New Scattering Matrix Model for Modeling Ferrite Media Using the TLM Method

Asmaa Zugari, Soufiane El Adraoui, Mohamed Iben Yaich, and Mohsine Khalladi

This paper aims to extend the transmission line matrix method with a hybrid symmetrical condensed node (HSCN) to model ferrite media in the time domain. To take into account the anisotropy and dispersive properties of ferrite media, equivalent current sources are incorporated into supplementary stubs of the original HSCN. The scattering matrix of the proposed HSCN is provided, and the validity of this approach is demonstrated for both transversely and longitudinally magnetized ferrites. Agreement is achieved between the results of this approach and those of the theoretical and the finite-difference time-domain method.

Keywords: Ferrite media, electromagnetics, TLM method time-domain.

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I. Introduction

There has been an increasing interest in recent years in using gyromagnetic media in microwave systems, mainly due to their unique properties and tuning capabilities [1]-[3]. For several microwave circuit applications, ferrite media offer more advantages than normal dielectric materials [4]-[6]. Their singular characteristics contribute to the design and performance of several microwave circuits, such as antennas, isolators, and circulators. However, due to the inherent anisotropy and nonlinearity of ferrites, the analytical study of electromagnetic (EM) interaction with these media is often a complex problem and their treatment in the literature is relatively scarce in spite of their importance. In light of this, numerical methods are a strong and inevitable alternative to deal with these problems. The transmission line matrix (TLM) method [7] is one among many time-domain numerical techniques that have been successfully used to study the interaction of EM waves with isotropic and anisotropic media [8]-[16]. Using different additional techniques to include frequency-dependent material properties in the time domain, this method is able to model several dispersive media.

Four main approaches are discussed in the literature. The first one, presented by de Menezes and Hoefer, describes the behavior of the medium by equivalent node sources [8]. The second one, presented by Paul and others, is based on the Z-transform technique [9], [10]. The third approach has been developed using the insertion of an equivalent circuit [1]. The last one models the dispersive media by adding voltage [11]-[16] and/or current sources [17], [18] in the TLM nodes.

In the present work, a novel TLM model using the hybrid symmetrical condensed node (HSCN) and current sources, allowing the time-domain simulation of anisotropic dispersive ferrite media, is developed. The scattering matrix of the proposed HSCN is presented, and two examples are given to verify the effectiveness and accuracy of this approach.

II. Formulation

Let us consider a homogeneous ferrite medium subjected to a static magnetic field H_0 acting along the z-direction. The relationship between the magnetic field H and the magnetic field density B is given in the frequency-domain by the following equation:

$$B(\omega)=\left[\mu(\omega)\right]\cdot H(\omega), \tag{1}$$

where $[\mu(\omega)]$ is the frequency domain permeability tensor for magnetized ferrites in the z-direction:

$$\left[\mu(\omega)\right]=\mu_0\begin{bmatrix}1+\chi_m(\omega) & -j\kappa(\omega) & 0\\ j\kappa(\omega) & 1+\chi_m(\omega) & 0\\ 0 & 0 & 1\end{bmatrix}.$$

The frequency susceptibilities are given by

$$\chi_m(\omega)=\frac{(\gamma H_0+j\omega\alpha)\gamma M_0}{(\gamma H_0+j\omega\alpha)^2-\omega^2}$$

and

$$\kappa(\omega)=\frac{-\omega\gamma M_0}{(\gamma H_0+j\omega\alpha)^2-\omega^2},$$

where γ is the gyromagnetic ratio, α is the damping constant, H_0 is the static magnetic biasing field, and M_0 is the static magnetization.

In the time domain, (1) becomes a convolution. This convolution can be computed since the susceptibility functions are complex and only the real parts of these functions are used to update the EM fields in the numerical algorithms [13], [17], [19], [20]. Assuming that the EM field components remain constant over each time interval $n\Delta t$ [19], the convolution integrals derived from (1) can be simplified, and these components can be easily obtained from Maxwell-Faraday equations at an instant $(n+1)\Delta t$. Complete details of the development of the solution of (1) in the time domain can be found in [17], [19].

