J Electr Eng Technol Vol. 7, No. 6: 911-917, 2012
http://dx.doi.org/10.5370/JEET.2012.7.6.911

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

# Digital Speed Regulator System Design for a Permanent Magnet Synchronous Motor

**Han Ho Choi***, **Jin-Woo Jung*** and **Tae-Heoung Kim**†

**Abstract** – In this paper, a digital speed regulator system design method is developed for a permanent magnet synchronous motor (PMSM). Firstly, an accurate approximate discrete-time model is proposed for a PMSM considering its inherent nonlinearities. Based on the discrete-time model, a digital acceleration observer as well as a digital speed regulator is designed. The exponential stability of the augmented control system is analyzed. The proposed digital speed regulator system is implemented by using a TMS320F28335 floating point DSP. Simulation and experimental results are given to verify the effectiveness of the proposed method.

**Keywords**: Permanent magnet synchronous motor (PMSM), Discrete-time system, Disturbance, Controller, Observer

## 1. Introduction

Rapid development of DSPs and power electronics has stimulated the widespread use of a permanent magnet synchronous motor (PMSM) in many industrial applications. A PMSM features low noise, low inertia, high efficiency, robustness, and low maintenance cost. Many researchers have developed various PMSM controller design methods, e.g., adaptive control [1-5], nonlinear feedback linearization control [6], fuzzy control [7, 8], disturbance-observer-based control [9, 10]. Almost all the previous PMSM control design methods are based on the controller emulation approach. Alternatively, the discrete-time approach can be used to design digital controllers for nonlinear systems. In view of stability and achievable performances, the discrete-time approach is typically better than the emulation approach [11]. This paper develops a digital controller design method for a PMSM based on the discrete-time approach. An accurate approximate discrete-time model is first derived. By using the discrete-time model a digital speed regulator as well as a digital acceleration observer is designed. The exponential stability of the augmented control system containing the regulator and observer is analytically proven in the discrete-time domain. Simulation and experimental results are given to verify that the proposed method can be successfully applied for PMSM speed control under load torque variations.

† Corresponding Author: Dept. of Electrical Engineering, Engineering Research Institute, Gyeongsang National University, Korea. (ktheoung@gnu.ac.kr)
* Division of Electronics and Electrical Engineering, Dongguk University, Korea. ({hhchoi, jinwjung}@dongguk.edu)
Received: November 9, 2011; Accepted: June 1, 2012

## 2. Continuous-Time Model of PMSM

A surface mounted PMSM can be represented by the following continuous-time nonlinear equation [10]:

$$\begin{aligned} \dot{\omega} &= k_1 i_{qs} - k_2\omega - k_3 T_L \\ \dot{i}_{qs} &= -k_4 i_{qs} - k_5\omega + k_6 V_{qs} - \omega i_{ds} \\ \dot{i}_{ds} &= -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs} \end{aligned} \tag{1}$$

where, $T_L$ denotes the load torque, $\omega$ is the electrical rotor angular speed, $i_{qs}$ is the $q$-axis current, $V_{qs}$ is the $q$-axis voltage, $i_{ds}$ is the $d$-axis current, $V_{ds}$ is the $d$-axis voltage, $p$ is the number of poles, $R_s$ is the stator resistance, $L_s$ is the stator inductance, $J$ is the rotor inertia, $B$ is the viscous friction coefficient, $\lambda_m$ is the magnetic flux, and $k_i > 0$, $i = 1, \cdots, 6$ are the parameter values given by

$$k_1 = \frac{3}{2}\frac{1}{J}\frac{p^2}{4}\lambda_m,\ k_2 = \frac{B}{J},\ k_3 = \frac{p}{2J}, k_4 = \frac{R_s}{L_s},\ k_5 = \frac{\lambda_m}{L_s},\ k_6 = \frac{1}{L_s} \tag{2}$$

## 3. Main Results

### 3.1 Discrete-Time Model of PMSM

Recently, several techniques have been developed to get an accurate approximate discrete-time model of a nonlinear system. Here, by using the method [11] we derive an accurate approximate discrete-time model of the nonlinear PMSM (1). The nonlinear PMSM (1) is first transformed into the normal form. Denote the electrical rotor angular acceleration by

$$\beta(t) = k_1 i_{qs}(t) - k_2\omega(t) - k_3 T_L = \dot{\omega}(t) \quad (3)$$

