# BLUE-Based Channel Estimation Technique for Amplify and Forward Wireless Relay Networks

M.PremKumar, V.N.SenthilKumaran, and S.J.Thiruvengadam

**The best linear unbiased estimator (BLUE) is most suitable for practical application and can be determined with knowledge of only the first and second moments of the probability density function. Although the BLUE is an existing algorithm, it is still largely unexplored and has not yet been applied to channel estimation in amplify and forward (AF)-based wireless relay networks (WRNs). In this paper, a BLUE-based algorithm is proposed to estimate the overall channel impulse response between the source and destination of AF strategy-based WRNs. Theoretical mean square error (MSE) performance for the BLUE is derived to show the accuracy of the proposed channel estimation algorithm. In addition, the Cramér-Rao lower bound (CRLB) is derived to validate the MSE performance. The proposed BLUE channel estimation algorithm approaches the CRLB as the length of the training sequence and number of relays increases. Further, the BLUE performs better than the linear minimum MSE estimator due to the minimum variance characteristic exhibited by the BLUE, which happens to be a function of signal-to-noise ratio.**

**Keywords: Cramér-Rao lower bound, best linear unbiased estimation, channel estimation, mean square error, training sequence, wireless relay networks.**

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M.PremKumar (phone: +91 9944565193, premkumar@tce.edu), V.N.SenthilKumaran (vnsenthilkumaran@tce.edu), and S.J.Thiruvengadam (sjtece@tce.edu) are with the Department of Electronics and Communication Engineering, Thiagarajar College of Engineering, Tamil Nadu, India.

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## I. Introduction

Wireless relay networks (WRNs) are comprised of spatially dispersed nodes, which are referred to as “relays” or “virtual antennas” and are located between the source node and destination node. These relays are specifically employed to exploit spatial diversity and to minimize the power requirements for transmission [1], [2]. In addition, such relays can be grouped as amplify and forward (AF), decode and forward (DF), demodulate and forward, and compress and forward (CF) [3], depending on the relay strategy. In AF, the relays amplify the received signal broadcast from the source node and forward the scaled signal to the destination node. In DF, the relays decode the entire received message, re-encode it, and send the resulting sequence to the destination node. In demodulate and forward, the relays demodulate every received symbol individually, modulate them, and then retransmit them to the destination node. In CF, the relays send a quantized version of the received signal to the destination node [3], [4]. In all the relay strategies of WRNs, an optimal training sequence is broadcast from the source node to all the relays during phase I, and the relays forward the received signals after performing a linear transformation to the destination node during phase II for estimating the overall channel impulse response between the source node and the destination node. In a WRN, accurate channel state information (CSI) is required at the destination node for decoding [4]. The scope of this paper is limited to AF strategy. In AF-based transmission, channel estimation must be separated into two phases. However, this separation has drawbacks because the relay must inform the destination node of the CSI of phase I, which not only reduces bandwidth efficiency but also consumes additional transmitting power. Moreover, transmitting the estimated channel contributes to further distortion.

Several research works have been carried out for channel estimation in AF-based WRNs [4]-[8]. The problems in estimating channel links in WRNs for AF topology are solved using least squares (LS) and minimum mean square error (MMSE) approaches accompanied by the design of optimal training sequences [4]. In [5], an amplification factor is introduced to amplify the pilot symbols transmitted by the source node. Studies on channel estimation for AF relay channels is carried out for selecting appropriate channel models based upon relay mobility, relay gain, impact of the underlying channel on the pilot insertion strategy, and the estimator design [6]. In [7], a training-based linear MMSE (LMMSE) channel estimator algorithm is proposed for time-division multiplex AF relay networks. A semi-blind channel estimation algorithm is developed in [8] in which the main advantage is that the required training sequence is very short. Thus, most of the existing channel estimation algorithms for AF WRNs are based on LS and MMSE approaches.

The LS approach reduces signal estimation error but not channel estimation error [9]. The MMSE estimator requires the complete knowledge of the probability density function (PDF), and it employs second order statistics of channel conditions to minimize MSE. When channel statistics are not available, it is difficult to estimate the channel. In addition, a major drawback of the MMSE estimator is its high complexity, which grows exponentially with observation samples [10]. Due to the aforementioned drawbacks of the LS and MMSE estimators, in this paper, the BLUE approach is used to estimate the channel impulse response in AF-based WRN. The BLUE is more suitable for practical implementation as it requires less knowledge of channel second-order statistics and does not require the complete knowledge of the PDF [10], [11]. Furthermore, an estimator based on the BLUE is unbiased and it exhibits linearity [11].

