Mode-SVD-Based Maximum Likelihood Source Localization Using Subspace Approach

Chee-Hyun Park and Kwang-Seok Hong

A mode-singular-value-decomposition (SVD) maximum likelihood (ML) estimation procedure is proposed for the source localization problem under an additive measurement error model. In a practical situation, the noise variance is usually unknown. In this paper, we propose an algorithm that does not require the noise covariance matrix as *a priori* knowledge. In the proposed method, the weight is derived by the inverse of the noise magnitude square in the ML criterion. The performance of the proposed method outperforms that of the existing methods and approximates the Taylor-series ML and Cramér-Rao lower bound.

Keywords: Singular value decomposition, maximum likelihood, weight, TOA, localization.

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I. Introduction

Source localization is a technique that finds a geometrical point of intersection using the measurements from each receiver, such as the time difference of arrival, the time of arrival (TOA), or the angle of arrival. Localizing point sources, using passive and stationary sensors, is of considerable interest, and this has been a repeated theme of research in radar, sonar, the Global Positioning System, video conferencing, and telecommunication areas.

In this paper, a new source position location algorithm is developed using mode-singular-value-decomposition (SVD)-based maximum likelihood (ML) to solve the TOA localization problem. The efficiency of the localization method based on the weights is dependent on the noise estimation accuracy since the weights are given as the inverse of the noise level. The existing ML uses the inverse of noise variance as the weights [1]. In the other weighted least squares (WLS)-based methods, the noise variance is required [2], [3]. However, in a practical situation, the noise covariance is usually unknown [2], [4]. In this paper, we propose an algorithm that does not require a noise covariance matrix as *a priori* knowledge. The measurement with noise is projected onto the mode matrix, and the signal component is then estimated by averaging the projected signal components. The difference between the measurement and estimated signal component is regarded as the noise magnitude. The weight, which is derived in the ML sense, is given by the inverse of the square difference.

The proposed method is effective only under the uncorrelated noise condition since the correlated weight must also be estimated in the correlated noise case. In the equation **Ax**=**b**, **A** is a mode matrix [5]. Since the measurement is projected onto the left singular vectors of the mode matrix to estimate the noise component, we named the method the "mode-SVD ML."

Several methods exist for multisensor localization. Torrieri [1] derived a principal algorithm to analyze the hyperbolic location systems and direction-finding location systems and used distance and angle information for the ML. Foy [6] adopted an iterative least squares (LS) method, which used a combination of range and angular information, and showed that the Taylor-series method works for a variety of problems, including the mixed-measurement mode. In [7], the source position was estimated using the adaptive filter theory presented therein. Performance analysis of source localization methods is included in the research results of [8]-[10].

The accuracies of LS and squared-range LS (SRLS) were compared in [9]. The researchers in [10] discussed whether the three source localization methods are minimum variance unbiased estimators and robust [10]. Research results regarding source localization using the subspace approach were presented in [11]-[13].

The orthogonal property of the signal subspace and noise subspace was used to form the estimating equation [11], [12]. In [13], the multidimensional scaling method was used for the mobile positioning, and the researchers assumed that the eigenvalue of the noise space was zero. Whereas the orthogonal property and eigenvalue decomposition were used in [11]-[13], the projection method and SVD are used to estimate the signal component in the proposed method.

The organization of this paper is as follows. Section II deals with the details of the proposed algorithm, which uses a new weighting method based on SVD and subspace theory. Section III discusses the experiment results to evaluate the estimation performance of the mode-SVD ML algorithm, comparing it with existing algorithms. Section IV presents conclusions and directions for future work.

II. Localization Method

The TOA source localization method finds the position of a source using multiple circles whose centers are sensors. The intersection of these circles is determined as the source position. At least three sensors are required to determine the source position.

1. Maximum Likelihood

ML is the method that finds the variable by maximizing the probability density function $p(\mathbf{z};\mathbf{x})$, assuming the measurement vector $\mathbf{z}$ is known. We modify the existing Taylor-series-based ML [1] and compare it with the proposed method. In this subsection, we describe the existing ML method since the proposed method is the improved version of [1].

