J Electr Eng Technol Vol. 7, No. 6: 1009-1013, 2012
http://dx.doi.org/10.5370/JEET.2012.7.6.1009

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

# Position DOP Analysis for Sensor Placement in the TDOA-based Localization System

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**Abstract** – A relationship between the sensor placement and the PDOP (Position Dilution of Precision) is derived in the TDOA-based localization system. And the geometric condition of the sensor placement is analyzed in order to get a minimum PDOP based on the derived relationship. Through computer simulations, effect of the sensor placement on the PDOP is observed.

**Keywords**: PDOP, TDOA, Sensor placement, Localization system

## 1. Introduction

Positioning error of the radio navigation systems is known to be influenced by not only the measurements quality but also the geometric placement of sensors. And the effects of the geometric placement on the positioning error is typically quantified as the dilution of precision (DOP) [1]. Some research results for the relationship between the geometric placement and the DOP of the time of arrival (TOA) method can be easily found in literatures [1, 2]. However, relationship for the time difference of arrival (TDOA) method cannot be found in literatures although the DOP of the TDOA method was derived in [3-6]. And there are simulation results on the position errors for several sensor placements of the TDOA-based systems [7, 8]. So, it is difficult to predict the positioning error by the sensor placement in a given work space.

In this paper, a relationship between sensor placement and PDOP of the TDOA method is derived in an inequality form. Using the relationship, a minimum PDOP can be calculated for a given sensor placement. Through computer simulations, effect of the sensor placement on the PDOP is shown.

## 2. Relationship Between the Sensor Placement and the PDOP of the TDOA Method

### 2.1 PDOP of the TDOA method

A radio localization system is generally composed of a tag to transmit RF signal and sensors to receive the signal. When the TDOA method is used, a reference sensor is required to calculate time differences. In the following, the reference sensor is notated in subscript r and sensors are in subscripted in $i$ $(i = 1, \cdots, N-1)$.

It is assumed that tag, the $i$ -th sensor and the reference sensor are located at $(x, y, z)$, $(x_i, y_i, z_i)$ and $(x_r, y_r, z_r)$, respectively. When the range difference between $i$ -th sensor and the reference sensor to the tag is linearized at $(\hat{x}, \hat{y}, \hat{z})$, the geometry matrix of the TDOA method with $N$ sensors can be expressed in Eq. (1) [3, 4].

$$H = \begin{bmatrix} h_{x,1} - h_{x,r} & h_{y,1} - h_{y,r} & h_{z,1} - h_{z,r} \\ h_{x,2} - h_{x,r} & h_{y,2} - h_{y,r} & h_{z,2} - h_{z,r} \\ \vdots & \vdots & \vdots \\ h_{x,N-1} - h_{x,r} & h_{y,N-1} - h_{y,r} & h_{z,N-1} - h_{z,r} \end{bmatrix}, \qquad (1)$$

where the line of sight vector between the tag and the reference sensor, $\begin{bmatrix} h_{x,r} & h_{y,r} & h_{z,r} \end{bmatrix}$, and the line of sight vector between the tag and the $i$ -th sensor, $\begin{bmatrix} h_{x,i} & h_{y,i} & h_{z,i} \end{bmatrix}$ are given by Eqs. (2) and (3), respectively.

$$h_{p,r} = \frac{\hat{p} - p_r}{\sqrt{(\hat{x} - x_r)^2 + (\hat{y} - y_r)^2 + (\hat{z} - z_r)^2}}, \text{ where } p \in \{x, y, z\} \qquad (2)$$

$$h_{p,i} = \frac{\hat{p} - p_i}{\sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2 + (\hat{z} - z_i)^2}}, \text{ where } p \in \{x, y, z\} \qquad (3)$$

The PDOP of the TDOA method is expressed in Eq. (4)

$$PDOP_{TDOA} \equiv \sqrt{((H^T H)^{-1})_{1,1} + ((H^T H)^{-1})_{2,2} + ((H^T H)^{-1})_{3,3}}, \qquad (4)$$

where $((H^T H)^{-1})_{1,1}$, $((H^T H)^{-1})_{2,2}$ and $((H^T H)^{-1})_{3,3}$ are the first, the second and the third diagonal term of $(H^T H)^{-1}$, respectively. And the error covariance matrix can be written in (5).

