Single Relay Selection for Bidirectional Cooperative Networks with Physical-Layer Network Coding

Yingting Liu, Hailin Zhang, Leifang Hui, Quanyang Liu, and Xiaofeng Lu

To serve the growing demand of the bidirectional information exchange, we propose a single relay selection (RS) scheme for physical-layer network coding (PNC) in a bidirectional cooperative network consisting of two sources and multiple relays. This RS scheme selects a single best relay by maximizing the bottleneck of the capacity region of both information flows in the bidirectional network. We show that the proposed RS rule minimizes the outage probability and that it can be used as a performance benchmark for any RS rules with PNC. We derive a closed-form exact expression of the outage probability for the proposed RS rule and show that it achieves full diversity gain. Finally, numerical results demonstrate the validity of our analysis.

Keywords: Bidirectional cooperative networks, physical-layer network coding (PNC), relay selection.

I. Introduction

Bidirectional cooperative networks can achieve higher spectral efficiency than unidirectional cooperative networks, and they have been extensively studied [1]-[3]. In particular, two bidirectional protocols, namely physical-layer network coding (PNC) [1] and time division broadcast (TDBC) [2] have been proposed. It was shown that PNC achieves higher spectral efficiency than TDBC, whereas TDBC achieves higher diversity gain than PNC [3]. This letter will focus on PNC due to its spectral efficiency benefit.

In bidirectional cooperative networks with multiple relays, relay selection (RS) was shown to be an effective technique to achieve diversity without sacrificing spectral efficiency [4], [5]. Specifically, a MinMax RS rule [4] was proposed for a bidirectional cooperative network with PNC. The MinMax rule [4] was developed to minimize the bit error rate of the worse user (between two end-sources), and it did not take into account the outage probability. Thus, the MinMax selection rule may lose outage optimality.

In this letter, we propose a new RS rule by maximizing the bottleneck of the capacity region of both information flows for PNC; hence, the proposed RS scheme minimizes the outage probability. We derive an exact and closed-form expression of the outage probability and analytically show that the proposed RS scheme achieves full diversity.

II. System Model

We consider a bidirectional cooperative network with two sources and K relays, where each terminal operates in a half duplex mode. We use f_k to denote the reciprocal channel between source 1 and relay k, and g_k to denote the channel between source 2 and relay k, for $k = 1, 2,\ldots, K$. Furthermore, we assume that f_k and g_k are independent and complex Gaussian random variables with zero mean and variances Ω_k and Φ_k, respectively. The additive noise associated with each channel is modeled as a complex Gaussian random variable with zero mean and unit variance.

The information exchange in the bidirectional cooperative network with single RS for PNC is accomplished in two time slots. In the first time slot, source 1 and source 2 transmit their

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Yingting Liu (phone: +86 29 88201357-8510, ytliu@xidian.edu.cn), Hailin Zhang (hailinzhang@mail.xidian.edu.cn), Leifang Hui (xileifang@gmail.com), and Xiaofeng Lu (luxf@xidian.edu.cn) are with the State Key Laboratory of Integrated Services Networks, Xidian University, Shaanxi, China.

Quanyang Liu (happylqy@163.com) is with Gansu Design & Research Institute of Environmental Science, Lanzhou, China.

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own messages simultaneously with transmission powers E_1 and E_2, respectively. In the second time slot, a single "best" relay selected beforehand tries to decode both symbols and then broadcasts an exclusive-ORed (XORed) version with power E_r to both sources. Each source then decodes the XORed message and obtains its desired message via self-interference cancelation. It is evident that the RS rule has a direct impact on the outage performance. We address this issue in the next section.

