# Moving-Target Tracking Based on Particle Filter with TDOA/FDOA Measurements

Jeong A Cho, Hanbyeul Na, Sunwoo Kim, and Chunsoo Ahn

*In this letter, we propose a moving-target tracking algorithm based on a particle filter that uses the time difference of arrival (TDOA)/frequency difference of arrival (FDOA) measurements acquired by distributed sensors. It is shown that the performance of the proposed algorithm, based on the particle filter, outperforms the one based on the extended Kalman filter. The use of both the TDOA and FDOA measurements is shown to be effective in the moving-target tracking. It is proven that the particle filter deals with the nonlinear nature of the moving-target tracking problem successfully.*

*Keywords: Time difference of arrival (TDOA), frequency difference of arrival (FDOA), particle filter, target tracking.*

## I. Introduction

Localization is one of the most important research issues in wireless communications. Moving-target tracking has particularly received significant attention in military surveillance applications. For many years, finding the location of a target exploited various metrics, such as angle of arrival, time of arrival, and received signal strength. The use of both the time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements between receiving sensors is one of the most effective methods in tracking a moving target [1]. The TDOA and FDOA measurements are complementary in that using both TDOA and FDOA measurements allows for bearing and range estimation as well as for a wide range of emitter speed and signal bandwidth [2], [3].

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Jeong A Cho (phone: +82 2 2220 4822, sinichi@hanyang.ac.kr), Hanbyeul Na (lhstar@hanyang.ac.kr), Sunwoo Kim (remero@hanyang.ac.kr), and Chunsoo Ahn (acsmmo1@gmail.com) are with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, Rep. of Korea.

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To estimate the position and velocity of a moving target with the TDOA/FDOA measurements, various approaches have been investigated [2], including the Kalman filter (EKF) [4]. The EKF has been widely used in nonlinear estimation for a long time. However, the EKF often exhibits poor performance when the nonlinearity in the process and measurement models is very severe [5]. Particle filtering [5] based on the Monte Carlo method solves a wide range of nonlinear filtering problems by approximating probability distributions with discrete random samples. Research has consistently shown that the particle filter is more suitable for practical tracking conditions than other nonlinear filters [6], [7].

In this letter, we propose a target tracking algorithm based on the particle filter with the TDOA/FDOA measurements. The proposed algorithm uses the combination of the TDOA/FDOA measurements obtained from distributed sensors to estimate the position and the velocity of a moving target in 3D space.

## II. System Model

We assume a 3D Euclidean space. Multiple sensors are located to determine the position and the velocity of a moving target. The state of the target $\mathbf{x}_k \in R^{9\times1}$ at $t_k$ is given as

$$\mathbf{x}_k = \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{\Gamma}\mathbf{w}_{k-1}, \tag{1}$$

where $\mathbf{x}_k = [\mathbf{u}_k^T, \dot{\mathbf{u}}_k^T, \ddot{\mathbf{u}}_k^T]$ consists of the 3D position vector $\mathbf{u}_k$, the velocity vector $\dot{\mathbf{u}}_k$, and the acceleration vector $\ddot{\mathbf{u}}_k$:

$$\mathbf{u}_k = [x_k \, y_k \, z_k]^T,\ \dot{\mathbf{u}}_k = [\dot{x}_k \, \dot{y}_k \, \dot{z}_k]^T,\ \ddot{\mathbf{u}}_k = [\ddot{x}_k \, \ddot{y}_k \, \ddot{z}_k]^T. \tag{2}$$

The state transition matrix $\mathbf{\Phi}$ is

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{I}_{3\times3} & \Delta\mathbf{I}_{3\times3} & \Delta^2/2\mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \Delta\mathbf{I}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \alpha\mathbf{I}_{3\times3} \end{bmatrix}, \tag{3}$$

  

where $\mathbf{I}_{n\times n}$ is an $n\times n$ identity matrix, $\mathbf{0}_{n\times n}$ is a $n\times n$ zero matrix, and $\alpha$ is a constant acceleration parameter. The transformation matrix of the process noise, $\mathbf{\Gamma}$, is

$$\mathbf{\Gamma} = \left[\frac{\Delta^2}{2}\mathbf{I}_{3\times3} \quad \Delta\mathbf{I}_{3\times3} \quad \mathbf{I}_{3\times3}\right]^T, \tag{4}$$

where $\Delta=t_k-t_{k-1}$ is a fixed time step and $\mathbf{w}_{k-1}$ is the following white Gaussian noise process:

$$\mathbf{w}_{k-1} = [w_{x,k-1}, w_{y,k-1}, w_{z,k-1},]^T, \tag{5}$$

$$E[\mathbf{w}_{k-1}] = \mathbf{0},\ E[\mathbf{w}_{k-1}\mathbf{w}_{k-1}^T] = \mathbf{Q}_w, \tag{6}$$

where the covariance matrix $\mathbf{Q}_w = \sigma_w^2\mathbf{I}_{3\times3}$ and $\sigma_w$ is the standard deviation of the process noise.