As the information follows a Gaussian distribution, it is represented as

$$z_i = f_i(\mathbf{x}) + n_i, \quad i=1,2,\ldots,N, \tag{1}$$

$$p(\mathbf{z};\mathbf{x}) = \frac{1}{(2\pi)^{N/2}|\mathbf{R}|^{1/2}} \exp\{-(1/2)[\mathbf{z}-\mathbf{f}(\mathbf{x})]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}-\mathbf{f}(\mathbf{x})]\}. \tag{2}$$

In this case, z_i is the i-th sensor measurement with the additive noise, $f_i(\mathbf{x})$ is the signal model represented as $\sqrt{(x-x_i)^2+(y-y_i)^2}$, and n_i is the i-th sensor measurement noise. Here, x and y are the source coordinates, x_i and y_i are the coordinates of the i-th sensor, i ($1\le i\le N$) is the sensor index, $\mathbf{z}=[z_1,\ldots, z_N]^{\mathrm{T}}$, $\mathbf{f}(\mathbf{x})=[f_1(\mathbf{x}),\ldots, f_N(\mathbf{x})]^{\mathrm{T}}$, $\mathbf{x}=[x, y]^{\mathrm{T}}$, $\mathbf{n}=[n_1,\ldots n_N]^{\mathrm{T}}$, and $\mathbf{R}$ is the covariance matrix of the measurement noise ($\mathbf{n}$). The measurement error $\mathbf{n}$ is assumed to be a multivariate random vector with a zero mean and Gaussian distribution. Its inverse exists because $\mathbf{R}$ is symmetric and positive definite.

The ML estimator is the value of $\mathbf{x}$, which maximizes the quadratic form:

$$Q(\mathbf{x}_{k+1}) = [\mathbf{z}-\mathbf{f}(\mathbf{x}_{k+1})]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}-\mathbf{f}(\mathbf{x}_{k+1})], \tag{3}$$

$$\mathbf{R} = \mathrm{diag}(\sigma_1^2,\cdots,\sigma_i^2,\cdots,\sigma_N^2), \tag{4}$$

where σ_i^2 is a variance of the measurement noise obtained from the i-th sensor and $\mathbf{z}$ is the range measurement vector $[z_1,\ldots, z_i,\ldots, z_N]^{\mathrm{T}}$ from sensors. The iterative source position estimate at the (k+1)th iteration, $\mathbf{x}_{k+1}$, is $[x_{k+1}, y_{k+1}]^{\mathrm{T}}$. The initial position estimate, $\mathbf{x}_0$, is set to be near the true source position.

In general, as $\mathbf{f}(\mathbf{x}_{k+1})$ is a nonlinear vector function, the nonlinear optimization methods can be applied to find the minimum point of $Q(\mathbf{x}_{k+1})$, but they suffer from high computational complexity and get stuck at local minima. We can remove these problems by linearizing $\mathbf{f}(\mathbf{x}_{k+1})$. When $\mathbf{f}(\mathbf{x}_{k+1})$ is linearized, (3) becomes the quadratic cost function and the local minima problem can be solved. The nonlinear function $\mathbf{f}(\mathbf{x}_{k+1})$ can be linearized by expanding it in a Taylor series at a reference point specified by $\mathbf{x}_k$ and by retaining the first two terms to determine a reasonably simple estimator as

$$\mathbf{f}(\mathbf{x}_{k+1}) \cong \mathbf{f}(\mathbf{x}_k) + \mathbf{G}_k(\mathbf{x}_{k+1}-\mathbf{x}_k), \tag{5}$$