III. Relay Selection and Outage Analysis

1. Relay Selection

To derive a proper RS rule, we first investigate the capacity region of the PNC protocol employing a single relay. Assume that only the k-th relay is allowed to participate in the second time slot of the information exchange, $1 \le k \le K$. The achievable rate region of the PNC protocol with the k-th relay is the closure of the convex hull of the set of points (R_1, R_2) satisfying the following inequalities [2]:

$$R_1 \le I_{1,k}, \quad (1)$$

$$R_2 \le I_{2,k}, \quad (2)$$

$$R_1 + R_2 \le I_{\text{sum},k}, \quad (3)$$

where

$$I_{1,k} = 1/2 \min\{\log_2(1+E_1X_{1,k}), \log_2(1+E_rX_{2,k})\}, \quad (4)$$

$$I_{2,k} = 1/2 \min\{\log_2(1+E_2X_{2,k}), \log_2(1+E_rX_{1,k})\}, \quad (5)$$

$$I_{\text{sum},k} = 1/2 \log_2(1+E_1X_{1,k}+E_2X_{2,k}). \quad (6)$$

In (4) through (6), $X_{1,k} = |f_k|^2$ and $X_{2,k} = |g_k|^2$. So, it is easy to see that $X_{1,k}$ and $X_{2,k}$ are exponentially distributed with parameters $\lambda_{1,k} = 1/\Omega_k$ and $\lambda_{2,k} = 1/\Phi_k$, respectively.

A wise RS rule must intelligently take into account the three mutual information expressions (4) through (6) simultaneously. To this end, we first evaluate the outage probability $P_{\text{out},k}$ for the k-th relay-branch. We let the target rate for each information flow be $R/2$ assuming that the total target rate of the whole network is R. Therefore, the outage probability for the k-th relay-branch is given by

$$\begin{aligned} P_{\text{out},k} &= \Pr(I_{1,k} < R/2 \ or\ I_{2,k} < R/2 \ or\ I_{\text{sum},k} < R) \\ &= \Pr(\min(I_{1,k}, I_{2,k}, I_{\text{sum},k}/2) < R/2). \end{aligned} \quad (7)$$

Note that the bidirectional cooperative network with a single best relay is in outage if and only if all relay-branches are in outage. Therefore, the outage probability of the PNC protocol with single RS is given by

$$\begin{aligned} P_{\text{out}} &= \Pr\left(\bigcap_{k=1}^{K}\left[\min(I_{1,k}, I_{2,k}, I_{\text{sum},k}/2) < R/2\right]\right) \\ &= \Pr\left(\max_{k=1,2,\cdots,K} \min(I_{1,k}, I_{2,k}, I_{\text{sum},k}/2) < R/2\right). \end{aligned} \quad (8)$$

From (8), therefore, we propose a single RS rule as

$$i = \arg\max_{k=1,2,\cdots,K} \min(I_{1,k}, I_{2,k}, I_{\text{sum},k}/2), \quad (9)$$

where i is the index of the selected relay. From (8) and (9), we can see that the single RS rule of (9) maximizes the minimum of the three mutual information expressions $I_{1,k}$, $I_{2,k}$, and $I_{\text{sum},k}/2$ over all relays. That is, a single best relay is selected over all relays such that the bottleneck of the capacity region of the PNC protocol is maximized. As a result, this RS rule also minimizes the outage probability. Thus, the single RS rule of (9) can be considered as a benchmark scheme for any suboptimum RS rules. In particular, the outage probability (to be derived) of the proposed RS rule can be used to evaluate the effectiveness of any suboptimum RS rules.

In comparison, we briefly describe the suboptimum MinMax selection rule of [4]. In [4], the single best relay was determined via $j = \arg\max_{k=1,2,\ldots,K} \min(E_1X_{1,k}, E_2X_{2,k})$. Therefore, it can be shown that the achievable rate corresponding to relay j is less than that associated with relay i, that is, $\min(I_{1,j}, I_{2,j}, I_{\text{sum},j}/2) < \min(I_{1,i}, I_{2,i}, I_{\text{sum},i}/2)$. Thus, the outage probability performance of the MinMax selection rule of [4] is strictly worse than that of the proposed RS scheme of (9). In section IV, we will further compare the MinMax selection rule with our proposed RS rule.