We will assume that $\dot{T}_L$ can be neglected. Then, (1) can be transformed into the following normal form [10]

$$\begin{aligned}
&\dot{\omega} = \beta \\
&\dot{\beta} = -k_2\beta - k_1 k_1 i_{qs} - k_1 k_2\omega - k_1\omega i_{ds} + k_1 k_6 V_{qs} \\
&\dot{i}_{ds} = -k_4 i_{ds} + k_6 V_{ds} + \omega i_{qs}
\end{aligned} \quad (4)$$

After all, by [11] we can get the following accurate approximate discrete-time model of the PMSM

$$\begin{aligned}
\omega(k+1) &= \omega(k) + T\beta(k) + \frac{T^2}{2}k_1 k_6 V_{qs}(k) - \frac{T^2}{2}k_2\beta(k) \\
&\quad -\frac{T^2}{2}k_1 k_4 i_{qs}(k) - \frac{T^2}{2}k_1 k_2\omega(k) - \frac{T^2}{2}k_1\omega(k) i_{ds}(k) \\
\beta(k+1) &= \beta(k) + Tk_1 k_6 V_{qs}(k) - Tk_2\beta(k) - Tk_1 k_4 i_{qs}(k) \\
&\quad - Tk_1 k_2\omega(k) - Tk_1\omega(k) i_{ds}(k) \\
i_{ds}(k+1) &= i_{ds}(k) + Tk_6 V_{ds}(k) - Tk_4 i_{ds}(k) + T\omega(k) i_{qs}(k)
\end{aligned} \quad (5)$$

It should be noted that a discrete-time model by an Euler approximation method or simple derivative replacement method would not contain the term $\frac{T^2}{2}k_1 k_6 V_{qs}(k) - \frac{T^2}{2}k_2\beta(k) - \frac{T^2}{2}k_1 k_1 i_{qs}(k) - \frac{T^2}{2}k_1 k_2\omega(k) - \frac{T^2}{2}k_1\omega(k) i_{ds}(k)$ of (5). By introducing the speed error $\omega_e = \omega - \omega_d$ the following error dynamics can be obtained

$$x(k+1) = Ax(k) + Bg(\omega, \omega_d, i_{ds}, i_{qs}) + Bv_s(k) \quad (6)$$

where

$$A = \begin{bmatrix} 1-\frac{T^2}{2}k_1 k_5 & T(1-\frac{T}{2}k_2) & 0 \\ -Tk_1 k_5 & 1-Tk_2 & 0 \\ 0 & 0 & 1-Tk_4 \end{bmatrix}, B = \begin{bmatrix} \frac{T^2}{2}k_1 k_6 & 0 \\ Tk_1 k_6 & 0 \\ 0 & Tk_6 \end{bmatrix},$$

$$x(k) = \begin{bmatrix} \omega_e(k) \\ \beta(k) \\ i_{ds}(k) \end{bmatrix}, v_s(k) = \begin{bmatrix} V_{qs}(k) \\ V_{ds}(k) \end{bmatrix},$$

$$g(\omega, \omega_d, i_{ds}, i_{qs}) = -\frac{1}{k_6}\begin{bmatrix} k_5\omega_d + i_{ds}(k)\omega(k) + k_4 i_{qs}(k) \\ -i_{qs}(k)\omega(k) \end{bmatrix} \quad (7)$$

and $T$ is the sampling period.

### 3.2 Discrete-time speed controller design

Let the control input variables $V_{qs}$ and $V_{ds}$ be decomposed as

$$V_{qs}(k) = u_q(k) + u_{qf}(k),\ V_{ds}(k) = u_d(k) + u_{df}(k) \quad (8)$$

and as shown in Fig. 1 let the control law be given by

$$\begin{aligned}
u_q(k) &= \frac{1}{k_6}\left[k_5\omega_d + \omega(k) i_{ds}(k) + k_4 i_{qs}(k)\right] \\
u_q(k) &= -\frac{1}{k_6}\omega(k) i_{qs}(k) \\
u_{qdf}(k) &= Kx(k)
\end{aligned} \quad (9)$$

where $u_{qdf}(k) = [u_{qf}, u_{df}]^T$, and $K \in R^{2\times 3}$ is a gain matrix.