$$\mathbf{G}_k = \begin{bmatrix} \partial f_1(\mathbf{x})/\partial x|_{\mathbf{x}=\mathbf{x}_k} & \partial f_1(\mathbf{x})/\partial y|_{\mathbf{x}=\mathbf{x}_k} \\ \vdots & \vdots \\ \partial f_i(\mathbf{x})/\partial x|_{\mathbf{x}=\mathbf{x}_k} & \partial f_i(\mathbf{x})/\partial y|_{\mathbf{x}=\mathbf{x}_k} \\ \vdots & \vdots \\ \partial f_N(\mathbf{x})/\partial x|_{\mathbf{x}=\mathbf{x}_k} & \partial f_N(\mathbf{x})/\partial y|_{\mathbf{x}=\mathbf{x}_k} \end{bmatrix}, \tag{6}$$

where $\partial f_i(\mathbf{x})/\partial x\big|_{\mathbf{x}=\mathbf{x}_k}$ and $\partial f_i(\mathbf{x})\partial y\big|_{\mathbf{x}=\mathbf{x}_k}$ mean $\partial f_i(\mathbf{x})/\partial x$ and $\partial f_i(\mathbf{x})/\partial y$ at $\mathbf{x}=\mathbf{x}_k$, respectively.

In (6), each row of this matrix is the gradient vector of one of the components of $\mathbf{f}(\mathbf{x})$. It is assumed in the subsequent analysis that $\mathbf{x}_{k+1}$ is sufficiently close to $\mathbf{x}_k$ so that (5) and (6) are accurate approximations.

Substituting (5) into (3) and finding the minimum point of $Q(\mathbf{x}_{k+1})$ result in

$$\begin{aligned}\mathbf{x}_{k+1} &= (\mathbf{G}_k^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)+\mathbf{G}_k\mathbf{x}_k] \\ &= \mathbf{x}_k + (\mathbf{G}_k^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)]. \end{aligned} \tag{7}$$

In this case, the position estimate is iteratively updated by (7).

2. Mode-SVD Maximum Likelihood

We let $\mathbf{z}$–$\mathbf{f}(\mathbf{x}_k)$+$\mathbf{G}_k\mathbf{x}_k$ be $\mathbf{r}_k$. By SVD, $\mathbf{G}_k$ can be decomposed as [14]

$$\mathbf{G}_k = \begin{bmatrix}\mathbf{u}_{k,1} & \mathbf{u}_{k,2} & \cdots & \mathbf{u}_{k,N}\end{bmatrix}\begin{bmatrix} q_{k,1} & 0 \\ 0 & q_{k,2} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_{k,1}^{\mathrm{T}} \\ \mathbf{v}_{k,2}^{\mathrm{T}}\end{bmatrix}, \tag{8}$$

where $q_{k,1}>q_{k,2}>0$, $\mathbf{u}_{k,h}$ is the h-th left singular vector of $\mathbf{G}_k$, $q_{k,h}$ is the h-th singular value, and $\mathbf{v}_{k,h}$ is the h-th right singular vector at iteration number k. Clearly, the rank of $\mathbf{G}_k$ is two from (8) if N is larger than three. Therefore, the estimation of rank can be omitted. Since $\mathbf{u}_{k,1}$ and $\mathbf{u}_{k,2}$ span the column space of $\mathbf{G}_k$, from (8), $\mathbf{r}_k$ can be represented as

$$\mathbf{r}_k = \mathbf{G}_k\mathbf{x}_k + \mathbf{w}_k = \hat{\mathbf{r}}_k + \mathbf{w}_k = \gamma_{k,1}\mathbf{u}_{k,1} + \gamma_{k,2}\mathbf{u}_{k,2} + \mathbf{w}_k, \tag{9}$$

where the Taylor-series approximation error is assumed sufficiently small and $\mathbf{w}_k$ is the estimated noise vector at the k-th iteration, represented as

$$\mathbf{w}_k = \mathbf{z} - \mathbf{f}(\mathbf{x}_k) + \mathbf{G}_k\mathbf{x}_k - (\mathbf{z}^{\mathrm{o}} - \mathbf{f}(\mathbf{x}^{\mathrm{o}}) + \mathbf{G}^{\mathrm{o}}\mathbf{x}^{\mathrm{o}}), \tag{10}$$

