol{N}_{2n+2}$ are white Gaussian noise, the distribution of $\boldsymbol{R}_{2n+2}$ is also Gaussian and has the dimension of $2N$. Hence, $f(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon)$ is the Gaussian distribution probability density function, which can be written as

$$f(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon) = \frac{1}{(2\pi)^{2N}\det(\mathbf{P})} e^{-\frac{1}{2}(\boldsymbol{R}_{2n+2}-\boldsymbol{M})^H \mathbf{P}^{-1}(\boldsymbol{R}_{2n+2}-\boldsymbol{M})},$$

where det(.) denotes the determinant of matrix (.) and $\boldsymbol{M}$ and $\mathbf{P}$ are the expectation and variance, respectively, as

$$\boldsymbol{M} = E(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon) = e^{j(2\theta + \frac{2\pi}{N}\varepsilon(N+N_g))}\bar{\boldsymbol{R}}_{2n+1},$$
$$\mathbf{P} = Var(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon) = \sigma^2(\mathbf{I}_{2N\times 2N} + \bar{\mathbf{H}}\bar{\mathbf{H}}^H),$$
$$\bar{\boldsymbol{R}}_{2n+1} = \bar{\mathbf{H}}\boldsymbol{R}_{2n+1}^*, \quad \bar{\mathbf{H}} = \mathbf{H}^1(\mathbf{H}^{0*})^{-1},$$

where σ^2 is the noise power of subcarriers at each antenna, $\boldsymbol{M}$ is a $2N\times 1$ vector, and $\mathbf{P}$ is a $2N\times 2N$ matrix, respectively. Consequently, the estimation of the FO ε can be given by

$$\hat{\varepsilon} = \arg\max_{\varepsilon}(f(\boldsymbol{R}_{2n+2} \mid \boldsymbol{R}_{2n+1}, \varepsilon)) = \arg\min_{\varepsilon}((\boldsymbol{R}_{2n+2} - \boldsymbol{M})^H \mathbf{P}^{-1}(\boldsymbol{R}_{2n+2} - \boldsymbol{M})) = \arg\min_{\varepsilon}(\Lambda). \tag{3}$$

We define $\Lambda = (\boldsymbol{R}_{2n+2} - \boldsymbol{M})^H \mathbf{P}^{-1}(\boldsymbol{R}_{2n+2} - \boldsymbol{M})$ as the log-likelihood function. Since we assume that different receive antennas do not correlate to each other, it means that the matrix $\mathbf{P}$ is invertible. With the aid of the quasi-diagonal assumption of $\mathbf{H}^i$, it can be found that the correlation matrix $\mathbf{P}$ and its inverse are also quasi-diagonal. Therefore, Λ can be rewritten as a sum of some submatrices, which is expressed as

$$\Lambda = \sum_{k=0}^{N-1} (\boldsymbol{R}_{2n+2,k} - \boldsymbol{M}_k)^H \mathbf{P}_k^{-1} (\boldsymbol{R}_{2n+2,k} - \boldsymbol{M}_k), \quad (4)$$

where $\mathbf{P}_k$ is a 2×2 matrix and $\boldsymbol{M}_k$ is a 2×1 vector. They are defined as

$$\boldsymbol{M}_k = e^{j(2\theta + \frac{2\pi}{N}\varepsilon(N+N_g))} \overline{\boldsymbol{R}}_{2n+1,k}, \quad \mathbf{P}_k = \sigma^2 \overline{\mathbf{P}}_k, \quad (5)$$

where

$$\overline{\mathbf{P}}_k = \mathbf{I}_{2\times 2} + \overline{\mathbf{H}}_k \overline{\mathbf{H}}_k^H, \; \overline{\mathbf{H}}_k = \mathbf{H}_k^1 (\mathbf{H}_k^{0*})^{-1}, \; \overline{\boldsymbol{R}}_{2n+1,k} = \overline{\mathbf{H}}_k \boldsymbol{R}_{2n+1,k}^*. \quad (6)$$

Substituting (4) into (3), by equating the derivative of Λ to zero[0] ($\frac{d\Lambda}{d\varepsilon} = 0$), the ML solution for the RFO can be obtained:

$$\tilde{\varepsilon}_1 = 2\theta + \frac{2\pi(N+N_g)}{N}\hat{\varepsilon} = \tan^{-1}\left(\frac{\text{imag}(\sum_{k=0}^{N-1} \overline{\boldsymbol{R}}_{2n+1,k}^H \overline{\mathbf{P}}_k^{-1} \boldsymbol{R}_{2n+2,k})}{\text{real}(\sum_{k=0}^{N-1} \overline{\boldsymbol{R}}_{2n+1,k}^H \overline{\mathbf{P}}_k^{-1} \boldsymbol{R}_{2n+2,k})}\right). \quad (7)$$