To find the scattering matrix of the new HSCN modeling magnetized ferrite, three supplementary stubs (16, 17, 18) are added to the standard (15×15) HSCN. In these stubs, equivalent current sources (I_{sx}, I_{sy}, I_{sz}) are respectively included to model the gyromagnetic properties of the ferrite [17]. To determine these equivalent current sources, we start by enforcing charge conservation and imposing the magnetic flux conservation condition in the HSCN. Then, we use the equivalence between voltages and EM field components, the equations relating incident and reflected pulses for the EM field components so as to obtain the equivalent currents at time $(n+1)\Delta t$. Finally, we couple the equivalent currents located in the xoy plane. The obtained expressions of the equivalent current sources modeling ferrite media are given by

$$I_{su}^{n+1}=-I_{su}^{n}+(4+Z_{su})\cdot\left(-\alpha_u I_u^n+\beta_u\psi_{mu}^n\right)$$

and $u=x, y,$ or z, where

$$\begin{pmatrix}\psi_{mx}^n\\ \psi_{my}^n\\ \psi_{mz}^n\end{pmatrix}=\mathrm{Re}\begin{pmatrix}\left[\sum_m^{n-1}\Delta\chi_{xx}^m H_x^{n-m}-\sum_m^{n-1}\Delta\kappa_{xy}^m H_y^{n-m}\right]\\ \left[\sum_m^{n-1}\Delta\chi_{yy}^m H_y^{n-m}+\sum_m^{n-1}\Delta\kappa_{yx}^m H_x^{n-m}\right]\\ \left[\sum_m^{n-1}\Delta\chi_{zz}^m H_z^{n-m}\right]\end{pmatrix}$$

are the discrete convolutions, based on the constant recursive convolution (CRC) [11], [21] at the time $n\Delta t$ and can be computed recursively due to the exponential nature of the complex time-domain susceptibility functions. I_u^n are the equivalent currents in the HSCN at the time $n\Delta t$, Re is the real part, and (χ^m, κ^m) are the generalized susceptibilities. The other parameters are defined by the following equations [17]:

$$Z_{su}=\frac{4}{\beta_u}-4,$$

where Z_{su} is the impedance of ferrite media,

$$\begin{pmatrix}\alpha_x\\ \alpha_y\\ \alpha_z\end{pmatrix}=\begin{pmatrix}\left(\chi_{xx}^{0^2}+\chi_{xx}^0+\kappa_{xy}^{0^2}\right)\Big/\left(\left(1+\chi_{xx}^0\right)^2+\kappa_{xy}^{0^2}\right)\\ \left(\chi_{yy}^{0^2}+\chi_{yy}^0+\kappa_{yx}^{0^2}\right)\Big/\left(\left(1+\chi_{yy}^0\right)^2+\kappa_{yx}^{0^2}\right)\\ \chi_{zz}^0\Big/\left(1+\chi_{zz}^0\right)\end{pmatrix},$$

and

$$\begin{pmatrix}\beta_x\\ \beta_y\\ \beta_z\end{pmatrix}=\begin{pmatrix}\left(1+\chi_{xx}^0\right)\Big/\left(\left(1+\chi_{xx}^0\right)^2+\kappa_{xy}^{0^2}\right)\\ \left(1+\chi_{yy}^0\right)\Big/\left(\left(1+\chi_{yy}^0\right)^2+\kappa_{yx}^{0^2}\right)\\ 1\Big/\left(1+\chi_{zz}^0\right)\end{pmatrix}.$$

The use of the equivalence between currents obtained from the transmission line equations and magnetic field components in the Maxwell equations, the recursive evaluation of ψ_{mu}, and the determination of α and β from the previous equations, give the equivalent current sources I_{su}.

In the HSCN, the reflected and incident voltages, the equivalent current sources, and the equivalent voltages [9], [22] are related by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \\ V_{13} \\ V_{14} \\ V_{15} \end{bmatrix}^r = \begin{bmatrix} V_x \\ V_x \\ V_y \\ V_y \\ V_z \\ V_z \\ V_z \\ V_y \\ V_x \\ V_z \\ V_y \\ V_x \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} I_z \\ -I_y \\ -I_z \\ I_x \\ -I_x \\ I_y \\ I_x \\ -I_x \\ I_y \\ -I_y \\ I_z \\ -I_z \\ I_x \\ I_y \\ I_z \end{bmatrix} + \begin{bmatrix} V_2 \\ V_9 \\ V_{11} \\ V_8 \\ V_7 \\ V_{10} \\ V_5 \\ V_4 \\ V_2 \\ V_6 \\ V_3 \\ V_1 \\ V_{13} \\ V_{14} \\ V_{15} \end{bmatrix}^i .$$

Thus, we obtain at a time $n\Delta t$ the scattering matrix of the new HSCN modeling ferrite media as shown in (2), where

$$a_u = \frac{Z_{su}}{2(4+Z_{su})}, \quad b_u = \frac{1}{2}, \quad c_u = -\frac{Z_{su}}{2(4+Z_{su})},$$

$$d_u = \frac{2}{(4+Z_{su})}, \quad i_u = \frac{2Z_{su}}{(4+Z_{su})}, \quad g_u = \frac{Z_{su}-4}{Z_{su}+4},$$

and Z_{su} is the impedance of ferrite media, $u = x, y,$ or z.