The rest of the paper is organized as follows. Section II presents a system model for AF WRNs and its receiver processing. Section III presents the approach for the BLUE channel estimation in a WRN and analyzes the MSE performance of the algorithm. In section IV, the Cramér-Rao lower bound (CRLB) is derived for an AF-based WRN. Simulation results for the MSE performance of the proposed BLUE algorithm are analyzed in section V. Section VI concludes the paper.

**Notations:** Vectors and matrices are represented as boldface lowercase and capital letters, respectively. The transpose, complex conjugate, Hermitian, and inverse of the matrix $\mathbf{C}$ are denoted by $\mathbf{C}^T$, $\mathbf{C}^*$, $\mathbf{C}^H$, and $\mathbf{C}^{-1}$, respectively. The $(i, j)$th entry of matrix $\mathbf{C}$ is $[\mathbf{C}]_{ij}$, and diag$\{\mathbf{c}\}$ denotes a diagonal matrix with the diagonal element constructed from $\mathbf{c}$; $\mathbf{I}$ is the identity matrix; $\mathrm{E}\{\cdot\}$ denotes statistical expectation; cov($\mathbf{c}$) denotes the covariance of the vector $\mathbf{c}$; and $CN(0,\sigma_f^2)$ represents the circular symmetric complex Gaussian random variable with zero mean and variance $\sigma_f^2$.

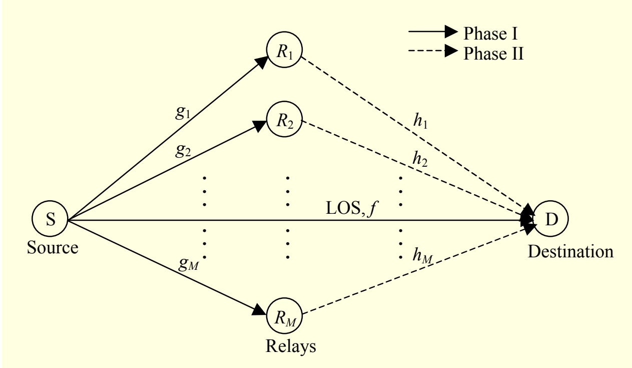


Fig. 1. WRN.

## II. System Model

Consider a WRN with a single source node $S$, a single destination node $D$, and $M$ randomly placed relays $R_i$; $i$=1,2,…,$M$, as shown in Fig. 1. The channel between each node pair is assumed to be quasi-stationary Rayleigh flat fading. It implies that the channel is constant within one frame but may vary from frame to frame. Denote the channel from source node to destination node as $f$, from source node to relays as $g_i$, and from relays to destination node as $h_i$. It is assumed that the channels are represented mathematically by $f \in CN(0,\sigma_f^2)$, $g_i \in CN(0,\sigma_{gi}^2)$, and $h_i \in CN(0,\sigma_{hi}^2)$.

If the source node intends to transmit a training data sequence through the AF mode to the destination node, the transmission is accomplished in two phases, namely phase I and phase II with each phase containing $T$ consecutive time slots for individual relays and then for relaying information toward the destination node. In phase I, the source node $S$ broadcasts the training data sequence $\mathbf{z}$ of size $N \times 1$ to all the relays $R_i; i = 1,2,...,M$. It is assumed that the training data sequence satisfies the power constraint, which is represented as $\mathbf{z}^H\mathbf{z} \le NP_S = E_S$, where $P_S$ is the source node power and $E_S$ is the source node energy. In this paper, the training data sequence is chosen to be $\mathbf{z} = \sqrt{P_S}\mathbf{1}_N$ since it reduces the peak to average power ratio (PAPR) problem at the source node and satisfies the power constraint [4]. The vector $\mathbf{1}_N$ is an $N \times 1$ vector with all its elements in unity. In phase I, the received signal at the relays $R_i$ $(i = 1,2,...,M)$ is represented by

$$\mathbf{r}_i = g_i\mathbf{z} + \mathbf{n}_{ri}, \quad \text{for } 1 \le i \le M, \tag{1}$$

where $\mathbf{n}_{ri}$ is an $N \times 1$ circularly symmetric complex Gaussian white noise vector and $\mathbf{n}_{ri} \in CN(0, N_0\mathbf{I})$ with noise variance $N_0$.