Then the closed-loop control system of the discrete-time PMSM model (6) and the discrete-time state feedback controller (9) is given by

$$x(k+1) = A_c x(k) = [A + BK]x(k) \quad (10)$$

Assume that the following LMI is feasible for ($X$, $Y$)

$$\begin{bmatrix} -X & AX + BY \\ XA^T + Y^T B^T & -X \end{bmatrix} < 0 \quad (11)$$

And assume that the controller gain matrix $K$ is given by

$$K = YX^{-1} \quad (12)$$

Then by using Schur complement formula of [18] it can be shown that (11) is equivalent to

$$(AX + BY)^T X^{-1}(AX + BY) - X < 0 \quad (13)$$

Pre-multiplying and post-multiplying (13) by $X^{-1}$ the following can be obtained

$$(A + BYX^{-1})^T X^{-1}(A + BYX^{-1}) - X^{-1} < 0 \quad (14)$$

which implies the existence of a positive constant $\delta_c$ such that

$$A_c^T X^{-1} A_c - X^{-1} \le -\delta_c I < 0 \quad (15)$$

By using the Lyapunov function as $V_c(k) = x^T P_c x$ where $P_c = X^{-1}$ we can obtain

$$V_c(k+1) - V_c(k) \le -\delta_c \| x(k) \|^2 \le 0 \quad (16)$$

which implies that the origin $x = 0$ is exponentially stable.

*Theorem 1:* Assume that the LMI (11) is feasible for ($X,Y$), the digital control law is given by (9), and the gain matrix $K$ is given by (12). Then, $x$ converges exponentially to zero.

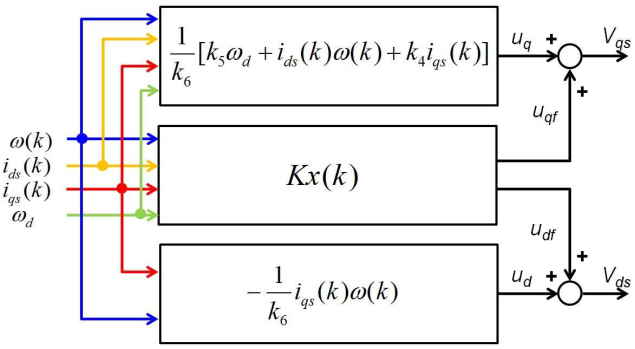


**Fig. 1.** Block diagram of the proposed digital speed regulator.

### 3.3 Discrete-time acceleration observer design

The proposed control law requires the acceleration information $\beta(k)$ in order to account for the load torque term $T_L$. Here, an observer is proposed to obtain the acceleration information $\beta(k)$. The following digital observer is used :

$$\begin{aligned} x_o(k+1) &= Ax_o(k) + B[g + v_s(k)] - L[y(k) - Cx_o(k)] \\ \beta_o(k) &= C_o x_o(k) \end{aligned} \tag{17}$$

where $\beta_o(k)$ is an estimate of $\beta(k)$, $L \in R^{3\times2}$ is a gain matrix, $y(k) = Cx(k)$, and

$$x_o = \begin{bmatrix} x_{o1} \\ x_{o2} \\ x_{o3} \end{bmatrix}, C_o = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Fig. 2 shows the block diagram of the proposed observer. Using the discrete-time PMSM model (5), the following observer error dynamics can be obtained

$$\chi(k+1) = A_o\chi(k) = [A + LC]\chi(k) \tag{19}$$

where $\chi(k)=x(k)-x_o(k)$.

*Theorem 2:* Assume that the following LMI is feasible for *(P, H)*

$$\begin{bmatrix} -P & PA + HC \\ A^T P + C^T H^T & -P \end{bmatrix} < 0 \tag{20}$$

And assume that the observer gain matrix $L$ is given by

$$L = P^{-1}H \tag{21}$$

Then, the estimation error converges exponentially to zero.

*Proof :* Assume that (20) is feasible. Then by using Schur complement formula of [13] it can be shown that there exists a positive constant $\delta_o$ such that

$$A_o^T PA_o - P \le -\delta_o I < 0 \tag{22}$$

Let us define the Lyapunov function as $V_o(k) = \chi^T P\chi$. Its time derivative along the error dynamics (19) is given by

$$V_o(k+1) - V_o(k) \le -\delta_o \| \chi(k) \|^2 \le 0 \tag{23}$$

which implies that the origin $\chi = 0$ is exponentially stable.

