Denoising Mapping Utilizing Constellation Symmetry in Denoise-and-Forward Two-Way Relay Channels

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The denoising mapping with the closest-neighbor clustering (CNC) method in denoise-and-forward two-way relay channels is studied. Specifically, the symmetry of the constellations in source terminals A and B is utilized to reduce the complexity of the CNC method. The specific case considered first to illustrate how the constellation symmetry works in the CNC method is the quadrature phase-shift keying constellation in A and B and the single-antenna deployment in all terminals. This case study shows that an enormous complexity reduction can be achieved. Next, the result is extended to multiple-antenna scenarios and square quadrature amplitude modulations.

Keywords: Two-way relaying, closest-neighbor clustering, constellation symmetry, complexity.

I. Introduction

The denoise-and-forward (DNF) scheme, originally introduced in [1], is a promising two-way (bidirectional) relay scenario due to its theoretic potential [2], [3]. It consists of two stages. The first stage is the multiple access (MA) stage, during which the source terminals *A* and *B* simultaneously transmit to the relay *R*, which then maps the received signals to symbols from a discrete constellation and broadcasts. In the second stage, the broadcast (BC) stage, these symbols are broadcast to *A* and *B*. Then, the terminal *A* (*B*) uses the broadcast signal from *R* to decode the data from *B* (*A*) with its own information as the a priori information.

The feature of the DNF scheme is the denoising procedure in the relay *R*. The use of structured codes to enable efficient denoising has been considered in [4], [5]. In [6], the complex plane (all the channel realizations) is partitioned into a finite number of regions, with a specific network coding map in a practical region. In [7], the complex plane is classified into two regions: singularity-free and singularity. Further, in [8], a Latin square is used to design the network coding maps.

To design the optimal denoising map, the closest-neighbor clustering (CNC) method [6] includes two successive phases: the distance calculation phase and the clustering phase. Because the distance calculation phase is enforced in a brute-force fashion, this phase of the CNC method is highly complex.

In this letter, the constellation symmetry of the signal constellations in the terminals *A* and *B* is utilized in the distance calculation phase to reduce the complexity of the CNC method. First, to illustrate how to utilize the constellation symmetry in the distance calculation phase, a quadrature phase-shift keying (QPSK) constellation and a single antenna are assumed. The main result is given in Proposition 1; this case study shows that complexity can be greatly reduced. Next, the extensions to the general square quadrature amplitude modulation (QAM) formats and multiple-antenna scenarios are given. Compared with the existing work in [9], where the constellation symmetry is used intuitionally in the QPSK and single-antenna scenario, the main contribution of this letter lies in the detailed analysis and derivation on the universal employment of the constellation symmetry in the distance calculation phase of the CNC method. Therefore, its extension to any multiple-antenna and square QAM scenario is available.

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II. Two-Way Denoise-and-Forward Relaying

Letting $\mathcal{M}$ be the signal constellation, the transmitted signals from the source terminals A and B can be presented as $x_A = \mathcal{M}(s_A)$ and $x_B = \mathcal{M}(s_B)$. Here, s_A and s_B represent the digital data per symbol from A and B, respectively. Over the MA channel, the received signal at relay R is expressed as

$$y_R = h_A x_A + h_B x_B + w_R, \qquad (1)$$

where h_A and h_B are the channel coefficients from terminals A and B, respectively, to relay R and are assumed to be known completely at the relay terminal, and w_R is the additive white Gaussian noise (AWGN) with zero mean and a variance of σ^2. In the BC stage, the received signals at A and B are as

$$y_A = h_A x_R + w_A, \quad y_B = h_B x_R + w_B. \qquad (2)$$

Here, we assume the reciprocal channel in the MA and BC stages and identical noise variance σ^2 for the AWGN w_A and w_B. Then, terminal A (B) can detect the desired signal s_B (s_A) by using its own information s_A (s_B).

The relay transmitted signal x_R is formed as follows. First, calculate the network-coded signal $s_R = \mathcal{C}(\hat{s}_A, \hat{s}_B)$ by maximum likelihood detection from the received signal y_R, where $\mathcal{C}(\cdot)$ is the denoising map. Next, the relay signal is formed by $x_R = \mathcal{M}_R(s_R)$ with $\mathcal{M}_R$ denoting the relay signal constellation. For successful decoding in terminals A and B, the exclusive law: 1) $\mathcal{C}(s_1, s_2) \neq \mathcal{C}(s_1', s_2)$ for any $s_1 \neq s_1'$; 2) $\mathcal{C}(s_1, s_2) \neq \mathcal{C}(s_1, s_2')$ for any $s_2 \neq s_2'$ should be satisfied.

