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Observer Based Nonlinear State Feedback Control of PEM Fuel Cell Systems

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Abstract – In this paper, the observer based nonlinear state feedback controller has been developed to control the pressures of the oxygen and the hydrogen in the PEM(Proton Exchange Membrane) fuel cell system. Nonlinear model of the PEM fuel cell system was introduced to study the design problems of the state observer and model based controller. A cascade observer using the filtering technique was used to estimate the pressure derivatives of the cathode and the anode in the system. In order to estimate the pressures of the cathode and the anode, the sliding mode observer was designed by using these pressure derivatives. To estimate the oxygen pressure and the hydrogen pressure in the system, the nonlinear state observer was designed by using the cathode pressure estimates and the anode it. These results will be very useful to design the state feedback controller. The validity of the proposed observers and the controller has been investigated by using the Lyapunov's stability analysis strategy.

Keywords: PEM fuel cell, Sliding mode observer, State feedback control.

1. Introduction

Fuel cell systems are under intensive development for mobile and stationary power applications. In particular, PEM fuel cells are currently in a relatively more mature stage for ground vehicle and stationary power applications [1, 5]. Despite a large number of studies on fuel cell modeling, relatively few are suitable for control and observation studies. The transient phenomena captured in the model include the flow and inertia dynamics of the compressor, the manifold filling dynamics (both anode and cathode), and membrane humidity. These variables affect the fuel cell stack voltage, and thus fuel cell efficiency and power [1, 5]. A two-dimensional along-the-channel mass and heat transfer model for a PEMFC(Proton Exchange Membrane Fuel Cell) is described in [1]. This model is used for calculation of cell performance (i.e., cell voltage against current density), ohmic resistance and water profile in the membrane, current distribution and variation of temperature along the gas channel. This model is useful for the analysis of cell performance. In [6], an adaptive nonlinear observer was designed to estimate the partial pressure of hydrogen in the anode channel of a fuel cell. By treating the slowly varying inlet partial pressure as an unknown parameter, an adaptive observer was developed that employs a nonlinear voltage injection term. However, this study does not treat an overall system dynamics of PEMFC.

In this paper, a nonlinear fuel cell system model suitable for designing the controller and the observer is introduced to estimate the transient response and also the steady state response. A cascade observer [2] with filtering technique is designed to estimate the pressures of the cathode and the anode. The oxygen pressure in the cathode and the hydrogen pressure in the anode will be estimate by using nonlinear feedback observer. The validity of the proposed observers will be investigated by using a Lyapunov's stability analysis method. Nonlinear state feedback controller will be designed to regulate each pressure.

2. System Dynamics of PEMFC

The system studied in this paper is shown in Fig. 1. It is assumed that the cathode and anode volumes of the multiple fuel cells are lumped as a single stack cathode and anode volumes.

2.1 Cathode pressure model

This model includes the air compressor dynamics, the supply manifold dynamics and the cathode dynamics. The cathode dynamics is developed using the mass conservation principle and the thermodynamic and psychrometric properties of air [4, 7, 8].

$$\begin{aligned}\frac{d\omega_{cp}}{dt} &= -\eta_{cm}\frac{k_t k_v}{R_{cm}J_{cp}}\omega_{cp} + \eta_{cm}\frac{k_t}{R_{cm}J_{cp}}v_{cm} \\ &\quad -\frac{c_p T_{atm} k_{cp}}{\eta_{cp}}\left[\left(\frac{p_{sm}}{p_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\Phi \\ \frac{dp_{sm}}{dt} &= -\frac{\gamma R_a T_{sm} k_{sm,out}}{V_{sm}}\left(p_{sm} - p_{ca}\right)\end{aligned} \tag{1}$$

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$$
+\frac{\gamma R_a T_{cp}}{V_{sm}} k_{cp}\Phi\omega_{cp} \tag{2}
$$