### 3.4 Stability of closed-loop system

*Theorem 3 :* Assume that the LMIs (11) and (20) are feasible, and the following observer-based control law is used instead of (9)

$$\begin{aligned} V_{qs}(k) &= \frac{1}{k_6}\left[k_5\omega_d + \omega(k)i_{ds}(k) + k_4 i_{qs}(k) + u_{qf}(k)\right] \\ V_{ds}(k) &= \frac{1}{k_6}\left[-\omega(k)i_{qs}(k) + u_{df}(k)\right] \\ u_{qdf}(k) &= \begin{bmatrix} u_{qf}(k) \\ u_{df}(k) \end{bmatrix} = Kx_e(k) \end{aligned} \tag{24}$$

where $x_e(k) = [\omega_e(k), x_{o2}(k), i_{ds}(k)]^T = [\omega_e(k), \beta_o(k), i_{ds}(k)]^T$, $x_{o2} = \beta_o$ is the estimated acceleration via the digital observer (17). Then $x$ and $\chi$ converge exponentially to zero.

*Proof :* Because $\beta - x_{o2} = \beta - \beta_o = [0, 1, 0]\chi$, the vector $x_e$ can be rewritten as $x_e = x - E\chi$ where

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Let us define the Lyapunov function as $V(k) = x^T X^{-1} x + \zeta\chi^T P\chi$ where $\zeta$ is a sufficiently large scalar, $X$ and $P$ satisfy the LMIs (11) and (20). Then, the following inequality is satisfied

$$V(k+1) - V(k) \le -w^T Qw$$

where $\delta_1 = ||E^T K^T B^T X^{-1} BKE||$, $\delta_2 = ||E^T K^T B^T X^{-1} A_c||$, and

$$w = \begin{bmatrix} \| x \| \\ \| \chi \| \end{bmatrix}, Q = \begin{bmatrix} \delta_c & -\delta_2 \\ -\delta_2 & \zeta\delta_o - \delta_1 \end{bmatrix}$$

If $\zeta$ is large enough to guarantee $\zeta > (\delta_2^2/\delta_c + \delta_1)/\delta_o$ , then $Q > 0$ and $V(k+1) < V(k)$ for all $(x, \chi) \neq 0$.

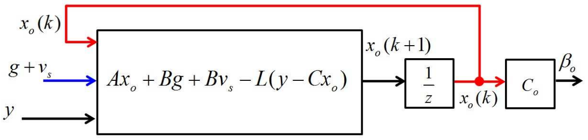


**Fig. 2.** Block diagram of the proposed digital acceleration observer.

## 4. Simulation and Experiment

Let us consider a prototype PMSM with the following nominal parameters: rated power $P_{rated}$ = 1 HP; rated phase current $I_{rated}$ = 3.94 A; rated torque $T_{rated}$ = 3.9 N·m; $p$ = 12; $R_s$ = 0.99 Ω; $L_s$ = 5.82 mH; $\lambda_m$ = 7.92×10$^{-2}$ V·sec/rad; $J$ = 12.08×10$^{-4}$ kg·m$^2$; $B$ = 3×10$^{-4}$ N·m·sec/rad. Under the sampling period $T$ = 1/5000, the discrete-time model (6) with the following data can be obtained

$$A = \begin{bmatrix} 0.9981 & 0.0002 & 0 \\ -9.6280 & 1.0000 & 0 \\ 0 & 0 & 0.9660 \end{bmatrix}, B = \begin{bmatrix} 0.0122 & 0 \\ 121.64 & 0 \\ 0 & 0.0344 \end{bmatrix},$$

$$g(\omega, \omega_d, i_{ds}, i_{qs}) = -\frac{1}{171.82}\begin{bmatrix} 13.6\omega_d + i_{ds}(k)\omega(k) + 170.1 i_{qs}(k) \\ -i_{qs}(k)\omega(k) \end{bmatrix} \quad (25)$$

By referring to the previous section, the following gain matrices can be obtained

$$K = \begin{bmatrix} 0.016 & -0.0082 & 0 \\ 0 & 0 & -28.11 \end{bmatrix}, \; L = \begin{bmatrix} -0.7914 & -0.0026 \\ -863.45 & 10.911 \\ -0.0046 & -0.9657 \end{bmatrix} \quad (26)$$

