

# Noncooperative Multimode Precoding with Limited Feedback in MIMO Interference Channels

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*This letter proposes an iterative multimode precoding scheme with limited feedback for nonreciprocal MIMO interference channels. Based on analysis of game theory, we model the iterative multimode precoding as a noncooperative game with a finite set of strategies. Numerical results are presented to verify the sum rate performance of the proposed scheme.*

*Keywords: Precoding, interference channel, limited feedback.*

## I. Introduction

In a  $K$ -user  $N \times N$  MIMO interference channel, each user achieves  $N/2$  degrees of freedom through the concept of interference alignment [1], which utilizes the reciprocity of wireless channels. In nonreciprocal MIMO interference channels, [1] needs huge feedback overhead since each receiver should inform each transmitter how much interference it induces. When the bandwidth of the feedback channel is limited, we can employ the concept of limited feedback. We may also allow each user to determine the number of transmitted data streams adaptively according to the given channel condition. In a single-user MIMO system, this concept has been referred to as multimode precoding [2].

In this work, we consider a nonreciprocal MIMO interference channel where multiple transmitter-receiver pairs exist and propose an iterative multimode precoding scheme where each pair iteratively updates the number of transmitted data streams and the corresponding precoder without cooperation. It is also found that the iterative multimode

precoding constructs a noncooperative game [3].

The previous works on noncooperative games assume an infinite set of continuous strategies and prove the existence and uniqueness of the Nash equilibrium (NE) in this type of game [4]. The noncooperative multimode precoding game with finite strategies has been studied in a multiuser uplink MIMO multiple access channel [5] in which multiple transmitters (that is, mobile stations) access a common receiver (that is, a base station). However, [5] operates under a zero interference condition in which the sum of the number of data streams for all transmitters is less than or equal to the number of receive antennas at the receiver. Therefore, the change of one of the rates of the users is equal to the change of the total sum of the rates, which implies a potential game with pure strategy NEs [6]. Note that the multimode precoding in MIMO interference channels never leads to a potential game and may have no pure strategy NE [3]. Here, we suggest a modified equilibrium that allows each player to have a suitable response to the other players' strategies.

## II. System Model

Let us consider a  $K$ -user MIMO interference channel with  $K$  transmitter-receiver pairs (see Fig. 1). Each node is equipped with  $N$  antennas. We define  $\mathbf{H}^{[kl]}$  as an  $N \times N$  MIMO channel matrix between the  $l$ -th transmitter and the  $k$ -th receiver. The  $N \times 1$  received signal vector at the  $k$ -th user is given as

$$\mathbf{y}^{[k]} = \mathbf{H}^{[kk]} \mathbf{V}^{[k]} \mathbf{s}^{[k]} + \sum_{l \neq k} \mathbf{H}^{[kl]} \mathbf{V}^{[l]} \mathbf{s}^{[l]} + \mathbf{w}^{[k]}, \quad (1)$$

where  $\mathbf{V}^{[k]}$  denotes the  $N \times M^{[k]}$  precoding matrix of the  $k$ -th transmitter,  $\mathbf{s}^{[k]}$  is the  $M^{[k]} \times 1$  transmit symbol vector with

$$E \left[ \mathbf{s}^{[k]} \left( \mathbf{s}^{[k]} \right)^\dagger \right] = \frac{P}{M^{[k]}} \mathbf{I}_{M^{[k]}}, \quad \text{and } \mathbf{w}^{[k]} \text{ is the additive white}$$

Manuscript received Mar. 1, 2012; revised Mar. 26, 2012; accepted Apr. 12, 2012.

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011-0005638).

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<http://dx.doi.org/10.4218/etrij.12.0212.0100>

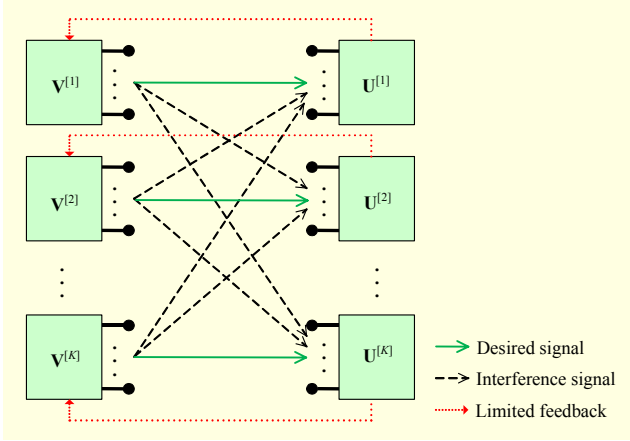


Fig. 1. MIMO interference channel with limited feedback.