where $\mathbf{z}^{\mathrm{o}}$ is the true range, $\mathbf{f}(\mathbf{x}^{\mathrm{o}})$ is $[f_1(\mathbf{x}^{\mathrm{o}}),\ldots, f_N(\mathbf{x}^{\mathrm{o}})]^{\mathrm{T}}$, $f_i(\mathbf{x}^{\mathrm{o}}) = \sqrt{(x^{\mathrm{o}}-x_i)^2 + (y^{\mathrm{o}}-y_i)^2}$, $\mathbf{x}^{\mathrm{o}}$ is the true source location composed of $(x^{\mathrm{o}}, y^{\mathrm{o}})$, and $\mathbf{G}^{\mathrm{o}}$ is the (6) calculated at $\mathbf{x}=\mathbf{x}^{\mathrm{o}}$.

The estimate $\hat{\mathbf{r}}_k$ can be represented as follows [14] since it should exist in the column space of $\mathbf{G}_k$:

$$\hat{\mathbf{r}}_k = \gamma_{k,1}\mathbf{u}_{k,1} + \gamma_{k,2}\mathbf{u}_{k,2}. \tag{11}$$

If it is assumed that $\hat{\mathbf{r}}_k$ at the steady state is equal to $\mathbf{a}$, that is, $\hat{\mathbf{r}}_s = \mathbf{a}$, our aim is to find $\mathbf{a}$. Here, $\hat{\mathbf{r}}_s$ is the $\hat{\mathbf{r}}$ at the steady state. The following equation is assumed, and $\mathbf{a}$ is the constant within the estimation period.

$$\mathbf{r}_s(l) = \mathbf{a} + \mathbf{w}_s(l), \tag{12}$$

where $\mathbf{r}_s(l)$=$[r_{s,1}(l),\ldots, r_{s,i}(l),\ldots, r_{s,N}(l)]^{\mathrm{T}}$, l ($1\le l\le L$) is the sample (time) index, and subscript i and s denote the sensor index and the steady state, respectively. Due to the reason given below, $\mathbf{w}_s$ is modeled as a Gaussian distribution. Equation (7) can be restated as

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{x}_k + (\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)] \\ &= \mathbf{x}_k + (\mathbf{G}_k^{\mathrm{T}}\mathbf{Q}_k^{\mathrm{T}}\mathbf{Q}_k\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{Q}_k^{\mathrm{T}}\mathbf{Q}_k[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)] \\ &= \mathbf{x}_k + (\mathbf{B}_k^{\mathrm{T}}\mathbf{B}_k)^{-1}\mathbf{B}_k^{\mathrm{T}}[\boldsymbol{\delta}_k - \mathbf{g}(\mathbf{x}_k)], \end{aligned} \tag{13}$$

where $\mathbf{B}_k$=$\mathbf{Q}_k\mathbf{G}_k$, $\boldsymbol{\delta}_k$=$\mathbf{Q}_k\mathbf{z}$, $\mathbf{g}(\mathbf{x}_k)$=$\mathbf{Q}_k\cdot\mathbf{f}(\mathbf{x}_k)$, and $\mathbf{W}_{\mathrm{svd},k}$=$\mathbf{Q}_k^{\mathrm{T}}\mathbf{Q}_k$. In (21), $\mathbf{W}_{\mathrm{svd},k}$ is defined.