Fig. 3 shows the overall block diagram of the proposed digital PMSM control system. In simulations and experiments, a space vector PWM (SVPWM) technique is adopted. Figs. 4 and 5 show the simulation results using Matlab/Simulink about two cases : nominal parameters and 150% variations of some parameters ($L_s$, $J$, and $T_L$). In both cases, the desired motor speed ($\omega_d$) increases from 251.32 [*rad/sec*] to 502.64 [*rad/sec*] and then decreases from 502.64 [*rad/sec*] to 251.32 [*rad/sec*]. Fig. 4 shows the simulation results ($\omega_d$, $\omega$, $\omega_e$, $i_{qs}$, $i_{ds}$, $V_{an}$, $i_a$) when the digital controller with the gain (26) is applied to the PMSM model under nominal condition. Fig. 5 shows the simulation results under 150% variations of some parameters ($L_s$, $J$, and $T_L$). Fig. 6 shows the simulation results under 150% variations of some parameters ($L_s$, $J$) when the desired speed $\omega_d$ increases up to the rated value (1850 rpm). That is, the desired motor speed $\omega_d$ suddenly

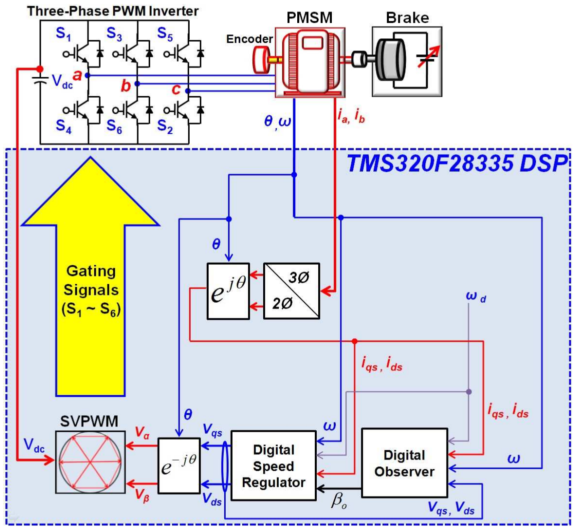


**Fig. 3.** Block diagram of the proposed digital PMSM control algorithm.

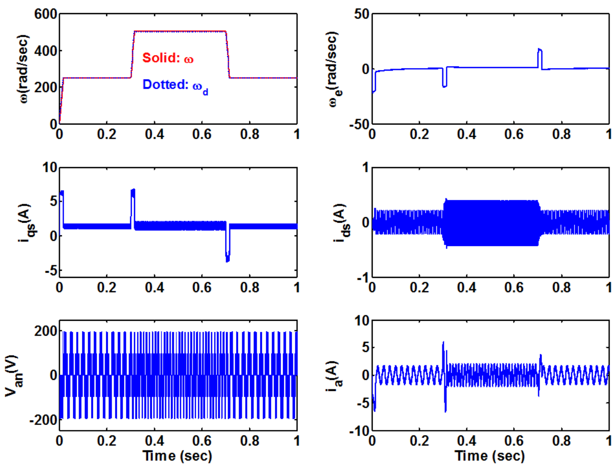


**Fig. 4.** Simulation results under nominal condition.

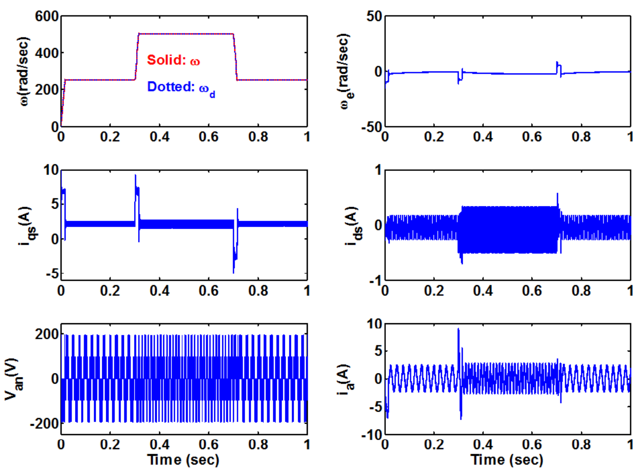


**Fig. 5.** Simulation results 150% variations of some parameters ($L_s$, $J$, and $T_L$).

increases from 565.49 to 1162.39 [rad/sec] and then decreases from 1162.39 to 565.49 [rad/sec]. Figs. 4 to 6, it can be observed that the proposed digital speed controller is very robust to model parameter and load torque variations.