In practice, the proposed RS rule can be implemented either in a centralized manner requiring global CSI of the network or a distributed manner requiring only local CSI. For details regarding these implementation issues, see [6].

2. Outage Analysis

We now study the outage probability and diversity performance of the proposed RS rule.

Theorem 1. The outage probability of the single RS rule of (9) is given by

$$P_{\text{out}} = \prod_{k=1}^{K} P_{\text{out},k}, \quad (10)$$

where $P_{\text{out},k}$ is given in (11).

In (11), $T_1 = 2^R - 1$; $T_2 = 2^{2R} - 1$; [1)] case 1 represents that $\max(E_1, E_2) \le E_r$ and $\lambda_{1,k}E_2 \ne \lambda_{2,k}E_1$; case 2: $\max(E_1, E_2) \le E_r$ and $\lambda_{1,k}E_2 = \lambda_{2,k}E_1$; case 3: $E_1 \le E_r \le E_2$, $\frac{T_2}{T_1} \ge 1 + \frac{E_2}{E_r}$, and

1) Note that for any rate $R>0$, we have $T_2>2T_1$. This can be easily shown by $T_2 - 2T_1 = 2^{2R} - 1 - 2(2^R - 1) = (2^R - 1)^2 \ge 0$.

$\lambda_{1,k}E_2 \neq \lambda_{2,k}E_1$; case 4: $E_1 \le E_r \le E_2$, $\frac{T_2}{T_1} \ge 1+\frac{E_2}{E_r}$, and $\lambda_{1,k}E_2 = \lambda_{2,k}E_1$; case 5: $E_1 \le E_r \le E_2$ and $\frac{T_2}{T_1} < 1+\frac{E_2}{E_r}$; case 6: $E_2 \le E_r \le E_1$, $\frac{T_2}{T_1} \ge 1+\frac{E_1}{E_r}$, and $\lambda_{1,k}E_2 \neq \lambda_{2,k}E_1$; case 7: $E_2 \le E_r \le E_1$, $\frac{T_2}{T_1} \ge 1+\frac{E_1}{E_r}$, and $\lambda_{1,k}E_2 = \lambda_{2,k}E_1$; case 8: $E_2 \le E_r \le E_1$ and $\frac{T_2}{T_1} < 1+\frac{E_1}{E_r}$; case 9: $\min(E_1,E_2) \ge E_r$, $\frac{T_2}{T_1} \ge \frac{E_1+E_2}{E_r}$, and $\lambda_{1,k}E_2 \neq \lambda_{2,k}E_1$; case 10: $\min(E_1,E_2) \ge E_r$, $\frac{T_2}{T_1} \ge \frac{E_1+E_2}{E_r}$, and $\lambda_{1,k}E_2 = \lambda_{2,k}E_1$; case 11: $\min(E_1,E_2) \ge E_r$ and $\frac{T_2}{T_1} < \frac{E_1+E_2}{E_r}$.

Proof. See Appendix A.