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Received: April 25, 2012; Accepted: August 7, 2012

$$H^T H = \begin{bmatrix} \sum_{i=1}^{N-1}(h_{x,i}-h_{x,r})^2 & \sum_{i=1}^{N-1}(h_{x,i}-h_{x,r})(h_{y,i}-h_{y,r}) & \sum_{i=1}^{N-1}(h_{x,i}-h_{x,r})(h_{z,i}-h_{z,r}) \\ \sum_{i=1}^{N-1}(h_{x,i}-h_{x,r})(h_{y,i}-h_{y,r}) & \sum_{i=1}^{N-1}(h_{y,i}-h_{y,r})^2 & \sum_{i=1}^{N-1}(h_{y,i}-h_{y,r})(h_{z,i}-h_{z,r}) \\ \sum_{i=1}^{N-1}(h_{x,i}-h_{x,r})(h_{z,i}-h_{z,r}) & \sum_{i=1}^{N-1}(h_{y,i}-h_{y,r})(h_{z,i}-h_{z,r}) & \sum_{i=1}^{N-1}(h_{z,i}-h_{z,r})^2 \end{bmatrix} \tag{5}$$

## 2.2 Derivation of the relationship

Since $H^T H$ is symmetric and positive definite, its eigenvalues, $\lambda_1, \lambda_2$ and $\lambda_3$, are real and positive [9]. Trace of $H^T H$ is equal to sum of eigenvalues as in Eq. (6).

$$\begin{aligned} & trace(H^T H) \\ & = \sum_{i=1}^{N-1}\left[(h_{x,i}-h_{x,r})^2+(h_{y,i}-h_{y,r})^2+(h_{z,i}-h_{z,r})^2\right] \\ & = \lambda_1+\lambda_2+\lambda_3 \end{aligned} \tag{6}$$

Letting $\alpha_i = (h_{x,i}-h_{x,r})^2+(h_{y,i}-h_{y,r})^2+(h_{z,i}-h_{z,r})^2$, the trace of $H^T H$ can be expressed in Eq. (7).

$$trace(H^T H) = \sum_{i=1}^{N-1}\alpha_i \tag{7}$$

The PDOP of the TDOA method in Eq. (4) can be expressed in Eq. (8) [9].

$$\left(PDOP_{TDOA}\right)^2 = \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3} \tag{8}$$

Assuming $0<\lambda_1 \le \lambda_2 \le \lambda_3$, the following inequality (9) can be obtained from the definition of the maximum eigenvalue [9].

$$\begin{aligned} \lambda_3 = \max_{\|x\|=1} x^T (H^T H) x &\ge \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} (H^T H) \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \\ &= \frac{1}{3}\sum_{i=1}^{N-1}\left(\alpha_i+2\beta_i\right) \end{aligned} \tag{9}$$

Here, $\beta_i$ is given in Eq. (10).

$$\begin{aligned} \beta_i = & (h_{x,i}-h_{x,r})(h_{y,i}-h_{y,r}) \\ & +(h_{y,i}-h_{y,r})(h_{z,i}-h_{z,r})+(h_{z,i}-h_{z,r})(h_{x,i}-h_{x,r}) \end{aligned} \tag{10}$$

Using Eqs. (6) and (7), the following inequality (11) can be obtained from the inequality (9).

$$\lambda_1+\lambda_2 = \sum_{i=1}^{N-1}\alpha_i - \lambda_3 \le \sum_{i=1}^{N-1}\alpha_i - \frac{1}{3}\sum_{i=1}^{N-1}\left(\alpha_i+2\beta_i\right) \tag{11}$$

Letting $\lambda_1 = \lambda_2$ in inequality (11), the following inequality (12) can be obtained.

$$\lambda_1 \le \frac{1}{2}\left(\sum_{i=1}^{N-1}\alpha_i - \frac{1}{3}\sum_{i=1}^{N-1}\alpha_i - \frac{2}{3}\sum_{i=1}^{N-1}\beta_i\right) = \frac{1}{3}\sum_{i=1}^{N-1}\left(\alpha_i-\beta_i\right) \tag{12}$$

In inequality (12), if $\lambda_3$ is $\frac{1}{3}\sum_{i=1}^{N-1}\left(\alpha_i+2\beta_i\right)$ and $\lambda_1 = \lambda_2 = \frac{1}{3}\sum_{i=1}^{N-1}\left(\alpha_i-\beta_i\right)$, the minimum PDOP of the TDOA method can be obtained as Eq. (13).

$$PDOP_{TDOA}\big|_{\min} = 3\sqrt{\frac{\sum_{i=1}^{N-1}\left(\alpha_i+\beta_i\right)}{\sum_{i=1}^{N-1}\left(\alpha_i-\beta_i\right)\sum_{i=1}^{N-1}\left(\alpha_i+2\beta_i\right)}} \tag{13}$$

Eq. (13) shows that the minimum PDOP of the TDOA method is affected by the number of sensors, $\alpha_i$ and $\beta_i$ which are calculated by the sensor placement. As a result of this, it is possible to predict the minimum PDOP of the TDOA-based system from the placement of the sensors.

