

Partial Shift Mapping for PAPR Reduction with Low Complexity in OFDM Systems

Xing Ouyang, Jiyu Jin, Guiyue Jin, and Zhisen Wang

The high peak-to-average power is one of the main drawbacks in OFDM systems. This letter proposes a partial shift mapping (PSM) method for peak power reduction in OFDM systems. By utilizing the properties of the discrete Fourier transform, the proposed method generates a set of candidate signals without additional complex multiplication and selects the one with minimum peak power for transmission. Analyses and simulations confirm that the PSM method achieves satisfactory peak power reduction performance and low complexity compared with other kindred methods, for example, selected mapping and partial transmit sequences.

Keywords: Peak-to-average power ratios, OFDM, discrete Fourier transform, partial shift mapping, selected mapping.

I. Introduction

OFDM is an attractive technique because of the advantages of high spectral efficiency and robustness to frequency selective fading. However, high peak-to-average power ratio (PAPR) of the transmit signal is a major drawback of OFDM systems [1]. The PAPR problem burdens the power amplifier and device battery life.

Numerous methods have been proposed to counteract the peak power problem in OFDM systems. As proposed in [2], [3], selected mapping (SLM) and partial transmit sequence (PTS) are promising techniques because they introduce no interference and noise power enlargement as opposed to clipping and companding. However, one of the disadvantages of SLM and PTS is that numerous inverse fast Fourier

transforms (IFFTs) or search operations are required to generate the candidate signals. Some reduced complexity methods proposed for SLM come at the expense of the PAPR reduction performance deterioration. For example, a modified IFFT structure was presented in [4] and conversion matrices were proposed in [5]. However, the system performance is degraded because some of those conversion matrices change the power of subchannels [5]. Of the matrices proposed in [5], Wang and others selected 12 conversion matrices in [6] that will not change the subcarrier power. However, the number of candidate signals is strictly limited to 12, and the PAPR reduction performance is somewhat discouraged. In [7], authors utilize the properties of the discrete Fourier transform (DFT) to avoid IFFT, and [8] proposed the cyclically shifted sequences in PTS to exploit more degrees with a similar performance to PTS, but there is still a mass of multiplications that mainly aggravate the complexity.

In this letter, we propose a low complexity method, named as partial shift mapping (PSM), which is similar to SLM that generates candidate signals with low peak power. By exploiting the properties of DFT, PSM needs only one IFFT and no additional complex multiplication to generate those candidate signals, which means significant reduction on the computational complexity. Analyses and simulations show that the proposed method achieves satisfied peak power reduction performance.

II. System Model and Properties of DFT

Let $X(k)$ denote the symbol on the k -th subchannel, $k=0, 1, \dots, N-1$, and N is the number of subchannels. The OFDM signals modulated by IFFT with oversampling can be given as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi}{LN} nk}, \quad 0 \leq n \leq LN-1, \quad (1)$$

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where L is the oversampling factor for accurately measuring the peak power. The PAPR of the OFDM signal is defined as the ratio of maximum instantaneous power to average power:

$$PAPR = \frac{\max_{0 \leq n \leq LN-1} |x(n)|^2}{\frac{1}{LN} \sum_{n=0}^{LN-1} |x(n)|^2}. \quad (2)$$

To introduce the properties of DFT in terms of the OFDM signals, some preprocesses should be performed. Let f_i , $i=0, 1, 2, 3$, denote four sequences with length LN as

$$f_i(k) = \frac{1}{4} \left(1 + j^{k-i} + (-1)^{k-i} + (-j)^{k-i} \right), \quad k=0, 1, \dots, LN-1, \quad (3)$$

where $f_i(k)$ is the k -th element of f_i . It can be deduced that $f_0 = [1, 0, 0, 0, 1, 0, 0, 0, \dots, 1, 0, 0, 0]$, and f_i is the sequence of f_0 right cyclically shifted with i steps. Therefore, the sum of f_i is all one sequence, and the original OFDM symbol sequence, $X(k)$, can be divided into four disjoint subblocks, each beginning with the non-zero elements, by weighting the sequence by $f_i(k)$:

$$X_i(k) = f_i(k) \cdot X(k). \quad (4)$$

Property 1. The time domain signal, $x_i(n)$, of $X_i(k)$ generated by performing IFFT on (4) can be given by

$$x_i(n) = \frac{1}{4} [x(n) + (-j)^i x((n+LN/4))_{LN} + (-1)^i x((n+LN/2))_{LN} + j^i x((n+3LN/4))_{LN}], \quad (5)$$

where $(\cdot)_{LN}$ denotes the modulo- LN operator. Based on property 1, the time domain signals of each subblock, $X_i(k)$, can be obtained by combining those copies of $x(n)$ with different circular shift steps ($LN/4$, $LN/2$, and $3 \cdot LN/4$) and different phases without any multiplication operation. However, in PTS-based approaches [3], [8], the time domain signals of those subblocks are generated by performing IFFTs on the subblocks individually. Hence, we can get the time domain signals of the subblocks with only one IFFT compared with PTS.