$$
\begin{aligned}
\frac{dp_{O_2}}{dt} = &-\frac{R_{O_2}T_{ca}}{V_{ca}}\left(\frac{x_{O_2,in}k_{ca,in}}{1+w_{ca,in}}+\frac{x_{O_2,out}k_{ca,out}}{1+w_{ca,out}}\right)\\
&\left(p_{O_2}+p_{N_2}+p_{v,ca}\right)+\frac{R_{O_2}T_{ca}}{V_{ca}}\frac{x_{O_2,in}k_{ca,in}}{1+w_{ca,in}}p_{sm}\\
&+\frac{R_{O_2}T_{ca}}{V_{ca}}\frac{x_{O_2,out}k_{ca,out}}{1+w_{ca,out}}p_{rm}-\frac{R_{O_2}T_{ca}}{V_{ca}}M_{O_2}\frac{n}{4F}I_{st}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\frac{dp_{N_2}}{dt} = &-\frac{R_{N_2}T_{ca}}{V_{ca}}\left(\frac{\left(1-x_{O_2,in}\right)k_{ca,1}}{1+w_{ca,in}}+\frac{\left(1-x_{O_2,out}\right)k_{ca,out}}{1+w_{ca,out}}\right)\\
&\left(p_{O_2}+p_{N_2}+p_{v,ca}\right)+\frac{R_{N_2}T_{ca}}{V_{ca}}\frac{\left(1-x_{O_2,in}\right)k_{ca,1}}{1+w_{ca,in}}p_{sm}\\
&+\frac{R_{N_2}T_{ca}}{V_{ca}}\frac{\left(1-x_{O_2,out}\right)k_{ca,out}}{1+w_{ca,out}}p_{rm}
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\frac{dp_{v,ca}}{dt} = &-\frac{R_{v,ca}T_{ca}}{V_{ca}}\left(\frac{w_{ca,in}k_{ca,1}}{1+w_{ca,in}}+\frac{w_{ca,out}k_{ca,out}}{1+w_{ca,out}}\right)\\
&\left(p_{O_2}+p_{N_2}+p_{v,ca}\right)+\frac{R_{v,ca}T_{ca}}{V_{ca}}\frac{w_{ca,in}k_{ca,1}}{1+w_{ca,in}}p_{sm}\\
&+\frac{R_{v,ca}T_{ca}}{V_{ca}}\frac{w_{ca,out}k_{ca,out}}{1+w_{ca,out}}p_{rm}+\frac{R_{v,ca}T_{ca}}{V_{ca}}\frac{M_v n\left(1+2A_{fc}n_d\right)}{2F}I_{st}\\
&-\frac{R_{v,ca}T_{ca}}{V_{ca}}\frac{M_v nA_{fc}D_w}{t_m}\left[f\left(p_{v,ca}\right)p_{v,ca}-f\left(p_{v,an}\right)p_{v,an}\right]
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\frac{dp_{rm}}{dt} = &-\left(\frac{R_a T_{rm}k_{ca,out}}{V_{rm}}+\frac{C_{D,rm}A_{T,rm}\zeta_{rm}}{\sqrt{RT_{rm}}}\right)p_{rm}\\
&+\frac{R_a T_{rm}k_{ca,out}}{V_{rm}}\left(p_{O_2}+p_{N_2}+p_{v,ca}\right)
\end{aligned} \tag{6}
$$

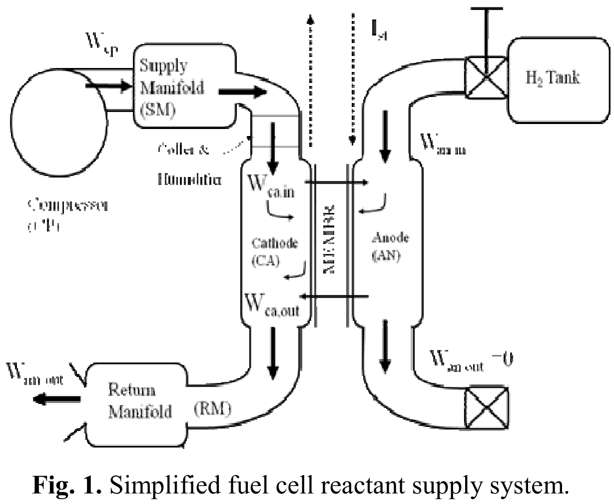

Fig. 1. Simplified fuel cell reactant supply system.

2.2 Anode pressure model

This model is quite similar to the cathode pressure model. In this model, it is assumed that pure hydrogen gas is supplied to the anode from a hydrogen tank [4, 7, 8].