The convergence of the Gauss-Newton method to the true solution is guaranteed if the function $\mathbf{g}(\mathbf{x})$ satisfies the Lipschitz condition and the initial point is sufficiently near the solution [15]. Function $\mathbf{g}(\mathbf{x})$ is defined as

$$\mathbf{g}(\mathbf{x}) = \mathbf{Q}^{\mathrm{o}}\cdot\mathbf{f}(\mathbf{x}^{\mathrm{o}}), \tag{14}$$

where $\mathbf{Q}^{\mathrm{o}} = \mathrm{diag}\left(\frac{1}{n_1},\cdots,\frac{1}{n_N}\right)$, n_i is defined in (1), and $\mathbf{f}(\mathbf{x}^{\mathrm{o}})$ is defined in (10). Hence, (10) is condensed to $\mathbf{w}_s$=$\mathbf{z}$–$\mathbf{z}^{\mathrm{o}}$ in the steady state. Since $\mathbf{z}$ was modeled as a Gaussian measurement, $\mathbf{w}_s$ can be modeled as a Gaussian distribution.

In the ML criterion, we should find that $\mathbf{a}$=$[a_1,\ldots, a_i,\ldots, a_N]^{\mathrm{T}}$, which maximizes the log-likelihood so that

$$\frac{\partial \ln p(\mathbf{r}_s;\mathbf{a})}{\partial a_i} = 0, \quad i=1,\ldots,N, \tag{15}$$

where

$$p(\mathbf{r}_s;\mathbf{a}) = \frac{1}{(2\pi)^{LN/2}\left(\prod_{i=1}^{N}\sigma_i^2\right)^{L/2}}\exp\left\{-\frac{1}{2}\sum_{l=1}^{L}\sum_{i=1}^{N}\frac{(r_{s,i}(l)-a_i)^2}{\sigma_i^2}\right\}, \tag{16}$$

where σ_i^2 is a variance of $r_{s,i}$, $r_{s,i}$ is the i-th element of $\mathbf{r}_s$, and a_i is the i-th element of $\mathbf{a}$. Equation (15) results in

$$a_i = \frac{1}{L}\sum_{l=1}^{L} r_{s,i}(l), \quad i=1,\ldots,N. \tag{17}$$

Representing (17) in vector form yields

$$\mathbf{a} = \hat{\mathbf{r}}_s = \frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_s(l). \tag{18}$$

From (9) and (18),

$$\hat{\mathbf{r}}_s = \gamma_{s,1}\mathbf{u}_{s,1} + \gamma_{s,2}\mathbf{u}_{s,2} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_s(l), \tag{19}$$

where

$$\gamma_{s,h}=\mathbf{u}_{s,h}^{\mathrm{T}}\left(\frac{1}{L}\sum_{l=1}^{L}\mathbf{r}_s(l)\right), h{=}1, 2, \tag{20}$$

$$\mathbf{u}_{s,p}^{\mathrm{T}}\mathbf{u}_{s,r}=1, \quad \text{if} \quad p=r,$$

$$\mathbf{u}_{s,p}^{\mathrm{T}}\mathbf{u}_{s,r}=0, \quad \text{if} \quad p\neq r.$$

The sample length (L) is set to two in the proposed method.

The source coordinates can be estimated as

$$\mathbf{x}_{k+1}=(\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)+\mathbf{G}_k\mathbf{x}_k], \tag{21}$$

$$=\mathbf{x}_k+(\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}\mathbf{G}_k)^{-1}\mathbf{G}_k^{\mathrm{T}}\mathbf{W}_{\mathrm{svd},k}[\mathbf{z}-\mathbf{f}(\mathbf{x}_k)], \tag{22}$$

where $\mathbf{W}_{\mathrm{svd},k}=\mathrm{diag}\left(\frac{1}{(r_{k,1}-\hat{r}_{k,1})^2},\cdots,\frac{1}{(r_{k,N}-\hat{r}_{k,N})^2}\right)$.