In [6], [7], the denoising map $\mathcal{C}$ is optimized to maximize the minimal squared Euclidean distance and an optimal mapping with the CNC method is proposed. Assuming $h_B = h_A \cdot \gamma e^{j\varphi}$, the squared Euclidean distance of the transmit signal pair $(x_A^1, x_B^1)(x_A^2, x_B^2)$ is defined as

$$d^2_{(x_A^1, x_B^1)(x_A^2, x_B^2)} = \|h_A\|^2 \cdot \left\| x_A^1 - x_A^2 + \gamma e^{j\varphi}\left(x_B^1 - x_B^2\right)\right\|^2. \qquad (3)$$

The CNC method includes two successive phases: the distance calculation phase and the clustering phase. In the first phase, all the $|\mathcal{M}|^4$ squared Euclidean distances of the signal pairs $(x_A^1, x_B^1)(x_A^2, x_B^2)$, $x_i^k \in \mathcal{M}$, $k = 1, 2$, $i = A, B$, are calculated and the resultant distance set is denoted as $\mathcal{D}$, with $|\mathcal{M}|$ being the cardinality of constellation $\mathcal{M}$. In the second phase, the denoising map is generated successively through testing all the members in the distance set $\mathcal{D}$ from the minimum member to the maximum member and checking the satisfaction of the exclusive raw. It is obvious that the complexity of the CNC method is dominated by that in the first phase, which needs to perform (3) $|\mathcal{M}|^4$ times.

When $\mathcal{M}$ is the QPSK constellation, with the CNC method, the size of the denoising map $\mathcal{C}$ (number of clusters in $\mathcal{C}$) will be 4 or 5 according to the channel condition. The relay constellation $\mathcal{M}_R$ will be the QPSK if the size is 4 and the sphere packing 5-QAM otherwise [6], [7].

III. Constellation Symmetry in CNC Method

In this section, we show how the symmetry of the signal constellation $\mathcal{M}$ in terminals A and B is utilized in the distance calculation phase to reduce the complexity of the CNC method when $\mathcal{M}$ is the QPSK modulation and all the terminals A, B, and R are deployed with one antenna. The extensions of the result in this section to other modulation forms and multi-antenna bidirectional relay channels are given in section IV.

The QPSK signal set with Gray mapping is $\{\mathcal{M}(0)=e^{j\pi/4}, \mathcal{M}(1) = e^{j3\pi/4}, \mathcal{M}(2) = e^{j7\pi/4}, \mathcal{M}(3) = e^{j5\pi/4}\}$. Rotating this constellation by an angle $k \cdot \pi/2$, $k = 1, 2, 3$, the new constellation is obviously the same as the original one except that the signal labels have been changed, which is referred to as the constellation symmetry.

Prior to considering the application of the constellation symmetry, the following lemma is introduced.

Lemma 1. If $\left(x_A^4 - x_A^3\right)/\left(x_A^2 - x_A^1\right) = \left(x_B^4 - x_B^3\right)/\left(x_B^2 - x_B^1\right) = e^{j\theta}, \theta \in (0, 2\pi]$, the squared Euclidean distance of signal pair $(x_A^1, x_B^1)(x_A^2, x_B^2)$ will be equal to that of $(x_A^3, x_B^3)(x_A^4, x_B^4)$, that is, $d^2_{(x_A^1, x_B^1)(x_A^2, x_B^2)} = d^2_{(x_A^3, x_B^3)(x_A^4, x_B^4)}$.

