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세 종류의 고장형태를 지닌 대기부품에 대한 주기적 검사정책 분석

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Analysis of Periodic Test Policy for a Standby Unit under Three Types of Failures

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대기부품은 대기기간 중에 우발적 고장이 발생할 수 있으며(type I failure), 해당상황이 장기간 방치되는 것을 방지하기 위해 주기적인 검사를 하는 것이 일반적이다. 그러나 검사가 대기기간 중 발생한 고장을 확인할 수 있게 하는 반면, 검사를 시작할 때 대기하던 부품에 부하를 가하는 과정에서 고장을 유발할 가능성이 존재하며(type Ⅱ failure), 검사시간동안 대기부품을 작동시킴으로써 열화에 의한 고장발생(type Ⅲ failure)의 가능성을 증가시키는 효과도 존재한다. 이에 본 논문은 주기적 검사정책을 갖는 대기부품을 대상으로 세 종류의 고장 가능성을 확률적으로 고려하여 성능분석을 실시하였으며, 성능을 평가하는 척도로 극한가용성을 사용하였다. 특히 type Ⅲ failure를 고려하는 것은 기존에 연구되지 않은 부분으로 본 논문의 기여점이라 할 수 있겠다. 또한 수치해석을 통해 가용성의 관점에서 전술한 세 가지 유형의 고장특성과 검사주기와의 관계를 파악할 수 있도록 하였으며, 그 결과를 통해 높은 수준의 신뢰성 확보가 목적인 대기시스템의 효율적인 운영을 위한 의사결정시 도움이 될 수 있도록 하였다.

Keywords : Standby Unit, Periodic Test, Availability

1. Introduction

In engineering field, we can find such kinds of systems that need to be kept on standby for operation until needed. For examples, a standby diesel generator of the cooling system in the nuclear power plant should always be ready only for when a priority diesel generator is failed. A fire protection system also should be kept on standby for operation whenever fire occurs. Missiles and spare parts of aircraft is also examples of such systems in that they should remain in storage state for operation whenever required. Such systems are usually called standby units and frequently hired for high mission reliability.

Comparing with the time on standby, the operation time of the standby units are very short, even nobody knows when the units are called. But failing to operate their functions when they are called may lead to catastrophic consequences, therefore standbys unit have been intensively investigated in reliability engineering fields [1, 4, 5, 7~12, 15~17].

In particular, regular inspection and maintenance policies for the standby units to avoid the occurrence of catastrophic consequences have attracted much attention to many researchers [4, 5, 10~12, 15]. More frequent tests can increase the likelihood of disclosing a failure, however, they may also deteriorate

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the unit faster [7]. Therefore, unit deterioration by test should be considered when test scheduling is determined.

This paper considers a standby unit which should be ready for operation for a long time whenever needed. Assuming the standby unit can fail during the standby period, the period test is considered for the standby unit [2, 3, 5, 10~14]. Although the periodic test is capable of detecting failures during the standby period, it can cause test-induced failures for the standby unit at the start of the test [6, 10, 12~15]. Also, the aging process of the standby unit during the test period needs to be taken into account, which is not treated in the existing studies.

This study adopts an availability of the standby unit as a meaningful measure of unit performance. The limiting availability is derived for the standby unit by incorporating three failure types : (i) type I failure; failure during availability for the standby period, (ii) type Ⅱ failure; test-induced failure at the start of the test and (iii) type Ⅲ failure; operating failures during the test period due to the aging effect.

2. Limiting Availability of the Standby Unit

The following assumptions and notations are used throughout this paper.

<Assumptions>

(1) Three failure types of a standby unit are independent of each other

(2) Type I failure rate is constant, i.e., type I failure occurs according to the Homogeneous Poison Process.

(3) Probability of test-induced(type Ⅱ) failure is constant.