The weight matrix $\mathbf{W}_{\mathrm{svd},s}$ (the weight matrix in the steady state) is optimal in the ML sense and can be proven as follows. Differentiating $\log p(\mathbf{r}_s;\mathbf{a})$ with respect to σ_i^2 and setting it to zero, the variance estimate is

$$\hat{\sigma}_i^2=\frac{1}{L}\sum_{l=1}^{L}(r_{s,i}(l)-a_i)^2$$

$$=\frac{1}{L}\sum_{l=1}^{L}(r_{s,i}(l)-\hat{r}_{s,i})^2, i{=}1,\ldots,N, \tag{23}$$

where $\hat{r}_{s,i}$ is the i-th element of $\hat{\mathbf{r}}_s$. The weight in the steady state is given as

$$\frac{1}{\hat{\sigma}_i^2}=\left(\frac{1}{L}\sum_{l=1}^{L}(r_{s,i}(l)-\hat{r}_{s,i})^2\right)^{-1}, \quad i{=}1,\ldots,N. \tag{24}$$

The weight matrix of the proposed method, in the steady state, is the special case of (24) in which L is set to one.

The mean square error (MSE) of $\hat{\mathbf{x}}$ $(\hat{x},\hat{y})$ can be represented as

$$\begin{aligned}
&\mathrm{E}[(\mathbf{x}^{\mathrm{o}}-\hat{\mathbf{x}})^{\mathrm{T}}(\mathbf{x}^{\mathrm{o}}-\hat{\mathbf{x}})]\\
&=\mathrm{E}[\mathrm{tr}\{(\mathbf{x}^{\mathrm{o}}-\hat{\mathbf{x}})(\mathbf{x}^{\mathrm{o}}-\hat{\mathbf{x}})^{\mathrm{T}}\}]\\
&=\mathrm{E}[\mathrm{tr}\{((\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G}\mathbf{x}^{\mathrm{o}}-(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{r})\\
&\cdot((\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G}\mathbf{x}^{\mathrm{o}}-(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{r})^{\mathrm{T}}\}]\\
&=\mathrm{E}[\mathrm{tr}\{((\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}(\mathbf{G}\mathbf{x}^{\mathrm{o}}-\mathbf{r})(\mathbf{G}\mathbf{x}^{\mathrm{o}}-\mathbf{r})^{\mathrm{T}}\mathbf{W}\mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\}]\\
&=\mathrm{tr}\{(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{N}^{\mathrm{T}}\mathbf{W}(\mathbf{G}^{\mathrm{T}}\mathbf{W}\mathbf{G})^{-1}\},
\end{aligned} \tag{25}$$

where $\mathrm{tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$, $\mathbf{x}^{\mathrm{o}}$ is the true source position, $\mathbf{N}$ denotes $\mathrm{E}\{(\mathbf{G}\mathbf{x}^{\mathrm{o}}-\mathbf{r})(\mathbf{G}\mathbf{x}^{\mathrm{o}}-\mathbf{r})^{\mathrm{T}}]=\mathrm{E}[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{T}}]$, $r_{s,i}$ and $\hat{r}_{s,i}$ are r and $\hat{r}$ of the i-th sensor at the steady state, respectively, and $\mathbf{W}$ is defined as

$$\mathbf{W}=\mathrm{diag}\left(\frac{1}{(r_{s,1}-\hat{r}_{s,1})^2},\cdots,\frac{1}{(r_{s,N}-\hat{r}_{s,N})^2}\right). \tag{26}$$

III. Simulation Results

In this section, the performance of the proposed source localization algorithm is compared with that of the Taylor-series ML [1], the best linear unbiased estimator approach based on LS calibration (BLUE-LSC) [3], and SRLS [16]. A Monte-Carlo simulation of 2,000 trials is performed.

Figure 1 is the plot of the first scenario. The source is located at (–2, 7) m. Six sensors are used. The sensors are randomly located in (–7.4794, –2.6308) m, (–4.7423, 7.7841) m, (6.3204, –4.9151) m, (3.3896, –6.8147) m, (–7.8873, 5.3780) m, and (6.1723, 7.2727) m to model a general sensor distribution in Fig. 1.