Proof. According to (3),

$$\begin{aligned}
d^2_{(x_A^3, x_B^3)(x_A^4, x_B^4)} &= \|h_A\|^2 \cdot \left\| x_A^3 - x_A^4 + \gamma e^{j\varphi}\left(x_B^3 - x_B^4\right)\right\|^2 \\
&= \|h_A\|^2 \cdot \left\| e^{j\theta}\left(x_A^1 - x_A^2\right) + \gamma e^{j\varphi} e^{j\theta}\left(x_B^1 - x_B^2\right)\right\|^2 \\
&= \|h_A\|^2 \cdot \left\| e^{j\theta}\right\|^2 \cdot \left\| x_A^1 - x_A^2 + \gamma e^{j\varphi}\left(x_B^1 - x_B^2\right)\right\|^2 \\
&= \|h_A\|^2 \cdot \left\| x_A^1 - x_A^2 + \gamma e^{j\varphi}\left(x_B^1 - x_B^2\right)\right\|^2 \\
&= d^2_{(x_A^1, x_B^1)(x_A^2, x_B^2)}.
\end{aligned}$$

Here, the second equation uses the condition in Lemma 1. □

Considering the $\pi/2$-phase rotation invariance of QPSK, the following proposition can be obtained from Lemma 1.

Proposition 1. With the QPSK constellation $\mathcal{M}$ in terminals A and B, the two signal pairs $(x_A^1, x_B^1)(x_A^2, x_B^2)$ and $(x_A^3, x_B^3)(x_A^4, x_B^4)$ have the same squared Euclidean distance, that is, $d^2_{(x_A^1, x_B^1)(x_A^2, x_B^2)} = d^2_{(x_A^3, x_B^3)(x_A^4, x_B^4)}, x_i^k \in \mathcal{M}, i = A, B; k = 1, 2, 3, 4,$ if either of the following conditions is satisfied:

1) (Simultaneous phase rotation) $x_A^4/x_A^2 = x_A^3/x_A^1 = x_B^4/x_B^2 = x_B^3/x_B^1 = e^{jk\pi/2}$, $k = 1, 2, 3$.

2) (Part phase inverse) $x_A^3 = x_A^1$, $x_A^4 = x_A^2$, $x_B^3 = e^{j\pi} x_B^1$, $x_B^4 = e^{j\pi} x_B^2$ or $x_B^3 = x_B^1$, $x_B^4 = x_B^2$, $x_A^3 = e^{j\pi} x_A^1$, $x_A^4 = e^{j\pi} x_A^2$.

Proof. The proof is direct by applying the QPSK constellation into Lemma 1. □

Before applying Proposition 1 into the distance calculation phase of the CNC method, a slight modification to the original CNC method is made. Specifically, the exclusive raw is integrated in the distance calculation phase; that is, a distance calculation such as $d^2_{(x_A^1, x_B^1)(x_A^2, x_B^2)}$ with $x_A^1 = x_A^2$ or $x_B^1 = x_B^2$ will be excluded, since such a signal pair is contrary to the exclusive raw. Therefore, with QPSK, in the distance calculation phase, $1/2 \times |\mathcal{M}| \times |\mathcal{M}| \times (|\mathcal{M}|-1) \times (|\mathcal{M}|-1) = 72$ distances must be calculated, as in (3). Here, the factor 1/2 is used to exclude a repeat case such as $(x_A^1, x_B^1)(x_A^2, x_B^2)$ and $(x_A^2, x_B^2)(x_A^1, x_B^1)$.

Applying Proposition 1 into the distance calculation phase, it declares that only 16 distance calculations are needed. To see this, as shown in Table 1, the transmit signal pair $(x_A^1, x_B^1)(x_A^2, x_B^2)$ with $x_A^1 \neq x_A^2$ and $x_B^1 \neq x_B^2$ can be partitioned into the following three exclusive forms: 1) $x_A^1 = -x_A^2$ and $x_B^1 = -x_B^2$, 2) $x_A^1 = -x_A^2$, $x_B^1 \neq -x_B^2$ or $x_A^1 \neq -x_A^2$, $x_B^1 = -x_B^2$, and 3) $x_A^1 \neq -x_A^2$ and $x_B^1 \neq -x_B^2$. For succinctness, only the interpretation of the second row of Table 1, corresponding to the case of the first signal pair form, is given. The interpretations of the other two cases are similar.