Gaussian noise vector with the covariance matrix of  $\mathbf{I}_N$ . Here,  $P$  is the total transmitted power at each transmitter and  $\mathbf{I}_n$  denotes an  $n \times n$  identity matrix. Note that  $M^{[k]}$  is the number of transmitted data streams at the  $k$ -th transmitter.

### III. Iterative Multimode Precoding

#### 1. Multimode Precoding with Limited Feedback

In the multimode precoding system, the number of transmitted data streams is referred to as a *mode* [3]. Let  $\chi$  be the set of available modes in the system. When  $M \in \chi$ , we define a finite set of  $N \times M$  unitary precoder matrices (that is, a codebook) for the mode  $M$  given as

$$\Psi(M) = \{\mathbf{V}(M, 1), \mathbf{V}(M, 2), \dots, \mathbf{V}(M, t_M)\}. \quad (2)$$

Here,  $t_M$  denotes the cardinality of  $\Psi(M)$  and  $M$  varies between 1 and  $N$ . We assume that each transmitter-receiver pair chooses the best mode and precoder matrix among  $\sum_{M \in \chi} t_M$  candidates to maximize its own information rate. To identify each candidate and inform the transmitter of the selected candidate with limited feedback, each pair requires  $\log_2(\sum_{M \in \chi} t_M)$  bits.

Let us assume that the  $k$ -th user chooses  $\tau^{[k]} = (M^{[k]}, t^{[k]})$ , which indicates the  $t^{[k]}$ -th precoder in  $\Psi(M^{[k]})$ . Then,  $\mathbf{V}^{[k]} = \mathbf{V}(t^{[k]})$  and the SINR of the  $d$ -th data stream for the  $k$ -th user is computed as

$$\rho_d^{[k]}(\tau^{[k]}, \tau^{[-k]}) = \frac{\left(\mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]})\right)^\dagger \mathbf{\Gamma}_d^{[kk]}(\tau^{[k]}) \mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]}) P}{\Phi_d^{[k]}(\tau^{[k]}, \tau^{[-k]}) M^{[k]}}, \quad (3)$$

where

$$\mathbf{\Gamma}_d^{[kk]}(\tau^{[k]}) = \mathbf{H}^{[kk]} \mathbf{V}_{*d}(\tau^{[k]}) \left(\mathbf{H}^{[kk]} \mathbf{V}_{*d}(\tau^{[k]})\right)^\dagger, \quad (4)$$

$$\Phi_d^{[k]}(\tau^{[k]}, \tau^{[-k]}) = \left(\mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]})\right)^\dagger \mathbf{B}_d^{[k]}(\tau^{[k]}, \tau^{[-k]}) \mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]}). \quad (5)$$

Moreover,  $\mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]})$  and  $\mathbf{V}_{*d}(\tau^{[k]})$  denote the  $d$ -th columns of  $\mathbf{U}^{[k]}(\tau^{[k]}, \tau^{[-k]})$  and  $\mathbf{V}(\tau^{[k]})$ , respectively. Here,  $\mathbf{U}^{[k]}(\tau^{[k]}, \tau^{[-k]})$  is defined as the receive filter of the  $k$ -th user and  $\tau^{[-k]}$  stacks the modes and precoders of all the other users. Furthermore, the covariance matrix of interference and noise,  $\mathbf{B}_d^{[k]}(\tau^{[k]}, \tau^{[-k]})$ , can be computed as

$$\mathbf{B}_d^{[k]}(\tau^{[k]}, \tau^{[-k]}) = \sum_{l \neq k} \frac{P}{M^{[l]}} \sum_{e=1}^{M^{[l]}} \mathbf{\Gamma}_e^{[ll]}(\tau^{[l]}) + \sum_{e=1, e \neq d}^{M^{[k]}} \frac{P}{M^{[k]}} \mathbf{\Gamma}_e^{[kk]}(\tau^{[k]}) + \mathbf{I}_N. \quad (6)$$

Using (6), we obtain the receive filter for  $\tau^{[k]}$ , shown as

$$\mathbf{U}_{*d}^{[k]}(\tau^{[k]}, \tau^{[-k]}) = \frac{\mathbf{B}_d^{[k]}(\tau^{[k]}, \tau^{[-k]})^{-1} \mathbf{H}^{[kk]} \mathbf{V}_{*d}(\tau^{[k]})}{\|\mathbf{B}_d^{[k]}(\tau^{[k]}, \tau^{[-k]})^{-1} \mathbf{H}^{[kk]} \mathbf{V}_{*d}(\tau^{[k]})\|}. \quad (7)$$