To eliminate θ, the solution for the adjacent STBC group ($t = 2n+3, 2n+4$) is also derived as

$$\tilde{\varepsilon}_2 = 2\theta + 5\frac{2\pi(N+N_g)}{N}\hat{\varepsilon} = \tan^{-1}\left(\frac{\text{imag}(\sum_{k=0}^{N-1} \overline{\boldsymbol{R}}_{2n+3,k}^H \overline{\mathbf{P}}_k^{-1} \boldsymbol{R}_{2n+4,k})}{\text{real}(\sum_{k=0}^{N-1} \overline{\boldsymbol{R}}_{2n+3,k}^H \overline{\mathbf{P}}_k^{-1} \boldsymbol{R}_{2n+4,k})}\right). \quad (8)$$

By employing (7) and (8), we can simply obtain the proposed FO estimator as

$$\hat{\varepsilon} = \frac{N}{N+N_g} \frac{1}{2\pi} \frac{(\tilde{\varepsilon}_2 - \tilde{\varepsilon}_1)_{2\pi}}{4}, \quad (9)$$

where $(.)_{2\pi}$ denotes the remainder after a division to 2π, the range of which is $[-\pi, \pi]$. So, the estimation range of the proposed algorithm is [–1/8, 1/8], normalized to the band of the subcarrier.

Step 1. Getting the CSI matrix $\mathbf{H}$.
Step 2. Calculating $\overline{\boldsymbol{R}}_{2n+1,k}$, $\mathbf{P}_k$, and $\overline{\mathbf{P}}_k$ by (5) and (6).
Step 3. Computing $\tilde{\varepsilon}_1$ according to (7).
Step 4. Repeating step 2 and step 3 to get $\tilde{\varepsilon}_2$ by (8).
Step 5. Computing the result $\hat{\varepsilon}$ by (9).

Fig. 1. Proposed estimate algorithm.

Finally, we summarize the proposed algorithm in Fig. 1.

IV. Simulation and Discussion

In the considered system, each OFDM symbol is composed of 256 subcarriers and a CP with a length of 32 sampling periods. The system employs the first training symbol to get the CSI by using the simple least square method in [11]. The first simulation channel model has four delay paths, which are 0, 4, 12, and 24 sampling periods. The coefficient of each path is an independently generated random complex variable. The second simulation employs the standard channel models in [12]. We use the channel B in "outdoor to indoor and pedestrian environment," where we set the fading rate, that is, the cycles per symbol $f_d T_s = 5\times 10^{-6}$.

The simulation results illustrated in Fig. 2 are for a system with an RFO of 4% normalized to the subcarrier. We compare the proposed algorithm with the Cramer-Rao bound (CRB) proposed by Morellie & Mengali (M&M) in [13]. From Fig. 2, it can be seen that the performance of the proposed estimator is much better than the CRB of M&M. In particular, in the condition of the first channel model, the proposed algorithm has an 8 dB gain at the MSE level of 10^{-5}. In the second channel model, it still delivers a good performance despite the fading channel of the Doppler frequency. Both the proposed algorithm and the M&M algorithm require one training symbol in practice, as the proposed algorithm needs an estimate of the channel, and the M&M algorithm uses it for the FO computation itself. Therefore, the throughput and effective SNR between the two methods are directly comparable.

Figure 2 also shows that when the CSI is estimated perfectly, the performance of the algorithm will increase, especially in the case of low SNR. Thus, the estimation accuracy of CSI should

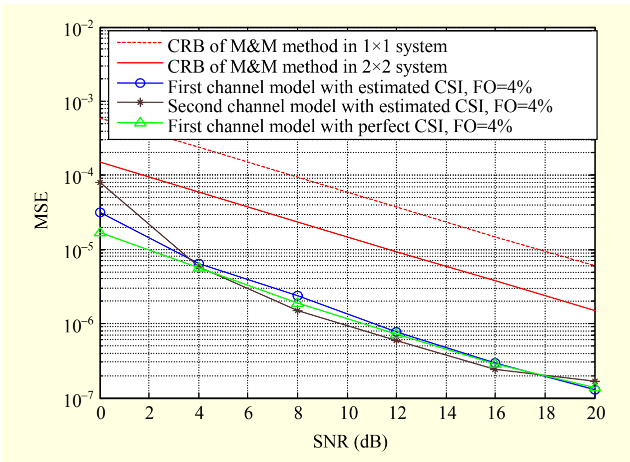

Fig. 2. MSE of proposed algorithm with normalized FO of 4%.