It is easy to show that there are $1/2 \times 4 \times 4 = 8$ different signal pairs in the form $(x_A^1, x_B^1)(-x_A^1, x_B^1)$. First, consider the simultaneous phase rotating with angle $k\pi/2$, $k = 1, 2, 3$. With this rotation, the signal pairs $(jx_A^1, jx_B^1)(-jx_A^1, -jx_B^1)$, $(-x_A^1, -x_B^1)(x_A^1, x_B^1)$, and $(-jx_A^1, -jx_B^1)(jx_A^1, jx_B^1)$ are obtained. Together with $(x_A^1, x_B^1)(-x_A^1, x_B^1)$, there exist only two different signal pairs, $(x_A^1, x_B^1)(-x_A^1, -x_B^1)$ and $(jx_A^1, jx_B^1)(-jx_A^1, -jx_B^1)$, excluding the repeat case. Furthermore, it is easy to find that the two distance pairs do not generate a new distance pair by the part phase inverse in Proposition 1. Therefore, for each signal pair in the form $(x_A^1, x_B^1)(-x_A^1, x_B^1)$, there is only one different signal pair that satisfies the condition of Proposition 1 and thereby has the same distance value as that of $(x_A^1, x_B^1)(-x_A^1, x_B^1)$, that is, the equal distance index is 2. So, only 8/2=4 different distance calculations are needed to obtain the distances of the eight different signal pairs in this form.

From Table 1, only 4+8+4=16 distance calculations are needed to obtain the squared Euclidean distances of all 72 different signal pairs. Therefore, a computational complexity saving factor $(72-16)/72 \approx 0.7778$ is achieved. Table 2 partitions all the 72 different signal pairs into 16 subsets with each subset having the same squared Euclidean distance. For notation simplicity, the modulation index, instead of the modulation signal, is used in this table.

To end this section, it should be emphasized that the same denoising map will be achieved by the original CNC method

Table 1. Number of different squared Euclidean distances in different signal pair forms.

Signal pair form	Number in this form	Equal distance index	Number of different distances
$x_A^1 = -x_A^2$ $x_B^1 = -x_B^2$	8	2	4
$x_A^1 = -x_A^2$, $x_B^1 \neq -x_B^2$ or $x_A^1 \neq -x_A^2$, $x_B^1 = -x_B^2$	32	4	8
$x_A^1 \neq -x_A^2$ and $x_B^1 \neq -x_B^2$	32	8	4

Table 2. Partitioning of all signal pairs into different subsets according to squared Euclidean distance.

	Subset leader	Simultaneous phase rotation	Part phase inverse
1	(0 0)(1 1)	(1 1)(3 3) (3 3)(2 2) (2 2)(0 0)	(0 2)(1 3) (1 0)(3 2) (3 1)(2 0) (2 3)(0 1)
2	(0 0)(1 2)	(1 1)(3 0) (3 3)(2 1) (2 2)(0 3)	(0 1)(1 3) (1 3)(3 2) (3 2)(2 0) (2 0)(0 1)
3	(0 0)(1 3)	(1 1)(3 2) (3 3)(2 0) (2 2)(0 1)	
4	(0 0)(2 1)	(1 1)(0 3) (3 3)(1 2) (2 2)(3 0)	(0 2)(2 3) (1 0)(0 2) (3 1)(1 0) (2 3)(3 1)
5	(0 0)(2 3)	(1 1)(0 2) (3 3)(1 0) (2 2)(3 1)	
6	(0 0)(3 1)	(1 1)(2 3) (3 3)(0 2) (2 2)(1 0)	
7	(0 0)(3 2)	(1 1)(2 0) (3 3)(0 1) (2 2)(1 3)	
8	(0 0)(3 3)	(1 1)(2 2)	
9	(0 1)(1 0)	(1 3)(3 1) (3 2)(2 3) (2 0)(0 2)	(0 3)(1 2) (1 2)(3 0) (3 0)(2 1) (2 1)(0 3)
10	(0 1)(1 2)	(1 3)(3 0) (3 2)(2 1) (2 0)(0 3)	
11	(0 1)(3 0)	(1 3)(2 1) (3 2)(0 3) (2 0)(1 2)	
12	(0 1)(3 2)	(1 3)(2 0)	
13	(0 2)(2 1)	(1 0)(0 3) (3 1)(1 2) (2 3)(3 0)	
14	(0 2)(3 0)	(1 0)(2 1) (3 1)(0 3) (2 3)(1 2)	
15	(0 2)(3 1)	(1 0)(2 3)	
16	(0 3)(3 0)	(1 2)(2 1)	

and the CNC method utilizing the constellation symmetry, since the same distance set $\mathcal{D}$ is obtained in both versions of the CNC method. Therefore, no performance loss results by utilizing the constellation symmetry in the CNC method.

