International Journal of Internet, Broadcasting and Communication Vol.4 No.1 18-22 (2012)
http://dx.doi.org/10.7236/IJIBC.2012.4.1.18

IJIBC 12-1-5

MMSE based Wiener-Hopf Equation

Juphil Cho[1*], Il Kyu Lee[2], Jae Sang Cha[3]

[1*]*Department of radio Communication Engineering, kunsan national university, Korea*
stefano@kunsan.ac.kr

[2]*Department of Information and Communications, Kongju Nat'l University, Korea*
leeik@kongju.ac.kr

[3]*Department of Electronic & IT Media Eng., Seoul National University, Korea*
chajs@seoultech.ac.kr

Abstract

In this paper, we propose an equivalent Wiener-Hopf equation. The proposed algorithm can obtain the weight vector of a TDL(tapped-delay-line) filter and the error simultaneously if the inputs are orthogonal to each other. The equivalent Wiener-Hopf equation was analyzed theoretically based on the MMSE(minimum mean square error) method. The results present that the proposed algorithm is equivalent to original Wiener-Hopf equation. In conclusion, our method can find the coefficient of the TDL (tapped-delay-line) filter where a lattice filter is used, and also when the process of Gram-Schmidt orthogonalization is used. Furthermore, a new cost function is suggested which may facilitate research in the adaptive signal processing area

Keywords: *Wiener-Hopf equation, equivalent Wiener-Hopf equation, MMSE, Gram-Schmidt orthogonalization.*

1. Introduction

The Wiener-Hopf equation is a core algorithm for developing new adaptation algorithms [1]-[3]. A lattice filter has generally been utilized for the modeling of linear time-varying systems[4],[5]. However, lattice filters have various constraints in these applications, since the coefficients of the lattice filter are not TDL filter coefficients[6],[7]. Generally, the lattice filter constitutes a prediction stage and a joint process estimation stage. The prediction stage produces reflection coefficients and the joint process estimation stage produce regression coefficients, respectively[1]-[3]. Therefore, the weakness of the lattice filter is that the coefficient of the TDL filter cannot be obtained directly.

In this paper, we present an advanced Wiener-Hopf equation for producing coefficients of TDL filters directly in a lattice filter (or adaptation filter that uses the Gram-Schmidt orthogonalization process, etc.). In this paper, the new algorithm of the Wiener-Hopf equation will be referred to as an equivalent Wiener-Hopf equation.

This experiment showed similar results for both methods. Firstly, it provides a simple method of finding the coefficient of a TDL (tapped-delay-line) filter in the case where a lattice filter or the process of Gram-Schmidt orthogonalization is used. Secondly, a new cost function is defined. This cost function may facilitate research in the adaptive signal processing area.

Manuscript Received: Dec 11, 2011 / Revised: Jan. 18, 2012 / Accepted: Feb. 13, 2012
Corresponding Author: stefano@kunsan.ac.kr
Tel:+82-63-469-4749, Fax: +82-63-469-4749
Dept. of radio Communication Engineering, kunsan national university, Korea

2. Derivation of equivalent Wiener-Hopf equation

In Fig. 1, the input signal vector $\boldsymbol{x}(\boldsymbol{n})$, optimal coefficient vector $\boldsymbol{w}_{opt}$, orthogonal input vector $\boldsymbol{b}(\boldsymbol{n})$ and regression coefficient $\boldsymbol{k}$ are vectors. And $\boldsymbol{Q}$ is an $M \times M$ orthogonal transform matrix.

The signal $\boldsymbol{d}(\boldsymbol{n})$ is the desired response, $\hat{\boldsymbol{d}}(n)$ is the estimated desired response and $\upsilon(n)$ is the measurement noise. As shown in Fig. 1, the system is excited by an input signal $\boldsymbol{x}(\boldsymbol{n})$, and we are looking for the estimated values of the M, the unknown tap coefficients.

That is,

$$J = E[e(n)e^*(n)] = E[|e(n)|^2] \tag{1}$$

is a minimum.

The error is defined as following

$$e(n) = d(n) - \boldsymbol{k}^H \boldsymbol{b}(n) \tag{2}$$

where, (H)is a Hermitian transpose. $\hat{\boldsymbol{d}}(n)$ and $\boldsymbol{b}(\boldsymbol{n})$ are expressed as follows

$$\hat{d}(n) = \boldsymbol{k}^H \boldsymbol{b}(n) = \boldsymbol{k}^H \boldsymbol{Q}\boldsymbol{x}(n) \tag{3}$$

$$\boldsymbol{b}(n) = \boldsymbol{Q}\boldsymbol{x}(n) \tag{4}$$

And we define the desired response $\boldsymbol{d}(\boldsymbol{n})$ as

$$d(n) = \boldsymbol{w}_{opt}^H \boldsymbol{x}(n) \tag{5}$$

The error $\boldsymbol{e}(\boldsymbol{n})$ reaches zero if $\boldsymbol{d}(\boldsymbol{n})$ is equal to $\hat{\boldsymbol{d}}(n)$ That is,

$$d(n) - \hat{d}(n) = [\boldsymbol{w}_{opt}^H - \boldsymbol{k}^H \boldsymbol{Q}]\boldsymbol{x}(n) = 0 \tag{6}$$

