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분산 제약을 갖는 비선형 시스템의 최적 퍼지 필터

Optimal Fuzzy Filter for Nonlinear Systems with Variance Constraints

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요 약

본 논문에서는 추정 분산 제약을 갖는 비선형 이산시간에 대한 최적의 퍼지 필터에 대한 내용을 다루고자 한다. 필터를 설계할 때, 추정오차의 분산값은 필터의 성능이 결정하는 변수중 하나다. 이런 분산값에 더욱 강인한 필터를 설계하고자, 분산 제약 조건을 주어 필터를 설계하고자 한다. 먼저, 비선형 모델을 Tagaki-Sugeno 퍼지 모델을 이용하여 선형 모델로 변형한 후, 이 모델을 기반으로 선형 필터를 디자인한다. 이때 필터설계 과정 중 필터의 각 파라미터값을 구하기 위해 상태 추정오차 값은 평균제곱에 제한되며, 상태오차의 정상상태 분산값은 각각의 미리 정한 상한 제한 값 보다 작은 조건에서 필터를 설계하여 선형행렬부등식과 대수 이차 행렬부등식을 이용하여 파라미터값을 구한다. 이렇게 설계된 퍼지 필터는 트럭트레일러 시뮬레이션을 통해 설계 과정과 성능을 보여준다.

키워드 : 최적퍼지필터, 분산제약, T-S 퍼지모델, 비선형성, 선형행렬부등식, 대수이차행렬부등식

Abstract

In this paper, we consider the optimal fuzzy filter of nonlinear discrete-time with estimation error variance constraint. First, the Takagi and Sugeno(T-S) fuzzy model is employed to approximate the nonlinear system. Next, the error state is mean square bounded, and the steady state variance of the estimation error of each state is not more than the individual predefined value. It is shown that, the addressed problem can be carried out by solving linear matrix inequality(LMI) and some algebraic quadratic matrix inequalities. Finally, some examples are provided to illustrate the design procedure and expected performance through simulations..

Key Words : Optimal fuzzy filter, Variance constraints, linear matrix inequality (LMI), T-S fuzzy model., algebraic quadratic matrix inequalities

1. Introduction

The well-known Kalman filter provides a recursive algorithm to minimize the variance of the state estimation error when the measurement noise is known. This estimation approach may not be robust against moldering uncertainty and disturbances[1,2]. In recent year, there has been considerable attention to robust estimation and H_∞filtering problems for dynamic systems[3]. For example, a robust Kalman filtering problem with bounded parameter uncertainty was considered only in the state matrix [4]. The H_∞ filtering approach aims at minimizing the H_∞ norm of the transfer function from noises to the estimation error, while the robust filtering approach guarantees an upper bound to the quadratic cost in spite of various parameter uncertainties, and then minimizes this upper bound locally[5]. On the other hand, the robust filtering problem for T-S fuzzy systems has been investigated in the past decades [6]. Based on the T-S fuzzy model, the H_∞ fuzzy filtering design for the dynamic systems can be characterized in terms of minimizing the attenuation level subject to some LMIs[7]. It requires a common solution to a set of Ricatti equations. Based on the T-S fuzzy model, motivated by the above observations, we develop a robust fuzzy filter of nonlinear

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이 논문은 2012년 정부(교육과학기술부)의 재원으로 한국연구재단의 지원을 받아 수행된 연구임(No. 2012014088).
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discrete-time with estimation error variance constraint. A state estimation approach called error covariance assignment theory was first proposed in [8] and then extended to the nonlinear case. This theory provides an alternative, more straightforward technique to meet the prespecified estimation error variance constraints. The main idea is to design a filter which directly assigns the prespecified steady-state estimation error covariance. However, there are few papers developing the robust estimation technique for uncertain systems subject to the simultaneous achievement of error variance upper-bound constraint and H_∞ disturbance attenuation constraint.