In the time-step algorithm of the TLM with the new HSCN, a set of pulses is incident on the node, formed by ports 1 to 15, and the supplementary stubs 16 to 18 contribute solely to the incident pulses associated with current sources characterizing the gyromagnetic media. They are reflected according to the above 15×18 scattering matrix. At the next time-step, the new incident voltages are obtained from the 'connection' process, which operates exactly like the standard HSCN [7]. This time-domain algorithm is depicted in Fig. 1.

III. Numerical Results

To validate the scattering matrix of the new HSCN, we study the reflection and transmission of a Gaussian plane wave

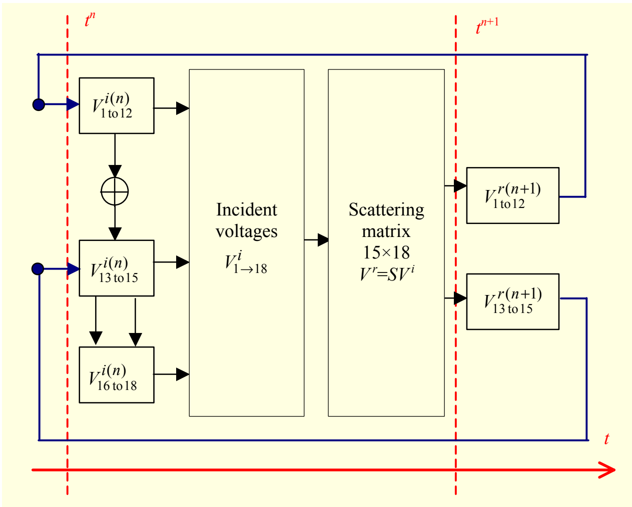

Fig. 1. Diagram of TLM process for ferrite dispersive media.

$$\left(\begin{array}{c|cccccccccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
 & z & y & z & x & x & y & x & x & y & y & z & z & x & y & z & x & y & z \\
1 & a & b & d & 0 & 0 & 0 & 0 & 0 & b & 0 & -d & c & 0 & 0 & i & 0 & 0 & d/2 \\
2 & b & a & 0 & 0 & 0 & d & 0 & 0 & c & -d & 0 & b & 0 & -i & 0 & 0 & -d/2 & 0 \\
3 & d & 0 & a & b & 0 & 0 & 0 & b & 0 & 0 & c & -d & 0 & 0 & -i & 0 & 0 & -d/2 \\
4 & 0 & 0 & b & a & d & 0 & -d & c & 0 & 0 & b & 0 & i & 0 & 0 & d/2 & 0 & 0 \\
5 & 0 & 0 & 0 & d & a & b & c & -d & 0 & b & 0 & 0 & -i & 0 & 0 & -d/2 & 0 & 0 \\
6 & 0 & d & 0 & 0 & b & a & b & 0 & -d & c & 0 & 0 & 0 & i & 0 & 0 & d/2 & 0 \\
7 & 0 & 0 & 0 & -d & c & b & a & d & 0 & b & 0 & 0 & i & 0 & 0 & d/2 & 0 & 0 \\
8 & 0 & 0 & b & c & -d & 0 & d & a & 0 & 0 & b & 0 & -i & 0 & 0 & -d/2 & 0 & 0 \\
9 & b & c & 0 & 0 & 0 & -d & 0 & 0 & a & d & 0 & b & 0 & i & 0 & 0 & d/2 & 0 \\
10 & 0 & -d & 0 & 0 & b & c & b & 0 & d & a & 0 & 0 & 0 & -i & 0 & 0 & -d/2 & 0 \\
11 & -d & 0 & c & b & 0 & 0 & 0 & b & 0 & 0 & a & d & 0 & 0 & i & 0 & 0 & d/2 \\
12 & c & b & -d & 0 & 0 & 0 & 0 & 0 & b & 0 & d & a & 0 & 0 & -i & 0 & 0 & -d/2 \\
13 & 0 & 0 & 0 & -d & d & 0 & d & 0 & b & 0 & 0 & 0 & g & 0 & 0 & d/2 & 0 & 0 \\
14 & 0 & d & 0 & 0 & 0 & -d & 0 & 0 & -d & d & 0 & 0 & 0 & g & 0 & 0 & d/2 & 0 \\
15 & -d & 0 & d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -d & d & 0 & 0 & g & 0 & 0 & d/2
\end{array}\right) \quad (2)$$