A signal is transmitted from the moving target, and the sensors receive the signal. Let $M$ be the number of sensors. In this circumstance, the distance between the target and the $a$-th sensor ($s_a$), $r_a(\mathbf{x}_k)$ is given as

$$r_a(\mathbf{x}_k) = |\mathbf{u}_k - \mathbf{s}_a|, \tag{7}$$

where $\mathbf{u}_k$ is the position vector of the target and $\mathbf{s}_a$ is the known position vector of $s_a$. Let $T_{ab}$ be the TDOA measurement between $s_a$ and $s_b$. If $c$ is the signal propagation speed, then the range difference of arrivals between $s_a$ and $s_b$ at time $t_k$ is

$$r_{ab}(\mathbf{x}_k) = cT_{ab}(\mathbf{x}_k) = r_a(\mathbf{x}_k) - r_b(\mathbf{x}_k), \tag{8}$$

where $a$=1, 2,…, $M$, and $b$=1, 2,…, $M$. To improve the tracking performance, we exploit not only the TDOA measurements but also the FDOA measurements. From the time derivative of (7), we can obtain

$$\dot{r}_a(\mathbf{x}_k) = \frac{(\dot{\mathbf{u}}_k - \dot{\mathbf{s}}_a)^T(\mathbf{u}_k - \mathbf{s}_a)}{r_a(\mathbf{x}_k)},\quad a = 1, 2, \cdots, M, \tag{9}$$

where $\dot{\mathbf{u}}_k$ is the velocity vector of the target and $\dot{\mathbf{s}}_a$ is the known velocity vector of $s_a$. The FDOA measurement is then

$$\dot{r}_{ab}(\mathbf{x}_k) = \frac{(\dot{\mathbf{u}}_k - \dot{\mathbf{s}}_a)^T(\mathbf{u}_k - \mathbf{s}_a)}{|\mathbf{u}_k - \mathbf{s}_a|} - \frac{(\dot{\mathbf{u}}_k - \dot{\mathbf{s}}_b)^T(\mathbf{u}_k - \mathbf{s}_b)}{|\mathbf{u}_k - \mathbf{s}_b|}. \tag{10}$$

The first sensor $s_1$ acts as a reference sensor, and the TDOA/FDOA measurements between $s_1$ and $s_l$, $l$=2,…, $M$, are stacked to yield a full measurement vector $\mathbf{Z}_k$ at time $t_k$:

$$\mathbf{Z}_k = \mathbf{H}_k(\mathbf{x}_k) + \mathbf{V}_k, \tag{11}$$

where

$$\mathbf{H}_k(\mathbf{x}_k) = [\mathbf{h}_{21}(\mathbf{x}_k)\mathbf{h}_{31}(\mathbf{x}_k)\cdots\mathbf{h}_{M1}(\mathbf{x}_k)]^T, \tag{12}$$

and $\mathbf{h}_{ab}(\mathbf{x}_k)$ is the true TDOA/FDOA measurement between $s_a$ and $s_b$ at $t_k$ as

$$\mathbf{h}_{ab}(\mathbf{x}_k) = \begin{bmatrix} r_{ab}(\mathbf{x}_k) \\ \dot{r}_{ab}(\mathbf{x}_k) \end{bmatrix}. \tag{13}$$

The white Gaussian noise process $\mathbf{V}_k$ is

$$E[\mathbf{V}_k] = \mathbf{0}, \tag{14}$$

$$E[\mathbf{V}_k(\mathbf{V}_k)^T] = diag[\sigma_v^2, \sigma_{\dot{v}}^2] \otimes \mathbf{I}_{M-1\times M-1}, \tag{15}$$

where the variances of TDOA measurement noise and FDOA measurement noise are denoted as $\sigma_v$ and $\sigma_{\dot{v}}$, respectively, and $\otimes$ represents the Kronecker product.

## III. Moving-Target Tracking Algorithm with TDOA/FDOA Measurements

As shown in Fig. 1, the distributed sensors receive signals from the target. The sensors are able to measure the TDOAs and the FDOAs between them. The TDOA/FDOA measurements in (13) are processed by the particle filter to yield the position estimate of the target.