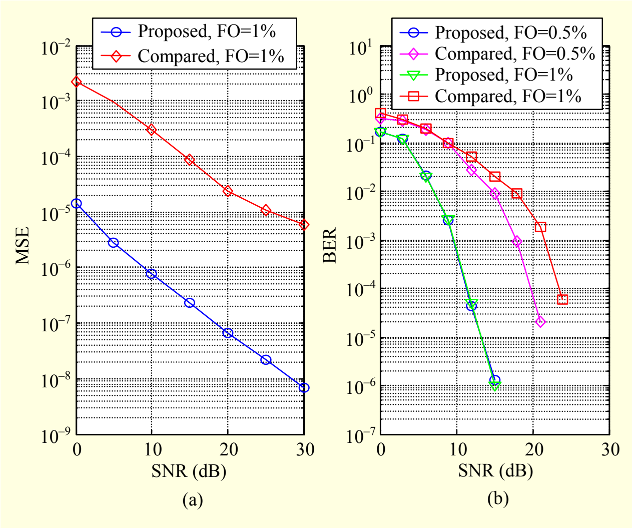

Fig. 3. (a) MSE and (b) BER performance of proposed algorithm and compared method.

be guaranteed to ensure the accuracy of the algorithm.

We also compare the performance of the proposed algorithm with the existing algorithm in [6], which proposed an averaging approach for FO estimation in a 2×2 STBC-OFDM system. The type of modulation is quadrature phase shift keying. Perfect timing synchronization is assumed in the simulation. We consider the Rayleigh fading channel, and its power delay profile follows the exponential model as mentioned in [6].

Figure 3(a) demonstrates the MSE of the proposed estimator and the compared method. The FO is 1% of the subcarrier. It can be seen that there is a great improvement when utilizing our method. This significant improvement comes from the fact that we use the whole data symbol to achieve the relevant computation while the compared method only uses the special pilots and requires a moving averaging scheme.

Figure 3(b) shows the system BER performance with a normalized FO of 0.5% and 1%, respectively. We use the first two successive OFDM data symbols of a frame for FO estimation. Once the result is known, the phase offset on each data subcarrier can be compensated. In the simulation, we use the Zero-Forcing detector. The system of the proposed algorithm outperforms that of the existing algorithm in [6]. With the SNR increases, the advantage becomes more evident. In our system, after the FO is corrected, the BER performance of the FO of 0.5% is nearly the same as that of the FO of 1%.

V. Conclusion

In this letter, we proposed an FO estimation method for STBC MIMO-OFDM systems. The estimator is derived on the basis of the ML theory. It exploits the data redundancy from STBC and considers the use of transmit data itself for estimation. Therefore, it does not need any additional pilots or sequences; hence, it results in a high efficiency in spectrum. Moreover, we have shown that it can provide a large estimation accuracy improvement compared to the classical pilot-aided algorithms and enhance the performance of the STBC-OFDM system.

References

[1] S.M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, Oct. 1998, pp. 1451-1458.

[2] K. Yan et al., "A Novel Adaptive Prefix Interval Scheme for MIMO OFDM Systems," *ISCAS*, 2009, p. 2798.

[3] M. Sandell, D. McNamara, and S. Parker, "Analysis of Frequency-Offset Tracking in MIMO OFDM Systems," *IEEE Trans. Commun.*, vol. 54, no. 8, Aug. 2006, pp. 1481-1489.

[4] T. Aoki and M. Sandell, "Pilot Design for Frequency Offset Tracking in MIMO OFDM Systems," *PIMRC*, 2006, p. 1.

[5] C. Oberli, "ML-Based Tracking Algorithms for MIMO-OFDM," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, July 2007, pp. 2630-2639.

[6] T.M. Mingqian et al., "Frequency Offset Correction for Space-Time Block Coded OFDM Systems Based on Maximum Likelihood Estimation," *ICICS*, 2003, p. 926.

[7] T.M. Mingqian, A.S. Madhukumar, and A.B. Premkumar, "Performance Investigation of STBC-OFDM Systems with Frequency Offset and a Semi-Blind Approach for the Correction," *VTC*, 2004-Spring, p. 1836.

[8] C. Oberli and B. Daneshrad, "Maximum Likelihood Tracking Algorithms for MIMO-OFDM," *ICC*, 2004, p. 2468.

[9] A. Saemi et al., "Joint ML Time-Frequency Synchronization and Channel Estimation Algorithm for MIMO-OFDM Systems," *IET Circuits Devices Syst.*, vol. 2, no. 1, Feb. 2008, pp. 103-111.

[10] P.H. Moose, "A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction," *IEEE Trans. Commun.*, vol. 42, no. 10, Oct. 1994, pp. 2908-2914.

[11] X. Ma, H. Kobayashi, and S.C. Schwartz, "Joint Frequency Offset and Channel Estimation for OFDM," *GLOBECOM*, 2003, p. 15.

[12] ITU Document, Rec. ITU-R M.1225, "Guidelines for the Evaluation of Radio Transmission Technologies for IMT-2000," ITU-R, 1997.

[13] M. Morelli and U. Mengali, "An Improved Frequency Offset Estimator for OFDM Applications," *IEEE Commun. Lett.*, vol. 3, no. 3, Mar. 1999, pp. 75-77.