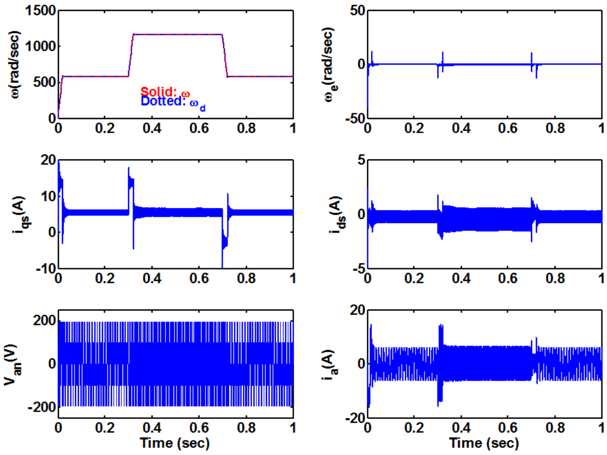


**Fig. 6.** Simulation results 150% variations of some parameters ($L_s$ and $J$) at the rated speed and rated torque.

Fig. 7 shows the experimental results about motor speed, voltage and current under the same condition as Fig. 4. Fig. 7(a) shows the desired speed ($\omega_d$), measured speed ($\omega$), speed error ($\omega_e$), and Fig. 8(b) shows the measured $q$-axis current ($i_{qs}$) and $d$-axis current ($i_{ds}$), and Fig. 8(c) shows the line to neutral voltage ($V_{an}$) and phase $a$ current ($i_a$). Fig. 8 also shows the experimental results about motor speed, voltage and current under the same condition as Fig. 5. The simulation and experimental results imply that the proposed digital controller can accurately control the speed of a PMSM under both model parameter and load torque variations.

## 4. Conclusion

This paper proposed a digital speed regulator design method for a PMSM based on the discrete-time approach. The proposed digital speed controller is robust because it does not depend on load torque variations. The closed-loop system stability was proven in the discrete-time domain. By using various simulation and experimental results, it was verified that the proposed method can be successfully applied for PMSM speed control under load torque variations.

## Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2012R1A1A2001439).