In phase II, the relays send the transformed training signal to the destination node $D$. At each relay, a linear transformation $\mathbf{t}_i$ is applied to exploit diversity, and it is expressed as

$$\mathbf{t}_i = \alpha_i \mathbf{A}_i \mathbf{r}_i \,. \tag{2}$$

The real scaling factor $\alpha_i = \sqrt{P_{ri} / (\sigma_{gi}^2 + N_0)}$ is introduced in the linear transformation to satisfy the individual power constraint at each relay $R_i$ as $P_{ri}$. This constraint becomes essential since the relay networks conduct distributed transmissions. At the $i$-th relay, $\mathbf{A}_i$ is an $\mathbf{N}\times\mathbf{N}$ unitary precoding matrix [4]. The precoding matrix $\mathbf{A}_i$ is designed such that it satisfies the following condition:

$$\mathbf{z}\,\mathbf{A}_i \mathbf{A}_j^H \mathbf{z} = \begin{cases} 0 & ;\ \text{if } i \neq j, \\ NP_S & ;\ \text{if } i = j. \end{cases}$$

In phase II, the $N \times 1$ signal vector received at the destination node D is represented as

$$\mathbf{d}_2 = \sum_{i=1}^{M} h_i \mathbf{t}_i + \mathbf{n}_{d2} \,, \tag{3}$$

where $\mathbf{n}_{d2} \in CN(0, N_0\mathbf{I})$ at destination node $D$. By using (2) as a substitute in (3), $\mathbf{d}_2$ is written as

$$\mathbf{d}_2 = \sum_{i=1}^{M} w_i \alpha_i \mathbf{A}_i \mathbf{z} + \sum_{i=1}^{M} h_i \alpha_i \mathbf{A}_i \mathbf{n}_{ri} + \mathbf{n}_{d2} \,,$$

where $w_i = h_i g_i;\ \ i = 1, 2, ...., M$. Let $\mathbf{w} = [w_1\ w_2\ ....\ w_M]^T$. In matrix form, $\mathbf{d}_2$ is given by

$$\mathbf{d}_2 = \mathbf{C}\boldsymbol{\Delta}\mathbf{w} + \mathbf{n_d} \,, \tag{4}$$

where $\mathbf{C} = [\mathbf{A}_1\mathbf{z}\ \mathbf{A}_2\mathbf{z}\ ....\mathbf{A}_M\mathbf{z}]$ is an $\mathbf{N}\times\mathbf{M}$ matrix, $\boldsymbol{\Delta} = diag\{\alpha_1, \alpha_2, ...., \alpha_M\}$ is an $\mathbf{M}\times\mathbf{M}$ matrix of scaling factors, and $\mathbf{n}_d = \sum_{i=1}^{M} h_i \alpha_i \mathbf{A}_i \mathbf{n}_{ri} + \mathbf{n}_{d2}$. Furthermore, it is easily checked that the covariance of $\mathbf{n}_d$, conditioned on a specific realization of $h_i$, is given as $\text{cov}(\mathbf{n}_d / h_i, i = 1, 2, ...., M) = (\sum_{i=1}^{M} |h_i|^2 \alpha_i^2 + 1) N_0 \mathbf{I}$, where the property $\mathbf{A}_i \mathbf{A}_i^H = \mathbf{I}$ is utilized. Therefore, the overall noise, under a specific realization of $h_i$, is still white Gaussian but with a scaled covariance of $N_0\mathbf{I}$ [4].

## III. BLUE Approach for Channel Estimation

The linear estimator for determining the $M\times1$ overall channel coefficient vector [11] is given by

$$\hat{w}_i = \sum_{n=0}^{N-1} b_{in} d_2(n) \quad \text{for} \quad i = 1, 2, ...., M \,, \tag{5}$$

where $b_{in}$ is the weight coefficient of the $n$-th sample in the $i$-th relay. The weight coefficients are represented in vector form as $\mathbf{b}_i = [b_{i0}\ \ b_{i1} \ldots\ \ b_{i(N-1)}]^T$. In matrix form, (5) is written as

$$\hat{\mathbf{w}} = \mathbf{B}\mathbf{d}_2 \,, \tag{6}$$

where $\mathbf{B}$ is an $\mathbf{M}\times\mathbf{N}$ matrix given by

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}_1^T & \mathbf{b}_2^T & . & . & \mathbf{b}_M^T \end{bmatrix}^T. \tag{7}$$