$$
\frac{dp_{sm,an}}{dt} = -\frac{R_{H_2}T_{sm,an}k_{sm,an,out}}{V_{sm,an}}\left(p_{sm,an}-p_{an}\right) \tag{7}
$$

$$
\begin{aligned}
\frac{dp_{H_2}}{dt} = &-\frac{R_{H_2}T_{an}}{V_{an}}\left(\frac{k_1}{1+\omega_{an,in}}+k_{H_2,out}\right)\\
&\left(p_{H_2}+p_{v,an}\right)+k_{H_2,out}\frac{R_{H_2}T_{an}}{V_{an}}p_{ca}\\
&+\frac{k_1}{1+\omega_{an,in}}\frac{R_{H_2}T_{an}}{V_{an}}p_{sm,an}-\frac{R_{H_2}T_{an}}{V_{an}}M_{H_2}\frac{n}{2F}I_{st}
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\frac{dp_{v,an}}{dt} = &-\frac{R_{v,an}T_{an}}{V_{an}}\left(\frac{\omega_{an,in}k_1}{1+\omega_{an,in}}+k_{v,an,out}\right)\left(p_{H_2}+p_{v,an}\right)\\
&+k_{v,an,out}\frac{R_{v,an}T_{an}}{V_{an}}p_{ca}+\frac{\omega_{an,in}k_1}{1+\omega_{an,in}}\frac{R_{v,an}T_{an}}{V_{an}}p_{sm,an}\\
&-\frac{R_{v,an}T_{an}}{V_{an}}\frac{M_v A_{fc}n_d n}{F}I_{st}+\frac{R_{v,an}T_{an}}{V_{an}}\frac{M_v A_{fc}nD_w}{t_m}\\
&\left[f\left(p_{v,ca}\right)p_{v,ca}-f\left(p_{v,an}\right)p_{v,an}\right]
\end{aligned} \tag{9}
$$

3. Observer based controller design

The cathode and anode pressures influence the voltage generated in fuel cell stack. Those also affect the efficiency and the power of the fuel cell. However, it is difficult to directly measure these variables [1]. This problem can be solved using a nonlinear observer. The observation of those variables is needed to design of the suitable controller. The filtered supply manifold pressure is used to design the sliding mode observer for the cathode and anode pressures. The nonlinear state observer for Oxygen and air pressures in the cathode is designed using the estimated cathode pressure. The estimates of the anode pressures are similar to the cathode it.

3.1 Cathode pressure observer

In order to estimate the cathode pressure, we rewrite the eq. (2) as follows:

$$
\begin{aligned}
p_{ca} &= \frac{V_{sm}}{\gamma R_a T_{sm}k_{sm,out}}\frac{dp_{sm}}{dt}+p_{sm}-\frac{T_{cp}}{T_{sm}k_{sm,out}}k_{cp}\Phi\omega_{cp}\\
&= a_{ca,1}\frac{dp_{sm}}{dt}+p_{sm}-a_{ca,2}W_{cp}
\end{aligned} \tag{10}
$$

where $a_{ca,1}\left(=V_{sm}/\gamma R_a T_{sm}k_{sm,out}\right)$, $a_{ca,2}\left(=T_{cp}/T_{sm}k_{sm,out}\right)$

are known parameters. The supply manifold pressure p_{sm} and the mass flow rate of compressor $W_{cp}\left(=k_{cp}\Phi\omega_{cp}\right)$ are also known variables measured via those sensors. However, their derivatives may not be known directly. To solve this problem, we use the cascade observer proposed in [2].

Using the cascade observer [2], we can estimate the derivative of the supply manifold pressure p_{sm} and also design the open loop observer for the cathode pressure p_{ca} as follows :

$$\hat{p}_{ca}=a_{ca,1}\frac{d\hat{p}_{sm}}{dt}+p_{sm}-a_{ca,2}W_{cp} \tag{11}$$

Defining the observation error as $\tilde{p}_{ca}=\hat{p}_{ca}-p_{ca}$, we obtain the equation as follows :

$$\tilde{p}_{ca}=a_{ca,1}\frac{d\tilde{p}_{sm}}{dt} \tag{12}$$

As shown in the above, the observation error may not converge to zero, even though the cascade observer [2] guarantees an asymptotic stability. Since the open-loop observer dose not guarantees the asymptotic stability, we construct the cathode pressure model as follows :

$$p_{f,ca}=\frac{1}{\mu_f s+1}p_{ca} \tag{13}$$

where 's' denotes the Laplace transform. Using the Eq. (10), we obtain the cathode pressure observer model as follows [7, 8]:

$$\mu_f\frac{dp_{f,ca}}{dt}=-p_{f,ca}+a_{ca,1}\frac{dp_{sm}}{dt}+p_{sm}-a_{ca,2}W_{cp} \tag{14}$$