The state of target $\mathbf{x}_k$ is estimated at time $t_k$ from the conditional probability density function $p(\mathbf{x}_k|\mathbf{Z}_{1:k})$, where $\mathbf{Z}_{1:k}$ is the set of measurements $\mathbf{Z}_{1:k}=\{\mathbf{Z}_1,\ldots,\mathbf{Z}_k\}$ up to time $t_k$. The posterior density function $p(\mathbf{x}_k|\mathbf{Z}_{1:k})$ can be inferred from $p(\mathbf{x}_k|\mathbf{Z}_{1:k-1})$ by

$$p(\mathbf{x}_k \mid \mathbf{Z}_{1:k}) = \frac{p(\mathbf{Z}_k \mid \mathbf{x}_k)p(\mathbf{x}_k \mid \mathbf{Z}_{1:k-1})}{p(\mathbf{Z}_k \mid \mathbf{Z}_{1:k-1})}. \tag{16}$$

The particle filter then performs the sequential Monte Carlo estimation to approximate the posterior density with $N$ particles and the associated weights:

$$p(\mathbf{x}_k \mid \mathbf{Z}_{1:k}) \approx \sum_{i=1}^{N} w_{k_i}\delta(\mathbf{x}_k - \mathbf{x}_{k_i}), \tag{17}$$

where $w_{k_i}$ is the weight of the $i$-th particle $\mathbf{x}_{k_i}$ and δ() denotes the Dirac delta function. The $i$-th particle $\mathbf{x}_{k_i}$ is obtained from the importance density $p(\mathbf{x}_k \mid \mathbf{x}_{k-1_i})$. The weights $w_{k_i}$ are updated by

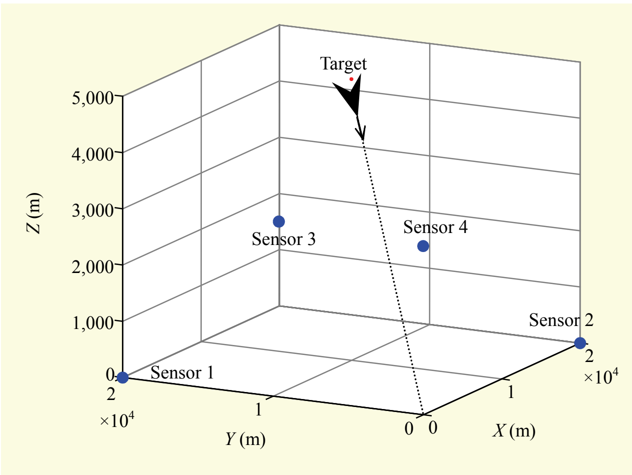


Fig. 1. Geometry for moving-target tracking with sensors.

Initialize the state $\mathbf{x}_0$

Generate $N$ particles $\mathbf{x}_{0_j} \sim \mathcal{N}(\mathbf{x}_0, \sigma^2 \mathbf{I}_0)$

for $j$=1, 2,…, $N$

Set initial weights $w_{0_j} = 1/N$ for $j$=1, 2,…, $N$

For $k$=1, 2,…

$\mathbf{x}_{k_j} = \mathbf{\Phi}\mathbf{x}_{k-1_j} + \mathbf{\Gamma}\mathbf{w}_{k-1_j}$ for $j$=1, 2,…, $N$

Where $\mathbf{w}_{k-1_j} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_w)$

$w_{k_j} = w_{k-1_j} p(\mathbf{Z}_k | \mathbf{x}_{k_j})$

Where $p(\mathbf{Z}_k | \mathbf{x}_{k_j}) \sim \mathcal{N}(\mathbf{H}(\mathbf{x}_{k_j}), \mathbf{Q}_v)$

Normalize weights $\hat{w}_{k_j} = w_{k_j} / \sum_{i=1}^{N} w_{k_i}$

If $N_{\text{eff}}$<$N_{\text{th}}$ where $N_{\text{eff}} = \left( \sum_{j=1}^{N} (\hat{w}_{k_j})^2 \right)^{-1}$

Resampling and $\hat{w}_{k_j} = 1/N$ for $j$=1, …, $N$

End

End

Fig. 2. Particle-filter-based tracking algorithm.

$$w_{k_j} = w_{k-1_j} p(\mathbf{Z}_k | \mathbf{x}_{k_j}), \tag{18}$$

where $p(\mathbf{Z}_k | \mathbf{x}_{k_j})$ is simply

$$p(\mathbf{Z}_k | \mathbf{x}_{k_j}) \sim \mathcal{N}(\mathbf{H}(\mathbf{x}_{k_j}), \mathbf{Q}_v). \tag{19}$$