By using (4) as a substitute in (6), we get $\hat{\mathbf{w}} = \mathbf{B}\mathbf{C}\boldsymbol{\Delta}\mathbf{w} + \mathbf{B}\mathbf{n}_d$. It is known that the condition for the unbiased estimation of $\mathbf{w}$ is $E(\hat{\mathbf{w}}) = \mathbf{w}$. This is satisfied only if $\mathbf{BU} = \mathbf{I}$, where $\mathbf{U} = \mathbf{C}\boldsymbol{\Delta}$. Let $\mathbf{b}_i$ and $\mathbf{u}_j$ denote the $i$-th and $j$-th columns of $\mathbf{B}$ and $\mathbf{U}$, respectively. Then, the constraint for unbiased estimation is reduced to

$$\mathbf{b}_i^T \mathbf{u}_j = \delta_{ij} \,; \ \text{for}\ \ i = 1, 2, ...., M \,, j = 1, 2, ...., M \,. \tag{8}$$

The variance of the estimator is

$$\sigma_{\hat{w}_i}^2 = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i \,, \tag{9}$$

where $\sigma_{\hat{w}_i}^2$ is the variance of the estimate of the $i$-th coefficient $(\sigma_{\hat{w}_i}^2 = \text{var}(\hat{w}_i))$ and $\mathbf{R}_n = E[\mathbf{n}_d \mathbf{n}_d^H]$ is the $\mathbf{N}\times\mathbf{N}$ noise covariance matrix. In addition to the unbiased constraint, the expression $\mathbf{BU}=\mathbf{I}$ also satisfies the individual relay power constraint of the relays. The estimate of $\mathbf{w}$ is derived by solving the linear constraint problem [11] by using the Lagrangian method,

$$\begin{aligned} &\text{Minimize } \sigma_{\hat{w}_i}^2 = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i, \\ &\text{subject to } \mathbf{b}_i^T \mathbf{u}_j = \delta_{ij}; i = 1, 2, ...., M, j = 1, 2, ...., M. \end{aligned} \tag{10}$$

This constraint optimization problem is solved using the Lagrangian method by minimizing the cost function, which is given as

$$J_i = \mathbf{b}_i^T \mathbf{R}_n \mathbf{b}_i + \sum_{j=1}^{M} \lambda_j^{(i)} (\mathbf{b}_i^T \mathbf{u}_j - \delta_{ij}) \,. \tag{11}$$

The gradient of (11) with respect to $\mathbf{b}_i$ is derived to be

$$\frac{\partial J_i}{\partial \mathbf{b}_i} = 2\mathbf{R}_n^{-1} \mathbf{b}_i + \mathbf{C}\boldsymbol{\Delta}\boldsymbol{\lambda}_j \,.$$

By equating the gradient to zero, $\mathbf{b}_i$ is determined as

$$\mathbf{b}_i = -\frac{1}{2} \mathbf{R}_n^{-1} \mathbf{C}\boldsymbol{\Delta}\boldsymbol{\lambda}_i \,. \tag{12}$$

Let $\mathbf{e}_i$ represent the vector of all zeros except in the $i$-th place, and, using (12), the constraint equation (10) is written as

$$(\mathbf{C}\boldsymbol{\Delta})^T (-\frac{1}{2} \mathbf{R}_n^{-1} \mathbf{C}\boldsymbol{\lambda}_i) = \mathbf{e}_i \,. \tag{13}$$

By solving (13), $\boldsymbol{\lambda}_i$ is given by

$$\lambda_i = -2((\mathbf{C\Delta})^T \mathbf{R}_n^{-1}(\mathbf{C\Delta}))^{-1}\mathbf{e}_i . \quad (14)$$

By using (14) as a substitute in (12), $\mathbf{b}_i$ is written as

$$\mathbf{b}_i = \mathbf{R}_n^{-1}\mathbf{C\Delta}((\mathbf{C\Delta})^T \mathbf{R}_n^{-1}(\mathbf{C\Delta}))^{-1}\mathbf{e}_i . \quad (15)$$

By using (15) as a substitute in (6) and using (7), BLUE is given by

$$\hat{\mathbf{w}}_{\text{BLUE}} = \begin{bmatrix} [\mathbf{R}_n^{-1}\mathbf{C\Delta}[(\mathbf{C\Delta})^T\mathbf{R}_n^{-1}\mathbf{C\Delta}]^{-1}]^T\mathbf{e}_1^T\mathbf{d}_2 \\ [\mathbf{R}_n^{-1}\mathbf{C\Delta}[(\mathbf{C\Delta})^T\mathbf{R}_n^{-1}\mathbf{C\Delta}]^{-1}]^T\mathbf{e}_2^T\mathbf{d}_2 \\ . \\ . \\ [\mathbf{R}_n^{-1}\mathbf{C\Delta}[(\mathbf{C\Delta})^T\mathbf{R}_n^{-1}\mathbf{C\Delta}]^{-1}]^T\mathbf{e}_M^T\mathbf{d}_2 \end{bmatrix}. \quad (16)$$