In this paper, we consider the optimal fuzzy filter of nonlinear discrete-time with estimation error variance constraint. An error variance constrained filter admits the system to have invariant measurement noise with intensity as large as possible. First, the Takagi and Sugeno(T-S) fuzzy model is employed to approximate the nonlinear system. Next, the error state is mean square bounded, and the steady state variance of the estimation error of each state is not more than the individual prespecified value. It is shown that, the addressed problem can be carried out by solving linear matrix inequality (LMI) and some algebraic quadratic matrix inequalities. This paper is organized as follows: Section 2 formulate the problem under consideration. Optimal fuzzy filtering is introduced in Section 3. Simulation results is presented in Section 4, and finally we conclude the paper in Section 5.

2. Problem Statement

The system plant with missing measurements is represented by th T-S fuzzy model. This is described by the following IF-THEN rules and will be employed here to deal with the estimation design problem for the nonlinear system. The ith rule of the fuzzy linear model for the nonlinear system is of the following form:

$$\begin{aligned} R^i:\ & IF\ z_i(k)\ is\ \Gamma_1^i \text{ and ...and } z_q(t)\ is\ \Gamma_q^i \\ & THEN\ x(k+1) = A_i x(k) + w(k) \\ & \qquad y(k) = C_i x(k) + v(k), \qquad i = 1,2,...,p \end{aligned} \tag{1}$$

where Γ_{ij} is fuzzy set of $Z_g(k)$ in R^i; $x(k)$ denotes the ith fuzzy rule; $y(k)$ denotes the vector of the outputs; $z_q(k)$ are the premise variables. $w(k)$ and $v(k)$ are uncorrelated stationary zero mean white noise sequences with respective covariance Q and R. Using the center average defuzzifer, product inference, and singleton fuzzifier, the out put of fuzzy systems is inferred as follows:

$$\begin{aligned} x(k) &= \sum_{i=1}^{L} h_i(z(k))\,[A_i x(k) + w(k)] \\ y(k) &= \sum_{i=1}^{L} h_i(z(k))\,[C_i x(k) + v(k)] \end{aligned} \tag{2}$$

where $h_i(z(k)) = \dfrac{\mu_i(z(k))}{\sum_{i=1}^{p}\mu_i(z(k))}$, $\mu_i(z(k)) = \prod_j^q \Gamma_{ij} z_j(k)$

and $\Gamma_{ij}(z(k))$ is the grade of membership of $z_i(k)$ in Γ_{ij}. For For notational convenience in the following discussions, we denote that

$$A_i(h) = \sum_{i=1}^{p} A_i,\ \ B_i(h) = \sum_{i=1}^{p} B_i,\ \ C_i(h) = \sum_{i=1}^{p} C_i \tag{3}$$

Based on the fuzzy model (2), in this paper, the following fuzzy estimator is adopted

$$\hat{x}(k+1) = G\hat{x}(k) + Ky(k) \tag{4}$$

where $\hat{x}(k)$ stands for the state estimate, G and K are filter parameters to be designed.
Defined the estimation error and the estimation error covariance, respectively, as follows.

$$e(k) := x(k) - \hat{x}(k) \tag{5}$$

$$P(k) := E[e(k)e^T(k)] \tag{6}$$

Then, it follows from (2) and (4) and from (6) and (7) that

$$\begin{aligned} e(k+1) = &[A_i(h) - G - KC_i(h)]x(k) + Ge(k) \\ &+ w(k) - Kv(k) \end{aligned} \tag{7}$$

Define

$$x_f(k) = [x(k)\ \ e(k)]^T \tag{8}$$

$$A_f = \begin{bmatrix} A_i(h) & 0 \\ A_i(h) - G - KC_i(h) & G \end{bmatrix}, \tag{9}$$

$$B_f = \begin{bmatrix} I & 0 \\ I & -K \end{bmatrix}, \tag{10}$$

$$\begin{aligned} W_f &:= B_f B_f^T \\ &:= \begin{bmatrix} Q & Q \\ Q & KRK^T \end{bmatrix} \end{aligned} \tag{11}$$

$$\begin{aligned} X(k) &:= E[x_f(k) x_f^T(k)] \\ &:= \begin{bmatrix} X_{xx}(k) & X_{xe}(k) \\ X_{xe}^T(k) & X_{ee}(k) \end{bmatrix}. \end{aligned} \tag{12}$$

Considering (2) and (4), we obtain the following augmented system.

