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Modeling of a Continuous-Time System with Time-delay

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Abstract

Control Theory for continuous-time system has been well developed. Due to the development of computer technology, digital control scheme are employed in many areas. When delays are in control systems, it is hard to control the system efficiently. Delays by controller-to-actuator and sensor-to-controller deteriorate control performance and could possibly destabilize the overall system. In this paper, a new approximated discretization method and digital design for control systems with multiple state, input and output delays and a generalized bilinear transformation method with a tunable parameter are also provided, which can re-transform the integer time-delayed discrete-time model to its continuous-time model. Illustrative examples are given to demonstrate the effectiveness of the developed method.

Keywords: *time-delay, Hankel matrix, Singular Value Decomposition, Generalized Bilinear Transformation*

1. Introduction

Time delay is one of the key factors influencing the overall system stability and performance. In particular, as the different effects of actuator, sensor and controllerexist in control systems, delays are often formulated as state time delays, input time delays as well as output time delays in a continuous-time or discrete-time framework [1], [2-4]. To digitallysimulate and design a continuous-time delayed control system, it is often required to obtain an equivalent discrete-time model. The digital modeling of continuous-time systems with input delays can be found in a standard textbook [5]. For improving the performance of a continuous-time system with multiple time delays, several advanced control theories and practical design techniques have been proposed [6-8]. Recently, a discretization method via the Chebyshev quadrature formula together with a linear interpolation method to construct an equivalent discrete-time model from the continuous time multiple time-delayed system was developed in [9]. Despite the significant progress that has been made on continuous/discrete time systems with multiple time delays, yet the digital modeling of a continuous-time system with multiple fractional/integer time delays in state, input and output is far from fully explored [4]. In this paper, we propose a new approximated discretization method and a generalized bilinear transformation method.

2. Modeling of Continuous-Time System using Generalized Bilinear Transformation

2.1 Problem formulation and model transformation

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Consider a controllable, observable and stable [3] continuous-time multiple-input, multiple-output (MIMO) system with multiple state, input and output time delays described by

$$\dot{x}(t) = \sum_{i=0}^{J} A_i x(t-\delta_i) + \sum_{i=0}^{K} B_i u(t-\gamma_i),$$
$$y(t) = \sum_{i=0}^{M} C_i x(t-\zeta_i) \quad (1)$$

where $x(t) \in R^n$ is the state with $x(t) = \varphi(t)$ for $t \in [t_0 - \max\{\delta_i, \zeta_i\}, t_0]$, $u(t) \in R^m$ is the control input and $y(t) \in R^p$ is the output of the system. Here n is the order of the system, m is number of inputs and p is number of outputs of the system (1). Also, $\delta_i \geq 0$, i= 0, 1, $\cdots, J$, are the state delays, $\gamma_i \geq 0$, i= 0, 1, $\cdots, K$, are the input delays and $\zeta_i \geq 0$, i= 0, 1, $\cdots, M$, are the output delays. These delays can be fractional or integer multiple of the sampling time T. The function $\varphi(t) \in C[t_0 - \max\{\delta_i, \zeta_i\}, t_0]$ is a continuous vector-valued initial condition. The system matrices (A_i , B_i , C_i) are sets of real matrices defined with $A_i \in R^{n\times n}$, $i = 0,1,\ldots J$, $B_i \in R^{n\times m}, i = 0,1,\ldots K$, $C_i \in R^{p\times n}, i = 0,1,\ldots,M$.

The discrete-time model is shown in (2) as follows

$$x(kT+T) = Gx(kT) + Hu(kT)$$
$$y(kT) = \hat{C}x(kT) \quad (2)$$

where $G = \Sigma_{\mathcal{R}}^{-1/2} R_{\mathcal{R}}^T H_1 S_{\mathcal{R}} \Sigma_{\mathcal{R}}^{-1/2}$,
$H = \Sigma_{\mathcal{R}}^{1/2} S_{\mathcal{R}}^T E_m$, $E_m^T = [\boldsymbol{I_m} \ \boldsymbol{0} \ \ldots \ \boldsymbol{0}]$, and
$\hat{C} = \beta C$, $C = E_p^T R\tilde{n} \Sigma_{\mathcal{R}}^{1/2}$, $E_p^T = [\boldsymbol{Ip} \ \ \boldsymbol{0} \ \ldots \ \boldsymbol{0}]$,
$\Sigma_{\mathcal{R}} = diag\{\sigma_i, \ldots, \sigma_{\mathcal{R}}\}$.

β is a modification factor which can adjust the steady-state value of the system (2) to match the original continuous-time system (1).