Property 2. According to the shift theorem of DFT, circular shift of the time domain sequence is equivalent to weighting the frequency domain sequence by linear phases:

$$X(k) \cdot e^{-j \frac{2\pi}{LN} kl} = FFT[x((n-l))_{LN}], \quad (6)$$

where $e^{-j \frac{2\pi}{LN} kl}$ is the linear phase and l is the circular shift step. In the PSM scheme, the candidate signals are generated by combining the cyclically shifted time domain signals, $x_i(n)$, so that the candidate signals are virtually generated by weighting frequency domain subblocks with different phases as SLM does. However, the operations are performed in time domain without any complex multiplication, which means numbers of IFFT and additional complex multiplications are avoided.

III. Proposed PSM Method

Figure 1 depicts the block diagram of the PSM method, where the oversampled time domain signal $x(n)$ is obtained by LN -point IFFT after the serial-to-parallel conversion of the input data. The frequency domain symbol sequence $X(k)$ is divided into four disjoint subblocks in time domain. That is, after $x(n)$ is obtained by IFFT, the corresponding time domain signals $x_i(n)$ of the four subblocks are generated based on property 1. The details are provided in Fig. 2(a), where $x(n)$ is first scaled by $1/4$ and then fed into a cyclic shift module to generate signals with different shift steps, $LN/4$, $LN/2$, and $3 \cdot LN/4$. The time domain signals of the disjoint subblocks are obtained by combining those shifted time domain signals with the original one scaled by $1/4$ according to (5).

The time domain signals of those subblocks are fed into the combiners to get a set of candidate signals as shown in Fig. 2(b). The PSM scheme generates different candidate signals by cyclically shifting the signals, $x_i(n)$, $i=1, 2, 3$, in predetermined steps, $l_{m,i}$, which may be generated uniformly from 1 to LN and adds them up with $x_0(n)$. Therefore, the m -th

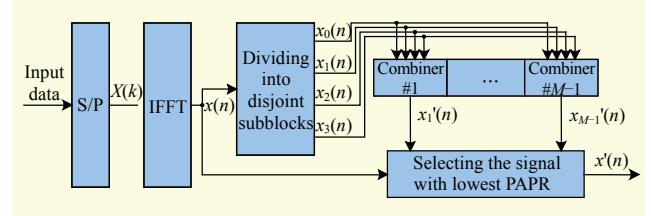


Fig. 1. Block diagram of PSM scheme.

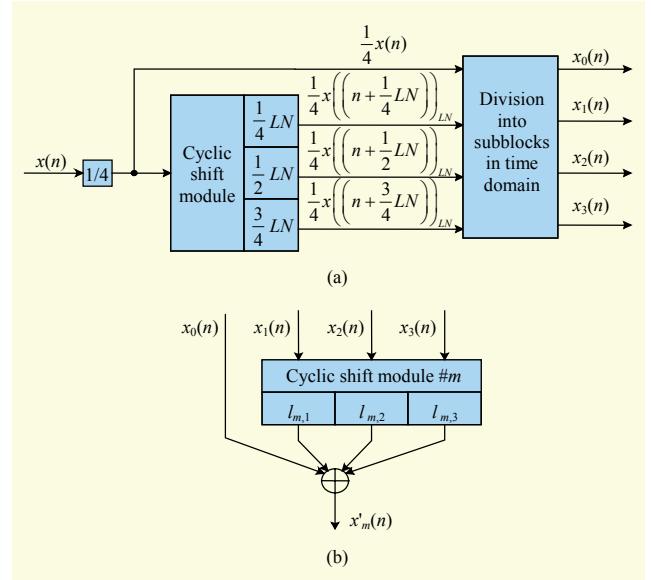


Fig. 2. (a) Architecture of block of dividing into disjoint subblocks and (b) architecture of m -th combiner.