## 3. Simulation Results

In order to show the effects of the sensor placement on the PDOP, simulations are performed in $W \times W$ space shown in Fig. 1. For a tag located in the workplace W= 400m, minimum PDOP for any sensor placement can be calculated using Eq. (13) and is shown in Fig. 2.

For the sensor placement that H=10m, z=25m, and N=5, PDOP is calculated by using Eq. (4) as shown in Fig. 3. From Fig. 2 and Fig. 3, it can be observed that PDOP for a

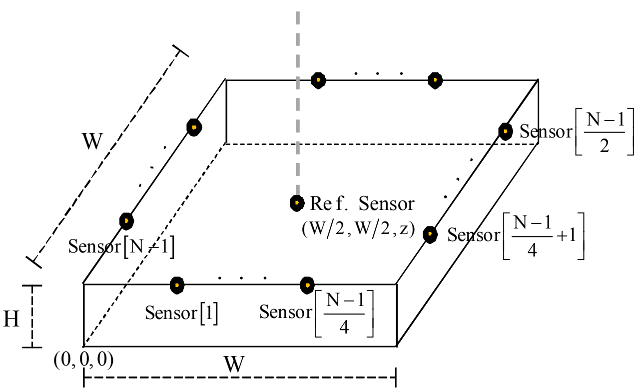



**Fig. 1.** Workspace and sensor placement for the simulation.

specific placement of sensors is higher than minimum PDOP for all possible sensor placements.

In Fig. 4, all values of $\sum_{i=1}^{N-1}(\alpha_i)$ and $\sum_{i=1}^{N-1}(\beta_i)$ for the workspace are shown. From Fig. 3, it can be seen that PDOP becomes smaller as value of $\sum_{i=1}^{N-1}(\alpha_i)$ increases.

In order to observe effect of the sensor placement on PDOP in the workplace, averaged value of PDOP over the workspace is observed when the height of the sensors and