The global optimum to maximize the sum of the information rates of all users can be given as

$$(\bar{\tau}^{[1]}, \bar{\tau}^{[2]}, \dots, \bar{\tau}^{[K]}) = \operatorname{argmax}_{\tau^{[1]}, \tau^{[2]}, \dots, \tau^{[K]}} \sum_{k=1}^K \sum_{d=1}^{M^{[k]}} \log_2(1 + \rho_d^{[k]}(\tau^{[k]}, \tau^{[-k]})). \quad (8)$$

To obtain the global optimum in (8), full cooperation between all users should be guaranteed since each transmitter-receiver pair has to acquire channel information between all nodes. In the next subsection, we propose a noncooperative iterative multimode precoding scheme.

#### 2. Iterative Update

At each iteration, we assume that each user updates its mode and precoder simultaneously. When the  $k$ -th user considers  $\tau^{[k]}$  as its updated mode and precoder at the  $i$ -th iteration, we can compute the SINR of the  $d$ -th data stream  $\rho_d^{[k]}(\tau^{[k]}, \tau^{[-k](i-1)})$  by replacing  $\tau^{[-k]}$  in (3) with  $\tau^{[-k](i-1)}$ . Here,  $\tau^{[-k](i-1)}$  stacks the modes and precoders of all the other users at the  $(i-1)$ th iteration. Then, the  $k$ -th user updates its mode and precoder to maximize its own information rate shown as

$$\tau^{[k](i)} = \operatorname{argmax}_{\tau^{[k]}} \sum_{d=1}^{M^{[k]}} \log_2(1 + \rho_d^{[k]}(\tau^{[k]}, \tau^{[-k](i-1)})). \quad (9)$$

After  $i$  reaches a predetermined limit,  $I$ , we have  $\tau^{[k](I)} = (M^{[k](I)}, t^{[k](I)})$  for all users. Then,  $\rho_d^{[k]}(\tau^{[k](I)}, \tau^{[-k](I)})$  can be obtained by replacing  $\tau^{[k]}$  and  $\tau^{[-k]}$  in (3) by  $\tau^{[k](I)}$  and  $\tau^{[-k](I)}$ , respectively. The final information rate for the  $k$ -th user after  $I$  iterations can be expressed as

$$R^{[k]}(\tau^{[k](I)}, \tau^{[-k](I)}) = \sum_{d=1}^{M^{[k](I)}} \log_2(1 + \rho_d^{[k]}(\tau^{[k](I)}, \tau^{[-k](I)})). \quad (10)$$

At the beginning of the iteration, we assume that all users randomly initialize their modes and precoders.

#### IV. Analysis of Game Theory

As mentioned in subsection III.1, each user can select one of the  $\sum_{M \in \mathcal{X}} \ell_M$  candidates as  $\tau^{[k]}$ . Here, let  $\tau^{[k]}$  be a strategy of the  $k$ -th user and  $R^{[k]}(\tau^{[k]}, \tau^{[-k]})$  be a payoff of the  $k$ -th user for the strategy  $\tau^{[k]}$ , where  $\tau^{[-k]}$  contains the other user's strategies. Then, the multimode precoding process can simply become a *noncooperative game* [3]. Thus, we refer to it as a *multimode precoding game*. In the game, an NE point is defined as a point where players do not need to change their current strategies because each player's strategy in the NE is the best response to the other players' strategies [3]. For all  $k$  and  $\tau^{[k]}$ , the NE in the multimode precoding game,  $\{\hat{\tau}^{[k]}; k=1, 2, \dots, K\}$ , should satisfy

$$R^{[k]}(\hat{\tau}^{[k]}, \hat{\tau}^{[-k]}) \geq R^{[k]}(\tau^{[k]}, \hat{\tau}^{[-k]}). \quad (11)$$