(4) Type Ⅲ failure rate is increasing over cumulative test time until the standby unit is called for operation in place of the priority unit, i.e.,

$$\overline{F_s}(t|\gamma) = \frac{\overline{F_s}(\gamma+t)}{\overline{F_s}(\gamma)}. \tag{1}$$

<Notations>

T : test interval for the standby unit

τ : test period for the standby unit

$F_s(t)$: operating time distribution of the standby unit

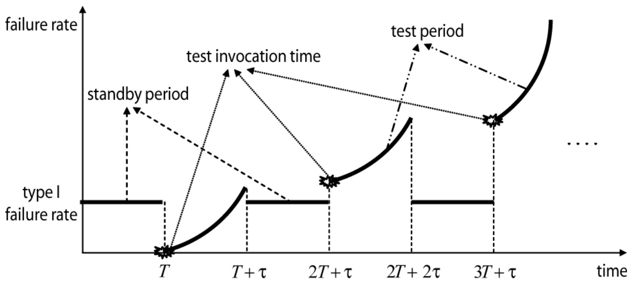
$G_s(t)$: repair time distribution of the standby unit

$H_s(t)$: standby failure time distribution of the standby unit

α : probability of type Ⅱ failure

$\beta = \int_0^T dH_s(x)$: probability of type I failure in each test interval

To derive availability of the standby unit, mean cycle time (MCT) and mean up time (MUT) must be calculated respectively. The standby unit undergoes standby and test states repeatedly until a failure for the standby unit occurs. The failure rate of the standby unit is constant during standby period. The failure rate function of the standby unit during test period is increasing function of the cumulative test duration. If no failure is found during a specific test period, the state of the standby unit at the beginning of the next test period is equal to that at the end of the previous test period. If any failure is found through the test, the standby unit undergoes repair action and is restored to the state of as-good-as-new. Therefore, from the initial standby status to the completion of the repair of the standby unit constitutes a cycle. <Figure 1> illustrates such aspect of the failure rate function of the standby unit.



<Figure 1> Failure Rate Function of the Standby Unit

<Figure 2>~<Figure 4> represent the mean cycle time and the mean up time for the following three failure type of the standby unit:

(i) type I failure; failure during standby period,

(ii) type Ⅱ failure; test-induced failure at the start of the test,

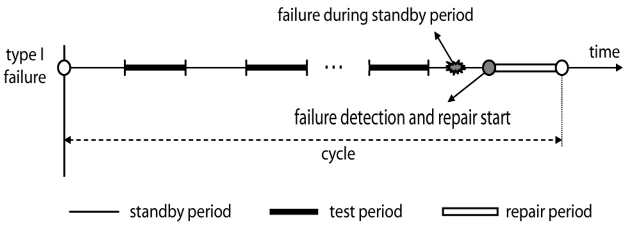
(iii) type Ⅲ failure; operating failure during the test period.

2.1 Mean Cycle Time (MCT)

To derive MCT of the standby unit, three failure types of the standby unit should be considered. Firstly, if the standby unit fails according to the type I, i.e., the standby unit has failed during the standby period and this failure is found at the beginning of the test as depicted in <Figure 2>, the mean cycle time due to type I failure can be written as

$$MCT_{type\,I} = \sum_{j=1}^{\infty}(1-\alpha)^{j-1}(1-\beta)^{j-1}\overline{F_s}[(j-1)\cdot\tau] \times \beta\cdot[j\cdot T+(j-1)\cdot\tau]. \quad (2)$$

Eq. (2) implies that there is no failure from the first standby period to $(j-1)^{th}$ test, and the failure occurring during j^{th} standby period is found at $[j\cdot T+(j-1)\cdot\tau]$.