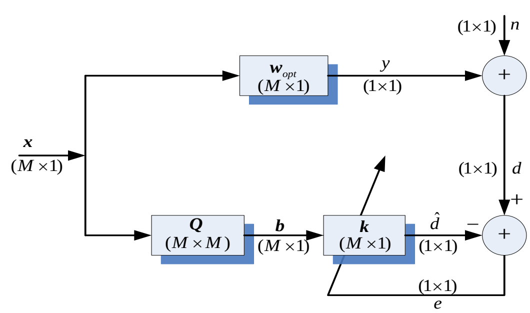


Fig. 1. The structure of an adaptive filter using an orthogonal input vector.

If $\boldsymbol{x}(n) \neq 0$, the solution of the above equation becomes

$$\boldsymbol{w}_{opt} = \boldsymbol{Q}^H \boldsymbol{k} \tag{7}$$

Generally, if the orthogonal input vector is used, the regression coefficient $\boldsymbol{k}$ and the coefficient of the TDL filter $\boldsymbol{w}_{opt}$ are related to each other, as presented in (7).

Using (1), the gradient of the mean-squared error can be obtained as following

$$\frac{\partial J}{\partial \boldsymbol{k}} = 2E[(d(n) - \boldsymbol{k}^H \boldsymbol{b}(n))(-\boldsymbol{b}^H(n))] \tag{8}$$

From the above equation, the vector $\boldsymbol{k}$ can be represented as

$$\boldsymbol{k} = E[\boldsymbol{b}(n)\boldsymbol{b}^H(n)]^{-1} E[\boldsymbol{b}(n)d^*(n)] \tag{9}$$

Rewriting the above equation using a matrix form, we obtain.

$$\boldsymbol{k} = \boldsymbol{R}_{bb}^{-1} \boldsymbol{p}_{bd} \tag{10}$$

$$\boldsymbol{R}_{bb} = E[\boldsymbol{b}(n)\boldsymbol{b}^H(n)] \tag{11}$$

$$\boldsymbol{p}_{bd} = E[\boldsymbol{b}(n)d^*(n)] \tag{12}$$

Where $\boldsymbol{R_{bb}}$ is an autocorrelation matrix of $\boldsymbol{b(n)}$, We have to assume that $\boldsymbol{R_{bb}}$is a nonsingular matrix. In this paper, we proposed the theorem that can determine $\boldsymbol{w_{opt}}$ directly without any calculation of $\mathcal{K}$.

Theorem 1. If the desired response $d(n)$ and the estimated desired response $\hat{d}(n)$ have so similar values, the relationship $\boldsymbol{R}_{bx}\boldsymbol{w}_{opt} = \boldsymbol{p}_{bd}$ is defined.

proof)

Substituting (4) and (7) into (8), we get

$$\frac{\partial J}{\partial \boldsymbol{k}} = 2E[(d(n) - \boldsymbol{w}_{opt}^H \boldsymbol{Q}^{-1}\boldsymbol{Q}\boldsymbol{x}(n))(-\boldsymbol{b}^H(n))] \tag{13}$$

The optimal coefficient $\boldsymbol{w_{opt}}$ can be obtained from (13) using $\boldsymbol{Q}^{-1}\boldsymbol{Q} = \boldsymbol{I}$.

$$E[\boldsymbol{b}(n)\boldsymbol{x}^H(n)]\boldsymbol{w}_{opt} = E[\boldsymbol{b}(n)d^*(n)] \tag{14}$$

The equation (14) can be simplified as $\boldsymbol{R}_{bx}\boldsymbol{w}_{opt} = \boldsymbol{p}_{bd}$ (15)

where

$$\boldsymbol{R}_{bx} = E[\boldsymbol{b}(n)\boldsymbol{x}^H(n)] \tag{16}$$

Q.E.D.

The matrix $\boldsymbol{R_{bX}}$ in (16) is the correlation matrix of the orthogonal input vector $\boldsymbol{b(n)}$ and nonorthogonal input vector $\boldsymbol{X(n)}$, and $\boldsymbol{R_{bX}}$ is nonsingular[1].

3. Computer Simulation

To evaluate the performance of the proposed algorithm, the identification of an unknown FIR system is performed. The input signal $x(n)$ to the adaptive filter was obtained from the output of a low pass filter which transfer function is as follows;

$$H(z) = \frac{1}{1 - 0.98z^{-1} + 0.693z^{-2} - 0.22z^{-3} + 0.309z^{-4} - 0.177z^{-5}} \tag{17}$$

where, its input is a white, zero mean and pseudorandom Gaussian noise. The response signal $y(n)$was obtained from the FIR system, as shown in (18).