$$P_{\text{out},k} =$$
$$\begin{cases}
1-\frac{\lambda_{2,k}E_1}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{1,k}}{E_1}-\frac{\lambda_{2,k}}{E_2}\right)T_1-\frac{\lambda_{1,k}T_2}{E_1}}+\frac{\lambda_{1,k}E_2}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{2,k}}{E_2}-\frac{\lambda_{1,k}}{E_1}\right)T_1-\frac{\lambda_{2,k}T_2}{E_2}}, & \text{case 1} \\
1-\left(1+\frac{\lambda_{1,k}T_2}{E_1}-\frac{2\lambda_{1,k}T_1}{E_1}\right)e^{-\frac{\lambda_{1,k}T_2}{E_1}}, & \text{case 2} \\
1-\frac{\lambda_{2,k}E_1}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{1,k}}{E_1}\frac{E_2}{Er}-\frac{\lambda_{2,k}}{E_r}\right)T_1-\frac{\lambda_{1,k}T_2}{E_1}}+\frac{\lambda_{1,k}E_2}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{2,k}}{E_2}-\frac{\lambda_{1,k}}{E_1}\right)T_1-\frac{\lambda_{2,k}T_2}{E2}}, & \text{case 3} \\
1-\left(1+\frac{\lambda_{1,k}T_2}{E_1}-\left(\frac{\lambda_{1,k}}{E_1}+\frac{\lambda_{2,k}}{E_r}\right)T_1\right)e^{-\frac{\lambda_{1,k}T_2}{E_1}}, & \text{case 4} \\
1-e^{-\left(\frac{\lambda_{1,k}}{E_1}+\frac{\lambda_{2,k}}{E_r}\right)T_1}, & \text{case 5} \\
1-\frac{\lambda_{2,k}E_1}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{1,k}}{E_1}-\frac{\lambda_{2,k}}{E_2}\right)T_1-\frac{\lambda_{1,k}T_2}{E_1}}+\frac{\lambda_{1,k}E_2}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{2,k}}{E2}\frac{E_1}{E_r}-\frac{\lambda_{1,k}}{E_r}\right)T_1-\frac{\lambda_{2,k}T_2}{E_2}}, & \text{case 6} \\
1-\left(1+\frac{\lambda_{1,k}T_2}{E_1}-\left(\frac{\lambda_{1,k}}{E_1}+\frac{\lambda_{1,k}}{E_r}\right)T_1\right)e^{-\frac{\lambda_{1,k}T_2}{E_1}}, & \text{case 7} \\
1-e^{-\left(\frac{\lambda_{1,k}}{E_r}+\frac{\lambda_{2,k}}{E_2}\right)T_1}, & \text{case 8} \\
1-\frac{\lambda_{2,k}E_1}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{1,k}}{E_1}\frac{E_2}{Er}-\frac{\lambda_{2,k}}{E_r}\right)T_1-\frac{\lambda_{1,k}T_2}{E_1}}+\frac{\lambda_{1,k}E_2}{\lambda_{2,k}E_1-\lambda_{1,k}E_2}e^{\left(\frac{\lambda_{2,k}}{E_2}\frac{E_1}{E_r}-\frac{\lambda_{1,k}}{E_r}\right)T_1-\frac{\lambda_{2,k}T_2}{E2}}, & \text{case 9} \\
1-\left(1+\frac{\lambda_{1,k}T_2}{E_1}-\frac{(\lambda_{1,k}+\lambda_{2,k})T_1}{E_r}\right)e^{-\frac{\lambda_{1,k}T_2}{E_1}}, & \text{case 10} \\
1-e^{-\frac{(\lambda_{1,k}+\lambda_{2,k})T_1}{E_r}}. & \text{case 11}
\end{cases} \quad (11)$$

Note that the outage probability of (10) along with (11) is an exact and truly closed-form expression, although it is rather tedious. Moreover, the obtained expression captures the outage probability of the RS with PNC for arbitrary power allocation across the sources and the selected relay. Furthermore, setting K=1 and E_1=E_2, one can verify that our expression of (10) reduces to the special case of PNC with a single fixed relay in (16) of [4]. Therefore, our result is a generalization of that of [3]. Finally, to provide more insight into the proposed RS rule for PNC, we now study the diversity performance.

Corollary 1. The single RS rule of (9) with PNC achieves full diversity order of K.

Proof. See Appendix B.