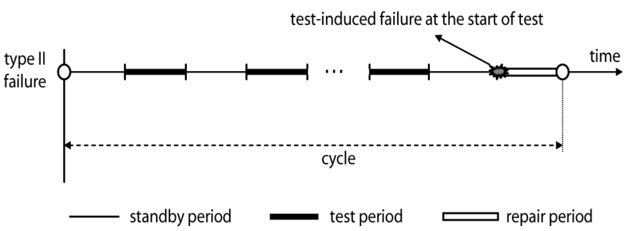


<Figure 2> Mean Cycle Time of Type I Failure Case

Secondly, if the standby unit fails according to the type II failure (test-induced failure case), i.e., the standby unit fails and is found at the start of the test as depicted in <Figure 3>, the mean time due to type II failure can be written as

$$MCT_{type\,II} = \sum_{j=1}^{\infty}(1-\alpha)^{j-1}(1-\beta)^{j-1}\overline{F_s}[(j-1)\cdot\tau] \times(1-\beta)\cdot\alpha\cdot[j\cdot T+(j-1)\cdot\tau]. \quad (3)$$

Eq. (3) can be obtained by similar way to obtain eq. (2), and note that the time to find the failure is same to the type I failure case.

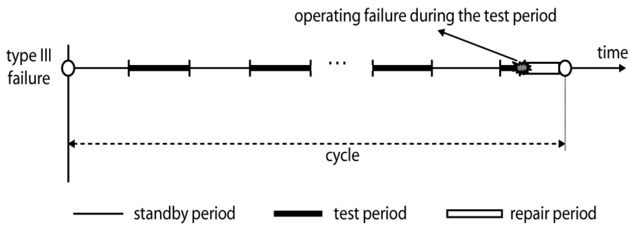


<Figure 3> Mean Cycle Time of Type II Failure Case

Thirdly, if the standby unit fails according to the type Ⅲ failure, i.e., the standby unit fails during the test operation and the failure is found immediately as depicted in <Figure 4>, the mean cycle time due to type Ⅲ failure can be written as

$$MCT_{type\,III} = \sum_{j=1}^{\infty}(1-\alpha)^{j}(1-\beta)^{j} \times\int_{(j-1)\cdot\tau}^{j\cdot\tau}(j\cdot T+x)dF_s(x). \quad (4)$$

Eq. (4) means that there is no failure from first standby period to j^{th} standby period, and j^{th} test starts successfully but the failure occurs during j^{th} test, i.e., between $[j\cdot T+(j-1)\cdot\tau]$ and $[j\cdot T+(j-1)\cdot\tau+\tau]$.



<Figure 4> Mean Cycle Time of Type III Failure Case

Lastly, we should consider mean repair time (MRT) by each failure types to compute the mean cycle time. the MRTs due to three failure types are all the same, and easily ob`tained by the distribution of repair time. Therefore, the mean cycle time can be obtained by

$$MCT = MCT_{type\,I}+MCT_{type\,II}+MCT_{type\,III} + \int_0^{\infty}x\,dG(x) \quad (5)$$

2.2 Mean Up Time (MUT)

In a similar way, the mean up time (MUT) of the standby unit during cycle is derived by considering three failure types as depicted in <Figure 2>~<Figure 4>. Note that only standby period is considered in MUT.

Firstly, if the standby unit fails according to the type I, the mean up time can be written as

$$MUT_{type\,I} = \sum_{j=1}^{\infty}(1-\alpha)^{j-1}(1-\beta)^{j-1}\overline{F_s}[(j-1)\cdot\tau] \times\int_0^{T}(j-1)\cdot T+x\,dH_s(x), \quad (6)$$

If the standby unit has failed during the j^{th} standby period, the up time of the standby unit is the sum $(j-1)\cdot T$ and working time during j^{th} standby period.

secondly, if the standby unit fails according to the type Ⅱ, the failure occurs at the beginning of test, i.e., at $[j\cdot T+(j-1)\cdot\tau]$, and the uptime of the standby is $j\cdot T$, therefore the means up time due to type Ⅱ failure can be written as

$$MUT_{type\,II} = \sum_{j=1}^{\infty} (1-\alpha)^{j-1}(1-\beta)^{j-1}\overline{F_s}[(j-1)\cdot\tau] \times (1-\beta)\cdot\alpha\cdot j\cdot T \quad (7)$$