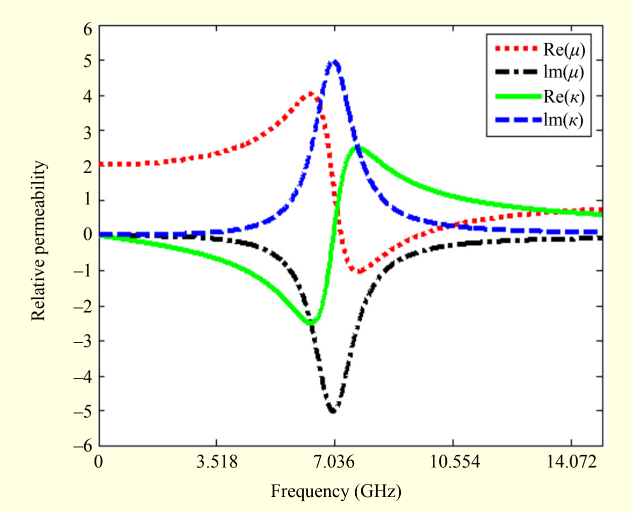

Fig. 2. Real and imaginary part of μ and κ for magnetized ferrite.

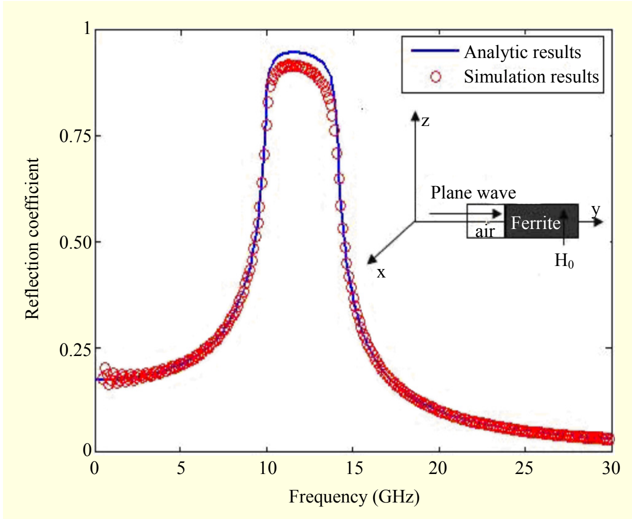

Fig. 3. Reflection coefficient versus frequency.

illuminating an air-ferrite interface. The real and imaginary parts of the diagonal and off-diagonal ($\mu = 1+\chi_m$ and κ) elements of the ferrite permeability tensor are represented versus frequency in Fig. 2, for $|H_0| = 200$ kA/m, $|M_0| = 200$ kA/m.

The gyromagnetic resonance frequency is 7.036 GHz. Hence, the resonant frequency of the ferrite is

$$f_{\text{res}} = \gamma\sqrt{H_0\left(H_0 + M_0\right)} = 9.95 \text{ GHz},$$

where γ is the gyromagnetic ratio.

To find out whether the field H_0 is perpendicular to or parallel to the propagation direction, two different examples are discussed in the following subsections.

1. Transversely Magnetized Ferrite

In simulation TLM mesh, the Gaussian wave travels 300Δl in free space along the OY axis before entering the z-direction magnetized ferrite layer with Δl = 25 μm and $\alpha = 0.01$.

The analytical relationship for a reflection coefficient is

$$R = \frac{Z_f - Z_0}{Z_f + Z_0},$$

with

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}, \qquad Z_f = Z_0\sqrt{\frac{\mu^2 - \kappa^2}{\mu\varepsilon_r}},$$

$$\kappa = \frac{-\omega\gamma M_0}{\left(\gamma H_0 + j\omega\alpha\right)^2 - \omega^2}, \quad \mu = 1 + \frac{\left(\gamma H_0 + j\omega\alpha\right)\gamma M_0}{\left(\gamma H_0 + j\omega\alpha\right)^2 - \omega^2}.$$

At each iteration, the reflection coefficient is calculated at a space-step before the air-ferrite interface. It is illustrated in the frequency domain in Fig. 3.