And we can design the sliding mode observer for the cathode pressure model (14) as follows [7, 8]:

$$\begin{aligned}\mu_f\frac{d\hat{p}_{f,ca}}{dt}&=-\hat{p}_{f,ca}+a_{ca,1}\frac{d\hat{p}_{sm}}{dt}+p_{sm}-a_{ca,2}W_{cp}\\&\quad-l_{f,ca}\tilde{p}_{f,ca}-\beta\xi\left(\tilde{p}_{f,ca}\right)\end{aligned} \tag{15}$$

$$\xi(z)=\begin{cases}1\ , & z>\delta\\ z/\delta, & \delta\geq z\geq-\delta\\ -1\ , & -\delta>z\end{cases} \tag{16}$$

where $\beta(>0)$ and $\delta(>0)$ are design parameters. Defining the observation errors as $\tilde{p}_{f,ca}=\hat{p}_{f,ca}-p_{f,ca}$, we obtain the error equation as follows :

$$\mu_f\frac{d\tilde{p}_{f,ca}}{dt}=-\left(1+l_{f,ca}\right)\tilde{p}_{f,ca}+a_{ca,1}\frac{d\tilde{p}_{sm}}{dt}-\beta\xi\left(\tilde{p}_{f,ca}\right) \tag{17}$$

Defining the Lyapunov function candidate as $V_{ca}=\frac{\mu_f}{2}\tilde{p}_{f,ca}{}^2$, the residual error set is as follows [7, [8] :

$$E_{R_{ca}}=\left\{V_{ca},\dot{V}_{ca},\tilde{p}_{f,ca}\ \middle|\ \begin{matrix}V_{ca}\leq\frac{\mu_f a_{ca,1}\gamma_{sm}\delta}{2(1+l_{f,ca}+(\beta/\delta))},\left|\dot{V}_{ca}\right|\leq a_{ca,1}\gamma_{sm}\delta\\ ,\left|\tilde{p}_{f,ca}\right|\leq\sqrt{\frac{\mu_f a_{ca,1}\gamma_{sm}\delta}{(1+l_{f,ca}+(\beta/\delta))}}\end{matrix}\right\} \tag{19}$$

As shown in the above, we can recognize that the sufficiently small μ_f,δ and reasonably large $\beta,l_{f,ca}$ guarantee the smaller error bounds. As the time increasing, we can also show that $\left|\hat{p}_{f,ca}-p_{f,ca}\right|=o\left(\mu_f\right)$, $\hat{p}_{f,ca}\rightarrow p_{f,ca}$ and also $\dot{\tilde{p}}_{f,ca}\rightarrow 0$.

3.2 Anode pressure observer

The anode pressure model is as follows [7, 8] :

$$\begin{aligned}p_{an}&=\frac{V_{sm,an}}{R_{H_2}T_{sm,an}k_{sm,an,out}}\frac{dp_{sm,an}}{dt}+p_{sm,an}\\&=a_{an,1}\frac{dp_{sm,an}}{dt}+p_{sm,an}\end{aligned} \tag{20}$$

The anode pressure observer model is as follows [7, 8] :

$$\mu_f\frac{dp_{f,an}}{dt}=-p_{f,an}+a_{an,1}\frac{dp_{sm,an}}{dt}+p_{sm,an} \tag{21}$$

Defining the observation errors as $\tilde{p}_{f,an}=\hat{p}_{f,an}-p_{f,an}$ and $\tilde{p}_{an}=\hat{p}_{an}-p_{an}$, we can design the sliding mode observer for the anode pressure model (21) as follows :

$$\begin{aligned}\mu_f\frac{d\hat{p}_{f,an}}{dt}&=-\hat{p}_{f,an}+a_{an,1}\frac{d\hat{p}_{sm,an}}{dt}+p_{sm,an}\\&\quad-l_{f,an}\tilde{p}_{f,an}-\beta\xi\left(\tilde{p}_{f,an}\right)\tilde{p}_{f,an}\end{aligned} \tag{22}$$

where $\beta(>0)$ and $\delta(>0)$ are design parameters, and $\xi\left(\tilde{p}_{f,an}\right)$ is the same as (16). Using the above observer (22) for the observer model (21), we obtain the error equation as follows :

$$\mu_f\frac{d\tilde{p}_{f,an}}{dt}=-\left(1+l_{f,an}\right)\tilde{p}_{f,an}+a_{an,1}\frac{d\tilde{p}_{sm,an}}{dt}-\beta\xi\left(\tilde{p}_{f,an}\right) \tag{23}$$