Further, (16) can be simplified as

$$\hat{\mathbf{w}}_{BLUE} = ((\mathbf{C\Delta})^T\mathbf{R}_n^{-1}\mathbf{C\Delta})^{-1}(\mathbf{C\Delta})^T\mathbf{R}_n^{-1}\mathbf{d}_2 . \quad (17)$$

By using (15) as a substitute in (9), the variance of the estimator is

$$\sigma_{\hat{w}_i}^2 = \mathbf{e}_i^T[(\mathbf{C\Delta})^T\mathbf{R}_n^{-1}(\mathbf{C\Delta})]^{-1}\mathbf{e}_i . \quad (18)$$

The MSE of the BLUE estimation algorithm is defined as $MSE = tr\left\{E\left\{(\hat{\mathbf{w}}_{\text{BLUE}} - \mathbf{w})(\hat{\mathbf{w}}_{\text{BLUE}} - \mathbf{w})^H\right\}\right\}$, and it is derived as

$$MSE = tr\left\{(\mathbf{C\Delta})^{-1}\left((\mathbf{C\Delta})^T\right)^H\left(\left((\mathbf{C\Delta})^T\,\mathbf{R}_n^{-1}\,\mathbf{C\Delta}\right)^{-1}\right)^H\right\}. \quad (19)$$

## IV. CRLB for WRN with AF Strategy

For coherent detection in AF mode, maximum likelihood detection is performed in destination node $D$, based only on a specific channel realization $w_i$ while treating $\mathbf{n}_d$ as the overall white Gaussian noise [4]. Hence, the likelihood function for the received signal model in (4) is determined as

$$p(\mathbf{d}_2;\mathbf{w}) = \frac{1}{(2\pi\det(\sigma^2))^{N/2}}\exp\left\{\frac{-1}{2\sigma^2}(\mathbf{x}^H\mathbf{x})\right\}, \quad (20)$$

where $\mathbf{x} = (\mathbf{d}_2 - \mathbf{C\Delta w})$ and the $\mathbf{N}\times\mathbf{N}$ covariance matrix of $\mathbf{n}_\mathrm{d}$ is $\mathbf{R}_n$. Taking a natural logarithm on both sides of the likelihood function of (20) results in

$$\ln(p(\mathbf{d}_2;\mathbf{w})) = \ln\left(\frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}\right) - \frac{1}{2\sigma^2}\left\{\mathbf{x}^H\mathbf{x}\right\}. \quad (21)$$

Omitting the first term in (21) and using the definition $\mathbf{x} = (\mathbf{d}_2 - \mathbf{C\Delta w})$, the log likelihood function is written as

$$\ln(p(\mathbf{d}_2;\mathbf{w})) = -\frac{1}{2\sigma^2}\left\{\mathbf{d}_2^H\mathbf{d}_2 - \mathbf{d}_2^H\mathbf{C\Delta w} - (\mathbf{C\Delta w})^H\mathbf{d}_2 + (\mathbf{C\Delta w})^H\mathbf{C\Delta w}\right\}. \quad (22)$$

The first term in (22) is independent of $\mathbf{w}$, and terms $\mathbf{d}_2^H\mathbf{C\Delta w}$, $(\mathbf{C\Delta w})^H\mathbf{d}_2$, and $(\mathbf{C\Delta w})^H\mathbf{C\Delta w}$ in (22) are expanded as follows, to find derivatives of individual weight coefficients:

$$\mathbf{d}_2^H\mathbf{C\Delta w} = \sum_{k=1}^{N}\sum_{i=1}^{N}\Delta_i c_{ki} w_i d_2^*(k), \quad (23)$$

$$(\mathbf{C\Delta w})^H\mathbf{d}_2 = \sum_{k=1}^{N}\sum_{i=1}^{N}\Delta_i^* c_{ki}^* w_i^* d_2(k), \quad (24)$$

and

$$(\mathbf{C\Delta w})^H\mathbf{C\Delta w} = \sum_{k=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\Delta_i\Delta_j^* c_{ki} c_{kj}^* w_i w_j^* . \quad (25)$$

Using (23), (24), and (25) as substitutes in (22), we obtain

$$\begin{aligned}\ln(p(\mathbf{d}_2;\mathbf{w})) = &-\frac{1}{2\sigma^2}\left\{\mathbf{d}_2^H\mathbf{d}_2 - \sum_{k=1}^{N}\sum_{i=1}^{N}\Delta_i c_{ki} w_i d_2^*(k)\right\} \\ &+\frac{1}{2\sigma^2}\left\{\sum_{k=1}^{N}\sum_{i=1}^{N}\Delta_i^* c_{ki}^* w_i^* d_2(k)\right\} \\ &-\frac{1}{2\sigma^2}\left\{\sum_{k=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\Delta_i\Delta_j^* c_{ki} c_{kj}^* w_i w_j^*\right\}.\end{aligned} \quad (26)$$