IV. Numerical Results

In this section, we perform Monte Carlo simulations to verify the analytical results. Figure 1 shows the outage probabilities for the RS with K=1, 2, 3, 4 relays. The outage probabilities are plotted against the total transmission power E of the whole network, where E=E_1+E_2+E_r. The target rate R is set to 1 bps/Hz. The channel statistics are chosen as $\{\Omega_k\}_{k=1}^{4} = \{4.2, 5.5, 0.4, 0.5\}$ and $\{\Phi_k\}_{k=1}^{4} = \{1, 3.5, 15, 2.5\}$ to model a general asymmetric network. The power is non-uniformly allocated as E_1=0.35E, E_2=0.4E, and E_r=0.25E. From Fig. 1, we observe that the diversity gain increases with K and the simulation outage probabilities perfectly match with our analysis. Moreover, our simulation result indicates that the MinMax selection rule of [4] performs almost as good as our proposed RS rule although the MinMax selection rule experiences certain loss of outage optimality. Thus, our derived outage expression of (10) provides a good approximation to the outage probability of MinMax selection rule of [4]. Finally, it is worth noting that our proposed RS rule minimizes the outage probability, and thus it can be used as a performance benchmark for any RS schemes with PNC.

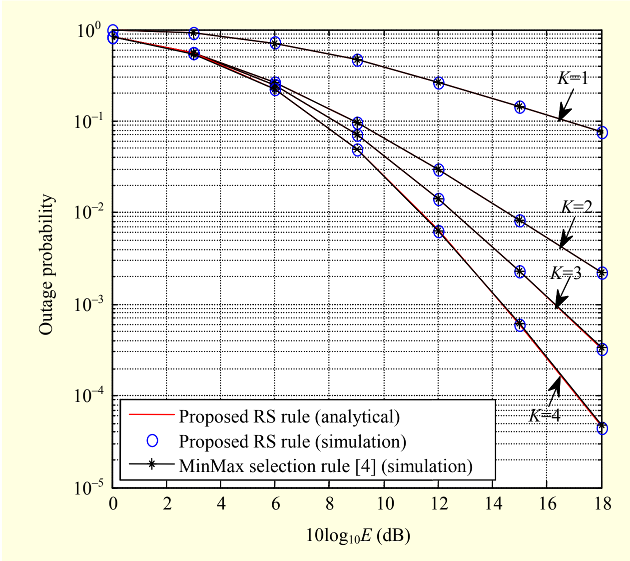

Fig. 1. Outage probabilities of RS rules with PNC.

V. Conclusion

We studied RS with PNC in a bidirectional cooperative network with multiple relays. Specifically, we proposed an RS scheme for PNC which selected a single best relay to maximize the bottleneck of the capacity region of both information flows in the bidirectional network. This RS scheme also minimized the outage probability. We derived an exact and closed-form expression for the outage probability of the proposed RS scheme. Finally, we analytically showed that our proposed RS scheme achieved full diversity.

Appendix A

From (7) and (8), it is not hard to rewrite the outage probability of the RS scheme as (10), where

$$P_{\text{out},k} = \Pr(\min(E_1,E_r)X_{1,k} < T_1 \text{ or } \min(E_2,E_r)X_{2,k} < T_1 \text{ or } E_1X_{1,k}+E_2X_{2,k} < T_2). \quad \text{(A.1)}$$

We now show that $P_{\text{out},k}$ can be finalized into (11). Since the expression of (11) is divided into 11 cases that are collectively exhaustive and mutually exclusive, we need to consider all the cases individually. Let us first consider case 1: $\max(E_1,E_2) \le E_r$. In this case, we can write

$$\begin{aligned} P_{\text{out},k} &= \Pr(E_1X_{1,k} < T_1 \text{ or } E_2X_{2,k} < T_1 \text{ or } E_1X_{1,k}+E_2X_{2,k} < T_2) \\ &= \int_{x_1=0}^{\infty}\int_{x_2=0}^{\frac{T_1}{E_2}} f_{X_{1,k}}(x_1) f_{X_{2,k}}(x_2)\,dx_1dx_2 \\ &+ \int_{x_1=0}^{\frac{T_1}{E1}}\int_{x_2=\frac{T_2}{E_2}}^{\infty} f_{X_{1,k}}(x_1) f_{X_{2,k}}(x_2)\,dx_1dx_2 \\ &+ \int_{x_1=\frac{T_1}{E1}}^{\frac{T_2-T_1}{E1}}\int_{x_2=\frac{T_1}{E_2}}^{\frac{T_1-E_1X_1}{E_2}} f_{X_{1,k}}(x_1) f_{X_{2,k}}(x_2)\,dx_1dx_2, \end{aligned} \quad \text{(A.2)}$$