After applying linear transformation, we get a Discrete-time Delay Difference Equation (DDDE) with integer delays as described in expressions shown below:

$$x_{01}(kT+T) = \hat{G}_0 x_{01}(kT) + \hat{G}_{01} x_{01}(kT-T) + \hat{H}_0 u(kT) + \hat{H}_{01} u(kT-T),$$
$$y(kT) = \hat{C}_0 x_{01}(kT) + \hat{C}_{01} x_{01}(kT-T) + \hat{D}_{01} u(kT-T), \quad (3)$$

where,
$\hat{G}_0 = -G_{01}, \hat{G}_{01} = -G_{02}, \hat{H}_0 = H_{01}, \hat{H}_{01} = H_{02}, \hat{C}_0 = C_{01}, \hat{C}_{01} = -C_{02}G_{02}, \hat{D}_{01} = C_{02}H_{02}$

2.2 Generalized Bilinear Transformation

From the discrete-time multiple integer time-delayed system represented by (3) we can obtain a continuous-time multiple integer time-delayed model via a generalized bilinear transformation method. Continuous-time systems with non-integer time-delays are not convenient for conventional multi-variable controller designing procedures and hence it is advantageous to obtain an equivalent continuous-time model with integer time-delays. Also it is easy to design controller and observer for continuous-time integer delay systems for digital control of sampled data systems. The procedure for computation of a continuous-time integer delay model from a given discrete-time integer time-delay state equation is discussed as below. For simplicity and better understanding, we consider a simple example of given a discrete-time system with a single state delay and appropriate system dimensions as represented in (4).

$$x(kT+T) = G_0 x(kT) + G_1 x(kT-T) + H_0 u(kT) \tag{4}$$

where, $u(kT)$ is a piecewise-constant input signal.

We want to find an equivalent continuous-time model with integer delay as shown in (5).

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-T) + B_0 u(t) \tag{5}$$

where, $u(t) = u(kT) \text{ for } kT \le t < (kT+T)$.

Integrating equation (5) with limits t=kT to t=kT+T gives:

$$\int_{kT}^{kT+T} \dot{x}(t)dt = \int_{kT}^{KT+T} A_0 x(\lambda)d\lambda + A_1 \int_{kT}^{kT+T} x(\lambda-T)d\lambda + B_0 \int_{kT}^{kT+T} u(\lambda)d\lambda = A_0 \int_{kT}^{kT+T} x(\lambda)d\lambda + A_1 \int_{kT-T}^{kT} x(\lambda)d\lambda + B_0 \int_{kT}^{kT+T} 1\, d\lambda \quad u(kt). \tag{6}$$

The first and second integral terms in the right-hand side of equation (6) can be approximated by the generalized bilinear transformation method or a generalized trapezoidal-rule method as follows:

$$\int_{kT}^{kT+T} x(\lambda)d\lambda = T[\alpha x(kT+T) + (1-\alpha)x(kT)]$$

and

$$\int_{kT-T}^{kT} x(\lambda)d\lambda = T[\alpha x(kT) + (1-\alpha)x(kT-T)] \tag{7}$$

where, $0 \le \alpha \le 1$. Note that when $\alpha = 0.5$, the generalized bilinear transformation method reduces to the standard bilinear transformation method. Substituting (6) and (7) into (5) gives:

$$x(kT+T) - x(kT) = A_0[\alpha x(kT+T) + (1-\alpha)x(kT)]T + A_1[\alpha x(kT) + (1-\alpha)x(kT-T)]T + B_0 T u(kT)$$

Comparing equations to find the expressions for $G_0, G_1, \text{and } H_0$ as

$$G_0 = (I - \alpha A_0 T)^{-1}[I + (1-\alpha)A_0 T + \alpha A_1 T],$$
$$G_1 = (I - \alpha A_0 T)^{-1}[1-\alpha]A_1 T,$$
$$H_0 = (I - \alpha A_0 T)^{-1} B_0 T \tag{8}$$

From above expressions, we now calculate the system matrices of the continuous-time system with integer delays for different values of α.

Remark 1: For 0 < α < 1, the system matricesfor continuous-time system with integer delays in (5) are as shown below.

$$A_0 = \frac{1}{T}\left\{\left[G_0 - \left(\frac{\alpha}{1-\alpha}\right)G_1 - I\right]\left[\alpha G_0 + (1-\alpha)I - \left(\frac{\alpha^2}{1-\alpha}\right)G_1\right]^{-1}\right\},$$
$$A_1 = \frac{1}{T}\left\{\left(\frac{1}{1-\alpha}\right)[I - \alpha A_0 T]G_1\right\},$$
$$B_0 = \frac{1}{T}[(I - \alpha A_0 T)H_0]. \tag{9}$$

Remark 2: If α=0.5, then the system matrices in (9) reduces to standard bilinear transform as shown in equations below.

$$A_0 = -\left(\frac{2}{T}\right)[I_2 - (G_0 - G_1)][I_2 + (G_0 - G_1)]^{-1},$$
$$A_1 = \frac{2}{T}\left[I_2 - \left(A_0\left(\frac{T}{2}\right)\right)\right]G_1,$$
$$B_0 = \frac{1}{T}\left[I_2 - \left(A_0\left(\frac{T}{2}\right)\right)\right]H_0. \quad (10)$$

The expressions represented in (10) are expressions for the system matrices of the continuous-time system with integer delays.