The CRLB for a vector parameter $\mathbf{w} = [w_1 \;\; w_2 \ldots w_M]^T$ is defined as

$$\sigma_{\hat{w}_i}^2 \geq [\mathbf{I}^{-1}(\mathbf{w})]_{ii} \quad \text{for } i = 1, 2\cdots M, \;\; [11] \quad (27)$$

where the Fisher information matrix (FIM) $\mathbf{I}(\mathbf{w})$ is defined as

$$[\mathbf{I}(\mathbf{w})]_{ij} = -E\left[\frac{\partial^2 \ln(p(\mathbf{d}_2;\mathbf{w}))}{\partial w_i \partial w_j}\right] \text{for } i, j = 1, 2, \cdots, M \quad (28)$$

and the values of $[\mathbf{I}(\mathbf{w})]_{ij}$ for $i, j = 1, 2\cdots M$ can be found using (26), and the FIM $\mathbf{I}(\mathbf{w})$, in general, is written as

$$\mathbf{I}(\mathbf{w}) = \begin{bmatrix} \frac{\Delta_1^2\left(\sum_{m=1}^{M} c_{m1}^2\right)}{\sigma^2} & 0 & \cdots & 0 \\ 0 & \frac{\Delta_2^2\left(\sum_{m=1}^{M} c_{m2}^2\right)}{\sigma^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\Delta_M^2\left(\sum_{m=1}^{M} c_{mM}^2\right)}{\sigma^2} \end{bmatrix}. \quad (29)$$

Since the FIM is a diagonal matrix, it can easily be inverted to obtain the CRLB for the vector parameter. Using (29) as a substitute in (27), the CRLB is given by

$$\sigma_{\hat{w}_i}^2 \geq \frac{\sigma^2}{\Delta_i^2 \left( \sum_{m=1}^{M} c_{mi}^2 \right)} \quad \text{for } i = 1, 2, \cdots, M . \tag{30}$$

## V. Results and Discussion

In this section, the MSE performance of the proposed BLUE channel estimation algorithm, theoretical MSE, and the CRLB are analyzed using an optimal training sequence and precoding matrix [4] with channels $g_i$, $h_i$, and noise assumed to be circularly symmetric Gaussian random variables with unit variances. The noise sequence is assumed to be circularly symmetric complex Gaussian with zero mean and unit variance. The signal-to-noise ratio (SNR) is defined as SNR = $(P_S \times 1)/N_0 = P_S$, where $P_S$ is the source node power. In all the simulations, 1,000 independent Monte-Carlo runs are used.

Figure 2 shows the MSE performance of the proposed algorithm with four relay nodes $M$=4 and the length of the optimal training sequence $N$=4, along with the relay powers as $\{0.8P_S, P_S, 0.8P_S, P_S\}$. The MSE performance of the LS and MMSE channel estimators are shown for comparison. It is observed that the proposed algorithm achieves an MSE of $10^{-2}$ at 15 dB, whereas conventional LS and MMSE estimators take 20 dB and 17 dB, respectively. The proposed BLUE algorithm shows an improvement in MSE performance since it gives a minimum variance on the estimation parameter, a quality that is not inherent in the LS estimator or the MMSE estimator. Also, having the optimal precoding matrix [4] employed at the relays and using the optimal training sequence, which minimizes the PAPR, results in superior performance. The simulation results are validated with the derived theoretical MSE of the BLUE algorithm. The CRLB reaches the MSE of $10^{-2}$ at 10 dB. This clearly shows the importance of the BLUE channel estimation algorithm for AF WRN in comparison to MMSE estimators.

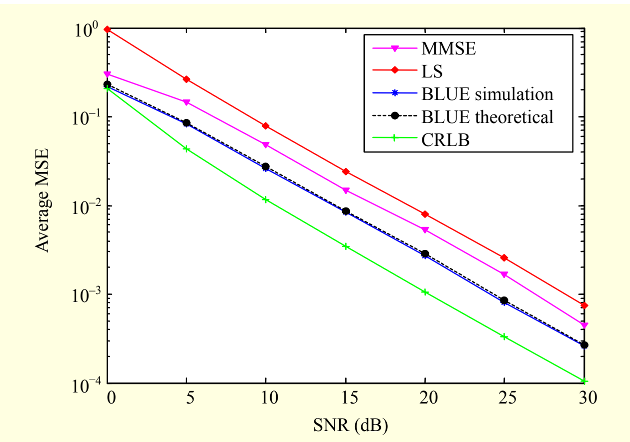


Fig. 2. MSE performance of channel estimation algorithms at $M$=4 ; $N$=4.