Lastly, if type III failure occurs at between $[j \cdot T+(j-1)\cdot\tau]$ and $[j\cdot T+(j-1)\cdot\tau+\tau]$, the up time of the standby unit is $j \cdot T$, therefore, the means up time due to type III failure can be written as

$$MUT_{type\,III} = \sum_{j=1}^{\infty} (1-\alpha)^{j}(1-\beta)^{j}\cdot j\cdot T \times \left[\overline{F_s}(j\cdot\tau) - \overline{F_s}(j\cdot\tau-\tau)\right]. \quad (8)$$

By eq(6)~(8), the mean up time can be obtained by

$$MUT = MUT_{type\,I} + MUT_{type\,II} + MUT_{type\,III}. \quad (9)$$

In conclusion, limiting availability of the standby unit can be derived as

$$A_{SB} = \frac{MUT_{type\,I} + MUT_{type\,II} + MUT_{type\,II}}{MCT_{type\,I} + MCT_{type\,II} + MCT_{type\,II} + \int_0^{\infty} x\, dG(x)}. \quad (10)$$

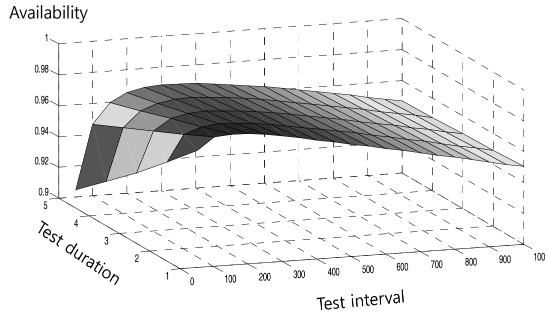
3. Numerical analysis

To identify relationships between the periodic test interval and various failure characteristics of the standby unit, several experiments are performed using the availability of the standby unit as measure of system performances.

In the experiments, life distribution of the standby unit in test operation is assumed to be Weibull with scale and shape parameters of 100 and 2 respectively. Repair time distribution of the standby unit is assumed to be the exponential with parameter of 0.2, i.e., MRT = 5. The type I failure rate is assumed to be constant, i.e., type I failure occurs according to the Homogeneous Poison Process, therefore, the failure time during each standby period follows exponential distribution with parameter of the type I failure (λ).

3.1 Sensitivity Analysis for Availability

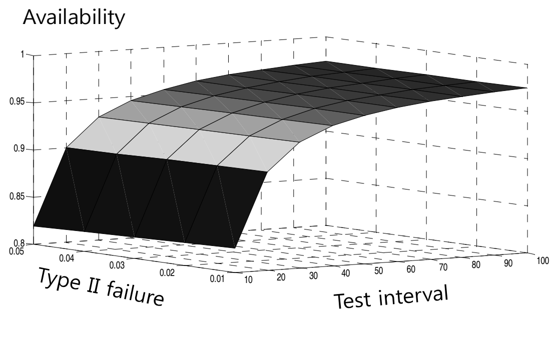
Variations of the test interval with respect to the changes of test duration (aging effect during test period) are observed for fixed $\alpha = 0.01$ and type I failure rate is 0.0001 ($H_s(t) = 1-e^{-t/10000}$, i.e., $h_s(t) = 0.0001e^{-t/10000}$). Result is depicted in <Figure 5> against performance measure of availability. For more comprehension, detailed values are provided in <Table 1> of appendix.



<Figure 5> Availability Versus T and τ

The availability is shown to be decreased as the test duration time τ increase (as the aging effect increases), especially, it is noticeable that availability is decreasing more rapidly when the test duration time τ is small. However, Availability with respect to increasing test interval T has turning point from increasing to decreasing as shown <Figure 5>. which means that the test interval has optimal point for maximizing availability of the standby unit.