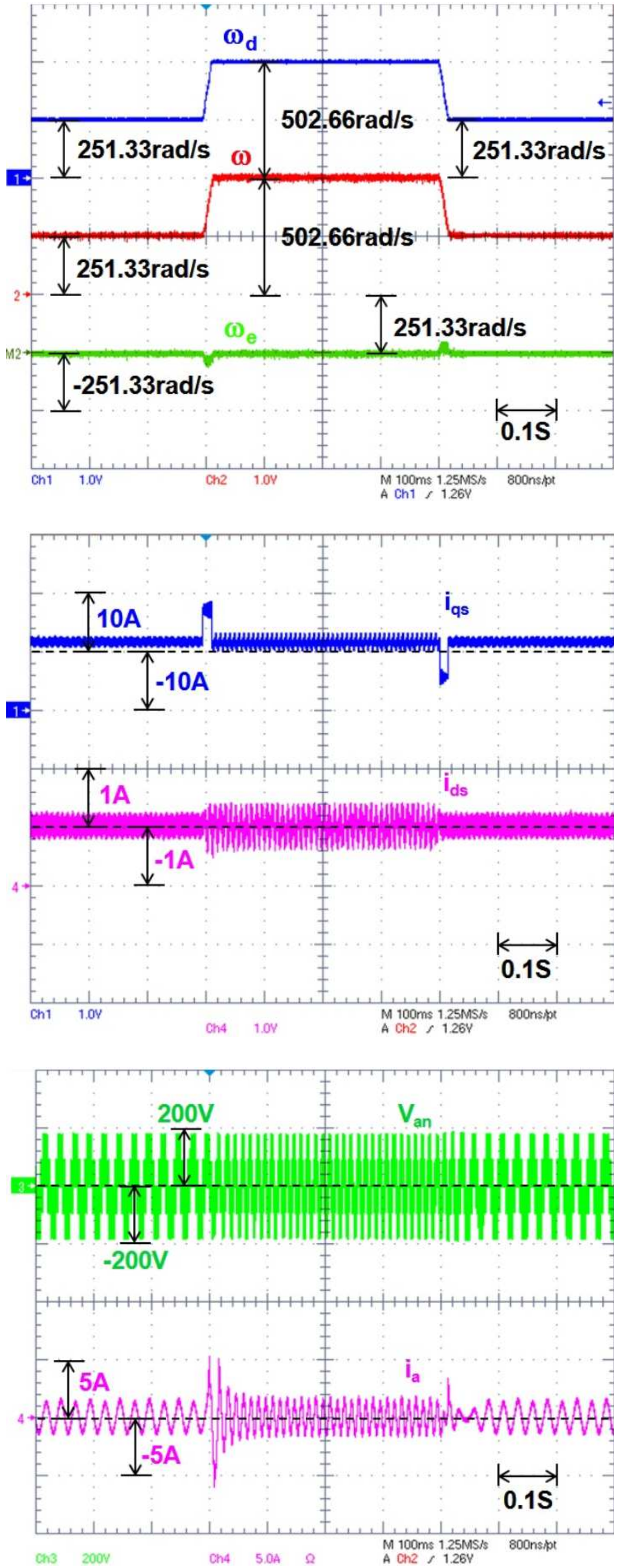


**Fig. 7.** Experimental results under nominal condition. (Top) $\omega_d$, $\omega$, $\omega_e$. (Middle) $i_{qs}$, $i_{ds}$. (Bottom) $V_{an}$, $i_a$

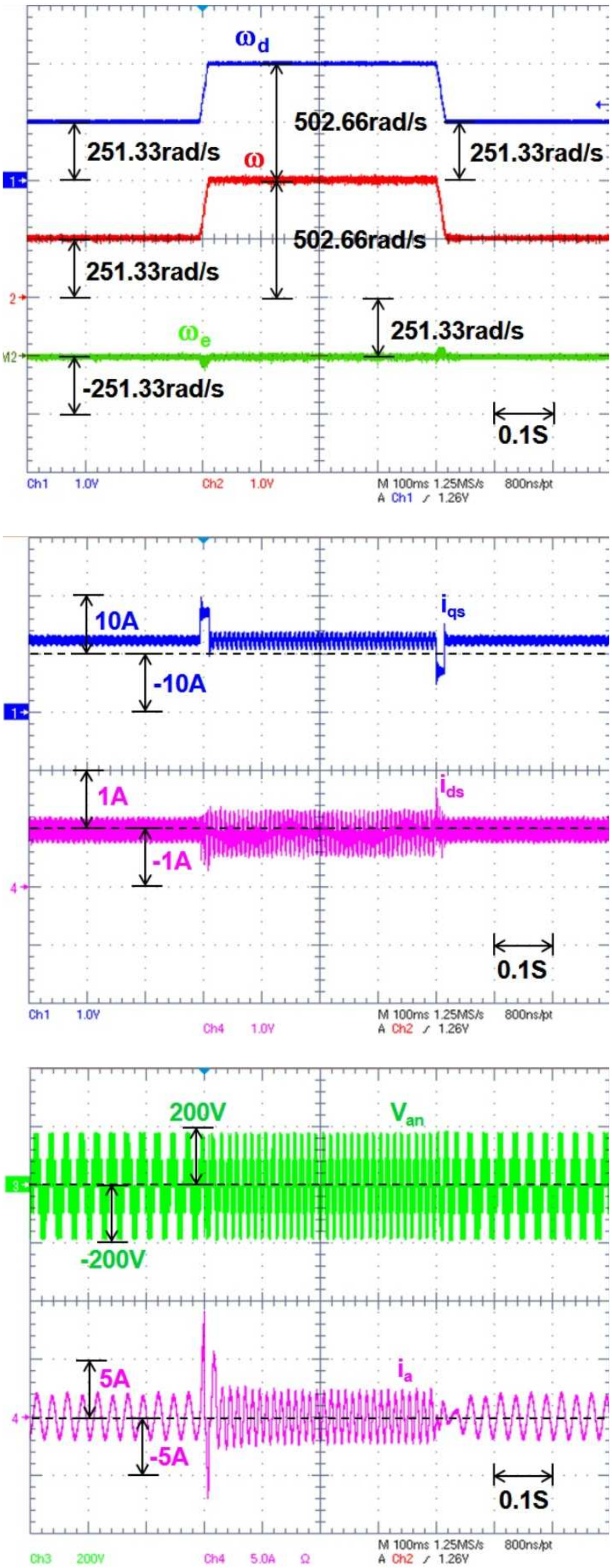


**Fig. 8.** Experimental results under 150% variations of some parameters ($L_s$, $J$, and $T_L$). (Top) $\omega_d$, $\omega$, $\omega_e$. (Middle) $i_{qs}$, $i_{ds}$. (Bottom) $V_{an}$, $i_a$

## References


[1] T.-H. Liu, H.-T. Pu, and C.-K. Lin, "Implementation of an adaptive position control system of a permanent-magnet synchronous motor and its application," *IET Electr. Power Appl.* vol. 4, no. 2, pp. 121-130, 2010.