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f w_f(k) \\ x_x(0) &= [x(0)\quad x(0) - \hat{x}(0)]^T \end{aligned} \tag{13}$$

where w_f stands for a zero mean Gaussian white noise sequence with covariance $I > 0$.

3. Optimal Fuzzy Filtering

In this section, a solution to the variance constrained state estimation problem formulated in the previous section will be obtained. The purpose of this paper is to design the filter parameters , G, and K. The following requirements are met simultaneously.

1) The state of the augmented system (13) is mean square bounded.

2) The steady-state erro covariance X_{ee} meets

$$[x_{ee}(k)] \le \alpha_i^2,\ i=1,2,...,n \quad (14)$$

where $[X_{ee}]_{ii}$ means the steady-state variance of the ith error state, and α_{ii}^2 denotes the prespecified steady-state error estimation variance constraint on the ith state.

The following lemma is useful in the proof of main results.

Lemma 1(Schur complement):Given constant matrices $\Omega_1, \Omega_2, \Omega_3,$ where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$,then, $\Omega_1 + \Omega_3^t \Omega_2^{-1}\Omega_3 < 0$ is and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0 \quad (15)$$

Lemma 2: [9] For a given negative definite matrix $\Pi < 0$, there always exists a matrix $L \in R^{n \times p}(p \le n)$ such that $\Pi + LL^T < 0$.

Lemma 3 (Matrix Inverse Lemma): Let A, B, C and D be given matrices of appropriate dimension with A, D, and $D^{-1}+CA^{-1}B$ being invertible;
$(A+BDC)^{-1} = A^{-1} - A^{-1}B(D^{-1}+CA^{-1}B)^{-1}CA^{-1}$
hold.

Lemma 4 [10]: Let a positive scalar $\epsilon > 0$ and a positive definite matrix $\theta_f > 0$ be such that

$$N_f \theta_f N_f^T < \epsilon I$$

Then, we have that

$$A_f \theta_f A_f^T \le \epsilon I. \quad (16)$$

Using the statistics of the noise $w(k)$, $v(k)$, $X(k+1)$ defined in (12) is found to satisfy.

We know from [11] that, if the state of (13) is mean square bounded, the steady-state covariance X of (13) defined by

$$X := \lim_{k \to \infty} X(k) \quad (17)$$

exists and satisfied the following discrete-time modified Lyapunov equation:

$$X = A_f X A_f^T + W_f \quad (18)$$

Theorem 1 Assume that there exists a positive scalar $\epsilon > 0$ such that following two quadratic matrix inequalities:

$$A_i(h) P_1 A_1^T(h) - P_1 + W < 0 \quad (19)$$

$$GP_2G^T - P_2 + (A_i(h) - G - KC_i(h))P_2^{-1}(A_i(h) - G - KC_i(h))^T < 0 \quad (20)$$

respectively have positive-definite solutions $P_1 < 0$ and $P_2 > 0$, where

$$G = A_i(h) + (\varepsilon I + W)A_i^{-1}(h)^T(P_1^{-1} - \varepsilon^{-1}I^T I) \quad (21)$$

And by using Lemma 2, $L \in R^{n \times p}(p \le n)$ and $U \in R^{p \times p}$ is an arbitrary orthogonal matrix. Then, the fuzzy estimatorr (4) with the parameters determined by (21) and

$$K = GP_2G^T R^{-1} + LU\Xi^{1/2} \quad (22)$$

Proof : Since $A_i(h)$ is assumed to be nonsingular. We set

$$P_f := \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}.$$

Then, by means of Lemma 4, it is easily verified that

$$A_f P_f A_f^T - P_f + W_f := \Psi := \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{12}^T & \Psi_{22} \end{bmatrix} \quad (23)$$

where

$$\Psi_{11} = A_i(h)(P_1^{-1} - \varepsilon^{-1}I^T I)A_i^T - P_1 + W \quad (24)$$

$$\Psi_{12} = A_i(h)(P_1^{-1} - \varepsilon^{-1}I^T)(A_i(h) - G - KC_i(h))^T + W \quad (25)$$

$$\begin{aligned} \Psi_{22} = {} & GP_2G^T - P_2 \\ & + (A_i(h) - G - KC_i(h))(P_1^{-1} - \varepsilon^{-1}I^T I)^{-1} \\ & \bullet (A_i(h) - G - KC_i(h))^T + W + KRK^T \end{aligned} \quad (26)$$