where $f_{X_{1,k}}(x_1) = \lambda_{1,k}e^{-\lambda_{1,k}x_1}$ and $f_{X_{2,k}}(x_2) = \lambda_{2,k}e^{-\lambda_{2,k}x_2}$ represent the probability density functions of $X_{1,k}$ and $X_{2,k}$, respectively. Since the integrands in (A.2) only involve elementary exponential functions, we can solve these integrals using standard mathematical manipulations, which eventually yield the expression of case 1 in (11). Note that we omit the technical details on solving these integrals due to a space limitation. For cases 2 through 11, we can take a similar approach as in (A.2). Then, using similar manipulations, we can show that $P_{\text{out},k}$ is given by (11). This completes the proof.

Appendix B

In this proof, we first derive lower and upper bounds on P_{out}. Let $E_1 = \alpha_1 E$, $E_2 = \alpha_2 E$, and $E_r = \alpha_3 E$, where $\alpha_i > 0$, $\alpha_1+\alpha_2+\alpha_3 = 1$, and E denotes the total transmission power of the whole network. From (A.2), a lower bound $P_{\text{out},k}^{\text{lower}}(E)$ and an upper bound $P_{\text{out},k}^{\text{upper}}(E)$ for $P_{\text{out},k}$ can be written as

$$\begin{aligned} P_{\text{out},k}^{\text{lower}}(E) &= \Pr(\min(E_1,E_r)X_{1,k} < T_1 \text{ or } \min(E_2,E_r)X_{2,k} < T_1) \\ &= 1 - e^{-\left(\frac{\lambda_{1,k}}{\min(\alpha_1,\alpha_3)}+\frac{\lambda_{2,k}}{\min(\alpha_2,\alpha_3)}\right)\frac{T_1}{E}}, \end{aligned} \quad \text{(B.1)}$$

$$\begin{aligned} P_{\text{out},k}^{\text{upper}}(E) &= \Pr(\min(E_1,E_r)X_{1,k} < T_2 \text{ or } \min(E_2,E_r)X_{2,k} < T_2 \text{ or } E_1X_{1,k}+E_2X_{2,k} < T_2) \\ &= 1 - e^{-\left(\frac{\lambda_{1,k}}{\min(\alpha_1,\alpha_3)}+\frac{\lambda_{2,k}}{\min(\alpha_2,\alpha_3)}\right)\frac{T_2}{E}}. \end{aligned} \quad \text{(B.2)}$$

Le $P_l(E)$ be defined as

$$P_l(E) = \prod_{k=1}^{K}\left(1-e^{-\frac{\beta_{k,l}}{E}}\right), \quad \text{(B.3)}$$

where $\beta_{k,l} = \left(\frac{\lambda_{1,k}}{\min(\alpha_1,\alpha_3)}+\frac{\lambda_{2,k}}{\min(\alpha_2,\alpha_3)}\right)T_l$, l=1, 2. It is evident that $P_1(E)$ provides a lower bound on P_{out}, and $P_2(E)$ provides an upper bound on P_{out}. We now show that both bounds yield the same diversity order of K. The diversity order d_l for both bounds can be calculated as

$$\begin{aligned} d_l &= \lim_{E\to\infty}\frac{\ln P_l(E)}{\ln 1/E} \\ &= \sum_{K=1}^{K}\lim_{E\to\infty}\frac{\beta_{k,l}/E}{1-e^{-\beta_{k,l}/E}} = \sum_{K=1}^{K}1 = K, \end{aligned} \quad \text{(B.4)}$$

where we have used L'Hôpital's rule to compute the limit. Therefore, both upper and lower bounds achieve diversity order of K, which implies that P_{out} achieves full diversity.

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