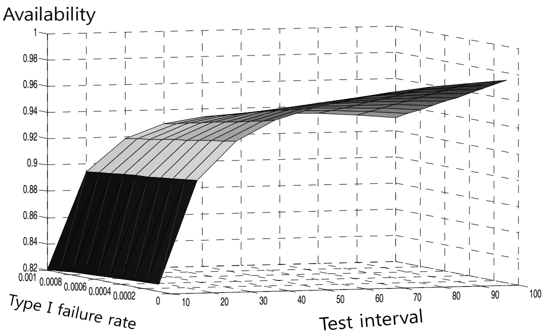
<Figure 6> shows the changes of availability with respect to the test interval and the probability of type II failure α (test-induced failure) for fixed $\tau = 2$ and type I failure rate = 0.0001. The availability is shown to be decreased as the probability of type II failure α (test-induced failure) increases, which is commonly predictable result (Detailed values are provided in <Table 2> of appendix).



<Figure 6> Availability Versus T and α

The changes of availability with respect to the test interval and the type I failure rate for fixed $\tau = 2$ and $\alpha = 0.01$ are shown in <Figure 7> (Detailed values are provided in <Table 3> of appendix). The type I failure rate is assumed to be constant but we should keep in mind that the probability of type I failure occurrence is obtained by $\beta = \int_0^T dH_s(x)$ where $H_s(x)$ is cumulative density function of exponential distribution with parameter of the type I failure rate.

<Figure 7> is commonly predictable such that the availability tends to decrease as increasing type I failure rate. However, It is somewhat interesting that the changes of the availability versus type I failure rate are more rapid with respect to increasing test interval.



<Figure 7> Availability Versus T and Type I Failure Rate

3.2 Sensitivity analysis for optimal test interval

Numerical analyses are performed to observe the change of optimal test interval by changing the test duration (τ), probability of type II failure (α) and type I failure rate.

<Table 1> shows the optimal test interval with respect to the test duration and the probability of type II failure. The optimal test interval tends to increase as test duration increase, which implies that the aging process of the standby unit during the test period affects strongly to the optimal test interval.

We can observe somewhat interesting point in the change of optimal test interval with respect to the probability of type II failure of <Table 1>. the optimal test interval is increasing with respect to the increasing α when $\tau \leq 4$, however, the optimal test interval is decreasing with respect to the increasing α when $\tau > 4$.

<Table 1> The Optimal Test Interval with Respect to α and τ

τ \ α	0.01	0.02	0.03	0.04	0.05
1	146.19	148.36	150.66	153.03	155.43
2	204.33	205.28	206.33	207.44	208.60
3	248.77	249.15	249.59	250.07	250.59
4	285.96	285.97	286.02	286.10	286.20
5	318.45	318.19	317.95	317.73	317.53
6	347.58	347.10	346.63	346.18	345.73

The optimal test interval with respect to the test duration and the type II failure rate is computed in <Table 2>. As expected, the optimal test interval decrease with respect to increasing type II failure rate.

<Table 2> The Optimal Test Interval with Respect to α and τ

λ \ τ	0.01	0.02	0.03	0.04	0.05
0.0001	146.19	204.33	248.77	285.96	318.45
0.0002	103.17	144.18	175.52	201.74	224.63
0.0003	84.11	117.53	143.06	164.42	183.06
0.0004	72.75	101.64	123.71	142.17	158.28
0.0005	65.00	90.79	110.51	126.99	141.37
0.0006	59.28	82.79	100.76	115.78	128.89
0.0007	54.83	76.57	93.18	107.07	119.18
0.0008	51.25	71.56	87.08	100.05	111.36
0.0009	48.28	67.41	82.02	94.23	104.88
0.001	45.77	63.89	77.74	89.31	99.40

4. Concluding Remarks

This paper analyzes the availability of the standby unit which should be ready for operation for a long time whenever needed. The periodic test is adopted to disclose a failure during standby period, however, it may also deteriorate the unit faster. Therefore, operating failure due to the aging effect during the test period, which is not treated in the previous works.