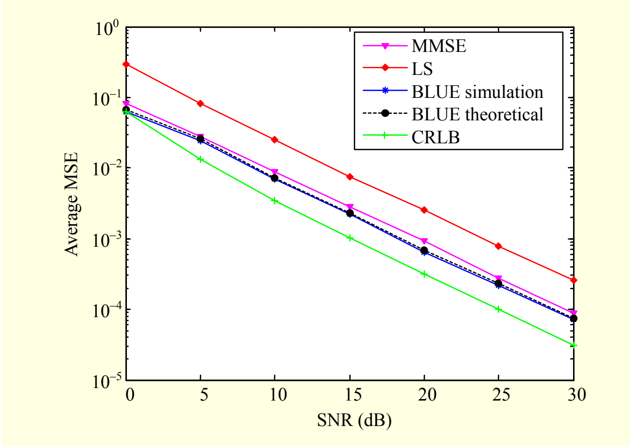


Fig. 3. MSE performance of channel estimation algorithms at $M$=8 ; $N$=8.

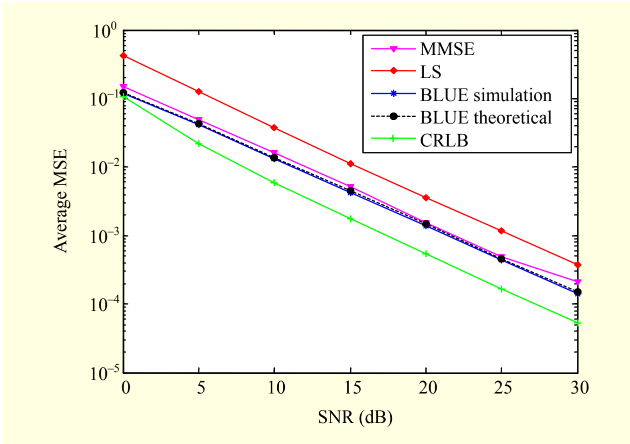


Fig. 4. MSE performance of channel estimation algorithms at $M$=4; $N$=8.

Figure 3 shows the MSE performance of the proposed algorithm with eight relay nodes $M$=8 and the length of optimal training sequence $N$=8. It is observed that the proposed algorithm requires 9 dB only, compared to 15 dB when $N$=4 and $M$=4, to achieve the MSE of $10^{-2}$. The CRLB reaches the MSE of $10^{-2}$ at 6 dB. This MSE performance improvement is due to an increase in the number of relays and the length of the training sequence, which validates simultaneous training for all relay channels instead of trivially training each relay channel individually. However, the deployment of an additional number of relays is not considered in practice due to its implementation aspects.

Figure 4 analyzes the significance of the increase in the length of the training sequence from $N$=4 to $N$=8 with $M$=4

Table 1. SNR requirement of channel estimation algorithms at MSE of $10^{-2}$.

(dB)

| | $M$=2, $N$=2 | $M$=4, $N$=4 | $M$=8, $N$=8 | $M$=2, $N$=8 | $M$=4, $N$=8 |
|---|---|---|---|---|---|
| LS | 24 | 20 | 15 | 15 | 16 |
| MMSE | 21 | 17 | 10 | 15 | 14 |
| BLUE | 20 | 15 | 9 | 14 | 11 |
| CRLB | 15 | 10 | 6 | 12 | 8 |

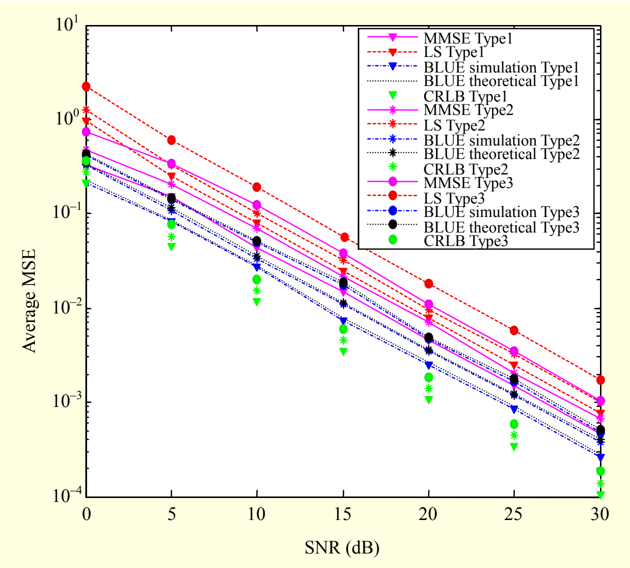


Fig. 5. MSE performance of channel estimation algorithms at $M$=4; $N$=4: unbalanced relay power distribution.