Defining the Lyapunov function candidate as $V_{an}=\frac{\mu_f}{2}\tilde{p}_{f,an}{}^2$, the residual error set is as follows [7, 8]:

$$E_{Ran} = \left\{ V_{an}, \dot{V}_{an}, \tilde{p}_{f,an} \middle| V_{an} \le \frac{\mu_f a_{an,1} \gamma_{sm,an} \delta}{2\left(1 + l_{f,an} + (\beta/\delta)\right)}, \right.$$
$$\left. \left|\dot{V}_{an}\right| \le a_{an,1}\gamma_{sm,an}\delta, \left|\tilde{p}_{f,an}\right| \le \sqrt{\frac{\mu_f a_{an,1}\gamma_{sm,an}\delta}{\left(1 + l_{f,an} + (\beta/\delta)\right)}} \right\} \quad (24)$$

where $\gamma_{sm,an} = \left\|\dot{\hat{p}}_{sm,an}\right\|_\infty$. As shown in the above, we can recognize that the sufficiently small μ_f, δ and reasonably large $\beta, l_{f,ca}$ guarantee the smaller error bounds. As the time increasing $\left|\hat{p}_{f,an} - p_{f,an}\right| = o(\mu_f)$, $\hat{p}_{f,an} \to p_{f,an}$ and also $\dot{\tilde{p}}_{f,an} \to 0$.

3.3 Nonlinear oxygen pressure observer

Defining $x_O{}^T = [x_3 \;\; x_4 \;\; x_5] = \left[p_{O_2} \;\; p_{N_2} \;\; p_{v,ca}\right]$, and $u^T = \left[p_{sm} \;\; I_{st}\right]$, we rewrite the eqs. (3-5) as follows :

$$\begin{aligned} \dot{x}_O &= A_1 x_O + B_1 u + d_1 \\ y_1 &= C x_O \end{aligned} \quad (25)$$

where $d_1{}^T = \left[a_{38} p_{rm} \;\; a_{48} p_{rm} \;\; a_{555} f(p_{v,an}) p_{v,an} + a_{58} p_{rm}\right]$, $A_1 \in R^{3\times3}$, $B_1 \in R^{3\times2}$ and $C = [1 \;\; 1 \;\; 1]$. We design the observer for the above model as follows :

$$\begin{aligned} \dot{\hat{x}}_O &= A_1 \hat{x}_O + B_1 u + \hat{d}_1 + \xi_1 + K(\hat{y}_1 - y_1) \\ \hat{y}_1 &= C \hat{x}_O \end{aligned} \quad (26)$$

where K is the observer gain matrix, $\xi_1 = \beta_1 C^T \xi(\tilde{y}_1)$, $\tilde{y}_1 = \hat{y}_1 - y_1$ and $\xi(\tilde{y}_1)$ is the same as (16). Defining the observation error $\tilde{x}_O = \hat{x}_O - x_O$, we obtain the error equation as follows :

$$\begin{aligned} \dot{\tilde{x}}_O &= A_c \tilde{x}_O + \tilde{d}_1 + \xi_1 \\ \tilde{y}_1 &= C \tilde{x}_O \end{aligned} \quad (27)$$

where $A_c = A_1 + KC$. We can assume that $\|d_1\|$ is bounded and its upper bound $\lambda_d (> \|d_1\|)$ is known. Defining $p_{\min}$, $p_{\max}$ are the minimum and the maximum eigenvalue of the matrix P, we get the following conclusions.

Theorem 1 **:** *If there exist* $P(= P^T > 0)$ *and* $Q(= Q^T > 0)$ *satisfying* $PA_c + A_c^T P = -Q$ *and* $3p_{\min}|\tilde{y}_1| \le \|P\tilde{x}_O\| \le 3p_{\max}|\tilde{y}_1|$*, then the observer in (26) for the system (25) guarantees the asymptotic stability of the observation error system (27), and the observation errors in (27) decay to zero exponentially fast.*

Proof : Defining the positive definite function $V_{O1} = \tilde{x}_O{}^T P \tilde{x}_O$, its 1st time derivative is as follows :

$$\dot{V}_{O1} \le -\tilde{x}_O{}^T Q \tilde{x}_O + 6\lambda_d p_{\max}|\tilde{y}_1| - 6\frac{\beta_1 p_{\min}}{\delta_1}|\tilde{y}_1| \quad (28)$$

where $\beta_1 (> 0)$ and $\delta_1 (> 0)$ are the design parameters satisfying the inequality $\beta_1/\delta_1 > p_{\max}\lambda_d / p_{\min}$. Using this inequality, we can rewrite the Eq. (28) as follows :

$$\dot{V}_{O1} \le -\tilde{x}_O{}^T Q \tilde{x}_O - 6p_{\min}\left\{\frac{\beta_1}{\delta_1} - \frac{\lambda_d p_{\max}}{p_{\min}}\right\} \le -\frac{\lambda_{Q_{\min}}}{\lambda_{P_{\max}}} V_{O1} \quad (29)$$

Therefore V_{O1}, $\dot{V}_{O1}$ are exponentially stable, and $\tilde{x}_O$, $\dot{\tilde{x}}_O$, $\tilde{y}_1$ decay to zero exponentially fast.