As shown in section II, if the reference sensor is $s_1$, $\mathbf{H}(\mathbf{x}_{k_j})$ is

$$\mathbf{H}(\mathbf{x}_{k_j}) = [\mathbf{h}_{21}(\mathbf{x}_{k_j}) \mathbf{h}_{31}(\mathbf{x}_{k_j}) \cdots \mathbf{h}_{M1}(\mathbf{x}_{k_j})]^T, \tag{20}$$

where

$$[\mathbf{h}_{ab}(\mathbf{x}_{k_j}) ] = \begin{bmatrix} r_{ab}(\mathbf{x}_{k_j}) \\ \dot{r}_{ab}(\mathbf{x}_{k_j}) \end{bmatrix}. \tag{21}$$

In the approximation of the posterior density function in (17), it is necessary to normalize the weights such that

$$\hat{w}_{k_j} = \frac{w_{k_j}}{\sum_{i=1}^{N} w_{k_j}}. \tag{22}$$

Finally, we can estimate the state of the target at time $t_k$ as

$$\hat{\mathbf{x}}_k = \sum_{j=1}^{N} \hat{w}_{k_i} \mathbf{x}_{k_j}. \tag{23}$$

The proposed tracking algorithm is summarized in Fig. 2.

The Cramér-Rao lower bound (CRLB) is known as the best achievable error bound [5]. The recursive computation of the CRLB for nonlinear filtering follows [5] and is briefly noted below:

The covariance matrix of $\hat{\mathbf{x}}_k$, $\mathbf{P}_k$ has a lower bound expressed as

$$\mathbf{P}_k \triangleq \mathbb{E}\{(\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^T\} \geq \mathbf{J}_k^{-1}, \tag{24}$$

and the information matrix $\mathbf{J}_{k+1}$ is obtained by

$$\mathbf{J}_{k+1} = (\mathbf{Q}_w + \mathbf{F}_k \mathbf{J}_k^{-1} \mathbf{F}_k^{-1})^{-1} + \mathbb{E}\{\tilde{\mathbf{H}}_{k+1}^T \mathbf{Q}_v^{-1} \tilde{\mathbf{H}}_{k+1}\}, \tag{25}$$

where

$$\tilde{\mathbf{H}}_{k+1} = [\nabla_{\mathbf{x}_{k+1}} \mathbf{H}_{k+1}^T (\mathbf{x}_{k+1})]^T \tag{26}$$

is the Jacobian of $\mathbf{H}_{k+1}(\mathbf{x}_{k+1})$ and $\mathbf{F}_k$ is $\mathbf{\Phi}$ in (1). $\mathbf{J}_0$ is the initial covariance matrix of the state estimate.

## IV. Simulation Results

The proposed algorithm was examined through computer simulations. The parameters used in the simulations are shown in Table 1. It is assumed that the target moves in a straight line with the velocity $\mathbf{v}$=[–30, –30, –30]$^T$ (m/s) and the acceleration $\mathbf{a}$=[0.1, 0.1, 0.1]$^T$ starting from the point $A$=[10000, 10000, 5000]$^T$ (m) for 100 s. As shown in Fig. 1, the sensors for the TDOA/FDOA measurements are located at the position $\mathbf{s}_1$=[0, 20000, 0]$^T$, $\mathbf{s}_2$=[20000, 0, 0]$^T$, $\mathbf{s}_3$=[20000, 20000, 1500]$^T$, and $\mathbf{s}_4$=[0, 0, 3000]$^T$. The reference sensor is $\mathbf{s}_1$.

The signal from the moving target is received by the four sensors, which are time-synchronized by GPS. Figure 3 shows the particle-filter-based tracking results. It is shown that tracking becomes more accurate as the number of particles increases.

We also considered a situation where only the TDOA measurements are available. In Fig. 4, the tracking results with only the TDOA and both TDOA/FDOA measurements are compared. In this simulation, 2,000 particles were generated. It is clearly seen that with the additional FDOA measurements, the tracking is more accurate.