relays. Now, the proposed algorithm requires 11 dB, compared to $N$=4, which requires 15 dB. In this scenario, half of the relays have a power of $\{0.8P_S\}$ and the other half have a power of $\{P_S\}$, which contributes to the decrease in channel estimation MSE when $N$ increases. In addition, the transmitting energies $E_S$ and $E_{ri}$ are linear functions of $N$. Hence, the requirement of employing more relays for channel estimation can also be minimized. The proposed algorithm BLUE interestingly outperforms LS and MMSE estimators by virtue of its minimum variance attribute, which happens to be a function of SNR. Table 1 shows the simulation results of the proposed algorithm for different lengths of the training sequence and the number of relays for the LS and MMSE algorithms and the CRLB.

Figure 5 shows the performance of the proposed BLUE channel estimation algorithm, assuming that the relays have different powers. Three "unbalanced" types of power distribution are considered: Type1, represented by $\{0.8P_S, 0.8P_S, P_S, P_S\}$; Type2, represented by $\{0.4P_S, 0.8P_S, P_S, 1.4P_S\}$; and Type3, represented by $\{0.2P_S, 0.4P_S, P_S, 2P_S\}$. Type1 exhibits the best MSE performance, since two of the relays have the same power. Note that unbalanced power distribution will deteriorate the accuracy of the channel estimation.

## VI. Conclusion

In this paper, a channel estimation algorithm based on the BLUE was developed for AF WRNs. It was proven by theoretical MSE derivation and simulation that the proposed algorithm shows better MSE performance than that of the LS and MMSE estimators for the same precoding matrix employed at relays and the same training data sequence broadcast from the source node. The BLUE's improvement in MSE performance is due to its inherent minimum variance characteristic, which makes it a minimum variance unbiased estimator. Although the BLUE is analogous to the LMMSE estimator, the BLUE does not require the knowledge channel covariance matrix, whereas the LMMSE does, and that matrix is difficult for the LMMSE to acquire in practice. Furthermore, the BLUE algorithm is simple in practical implementation in comparison to the MMSE estimator, as complete knowledge of the PDF is not essential. The CRLB for the channel estimation in an AF-based WRN is also derived, which acts as a benchmark for assessing the estimator performance. In future works, this paper can be extended to select the best number of AF relays for activation using the BLUE-based channel estimation technique.

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**M.PremKumar** graduated with a BE in electronics and communication engineering from K.L.N. College of Engineering, Madurai Kamaraj University, Madurai, Tamil Nadu, India, in 2001. He obtained his ME in communication systems from the MEPCO SCHLENK Engineering College, Anna University, Chennai, Tamil Nadu, India, in 2004. At present, he is working as an assistant professor in the Department of Electronics and Communication Engineering, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India. Additionally, he is working toward a PhD at Anna University. His current areas of research include signal processing for wireless communications and wireless relay networks.

**V.N.SenthilKumaran** graduated with a BE in electronics and communication engineering from Vel Tech Engineering College, University of Madras, Chennai, Tamil Nadu, India, in 2002. He obtained his ME in communication systems from S.R.M. Engineering College, Anna University, Chennai, Tamil Nadu, India, in 2004. At present, he is working as an assistant professor in the Department of Electronics and Communication Engineering, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India. Additionally, he is working toward a PhD at Anna University. His current areas of research include estimation algorithms, physical layer network coding, and capacity analysis of wireless networks.

**S.J.Thiruvengadam** received his BE in electronics and communication engineering from Thiagarajar College of Engineering, Madurai, Tamil Nadu, India, in 1991. He obtained his ME in applied electronics from Anna University, Chennai, Tamil Nadu, India, in 1994 and his PhD in the area of signal detection algorithms from Madurai Kamaraj University, Madurai, Tamil Nadu, India, in 2005. At present, he is working as a professor in the Department of Electronics and Communication Engineering, Thiagarajar College of Engineering. Previously, he was a visiting associate professor with the Department of Electrical Engineering, Stanford University, Stanford, CA, USA, under a postdoctoral fellowship sponsored by the Department of Science and Technology, Government of India, from January to December 2008. His current areas of research include statistical signal processing and multiple-input multiple-output wireless communications.