By considering three failure types of the standby unit, we can obtain limiting availability of the standby unit. Results of the experiments show that the availability decreases with respect to increasing test duration time and, probability of the test-induced failure and type I failure rate. Also, the availability increases rapidly to a certain value of the test interval and decreases slowly after this value. These results agree with what we have expected. However, an appropriate periodic test interval of the standby unit for maximizing the limiting availability can reasonably be identified under three types of failures of the standby unit. This paper illustrates the practical implementation of the optimal test interval by numerical examples.

The further study can be possible to determine optimal periodic test interval, and extend to the maintenance model considering cost variables. In addition, although three failure types of the standby unit are treated as independent in this study, the degree of dependency between three failures types need to be investigated further, which is left as future research works.

References

[1] Aven, T.; "Availability formulae for standby systems of similar units that are preventively maintained," *IEEE Transactions on Reliability*, 39(5) : 919-931, 1990.

[2] Bulter, D. A.; "A hazardous-inspection model," *Management Science*, 25(1) : 79-89, 1979.

[3] Ghou, C. G. and Bulter, D. A.; "Assessment of hazardous-inspection policies," *Naval Research Logistics Quarterly*, 30(1) : 171-177, 1983.

[4] Goel, L. R.; "A two-unit standby system with imperfect switch, preventive maintenance and correlated failures and repair," *Microelectronics Reliability*, 32(12) : 1687-1691, 1992.

[5] Ito, K. and Nakagawa, T.; "An optimal inspection policy for a storage system with high reliability," *Microelectronics Reliability*, 6(6) : 875-882, 1995.

[6] Kim, I. S., Martorell, S. A., Vesely, W. E., and Samanta, P. K.; "Risk analysis of surveillance requirements including their adverse effects," *Reliability Engineering and System Safety*, 45(3) : 225-234, 1994.

[7] Li, X., Yan, R., and Zuo, M. J.; "Evaluating a warm standby system with components having proportional hazard rates," *Operations Research Letters*, 37(1) : 56-60, 2009.

[8] Lu, L. and Jiang, J.; "Analysis of on-line maintenance strategies for k-out-of-n standby safety systems," *Reliability Engineering and System Safety*, 92(2) : 144-155, 2007.

[9] Merren, G. T.; "Dormant Storage Reliability Assessments-Data Based," *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, 4(4) : 446-454, 1981.

[10] Moon, J. P., Kim, Y. B., Park, J. H., and Lie, C. H.; "Investigation of Periodic Test Policy for Standby Redundant System," *6th APDSI Conference*, 1-10, 2001.

[11] Sim, S. H.; "Unavailability analysis of periodically tested components of dormant system," *IEEE Transactions on Reliability*, R-34(1) : 88-91, 1985.

[12] Sim, S. H.; "Reliability of standby equipment with periodic testing," *IEEE Transactions on Reliability*, R-36(1) : 117-123, 1987.

[13] Valdez-Flores, C. and Feldman, R. M.; "A survey of preventive maintenance model for stochastically deterring single-unit system," *Naval Research Logistics Quarterly*, 36(4) : 419-446, 1989.

[14] Vaurio, J. K.; "Optimization of test and maintenance interval based on risk and cost," *Reliability Engineering and System Safety*, 49(1) : 23-36, 1995.

[15] Wang, G. J. and Zhang, Y. L.; "An optimal replacement policy for repairable cold standby system with priority in use," *International Journal of Systems Science*, 38(12) : 1021-1027, 2007.

[16] Zhang, Y. L., Wang, G. J., and Ji, Z. C.; "Replacement problem for a cold standby repairable system," *International Journal of Systems Science*, 37(1) : 17-25, 2006.