It follows from the matrix inverse Lemma 3 that

$$(P_1^{-1} - \epsilon^{-1}I^T I)^{-1} = P_1 + P_1 I^T(\epsilon I - IP_1 I^T)^{-1} IP_1$$

and, therefore, equation (19) of Theorem1 implies that

$$\Psi_{11} < 0.$$

And substituting the expression of G in (21) into (25) leads to $\Psi_{12} = 0$ easily.

Next, we will consider Ψ_{22}. First, for presentation convenience, we denote

$$\Sigma = (A_i(h) - G)(P_1^{-1} - \epsilon^{-1}I^T I)^{-1}(A_i(h) - G)^T + W \quad (27)$$

$$\Xi = C_i(h) P_2 C_i^T + V \quad (28)$$

$$\Pi := \Sigma + GP_2G^T - P_2 - GP_2C_i^T X^{-1} C_i P_2 G^T \quad (29)$$

By using the definitions (27)-(29), we can rearrange (26) as follow:

$$\begin{aligned}\Psi_{22} &= \Sigma+(G-KC_i(h))P_2(G-KC_i(h))^T-P_2 \\ &\quad +KC_i(h)P_2C_i(h)^TK^T+KQK^T \\ &= \Sigma+GP_2G^T-P_2+K[C_i(h)P_2C_i(h)^T+R]K^T \\ &\quad -GP_2C_i^TK^T-KC_i(h)P_2G^T \\ &= \Sigma+GP_2G^T-P_2-GP_2C_i(h)^T\Xi^{-1}CP_2G^T \\ &\quad +(K\Xi^{1/2}-GP_2C_i(h)^T\Xi^{-1/2}) \\ &\quad \times(K\Xi^{1/2}-GP_2C_i(h)^T\Xi^{-1/2})^T \\ &= \Pi+(K\Xi^{1/2}-GP_2C_i(h)^T\Xi^{-1/2}) \\ &\quad \times(K\Xi^{1/2}-GP_2C_i(h)^T\Xi^{-1/2})^T\end{aligned} \quad (30)$$

Noticing the expression of $K=GP_2G^TR^{-1}+LU\Xi^{1/2}$ in (22) and the fact $UU^T=I$, we obtain

$$(K\Xi^{1/2}-GP_2C^TR^{-1/2})(K\Xi^{1/2}-GP_2C^TR^{-1/2})^T=LL^T \quad (31)$$

And it follows from (30), the definition of the matrix L and equation (20) of Theorem 1 that

$$\Psi_{22}=\Pi+LL^T<0 \quad (32)$$

The proof is completed.

4. Simulation results

In this section is present a example. This is the same example as that considered in [10]. The vehicle dynamics and measurements can be approximated by the following equations. The following noisy fuzzy model can be used to represent the system:

$$\begin{aligned}&\text{if } z(k) \text{ is } F_i \\ &\text{then } x(k+1)=A_ix(k)+B_1u(k)+w(k) \\ &\qquad y(k)=C_ix(k)+v(k) \qquad i=1,2\end{aligned} \quad (33)$$

The membership function are defined as $F_1=\{about\ 0\}$ and $F_1=\{about\ \pm\pi\}$.

The membership grades

$$h_1=\left(1-\frac{1}{1+\exp(-3(z-\pi/2))}\right)\times\left(\frac{1}{1+\exp(-3(z+\pi/2))}\right)$$
$$h_2=1-h_1$$
$$A_1=\begin{bmatrix}1-VT/L & 0 & 0\\ VT/L & 1 & 0\\ (VT)^2/(2/L) & 1 & 0\end{bmatrix},$$
$$A_1=\begin{bmatrix}1-VT/L & 0 & 0\\ VT/L & 1 & 0\\ (VT)^2/(2/L)(\pi/100) & V/(\pi/100) & 1\end{bmatrix},$$
$$B_1=B_2=[\,VT/l\ \ 0\ \ 0\,]^T.$$