[2] W.-T. Su, and C.-M. Liaw, "Adaptive positioning control for a LPMSM drive based on adapted inverse model and robust disturbance observer," *IEEE Trans. Power Electron.*, vol. 21, no. 2, pp. 505-517, 2007.

[3] Y.A.-R.I. Mohamed, "A hybrid-type variable-structure instantaneous torque control with a robust adaptive torque observer for a high-performance direct-drive PMSM," *IEEE Trans. Ind. Electron.,* vol. 54, pp. 2491-2499, 2007

[4] H.H. Choi, N.T.-T. Vu, and J.-W. Jung, "Digital implementation of an adaptive speed regulator for a PMSM," *IEEE Trans. Power Electron.*, vol. 26, no. 1, pp. 3-8, 2011

[5] K.-H. Kim, "Model reference adaptive control-based adaptive current control scheme of a PM synchronous motor with an improved servo performance," *IET Electr. Power Appl.*, vol. 3, no. 1, pp. 8-18, 2009

[6] C.-K. Lin, T.-H. Liu, and S.-H. Yang, "Nonlinear position controller design with input-output linearisation technique for an interior permanent magnet synchronous motor control system," *IET Power Electron.*, vol. 1, no. 1, pp. 14-26, 2008.

[7] M.N. Uddin, and M.A. Rahman, "High-speed control of IPMSM drives using improved fuzzy logic algorithms," *IEEE Trans. Ind. Electron.*, vol. 54, pp. 190-199, 2007.

[8] Y.-S. Kung, C.-C. Huang, and M.-H. Tsai, "FPGA realization of an adaptive fuzzy controller for PMLSM drive," *IEEE Trans. Ind. Electron.*, vol. 56, no. 8, pp. 2923-2932, 2009.

[9] S.-H. Choi, J.-S. Ko, J.-S. Park, and S.-C. Hong, "Precise position control using a PMSM with a disturbance observer containing a system parameter compensator," *IEE Proc.-Electr. Power Appl.*, vol. 152, pp. 1573-1577, 2005.

[10] V.Q. Leu, H.H. Choi, and J.-W. Jung, "Fuzzy sliding mode speed controller for PM synchronous motors with a load torque observer," *IEEE Trans. Power. Electron.*, vol. 27, no. 3, pp. 1530-1539, 2012

[11] J.I. Yuz, and G.C. Goodwin, "On sampled-data models for nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 50, no. 10, pp. 1477-1489, 2005

[12] K.J. Astrom, and B. Witternmark, *Computer-Controlled Systems - Theory and Design*, NJ : Prentice Hall Inc., 1990

[13] S. Boyd, S., L. El Ghaoui, F. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory,* PA : SIAM, 1994

**Han Ho Choi** received the B.S. degree in Control and Instrumentation Engineering from Seoul National Univ., Seoul, Korea, and the M.S. and Ph.D. degree in Electrical Engineering from KAIST in 1988, 1990, and 1994, respectively. From 1994 to 1998, he was a team leader with the Advanced Tech. Lab. of DaeWoo Elec. Company. He is now with the Div. of EEE, Dongguk Univ.-Seoul.

**Jin-Woo Jung** received his B.S. and M.S. in Electrical Engineering from Hanyang University, Seoul, Korea, in 1991 and 1997, respectively, and his Ph.D. in Electrical and Computer Engineering from The Ohio State University, Columbus, Ohio, USA, in 2005. From 1997 to 2000, he was with the Digital Appliance Research Laboratory, LG Electronics Co., Ltd., Seoul, Korea. From 2005 to 2008, he worked at the R&D Center and with the PDP Development Team, Samsung SDI Co., Ltd., Korea, as a Senior Engineer. Since 2008, he has been with the Division of Electronics and Electrical Engineering, Dongguk University, Seoul, Korea. His current research interests are in the area of DSP-based electric machine drives, control of distributed generation systems using renewable energy sources.

**Tae-Heoung Kim** received his B.S., M.S., and Ph.D. in Electrical Engineering from Hanyang University, Seoul, Korea, in 1993, 1995, and 2005, respectively. From 1995 to 2002, he worked for LG Electronics as a Senior Research Engineer. Since 2005, he has been a Professor in the Department of Electrical Engineering, Gyeongsang National University, Jinju, Korea. His research interests include the design of electric machines and control systems.