Figure 2 is the plot of the second scenario. The source is located at (9, –8) m. We start with four sensors with known positions at (–7.4794, –2.6308) m, (–4.7423, 7.7841) m,

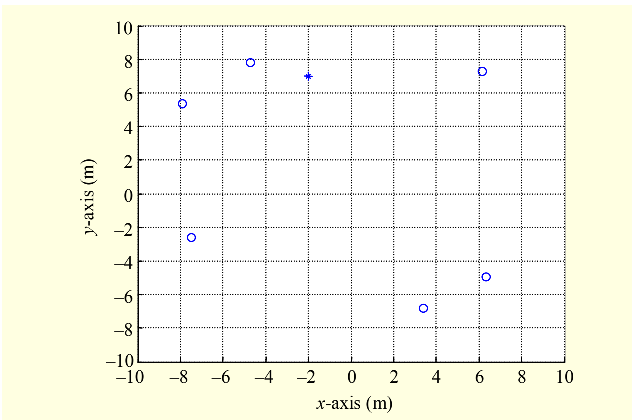

Fig. 1. Source and sensor arrangement (first scenario) (o: sensors, *: source).

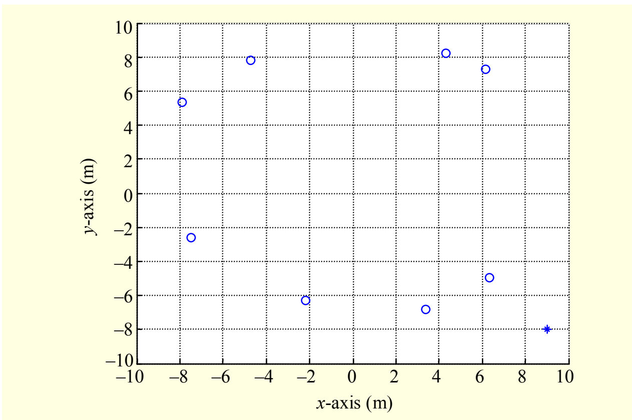

Fig. 2. Source and sensor arrangement (second scenario) (o: sensors, *: source).

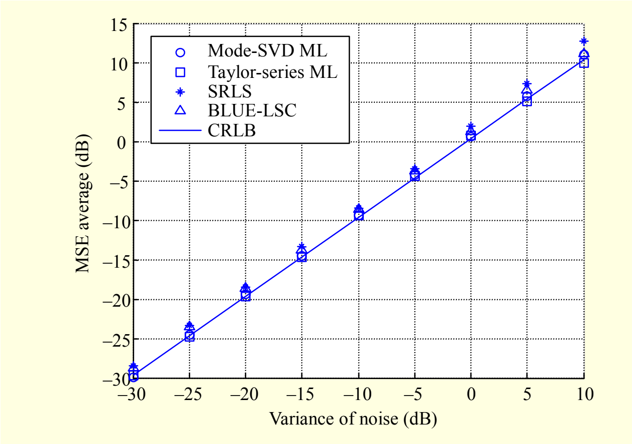

Fig. 3. Comparison of MSE average of respective algorithm with respect to various noise settings in first scenario.

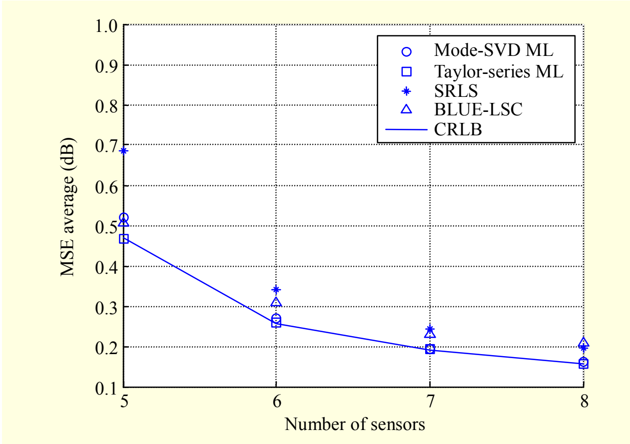

Fig. 4. Comparison of MSE average of respective algorithm with respect to number of sensors under second scenario.