3.4 Nonlinear hydrogen pressure observer

Defining $x_H{}^T = [x_7 \, x_8] = \left[p_{H_2} \, p_{v,an}\right]$, and $u_2{}^T = \left[p_{sm,an} \, I_{st}\right]$, we rewrite the Eqs. (7, 8) as follows :

$$\begin{aligned} \dot{x}_H &= A_2 x_H + B_2 u_2 + d_2 \\ y_2 &= C_2 x_H \end{aligned} \quad (30)$$

Where $A_2 \in R^{2\times2}$, $B_2 \in R^{2\times2}$ and $C_2 = [1 \;\; 1]$, $d_2{}^T = \left[a_{6,345} p_{ca} \;\; a_{777} f(p_{v,ca}) p_{v,ca} + a_{7,345} p_{ca}\right]$. We design the observer for the above model as follows :

$$\begin{aligned} \dot{\hat{x}}_H &= A_2 \hat{x}_H + B_2 u_2 + \hat{d}_2 + \xi_2 + K_2(\hat{y}_2 - y_2) \\ \hat{y}_2 &= C_2 \hat{x}_H \end{aligned} \quad (31)$$

where K_2 is the observer gain matrix, $\xi_2 = \beta_2 C_2{}^T \xi(\tilde{y}_2)$, $\tilde{y}_2 = \hat{y}_2 - y_2$ and $\xi(\tilde{y}_2)$ is the same as (16). Defining the observation error $\tilde{x}_H = \hat{x}_H - x_H$, we obtain the error equation as follows :

$$\begin{aligned} \dot{\tilde{x}}_H &= A_{c2} \tilde{x}_H + \tilde{d}_2 + \xi_2 \\ \tilde{y}_2 &= C_2 \tilde{x}_H \end{aligned} \quad (32)$$

where $A_{c2} = A_2 + K_2 C_2$. We can assume that $\|d_2\|$ is bounded and its upper bound $\lambda_{d2} (> \|d_2\|)$ is known. Defining $p_{\min}$, $p_{\max}$ are the minimum and the maximum eigenvalue of the matrix P, we get the following conclusions.

Theorem 2 **:** *If there exist* $P(= P^T > 0)$ *and* $Q(= Q^T > 0)$ *satisfying* $PA_{c2} + A_{c2}^T P = -Q$ *and* $3p_{\min}|\tilde{y}_2| \le \|P\tilde{x}_2\| \le 3p_{\max}|\tilde{y}_2|$*, then the observer in (31) for the system (30) guarantees the asymptotic stability of the observation error system (32), and the observation errors* *in (32) decay to zero exponentially fast.*

Proof : Defining the positive definite function $V_{H1} = \tilde{x}_H^T P \tilde{x}_H$, its 1st time derivative is as follows :

$$\dot{V}_{H1} \le -\tilde{x}_H^T Q\tilde{x}_H + 6\lambda_{d2} p_{\max} |\tilde{y}_2| - 6\frac{\beta_2 p_{\min}}{\delta_2}|\tilde{y}_2| \quad (33)$$

where $\beta_2(>0)$ and $\delta_2(>0)$ are the design parameters satisfying the inequality $\beta_2/\delta_2 > p_{\max}\lambda_{d2}/p_{\min}$.

$$\dot{V}_{H1} \le -\tilde{x}_H^T Q\tilde{x}_H - 6p_{\min}\left\{\frac{\beta_2}{\delta_2} - \frac{\lambda_{d2}p_{\max}}{p_{\min}}\right\} \le -\frac{\lambda_{Q_{\min}}}{\lambda_{P_{\max}}}V_{H1} \quad (34)$$

Therefore V_{H1}, $\dot{V}_{H1}$ are exponentially stable, and $\tilde{x}_H$, $\dot{\tilde{x}}_H$, $\tilde{y}_2$ decay to zero exponentially fast.