Let us assume that the iterative update in subsection III.2 converges after  $i$  iterations. Then, we have  $\tau^{[k(i)]} = \tau^{[k](i-1)}$  for all  $k$ . As in (9), the updated strategy,  $\tau^{[k(i)]}$ , provides the maximum information rate under the given other users' strategies,  $\tau^{[-k(i-1)]}$ , and this implies

$$R^{[k]}(\tau^{[k(i)]}, \tau^{[-k(i-1)]}) \geq R^{[k]}(\tau^{[k]}, \tau^{[-k(i-1)]}) \quad (12)$$

for all  $k$ . Due to the convergence assumption, we can replace  $\tau^{[-k(i-1)]}$  with  $\tau^{[-k(i)]}$ , which yields

$$R^{[k]}(\tau^{[k(i)]}, \tau^{[-k(i)]}) \geq R^{[k]}(\tau^{[k]}, \tau^{[-k(i)]}). \quad (13)$$

Comparing (11) and (13), we observe that the converged strategies satisfy the requirement of an NE. Therefore, the iterative update process can converge as long as an NE exists. However, it is not guaranteed that the iterative process always leads to an NE.

Even though an NE exists, the iterative process may not reach the NE due to the initial condition. Thus, we need to perform the iterative update with new initialization. Here, we refer to the iteration process presented in subsection III.2 as an *inner iteration* and employ an *outer iteration* to reset the initial mode and precoder for all users. We begin the inner iteration again when it cannot converge within the predetermined limit of iterations.

It is known that a noncooperative game with finite strategies can have several NEs or even no NE [3]. When there is no NE, the iterative update never converges. Thus, we suggest a concept of modified equilibrium, which is intended for the convergence of the iterative update in subsection III.2, even

when no NE is available.

In (11), the NE is defined as a group of the best response strategies of all the players in the game. That the response is "best" implies that each player considers only one strategy to provide the maximum payoff. Therefore, we lift the restriction of choosing only the best response, allowing each player to select an *adequate* strategy to provide the payoff beyond the given threshold. Here, a *modified equilibrium* is defined as a group of adequate responses from all players in the game. In the multimode precoding game, the modified equilibrium can be expressed as  $\{\hat{\tau}^{[k]}; k=1, 2, \dots, K\}$  to satisfy

$$R^{[k]}(\hat{\tau}^{[k]}, \hat{\tau}^{[-k]}) \geq R_{\text{Th}}^{[k]} \quad (14)$$

for all  $k$ , where  $R_{\text{Th}}^{[k]}$  denotes the predetermined rate threshold of the  $k$ -th user. We utilize the modified equilibrium in (14) for the iterative process in III.2. Before updating the mode and precoder, the  $k$ -th user computes  $R^{[k]}(\tau^{[k(i-1)]}, \tau^{[-k(i-1)]})$  at the  $i$ -th iteration. If  $R^{[k]}(\tau^{[k(i-1)]}, \tau^{[-k(i-1)]}) \geq R_{\text{Th}}^{[k]}$ , the  $k$ -th user keeps its current strategy such that  $\tau^{[k(i)]} = \tau^{[k(i-1)]}$ . Otherwise, it updates its mode and precoder to maximize the information rate as in (9).

The rate threshold can be considered an achievable lower bound because each user selects one of the strategies providing a rate beyond the threshold. If the threshold is set to a small value, the achievable rate becomes low. However, the large value of the threshold prevents some of the users with poor channel conditions from achieving the rate above the given threshold, in which case no modified equilibrium exists and the iterative process fails to converge. Therefore, we include a threshold adaptation scheme to update the threshold adaptively according to the channel condition. If the  $k$ -th user cannot achieve the rate above the threshold with  $\tau^{[k(i)]}$ , it reduces its own rate threshold, shown as

#### Algorithm 1. IMPC

Set the initial rate threshold.

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repeat                                     ◀ outer iteration
  initialize  $\tau^{[k(0)}$  randomly for all  $k$ .
  repeat                                     ◀ inner iteration
    at  $i$ -th iteration, compute  $R^{[k]}(\tau^{[k(i-1)]}, \tau^{[-k(i-1)]})$ .
    if  $R^{[k]}(\tau^{[k(i-1)]}, \tau^{[-k(i-1)]}) > R_{\text{Th}}^{[k]}$ ,
      set  $\tau^{[k(i)]} = \tau^{[k(i-1)]}$  and perform (16).
    else
      update  $\tau^{[k(i)]}$  as in (9).
      if  $R^{[k]}(\tau^{[k(i)]}, \tau^{[-k(i-1)]}) > R_{\text{Th}}^{[k]}$ , perform (16).
      else perform (15).
    end
  end
until convergence or  $i$  reaches  $I$ .         ◀ inner iteration
until convergence.                           ◀ outer iteration