(6.3204, –4.9151) m, and (3.3896, –6.8147) m, and the sensors with coordinates (–7.8873, 5.3780) m, (6.1723, 7.2727) m, (4.3, 8.2) m, and (–2.2, –6.3) m are then added successively.

The measurement noise variance of the first sensor (σ_1^2) is varied from –30 dB to 10 dB at an interval of 5 dB. The measurement noise variance of each sensor is assumed to be different and set to

$$\sigma_1^2 \times [1,\ 1{+}1.5\times\frac{1}{(N-1)}, 1{+}1.5\times\frac{2}{(N-1)},\ \cdots, 1{+}1.5\times\frac{(N-1)}{(N-1)}],$$

where σ_1^2 is the noise variance of the first sensor and N is the total number of sensors.

A single and omni-directional source is assumed in the stationary state. The initial position is set to the solution of the standard LS method [17]. The MSE average is calculated and can be represented as

$$\text{MSE average} = \frac{\sum_{k=1}^{2{,}000}\left\{(\hat{x}(k)-x)^2+(\hat{y}(k)-y)^2\right\}}{2{,}000}, \tag{27}$$

where $\hat{x}(k), \hat{y}(k)$ is the estimated position of the source in the k-th iteration, and x, y is the true position of the source.

Figure 3 compares the MSE average of each position estimation method under various range noise settings in the first scenario. The mode-SVD ML is superior to the other existing algorithms and approximates the Cramér-Rao lower bound (CRLB) [2], [18] and Taylor-series ML. The rationale for adopting weighting factors in the proposed method is to emphasize the contributions of those data samples that are deemed to be more reliable. The proposed method is seen as an instantaneous version of the conventional ML. This method can also be seen in many other algorithms (for example, the least mean square method), in which the MSE is substituted as an instantaneous squared error.

Figure 4 compares the MSE average of each position estimation method with regard to the number of sensors under the second scenario. Here, the variance of the sensor noise is –10 dB. The mode-SVD ML is superior to the other algorithms and approximates the CRLB and Taylor-series ML.

The horizontal dilution of precision (HDOP) is commonly used in geolocation using satellites [19]. It is defined as

$$\mathbf{A} = \begin{bmatrix} \frac{(x_1-x)}{R_1} & \frac{(y_1-y)}{R_1} & -1 \\ \frac{(x_2-x)}{R_2} & \frac{(y_2-y)}{R_2} & -1 \\ \vdots & \vdots & \vdots \\ \frac{(x_N-x)}{R_N} & \frac{(y_N-y)}{R_N} & -1 \end{bmatrix}, \tag{28}$$

$$\mathbf{P} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1} = \begin{bmatrix} d_x^2 & d_{xy}^2 & d_{xt}^2 \\ d_{xy}^2 & d_y^2 & d_{yt}^2 \\ d_{xt}^2 & d_{yt}^2 & d_t^2 \end{bmatrix}, \tag{29}$$

$$\text{HDOP} = \sqrt{d_x^2 + d_y^2}\ , \tag{30}$$

where x_i and y_i are the coordinates of the i-th sensor, x and y are the coordinates of the source, and R_i is the distance between the i-th sensor and the source. The larger the area formed by the sensors and source, the better (lower) the value of the HDOP; the smaller its area, the worse (higher) the value of the HDOP. Similarly, the greater the number of sensors, the better the value of the HDOP [19].

IV. Conclusion

In this paper, the existing methods and the proposed mode-SVD-based ML method were compared. The proposed method does not require *a priori* information, such as noise variance. In our simulation, after the noise magnitude was estimated by the SVD and subspace approach, the weighting matrix was determined as the inverse of the estimated noise magnitude square. The MSE performance of the proposed method was analyzed. The simulation showed that using the proposed method with a TOA measurement results in a superior MSE average to that of the existing methods and the ability to approximate the CRLB and Taylor-series ML under various noise conditions. The performance is expected to be improved in future work by using *a priori* information of the parameters.

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