3.5 Oxygen pressure state feedback controller

For the system (25), we choose the reference model [3] as follows:

$$\dot{x}_{mO} = A_{mO}x_{mO} + B_{mO}r_O \quad (35)$$

Where $A_{mO} \in R^{3\times3}$ is asymptotically stable, $B_{mO} \in R^{3\times2}$ is input matrix. We choose the control input as follows :

$$u = \Theta_O\hat{x}_O + Q_O^{*}r_O + z_1 \quad (36)$$

where $\Theta_O \in R^{2\times3}$ consists of adjustable parameters and $Q_O^{*} \in R^{2\times2}$ such that

$$B_1Q_O^{*} = B_{mO} \quad (37)$$

We assume that a constant matrix Θ_O exists such that

$$A_1 + B_1\Theta_O = A_{mO} \quad (38)$$

Substituting (36) into (25), we obtain the error equation as follows :

$$\dot{e}_O = A_{mO}e_O + d_1 + B_1z_1 \quad (39)$$

where $e_O = \hat{x}_O - x_m$, and z_1 is as follows :

$$z_1 = -\beta_1\xi(e_O)$$
$$\xi(e_O) = \begin{cases} 1 & , \ e_O > 0 \\ -1 & , \ e_O < 0 \end{cases} \quad (40)$$

where $\beta_1(>0)$ is design parameter. Defining $V_{O2} = e_O^T Pe_O$, its derivative is as follows :

$$\dot{V}_{O2} \le -e_O^T Qe_O + \|Pd_1\|\|e_O\| - \|PB_1d_z\|\|e_O\| \quad (41)$$

where $\|Pd_1\| \le \|PB_1d_z\|$, $\|z_1\| \le \|d_z\|$, and $A_{mO}^T P + PA_{mO} = -Q$.

$$\dot{V}_{O2} \le -e_O^T Qe_O \quad (42)$$

Therefore V_{O2}, $\dot{V}_{O2}$ are exponentially stable, and e_O, $\dot{e}_O$ decay to zero exponentially fast. However, e_O, $\dot{e}_O$ go to zero, $e_{mO}(= x_O - x_{mO})$ does not go to zero exponentially fast. Adding $(x_O - x_O)$ into $e_O(= \hat{x}_O - x_{mO} = e_{mO} + \tilde{x}_O)$, we can rewrite the Eq.(42) as follows :

$$\begin{aligned}\dot{V}_{O2} &\le -e_{mO}^T Qe_{mO} - \tilde{x}_O^T Q\tilde{x}_O + e_{mO}^T Q\tilde{x}_O + \tilde{x}_O^T Qe_{mO} \\ &\le -\lambda_{q\min}\|e_{mO}\|^2 - \lambda_{q\min}\|\tilde{x}_O\|^2 + 2\lambda_{q\max}\|e_{mO}\|\|\tilde{x}_O\| \\ &\le -\lambda_{q\min}\left\{\|e_{mO}\| - \frac{\lambda_{q\max}}{\lambda_{q\min}}\|\tilde{x}_O\|\right\}^2 + \left\{\frac{\lambda_{q\max}^2}{\lambda_{q\min}} - \lambda_{q\min}\right\}\|\tilde{x}_O\|^2\end{aligned} \quad (43)$$

where $\lambda_{q\min}$ is the minimum eigenvalue of matrix Q, and $\lambda_{q\max}$ is the maximum eigenvalue of it. Since $\|\tilde{x}_O\|$ decays to zero exponentially fast, we can prove the following inequality :

$$\gamma_{O1}\lambda_{qO}\|\tilde{x}_O\| \le \|e_{mO}\| \le \gamma_{O2}\lambda_{qO}\|\tilde{x}_O\| \quad (44)$$

where $\lambda_{qO} = \left\{\sqrt{(\lambda_{q\max}/\lambda_{q\min}+1)(\lambda_{q\max}/\lambda_{q\min}-1)} + \lambda_{q\max}/\lambda_{q\min}\right\}$, γ_{O1} and γ_{O2} are positive constants. Therefore we can show that the state feedback error $e_{mO}(= x_O - x_{mO})$ decreases to zero asymptotically as $\|\tilde{x}_O\|$ converges to zero exponentially.