We use the following system parameters:

$l=2.8m,\quad L=5.5m,\ V=-1m/s,\quad T=0.5s$,

The steady-state error covariance X_{ee} meets

$$[X_{ee}]_{11}\le\alpha_1^2=0.35,\ [X_{ee}]_{22}\le\alpha_2^2=0.8$$

By using the Schur Lemma 1, we can convert (19) into the following linear matrix inequality(LMI) and then we solve ϵ, P_1, and the standard Riccati-like matrix inequality for equation (20) for P_2. We get a solution:

$\varepsilon=0.574$

$$P_1=\begin{bmatrix}-0.0412 & 0.0041 & 0.001\\ 0.0037 & -0.0245 & 0.874\\ 0.0045 & -0.2472 & 0.485\end{bmatrix} P_2=\begin{bmatrix}-0.274 & 0.241 & 0.5274\\ 0.174 & -1.241 & 0.4265\\ 0.0485 & -0.257 & 0.8713\end{bmatrix}$$

One of the fuzzy estimator parameter, G is calculated form (21) as follows:

$$G=\begin{bmatrix}0.521 & 0.175 & 0.2458\\ 0.034 & 1.255 & 0.4125\\ 0.741 & 0.814 & 0.2745\end{bmatrix}$$

To obtain another parameter K, we choose $L=0.3I$. Then, it follows from (22) that

$$K=\begin{bmatrix}-1.212 & 0.712 & 0.5741\\ 0.052 & -0.792 & 0.2243\\ 0.9482 & -0.3050 & 08425\end{bmatrix}$$

Fig. 1 shows the position estimation error of the unconstrained Kalman filter, and Fig. 2 shows the position estimation error of the proposed method. It can be seen that the proposed constrained filter results in much more accurate estimates than unconstrained filter.

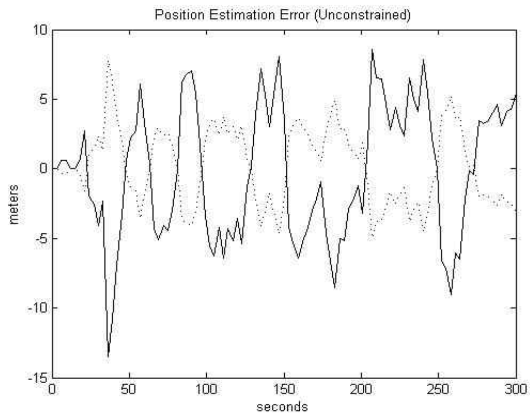

Fig. 1. Variance unconstrained position errors of 100 Monte Carlo simulations (Dotted line is north position, solid line is east position).

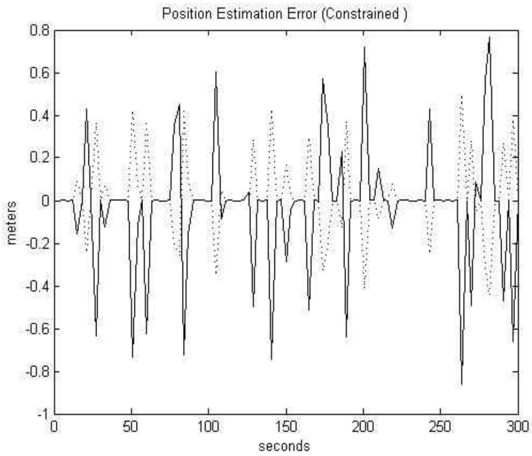

Fig. 2. Variance constrained position errors of 100 Monte Carlo simulations (Dotted line is north position, solid line is east position).

5. Conclusion

In this paper, we have considered the optimal fuzzy filter of nonlinear discrete-time with estimation error variance constraint. An error variance constrained filter admits the system to have invariant measurement noise with intensity as large as possible. First, the T-S fuzzy model has been employed to approximate the nonlinear system. Next, the error state was mean square bounded, and the steady state variance of the estimation error of each state was not more than the individual prespecified value. It has been shown that, the addressed problem can be carried out by solving linear matrix inequality (LMI) and some algebraic quadratic matrix inequalities.

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