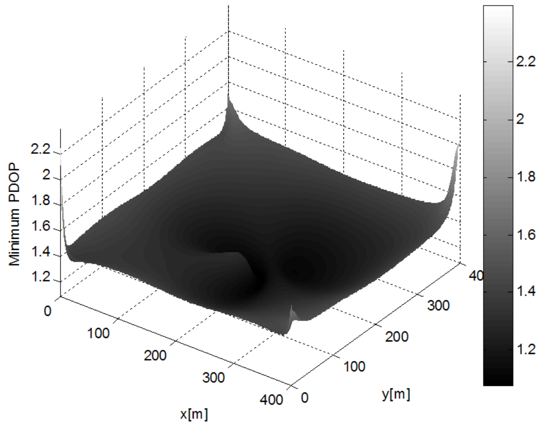



**Fig. 2.** Minimum PDOP for W=400m.

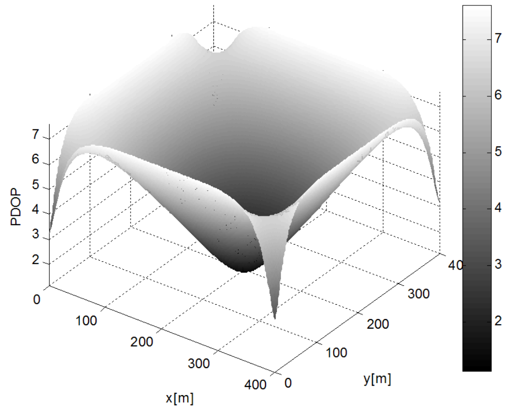
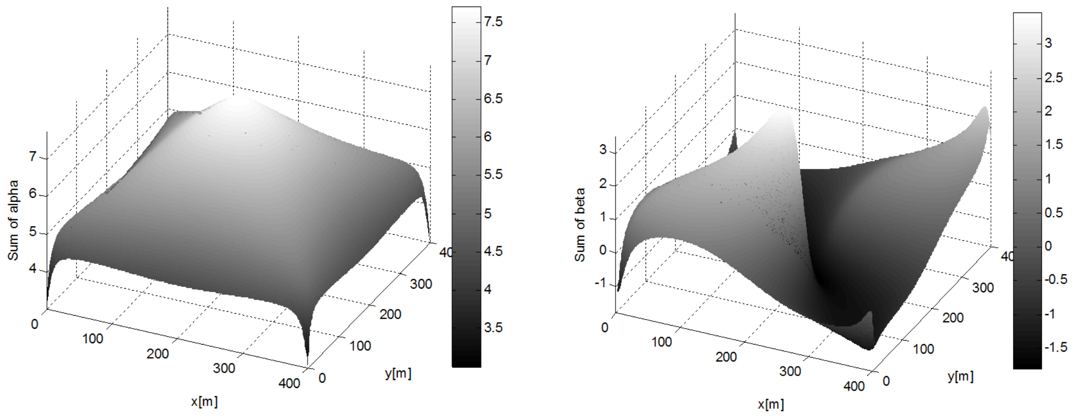