[17] Zhao, M. Xie, M., and Zhang, Y. T.; "A model of storage reliability with possible initial failures," *Reliability Engineering and System Safety*, 43(3) : 269-273, 1994.

<Appendix>

Detailed values of numerical analysis are given below.

<Table A1> Availability Versus T and τ

T \ τ	1	2	3	4	5
50	0.976189	0.957027	0.938769	0.921375	0.904791
100	0.984118	0.974319	0.96479	0.955539	0.946559
150	0.985157	0.978613	0.9722	0.965931	0.959806
200	0.984452	0.979562	0.974747	0.970024	0.965393
250	0.983051	0.979161	0.97532	0.971541	0.967829
300	0.981305	0.978085	0.974898	0.971758	0.968667
350	0.979365	0.976625	0.973908	0.971227	0.968586
400	0.977307	0.974928	0.972565	0.97023	0.967928
450	0.975172	0.973074	0.970987	0.968924	0.966887
500	0.972987	0.971113	0.969248	0.967402	0.965578
550	0.970767	0.969077	0.967393	0.965725	0.964077
600	0.968522	0.966986	0.965454	0.963935	0.962432
650	0.966261	0.964854	0.96345	0.962057	0.960679
700	0.963988	0.962692	0.961398	0.960114	0.958842
750	0.961708	0.960508	0.959309	0.958119	0.95694
800	0.959423	0.958307	0.957192	0.956084	0.954986
850	0.957135	0.956093	0.955052	0.954016	0.95299
900	0.954847	0.953871	0.952894	0.951923	0.950961
950	0.952559	0.951641	0.950724	0.94981	0.948905

<Table A2> Availability Versus T and α

α \ T	10	20	30	40	50	60	70	80	90	100
0.01	0.82552	0.90369	0.93285	0.94793	0.95702	0.96303	0.96723	0.97029	0.97257	0.97431
0.02	0.82422	0.90290	0.93229	0.94748	0.95666	0.96272	0.96696	0.97005	0.97236	0.97412
0.03	0.82278	0.90203	0.93166	0.947	0.95626	0.96238	0.96666	0.96978	0.97212	0.97390
0.04	0.82124	0.90110	0.93099	0.94648	0.95583	0.96202	0.96635	0.96951	0.97187	0.97368
0.05	0.81961	0.90019	0.93028	0.94594	0.95538	0.96164	0.96602	0.96922	0.97161	0.97344

<Table A3> Availability Versus T and α

λ \ T	10	20	30	40	50	60	70	80	90	100
0.0001	0.82552	0.90369	0.93285	0.94793	0.95702	0.96303	0.96723	0.97029	0.97257	0.97431
0.0002	0.82499	0.90264	0.93129	0.94586	0.95446	0.95997	0.96367	0.96623	0.96803	0.96927
0.0003	0.82445	0.90159	0.92973	0.94380	0.95190	0.95691	0.96012	0.96219	0.96349	0.96425
0.0004	0.82391	0.90053	0.92817	0.94174	0.94934	0.95386	0.95658	0.95816	0.95898	0.95926
0.0005	0.82338	0.89947	0.92661	0.93968	0.94678	0.95081	0.95305	0.95415	0.95448	0.95429
0.0006	0.82284	0.89841	0.92504	0.93761	0.94423	0.94777	0.94952	0.95015	0.95001	0.94935
0.0007	0.82230	0.89736	0.92348	0.93555	0.94168	0.94474	0.94601	0.94617	0.94557	0.94444
0.0008	0.82176	0.89629	0.92192	0.9335	0.93913	0.94171	0.94252	0.94220	0.94114	0.93956
0.0009	0.82122	0.89523	0.92035	0.93144	0.93660	0.93870	0.93904	0.93826	0.93674	0.93471
0.001	0.82067	0.89417	0.91879	0.92939	0.93406	0.93570	0.93557	0.93434	0.93237	0.92990