candidate signal can be denoted as

$$x'_m(n) = x_0(n) + x_1((n-l_{m,1}))_{LN} + x_2((n-l_{m,2}))_{LN} + x_3((n-l_{m,3}))_{LN}, \quad (7)$$

where $l_{m,i}$, $i=1, 2, 3$, are the shift steps in the m -th combiner. Based on property 2 and (6), the m -th candidate signal in the frequency domain can be obtained by performing FFT on both sides of (7) as

$$X'_m(k) = X_0(k) + X_1(k)e^{-j\frac{2\pi}{LN}kl_{m,1}} + X_2(k)e^{-j\frac{2\pi}{LN}kl_{m,2}} + X_3(k)e^{-j\frac{2\pi}{LN}kl_{m,3}}. \quad (8)$$

This indicates that those candidate signals are equivalent to weighting $X_1(k)$, $X_2(k)$, and $X_3(k)$ by different linear phases. It also means that the signal power on each subcarrier is constant because the subcarriers in different subblocks are separated from each other, and summing the subblocks will not affect the power of each subcarrier. Therefore, PSM may not degrade the bit error rate of the OFDM systems, contrary to the conversion matrices algorithm in [5]. Finally, the PAPR of the $M-1$ candidate signals and the original one should be measured, and the candidate signal with the lowest PAPR will be selected for transmission, where M is the total number of candidate signals.

As SLM, the index of the candidate signal selected for transmission must be sent to the receiver as side information (SI) for recovering the original data, since, according to (8), the frequency symbols are phase-rotated linearly. According to the SI, the original data can be recovered by dividing the symbols on the subchannel by linear phases. The SI is important for the receiver to recover the original data information, so the SI should be sent to the receiver in a robust way.

IV. Complexity Analysis and Simulations

From Fig. 1, the PSM method takes only one IFFT which requires $(LN/2)\cdot\log_2 LN$ complex multiplications and $LN\cdot\log_2 LN$ complex additions to generate $x(n)$; furthermore, it takes $12\cdot LN$ complex additions to obtain $x_i(n)$, $i=0, 1, 2, 3$, and $3(M-1)\cdot LN$ complex additions to get all of the candidate signals. As shown in Table 1, the number of complex multiplications of SLM or PTS is M or V times that of PSM, where V is the number of subblocks in PTS. In PTS, moreover, it can be seen that the complexity of searching candidate signals increases exponentially with respect to V . Figure 3 demonstrates the amount of complex operations of SLM and PSM in the case of $N=1,024$ and $L=4$. It can be seen that the PSM gets significant complexity reduction as M increases.

It should be noted that the OFDM symbols can be divided into more than four subblocks. However, this will introduce additional multiplications to generate the time domain signals in a more complicated way with negligible performance

Table 1. Computational complexity of SLM, PTS, and PSM.

PAPR reduction scheme	Number of complex multiplications	Number of complex additions
Conventional SLM	$M \frac{LN}{2} \log_2 LN$	$MLN \log_2 LN$
PTS	$V \frac{LN}{2} \log_2 LN$	$VLN \log_2 LN + (V-1)W^{V-1}LN$
PSM	$\frac{LN}{2} \log_2 LN$	$LN \log_2 LN + (3M+9)LN$

M is the number of candidate signals. V and W are the number of subblocks and phase factors in PTS, respectively.

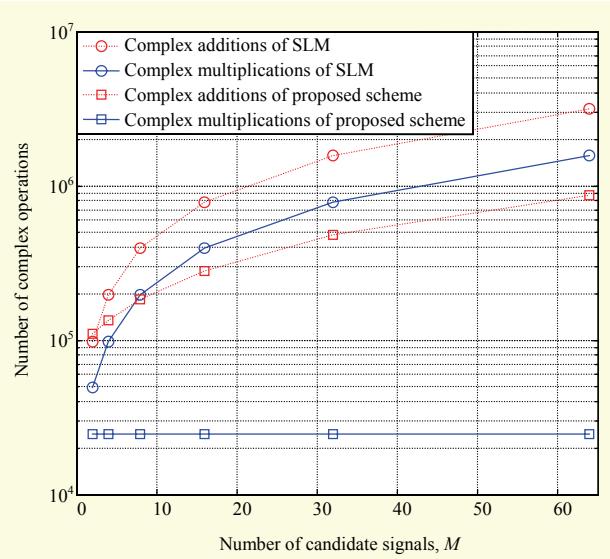


Fig. 3. Complex operations as a function of the number of candidate signals M ($LN=4,096$).

improvement. Thus, the number of subblocks is limited to four as a compromise between performance and complexity.