We computed the root mean squared error (RMSE) of the

Table 1. Simulation parameters.

| Parameter | Value |
|---|---|
| Number of sensors ($M$) | 4 |
| Number of particles ($N$) | 1,000, 2,000, 3,000 |
| Standard deviation of process noise ($\sigma_w$) | 10 m/s$^2$ |
| Standard deviation of TDOA measurement noise ($\sigma_v$) | 30 m |
| Standard deviation of FDOA measurement noise ($\sigma_{\dot{v}}$) | 10 m/s |
| Time step ($\Delta$) | 1 s |
| Number of runs ($N_r$) | 500 |
| Resampling threshold ($N_{\text{th}}$) | N/10 |

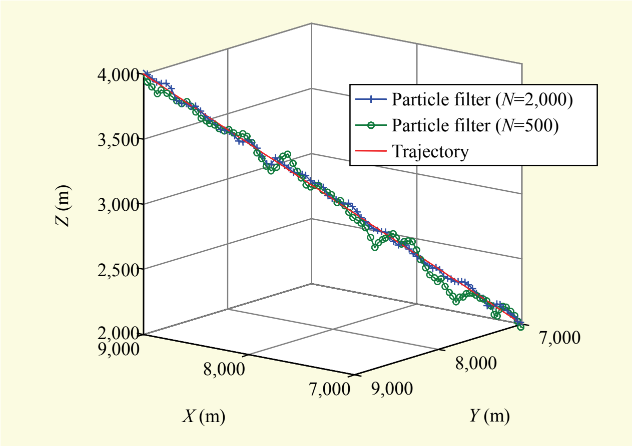

Fig. 3. Target tracking with TDOA/FDOA measurements.

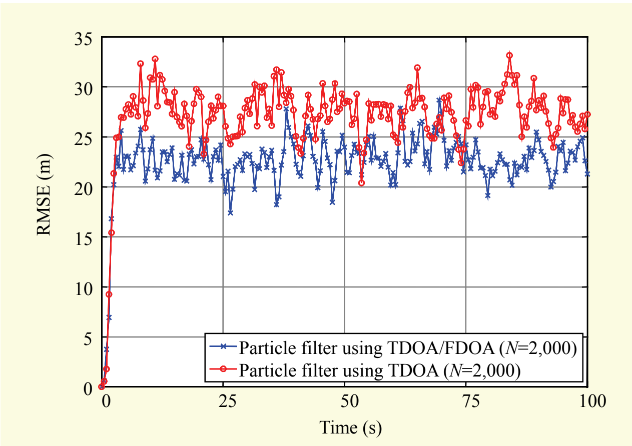

Fig. 4. Particle-filter-based moving-target tracking with TDOA and TDOA/FDOA measurements.

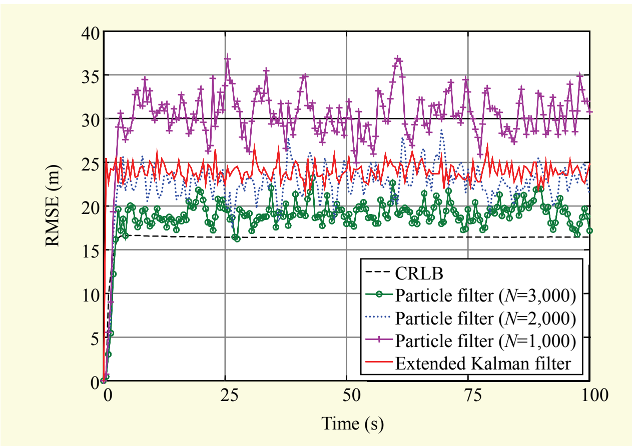

Fig. 5. RMSEs of tracking estimate and CRLB.

tracking results for different numbers of particles and compared them with the CRLB as shown in Fig. 5. As expected, we can observe that increasing the number of particles reduces the RMSE; hence, the target tracking performance approaches the CRLB. Note that when $N$=2,000, the performance of the proposed particle filter is slightly better than that of the EKF. Increasing $N$ further to 3,000 gives the RMSE close to the CRLB, but we expect the additional performance gain will be marginal. Also, when $N$ is reduced to 1,000, the PF performs worse than the EKF. This is due to the improper approximation of the true density. Therefore, we conclude empirically that $N$=3,000 is an appropriate quantity of particles without consuming too much computational power.

The results show that the particle-filter-based tracking outperforms that of the EKF, and the effectiveness of the particle filtering in the moving-target tracking is verified.

## V. Conclusion

In this letter, we proposed a moving-target tracking algorithm based on a particle filter that uses the TDOA/FDOA measurements acquired by the distributed sensors. It was shown that performance of the proposed algorithm, based on the particle filter, outperforms the one based on the EKF. The use of both the TDOA and FDOA measurements was shown to be effective in the moving-target tracking. It was proven that the particle filter deals with the nonlinear nature of the moving-target tracking problem successfully. The proposed algorithm is expected to be widely used in the surveillance applications.

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