**Fig. 3.** PDOP for W=400m, H=10m, z=25m, and N=5.




(a) Sum of alpha

(b) Sum of beta

**Fig. 4.** Sum of alpha and sum of beta over the workspace.

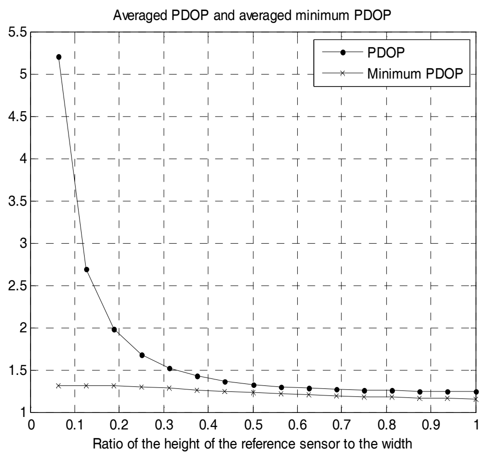



**Fig. 5.** Effect of the height of the reference sensor on averaged PDOP and averaged minimum PDOP.

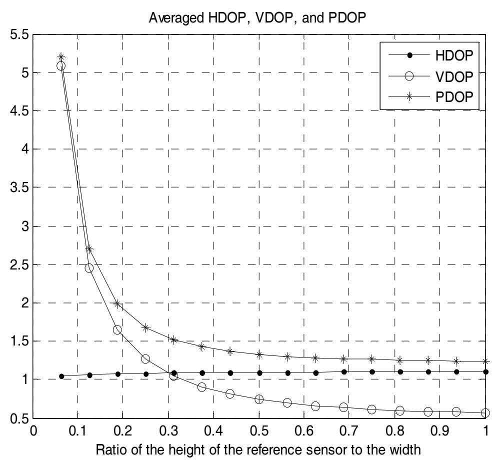



**Fig. 6.** Averaged HDOP, VDOP, and PDOP.

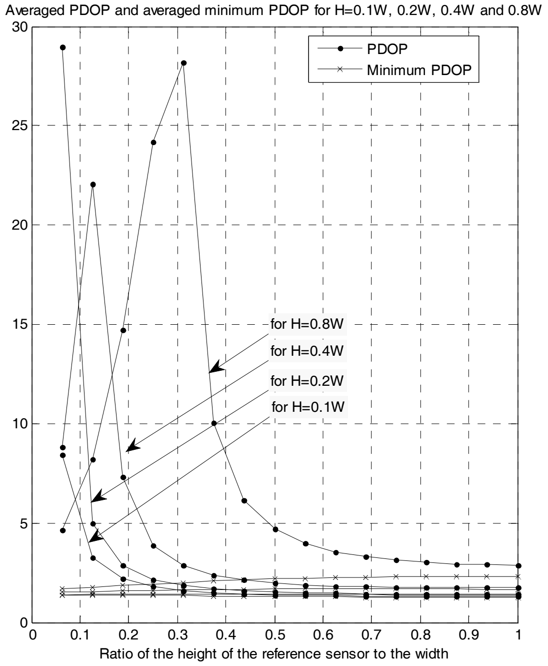



**Fig. 7.** Effect of the height of sensors except the reference sensor on averaged PDOP and minimum PDOP.

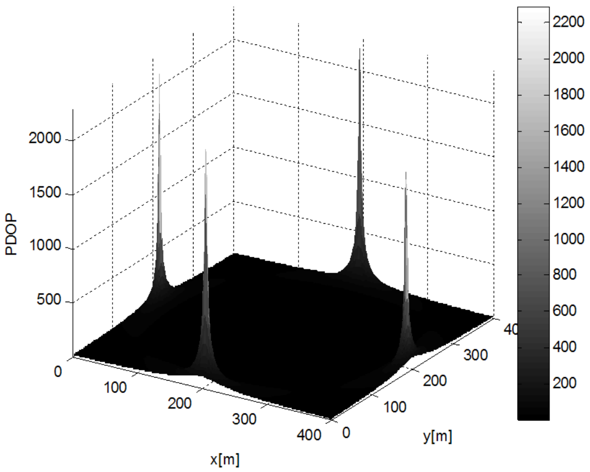



**Fig. 8.** PDOP over the workspace when H=0.8W and z= 0.3125W.

the number of the sensors vary.

Averaged PDOP and minimum PDOP over the workspace are shown in Fig. 5 when z is varied and H=10m. It can be seen from Fig. 5 that PDOP goes to the minimum PDOP as height of the reference sensor becomes larger. HDOP and VDOP are given in Fig. 6. From Fig. 6, it can be seen that VDOP is dominant in PDOP when the height of the reference sensor is less than 0.3W.

Fig. 7 shows the values of PDOP and minimum PDOP when H=0.1W, 0.2W, 0.4W, and 0.8W and N=5. It can be observed from Fig. 7 that the averaged minimum PDOP becomes larger as the height of the sensors except the reference sensor becomes higher.

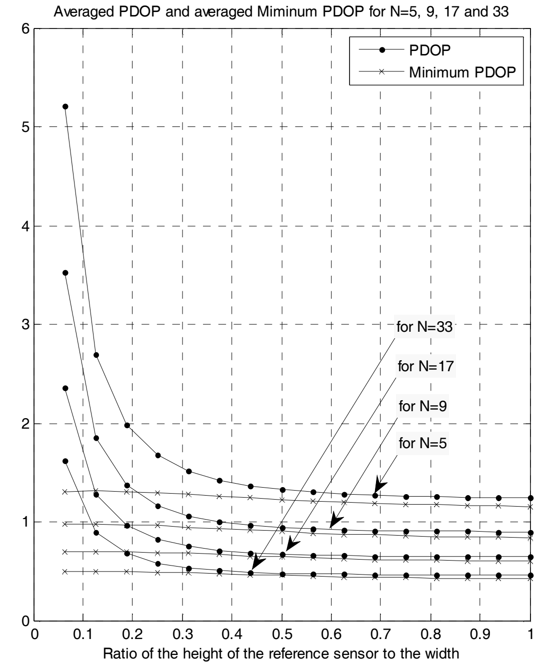



**Fig. 9.** Effect of the number of the sensors on averaged PDOP and averaged minimum PDOP.

PDOP over the workspace is plotted in Fig. 8 when H=0.8W and z=0.3125W. Four extremely large values can be observed in Fig. 8. Due to these values, large values around z=0.3125W are obtained in Fig. 7 when H=0.8W. Other cases in Fig. 7 can be similarly explained.

Fig. 9 shows simulation results for N=5, 9, 17, and 33 when H=10m. It can be observed From Fig. 9 that the averaged PDOP and averaged minimum PDOP becomes smaller as the number of the sensor increases and difference between averaged PDOP and averaged minimum PDOP becomes smaller as the number of the sensors increases

## 4. Conclusion

In this paper, a relationship between sensor placement and PDOP in the TDOA-based localization system has been derived and effect on the PDOP of the sensor placement has been observed through extensive computer simulations.

It is expected that the number of sensors and sensor placement can be determined from the results of this paper when a desired PDOP is given, and those results can be useful for developing the TDOA-based localization system

## Acknowledgements

This work was supported by a grant from Korea Research Council of Fundamental Science & Technology funded by Ministry of Education, Science and Technology in 2012. [Project: A Study on Satellite based Position Tracking Technology for Calamity Prevention and Public Safety Improvement].

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