3.6 Hydrogen pressure state feedback controller

For the system (30), we choose the reference model [3] as follows :

$$\dot{x}_{mH} = A_{mH}x_{mH} + B_{mH}r_H \quad (45)$$

Where $A_{mH} \in R^{2\times2}$ is asymptotically stable, $B_{mH} \in R^{2\times2}$ is input matrix. We choose the control input as follows :

$$u_2 = \Theta_H\hat{x}_H + Q_H^{*}r_H + z_2 \quad (46)$$

where $\Theta_H \in R^{2\times2}$ consists of adjustable parameters and $Q_H^{*} \in R^{2\times2}$ such that

$$B_2Q_H^{*} = B_{mH} \quad (47)$$

We assume that a constant matrix Θ_H exists such that

$$A_2 + B_2\Theta_H = A_{mH} \tag{48}$$

Substituting (48) into (30), we obtain the error equation as follows :

$$\dot{e}_H = A_{mH}e_H + d_2 + B_2 z_2 \tag{49}$$

where $e_H = \hat{x}_H - x_{mH}$, and z_2 is as follows :

$$z_2 = -\beta_2 \xi(e_H)$$
$$\xi(e_H) = \begin{cases} 1\ , e_H > 0 \\ -1, e_H < 0 \end{cases} \tag{50}$$

where $\beta_2(>0)$ is design parameter. Defining $V_{H2} = {e_H}^T P_{H2} e_H$, its derivative is as follows :

$$\dot{V}_{H2} \le -{e_H}^T Q_{H2} e_H + \|P_{H2}d_2\|\|e_H\| - \|P_{H2}B_2 d_{z2}\|\|e_H\| \tag{51}$$

where $\|P_{H2}d_2\| \le \|P_{H2}B_2 d_{z2}\|$, $\|z_2\| \le \|d_{z2}\|$, and $A_{mH}^T P_{H2} + P_{H2}A_{mH} = -Q_{H2}$.

$$\dot{V}_{H2} \le -{e_H}^T Q_{H2} e_H \tag{52}$$

Therefore V_{H2}, $\dot{V}_{H2}$ are exponentially stable, and e_H, $\dot{e}_H$ decay to zero exponentially fast. As the result, we can show that the system error converges to zero exponentially fast. However, e_H, $\dot{e}_H$ go to zero, $e_{mH}(= x_H - x_{mH})$ does not goes to zero exponentially fast. Adding $(x_H - x_H)$ into $e_H(= e_{mH} + \tilde{x}_H)$, we can rewrite the eq.(52) as follows :

$$\begin{aligned}\dot{V}_{H2} &\le -e_{mH}^T Q_{H2} e_{mH} - \tilde{x}_H^T Q_{H2} \tilde{x}_H + e_{mH}^T Q_{H2} \tilde{x}_H + \tilde{x}_H^T Q_{H2} e_{mH} \\ &\le -\lambda_{qH\min}\|e_{mH}\|^2 - \lambda_{qH\min}\|\tilde{x}_O\|^2 + 2\lambda_{qH\max}\|e_{mH}\|\|\tilde{x}_H\| \\ &\le -\lambda_{qH\min}\left\{\|e_{mH}\| - \frac{\lambda_{qH\max}}{\lambda_{qH\min}}\|\tilde{x}_H\|\right\}^2 + \left\{\frac{\lambda_{qH\max}^2}{\lambda_{qH\min}} - \lambda_{qH\min}\right\}\|\tilde{x}_H\|^2\end{aligned} \tag{53}$$

where $\lambda_{qH\min}$ is the minimum eigenvalue of matrix Q_{H2}, and $\lambda_{qH\max}$ is the maximum eigenvalue of it. Since $\|\tilde{x}_H\|$ decays to zero exponentially fast, we can prove the following inequality :

$$\gamma_{H1}\lambda_{qH}\|\tilde{x}_H\| \le \|e_{mH}\| \le \gamma_{H2}\lambda_{qH}\|\tilde{x}_H\| \tag{54}$$

where $\lambda_{qH} = \left\{\sqrt{\left(\lambda_{qH\max}/\lambda_{qH\min}+1\right)\left(\lambda_{qH\max}/\lambda_{qH\min}-1\right)} + \lambda_{qH\max}/\lambda_{qH\min}\right\}$, γ_{H1} and γ_{H2} are positive constants. Therefore we can show that the state feedback error $e_{mH}(= x_H - x_{mH})$ decreases to zero asymptotically as $\|\tilde{x}_H\|$ converges to zero exponentially.

4. Conclusion

In this paper, the sliding mode observer was designed to estimate the cathode and anode pressures of PEMFC system. A observer based nonlinear state feedback controller was designed to regulate the system states such as supply manifold pressure, O_2 pressure, H_2 pressure, return manifold pressure, etc. Exponential stability of the system was achieved for the nonlinear PEMFC system. Research is ongoing to address robustness issues for the proposed observer and controller in the presence of bounded disturbances and uncertainties.

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