The complementary cumulative density function of the PAPR is used to evaluate the PAPR reduction performance, and the PAPR reduction performance with QPSK modulation for SLM, PTS, and PSM is presented in Fig. 4. In PSM, the shift steps, $l_{m,i}$, are chosen from 1 to $LN-1$ randomly. For $M=8$ or 16, the performance of PSM is almost the same as that of SLM. However, the number of complex multiplications of SLM is 8 or 16 times that of the proposed scheme. For PTS, as $V=4$ and $M=16$, the crossover with PSM is about 10^{-2} in Fig. 4, but the number of multiplications of PTS is 4 times that of PSM. Moreover, it is required to generate 64 candidate signals to get the optimum one, which leads to numerous addition operations and needs more bits to index the candidate signals.

In Fig. 5, we provide the performance of both the PSM and the reduced complex SLM proposed in [4]. The authors

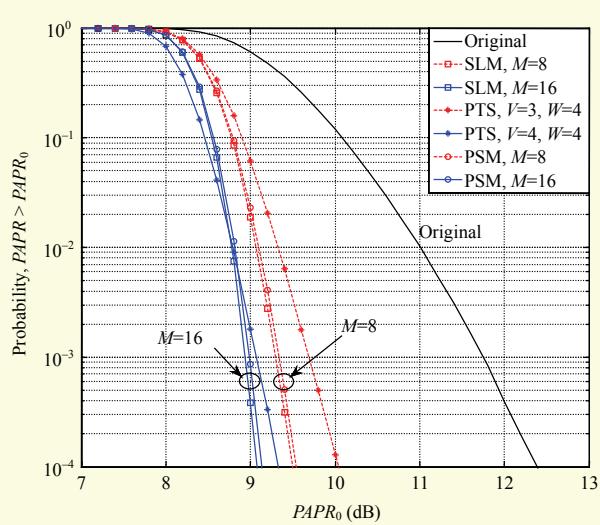


Fig. 4. PAPR reduction performance for SLM, PTS, and PSM ($N=1,024, L=4$).

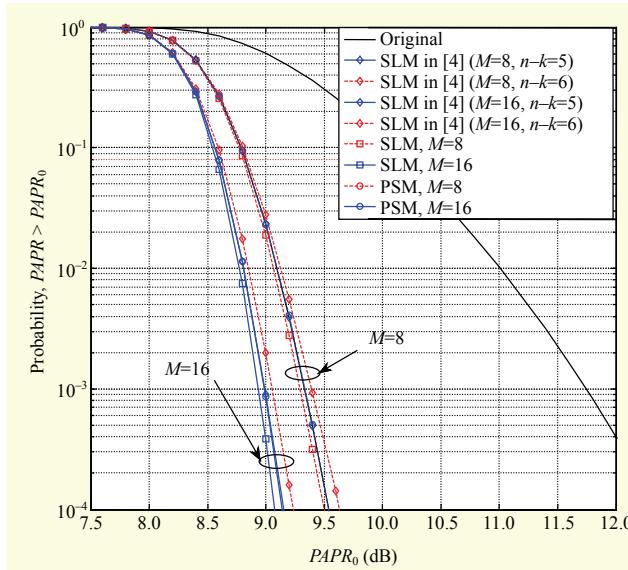


Fig. 5. PAPR reduction performance for reduced complex SLM, proposed in [4], and PSM ($N=1,024, L=4$).

modified the structure of IFFT in which the phase factor is weighted in the intermediate stage. As the intermediate stage k approaches the final stage n , the complexity is reduced (for $k=0$, it degenerated to SLM), but the performance is limited. In Fig. 5, when the intermediate stage is chosen to be 5, its performance is similar to PSM, whereas its complexity is 8.48 times that of PSM for $M=16$ (4.48 for $M=8$).

Additional simulation results, which are not shown herein, also confirmed that the PAPR reduction performances of PSM and SLM are similar but outperform PTS as the number of candidate signals is limited. The marginal degradation of PSM

results from that, (7) and (8), and the candidate signals are generated by weighting the symbols by linear phases. In PTS, the relativity of the candidate signals is the highest and leads to the most unsatisfactory performance.

V. Conclusion

This letter proposed a low-complexity PAPR reduction scheme for the OFDM systems. The proposed PSM scheme divides an OFDM symbol block into four subblocks and weighs these subblocks with different linear phases to get a set of candidate signals, but it utilizes the properties of DFT to generate the candidate signals without complex multiplication. Further, it will not degrade the system performance. From the analysis and simulation results, the PAPR reduction performance of the PSM scheme is almost the same as that of SLM and outperforms PTS. However, compared with SLM, for example, the number of complex multiplications and complex additions of the proposed scheme is only 6.25% and 36%, respectively, in the case of $M=16$ candidate signals.

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