IV. Extensions

1. Square QAM Constellations

In Section III, the symmetry is presented by the constellation invariance with a suitable phase rotation. Obviously, this character can also be found in the square QAM constellation. Specifically, a $k\pi/2$, $k=1,2,3$, phase rotation of any one point in the square QAM will move to another point in the same constellation. Therefore, Proposition 1 is generally true for any square QAM constellation, and, thereby, the results in section III can be extended directly to a general square QAM constellation. Consider, for example, a single-antenna scenario with a 16-QAM. It is easy to find from Proposition 1 that only 4,096 distance calculations are needed to obtain all the distances of $1/2\times16\times16\times15\times15=28,800$ signal pairs.

2. Multiple-Antenna Scenario

Here, we consider the case that all terminals A, B, and R are deployed with two antennas, to illustrate the use of constellation symmetry in the multiple-antenna scenario, with constellation $\mathcal{M}$ being QPSK. Let the channel coefficient matrices from A and B to R be $\boldsymbol{H}_A$ and $\boldsymbol{H}_B$, respectively. Additionally, assume $\boldsymbol{\Gamma}=\boldsymbol{H}_A^{-1}\boldsymbol{H}_B$. Then, the squared Euclidean distance of signal pair $(\boldsymbol{x}_A^1,\boldsymbol{x}_B^1)(\boldsymbol{x}_A^2,\boldsymbol{x}_B^2)$ is

$$d^2_{(\boldsymbol{x}_A^1,\boldsymbol{x}_B^1)(\boldsymbol{x}_A^2,\boldsymbol{x}_B^2)}=\left\|\boldsymbol{H}_A\left(\boldsymbol{x}_A^1-\boldsymbol{x}_A^2+\boldsymbol{\Gamma}\left(\boldsymbol{x}_B^1-\boldsymbol{x}_B^2\right)\right)\right\|^2, \tag{4}$$

with $\boldsymbol{x}_i^k=\left(x_i^k(1),x_i^k(2)\right)^T$ and $x_i^k(1),x_i^k(2)$, $i=A,B;\ k=1,2,$ belonging to the QPSK constellation. It is clear that there are $1/2\times16\times16\times15\times15=28,800$ different signal pairs that satisfy the exclusive raw.

Rewrite (4) as

$$d^2_{(\boldsymbol{x}_A^1,\boldsymbol{x}_B^1)(\boldsymbol{x}_A^2,\boldsymbol{x}_B^2)}=\left\|\boldsymbol{H}_A\left(\left(\begin{matrix}x_A^1(1)-x_A^2(1)\\0\end{matrix}\right)+\left(x_B^1(1)-x_B^2(1)\right)\boldsymbol{\Gamma}^1\right.\right.$$
$$\left.\left.+\left(\begin{matrix}0\\x_A^1(2)-x_A^2(2)\end{matrix}\right)+\left(x_B^1(2)-x_B^2(2)\right)\boldsymbol{\Gamma}^2\right)\right\|^2. \tag{5}$$

Here, $\boldsymbol{\Gamma}^k$, $k=1,2,$ is the k-th column of the matrix $\boldsymbol{\Gamma}$. Note that, from section III, there are 16 different values in the form $\left\|x_A^1(1)-x_A^2(1)+\Gamma^1(1)\left(x_B^1(1)-x_B^2(1)\right)\right\|$, with $\Gamma^1(1)$ being the first element of $\boldsymbol{\Gamma}^1$. Furthermore, considering the $\pi/2$-phase rotation invariance of the QPSK and $\left\|\{\pm1,\pm j\}\cdot c\right\|=\|c\|$ for any complex value c, there are 64 different values in the form $x_A^1(1)-x_A^2(1)+\Gamma^1(1)\left(x_B^1(1)-x_B^2(1)\right)$. Thus, there are 64 different values in the first argument on the RHS of (5).

$$\left(\begin{matrix}x_A^1(1)-x_A^2(1)\\0\end{matrix}\right)+\left(x_B^1(1)-x_B^2(1)\right)\boldsymbol{\Gamma}^1.$$

Similarly, there are 64 different values in the second argument on the RHS of (5). Therefore, there are 4,096 different squared Euclidean distance values, at most, in the total of 28,800 different signal pairs. A savings factor of 0.8578 can be achieved.

V. Conclusion

The utilization of constellation symmetry in the CNC relaying mapping method was considered. Analysis results showed that complexity can be significantly reduced by employing the constellation symmetry. This result can be achieved with QPSK and general square QAM modulations in both single- and multiple-antenna scenarios.

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