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$$R_{\text{Th}}^{[k]} \leftarrow R_{\text{Th}}^{[k]} - \lambda^{(-)} \left\{ R_{\text{Th}}^{[k]} - R^{[k]} \left( \tau^{[k](i)}, \tau^{[-k](i-1)} \right) \right\}, \quad (15)$$

where  $\lambda^{(-)}$  is a positive small number. When the  $k$ -th user can achieve a higher rate than the threshold,

$$R_{\text{Th}}^{[k]} \leftarrow R_{\text{Th}}^{[k]} + \lambda^{(+)} \left\{ R^{[k]} \left( \tau^{[k](i)}, \tau^{[-k](i-1)} \right) - R_{\text{Th}}^{[k]} \right\}, \quad (16)$$

where  $\lambda^{(+)}$  also denotes a positive small number.

The proposed iterative multimode precoding for convergence (IMPC) using outer iteration and modified equilibrium is summarized in Algorithm 1. In the proposed IMPC, each receiver feeds back its mode and precoder index only to its desired transmitter, such that the amount of feedback for each pair is the same as that of the single-user multimode precoding. Moreover, it requires that the interference covariance matrix in (6) can be learned at each receiver as in [1], which enables a noncooperative implementation of the proposed scheme.

## V. Numerical Results

The performance of the proposed scheme is evaluated by Monte Carlo simulation. The ergodic sum rate is presented in the following results. All channel coefficients are independent and identically distributed complex Gaussian with zero mean and unit variance. We assume that  $I=10$ ,  $N=4$ , and  $\chi=\{1, 2, 4\}$ , where  $t_1=10$ ,  $t_2=5$ , and  $t_4=1$ . The codebooks given in [3] are

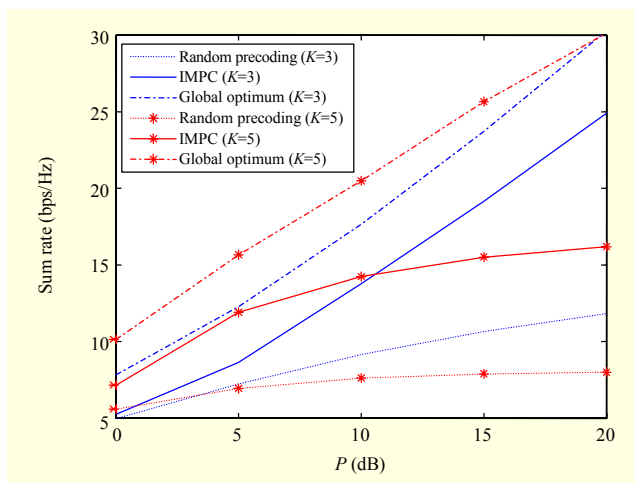


Fig. 2. Comparison of ergodic sum rates as function of  $P$ .

Table 1. Average number of outer iterations.

$P$ (dB)	0	5	10	15	20
$K=3$	1.002	1.062	1.133	1.094	1.056
$K=5$	1.075	1.434	1.603	1.815	2.057

used as  $\Psi(M)$ . We set the initial threshold to  $R_{\text{Th}}^{[k]} = \log_2(1+P)$ ,  $\lambda^{(-)}=0.2$ , and  $\lambda^{(+)}=0.1$ , which guarantees the convergence of the IMPC over 100,000 independent channel realizations.

Figure 2 compares how the ergodic sum rate varies with  $P$ . The sum rate of the IMPC with  $K=3$  increases almost linearly according to the increase of  $P$ , like the global optimum in (8). The IMPC achieves almost 85% of the sum rate of the global optimum, especially when  $P$  is large. When  $K=5$ , the sum rate of the IMPC saturates even though  $P$  becomes large. The sum rate of the IMPC is more than 55% of that of the global optimum.

Table 1 presents the average outer iterations required for the convergence of the proposed scheme. When  $K=3$ , the average number of outer iterations for convergence is almost constant for all ranges of  $P$ . However, the required number of iterations increases as  $P$  becomes large when  $K=5$ . This implies that the computational complexity of the proposed IMPC increases as  $K$  increases.

## VI. Conclusion

In this letter, we proposed an iterative multimode precoding scheme for nonreciprocal MIMO interference channels. We modeled the iterative multimode precoding as a noncooperative game with finite and discrete strategies. To guarantee the convergence, we defined a modified equilibrium and adopted the outer iteration and the rate threshold adaptation. Numerical results showed that the proposed scheme is effective in nonreciprocal MIMO interference channels.

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