It is worthwhile to mention that the simulation results are in agreement with those obtained from the analytical solutions.

2. Longitudinally Magnetized Ferrite

A second interesting case is when the direction of propagation coincides with the direction of the field H_0. Thus, the wave equation gives rise to two modes of propagation: right circular polarization (RCP) and left circular polarization (LCP) [19]. As the treated structure is described in [19], the spatial TLM lattice considered is (1×1×800)Δl in the z-direction, where Δl = 75 μm. The ferrite layer spans 50Δl.

The simulation results for the transmission and reflection coefficients, for the two propagation modes, are represented in Figs. 4 and 5, respectively. Comparing the obtained results, the results of the proposed algorithm strongly agree with those of the theoretical formulation given in [23] and those of the finite-difference time-domain (FDTD) method [19]. To model ferrite media, as with other dispersive material, the FDTD seems to be easier to implement and requires less computer memory. However, unlike the FDTD method, the TLM method solves the electric and magnetic fields at the same point in space and time. Thus, the TLM method is easier to work with and offers more straightforward solutions than the FDTD method.

IV. Conclusion

In this paper, we presented a novel modeling approach for ferrite media using the TLM method with an HSCN and current sources. This approach allows for the analysis of electromagnetic wave propagation and scattering by ferrite media. The scattering matrix of the HSCN was presented, and the results obtained with this model were compared to the analytical results and those obtained with the FDTD method, illustrating its validity and efficiency.

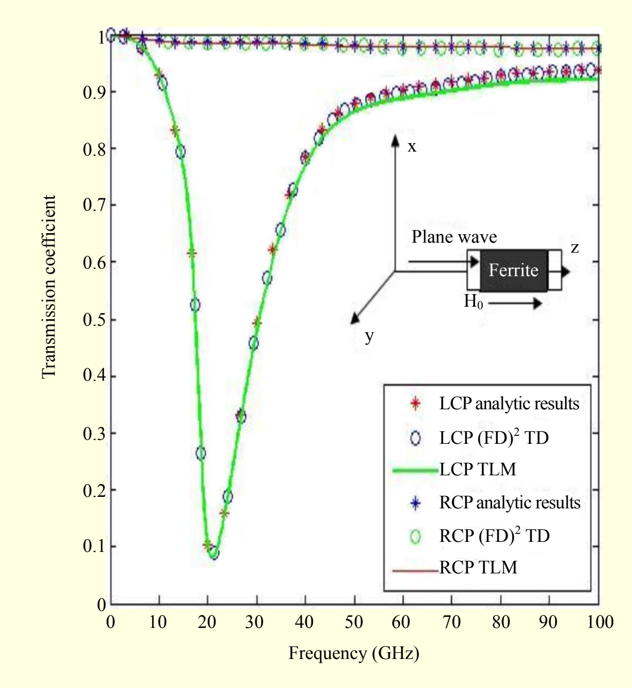

Fig. 4. Transmission coefficient versus frequency.

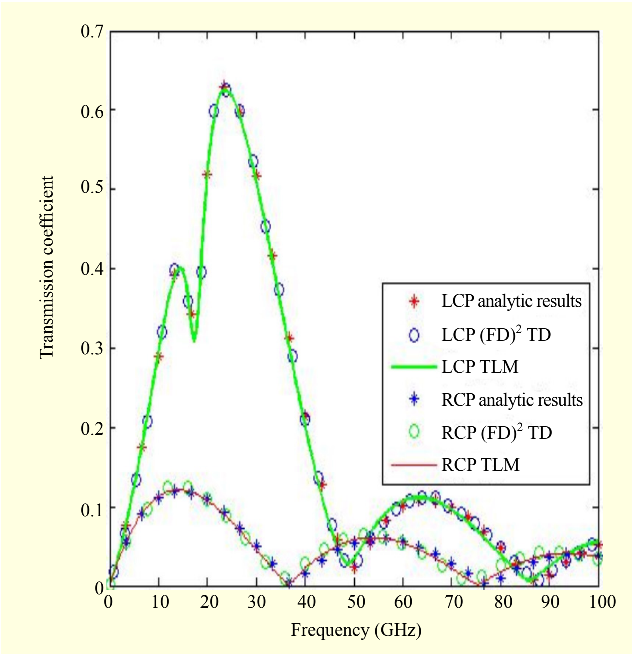

Fig. 5. Reflection coefficient versus frequency.

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