$$H(z) = 2.65 - 3.31z^{-1} + 2.24z^{-2} - 0.7z^{-3} \tag{18}$$

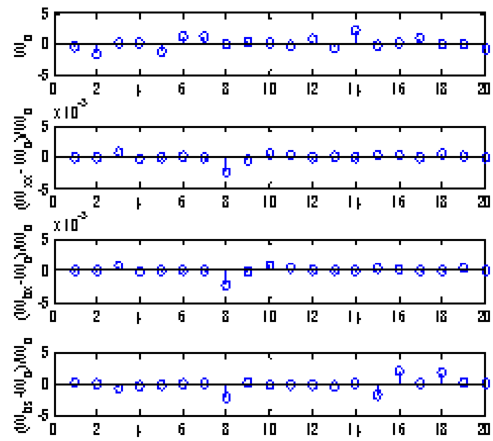
The measurement noise signal was an additive white, zero mean and pseudorandom Gaussian and was uncorrelated with the input signal. The computer simulation is designed to determine the similarities of the coefficient of the filters by solving the Wiener-Hopf equation and the equivalent Wiener-Hopf equation. All results presented in this paper are ensemble averages of 100 independent runs with 9000 data. Fig. 2 shows that the curves of the $(\boldsymbol{w}_{xx}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$, $(\boldsymbol{w}_{bx}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$ and $(\boldsymbol{w}_{bs}^{(i)} - \boldsymbol{w}_{opt}^{(i)}) / \boldsymbol{w}_{opt}^{(i)}$ obtained from the solutions of the Wiener-Hopf and equivalent Wiener-Hopf equation for an SNR of -90dB, -30dB, -10dB and 10dB, respectively. $\boldsymbol{w_{opt}}$ is the i-th optimal coefficient, $\boldsymbol{w_{xx}}$ is the i-th solution of original Wiener-Hopf equation and $\boldsymbol{w_{bx}^{(i)}}$ is the i-th solution of the proposed equivalent Wiener-Hopf equation.

The following simulation is made when the rank is deficient. Fig. 2 illustrates the results obtained by varying the rank of autocorrelation.

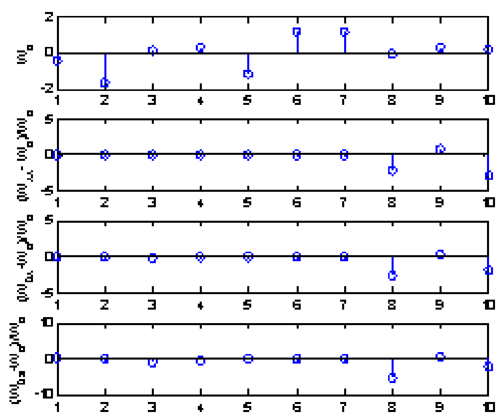
In Fig. 2, we set an order of unknown system to 20 and one of computer simulation to 10 under SNR of -30dB, respectively.

Fig. 2 shows the results of the coefficients for various ranks of the correlation matrix. The matrix is full rank. The figure shows that both the existing method and the proposed method have similar performance. Fig. 2 (b) is the case of a rank deficiency.

This figure shows the simulation results of an unknown system of order 20, while the order of



(a)



(b)

Fig. 2. Comparison of coefficients according to variation of correlation matrix.
(a) order of unknown system; 20, order of system identification; 20.
(b) order of unknown system;20, order of system identification; 10.

the simulated adaptive filter is 10. The figure shows that the proposed method and existing method produce similar results.

4. Conclusion

In this paper, an equivalent Wiener-Hopf equation is proposed. The proposed algorithm can determine the coefficient of a TDL filter directly when the adaptive filter has an orthogonal input signal. Also, a theoretical analysis was performed for the MMSE using a solution to the equivalent Wiener-Hopf equation. Furthermore, the proposed algorithm shows similar performance to that of the original Wiener-Hopf equation.

The proposed algorithm has the advantage of allowing the application area of adaptation filters which have an orthogonal input signal to be expanded.

References

[1] Haykin, Adaptive Filter Theory– Fourth Edition, PrenticeHall.

[2] J. G. Proakis, C. M. Rader, F. Ling and C. L. Nikias, Advanced Digital Signal Processing. Macmillan Publishing Company, 1992.
[3] P. S. R. Diniz, Adaptive Filtering -Algorithms and Practical Implementation, second edition. Kluwer Academic Publishers, 2002.
[4] J. Makhoul. "A Class of ALL-Zero Lattice Digital Filters: Properties" IEEE Trans. Acoustics, Speech, and Signal Processing, vol. ASSP-26, pp.304-314, Aug. 1978.
[5] E. Karlsson, M. Hayes, "Least squares ARMA modeling of linear time-varying systems: Lattice filter structures and fast RLS algorithms," IEEE Trans. Acoustics, Speech, and Signal Processing, vol.ASSP-35, NO.7, pp.994-1014, July1987.
[6] H. K. Baik, V. J. Mathews, "Adaptive lattice bilinear filters," IEEE Trans. on Signal Processing, vol.41, pp.2033-2046, June1993.
[7] L. Fuyun, J. Proakis, "A generalized multichannel least squares lattice algorithm based on sequential processing stages," IEEE Trans. on Signal Processing, vol.32, pp.381-389, Apr.1984.