3. Simulation and Results

A digital model obtained by the proposed SVD approach is compared with the digital model designed using the bilinear transformation method [9]. In this example, we discuss the accuracy of the proposed method (SVD approach) over the previous method of digital modeling (bilinear transformation). For comparison, we consider the following continuous time system with a state delay as described by

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - Tx) + Bu(t)\ y(t) = Cx(t) \quad (11)$$

where $A_0 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the delay time $T_x = 0.2s$, and the sampling period is T = 0.15s, n = 2.

Following the computation steps, let $E_{\sigma,\tilde{n}} < 1\%$, the singular value matrix $\Sigma_{\tilde{n}}$ is obtained as follows. $\Sigma_{\tilde{n}} = diag\{0.7884 \quad 0.3177 \quad 0.0012 \quad 0.0009\}$.

So the appropriate order for the digital model is $\tilde{n} = 2n$, and the extended discrete-time model (2) is obtained as

$$G= \begin{bmatrix} 0.9597 & 0.0492 & 0.0057 & 0.0001 \\ -0.1285 & 0.7745 & 0.0341 & 0.0003 \\ 0.0076 & 0.0387 & 0.0831 & -0.0926 \\ -0.0001 & -0.0003 & 0.0927 & -0.0923 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.2508 \\ 0.3357 \\ -0.0254 \\ 0.0002 \end{bmatrix},$$

$$\hat{C} = \begin{bmatrix} 0.2282 & -0.1405 & -0.0029 & -0.0001 \\ 0.0822 & 0.3213 & -0.0276 & -0.0002 \end{bmatrix}.$$

The observable canonical form transformation matrix T_0 is picked as $T_0 = [G\bar{C}_0^T \quad C_0^T]$, where $\bar{C}_0 = \begin{bmatrix} \hat{C}G \\ \hat{C} \end{bmatrix}^{-1} C_0^T$, $\hat{C}_0 = [\mathbf{I2} \quad \mathbf{02}]$, so that the output vector y(kT) is equal to the state vector x01(kT).

So the matrices of the integer time-delayed discrete-time model are

$$\hat{G}_0 = \begin{bmatrix} -2.6361 & 0.4752 \\ -38.1408 & 4.3611 \end{bmatrix},$$
$$\hat{G}_{01} = \begin{bmatrix} 3.6344 & 0.2155 \\ 38.1043 & 2.2601 \end{bmatrix},$$
$$\hat{H}_0 = [0.0101 \quad -0.0080],$$
$$\hat{H}_{01} = [0.1292 \quad -0.0835]\ \hat{C}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \hat{C}_{01} = 0_2, \hat{D}_{01} = 0_2.$$

The equivalent digital model of the given continuous-time system with a state delay represented by (11), is calculated based on the bilinear transformation method as

$$x_b(kT+T) = G_{b0}x_b(kT) + G_{b1}x_b(kT-T) + G_{b2}x_b(kT-2T) + H_{b0}u(kT)$$
$$y_b(kT) = x_b(kT) \quad (12)$$

with,

$G_{b0} = \begin{bmatrix} 0.9901 & 0.1293 \\ -0.1231 & 0.7346 \end{bmatrix}$,

$G_{b1} = \begin{bmatrix} 0.0015 & 0.0008 \\ 0.0185 & 0.0093 \end{bmatrix}$,

$Gb2 = \begin{bmatrix} 0.0002 & 0.0001 \\ 0.0013 & 0.0006 \end{bmatrix}$,

$Hb0 = \begin{bmatrix} 0.0102 \\ 0.1291 \end{bmatrix}$.

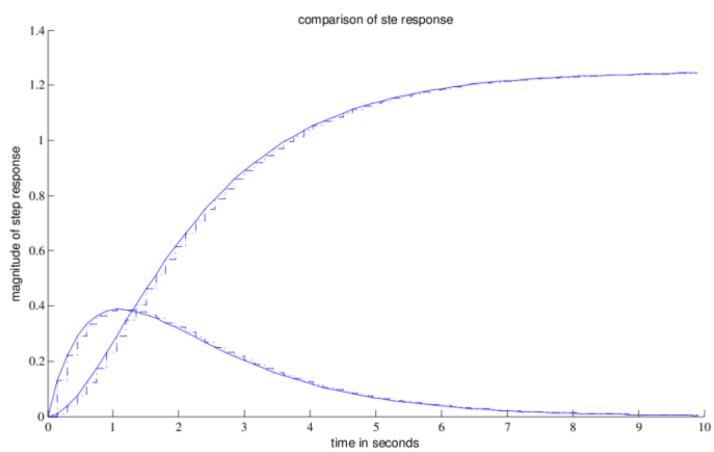

Fig. 1. Comparison of Step responses

4. Conclusion

In this paper, a new approximated state-space discretization scheme for a multivariable continuous-time system with multiple state, input and output delays has been presented. And a generalized bilinear transform method for a multiple integer time-delayed systems is also proposed. As a result, the infinite-dimensional continuous-time control system can be converted into a finite dimensional sampled-data system, and a direct digital design of the sampled-data closed-loop system can be adopted.

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