J Electr Eng Technol Vol. 7, No. 6: 884-890, 2012
http://dx.doi.org/10.5370/JEET.2012.7.6.884

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

Dynamic Hysteresis Model Based on Fuzzy Clustering Approach

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Abstract – Hysteretic behavior model of soft magnetic material usually used in electrical machines and electronic devices is necessary for numerical solution of Maxwell equation. In this study, a new dynamic hysteresis model is presented, based on the nonlinear dynamic system identification from measured data capabilities of fuzzy clustering algorithm. The developed model is based on a Gustafson-Kessel (GK) fuzzy approach used on a normalized gathered data from measured dynamic cycles on a C core transformer made of 0.33mm laminations of cold rolled SiFe. The number of fuzzy rules is optimized by some cluster validity measures like 'partition coefficient' and 'classification entropy'. The clustering results from the GK approach show that it is not only very accurate but also provides its effectiveness and potential for dynamic magnetic hysteresis modeling.

Keywords: Cluster validity, Dynamic magnetic hysteresis, Gustafson-kessel algorithm, Model identification

1. Introduction

The accuracy and dynamic performance of rotational electrical machines, magnetic actuators and transformers used in power plants and electronic devices are directly linked to the nonlinearity of the magnetic material used in their construction. Proper analysis of systems with hysteresis requires a model of this phenomenon. Recently, some papers have presented a variety of models for the representation of hysteresis loops [1-5] in theses papers the authors have introduced different linearization methods and mathematical functions that allow desired BH relationships to be predicted from observed loops. But these models cannot represent the nonlinear hysteresis characteristics with high precision. The subject of soft computing especially neural network, fuzzy logic and neuro-Fuzzy has received much attention in the field of function approximation. However, several papers have presented a diversity of models for the prediction of magnetic hysteresis especially by applying neural network trained by back propagation algorithm. It has the advantage of easy identification, and the resulting model can approximate the measured object well as a continuous function [6-12]. In, this paper we introduce a mapping model based on fuzzy clustering approach to characterize dynamic magnetic hysteresis of soft material vastly used in electrical devices. Four hysteresis loops and 101 data points in each loop are used as the input and desired output for learning and identification of fuzzy model based on Gustafson Kessel clustering Algorithm. These hysteresis loops are enough for the depiction of the highly nonlinear relationship between the magnetic induction B and the magnetic field H.

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Received: April 5, 2011; Accepted: July 1, 2012

2. Fuzzy Clustering Analysis

The objective of cluster analysis is the partitioning of data into subsets based on common similarities, from a large data set to produce a concise representation of a system's behavior. Clustering can be considered as the most important unsupervised learning method and do not rely on assumptions common to statistical classification methods. It can be used in nonlinear system modeling, identification and other scientific fields. A cluster can be defined as a collection of objects which are similar between them and dissimilar to the objects belonging to other clusters. Clustering algorithms are applied to data that could be quantitative, qualitative or a mixture of both. Our attention here will be focused on clustering of quantitative data which consist on physical process observations. Clustering algorithms may use an objective function to measure the desirability of partitions. Nonlinear optimization algorithms are used to search for local optima of the objective function. In fuzzy clustering methods can see as a generalization of hard clustering. A fuzzy partition of data set Z can be represented by a cxN matrix $U=[\mu_{i,k}]$ called fuzzy partition matrix, where $\mu_{i,k}$ denotes the degree of membership that the k^{th} observation belongs to the c^{th} cluster and the i^{th} row of U contains values of membership function of the i^{th} fuzzy subsets of Z.

2.1 Fuzzy c-means technique

A large family of fuzzy clustering algorithms is based on minimization of the fuzzy c-means objective function formulated as:

$$J_{FCM}(Z;U,V)= \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m D_{ikA}^2 \tag{1}$$

$$\mu_{ik}\in[0,1],\quad 0<\sum_{k=1}^{N}\mu_{ik}<N,\quad \sum_{i=1}^{c}\mu_{ik}=1 \tag{2}$$

$$V=[v_1,v_2,...,v_c]\in R^n$$

Where V is a matrix of cluster centers, which have to be determined, and the squared inner-product distance norm is done by the following equation:

$$D_{ikA}=\|z_k-v_i\|_A^2=(z_k-v_i)^T A(z_k-v_i) \tag{3}$$
$$1\le i\le c,\quad 1\le k\le N$$

It is defined by matrix A and $m\in[1,\infty]$ is a weighting exponent which determines the fuzziness of the resulting clusters.

2.2 Gustafson-Kessel clustering approach (GK)

The proposed approach by Gustafson and Kessel [19] is an FCM extension by employing an adaptive distance norm taking into account the accurate form of each cluster. It allows the detection of the hyper ellipsoids clusters adapted to the geometry of the observations. However, each cluster has its own norm-inducing matrix A_i, which yields the following inner-product norm:

$$D_{ikA_i}^2=(z_k-v_i)^T A_i(z_k-v_i) \tag{4}$$

In this way, quasi-linear behaviors of the existing operating regimes are detected quite correctly. Improved covariance estimation for Gustafson-Kessel algorithm has been introduced in [20]. The objective function of GK algorithm is described by:

$$J_{GK}(Z;U,V,A)=\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m D_{ikA_i}^2 \tag{5}$$

where $U\in R^{c\times N}$, $V\in R^{c\times N}$ and $A\in R^{n\times n}$ it is defined by :

$$A_i=(\psi_i.\det(F_i))^{1/n}F_i^{-1} \tag{6}$$

$$|A_i|=\psi_i \tag{7}$$

F_i is the fuzzy covariance matrix of i-th cluster defined by the following expression:

$$F_i=\frac{\sum_{k=1}^{N}(\mu_{ik})^m(z_k-v_i)(z_k-v_i)^T}{\sum_{k=1}^{N}(\mu_{ik})^m} \tag{8}$$

The structure of the fuzzy covariance matrix $F_i\in R^{n\times n}$ provides information on the form and orientation of the hyperellipsoïds clusters. in fact, the square root of each one of its eigenvalues λ_i, $j= 1... n$, correspond to the lengths of the axes in the hyperspace. The directions of the axes are given by the eigen vectors ϕ_j. The μ_{ik} cœfficients of fuzzy covariance matrix is:

$$\mu_{ik}=\frac{1}{\sum_{j=1}^{c}(D_{ikA}^2/D_{jkA}^2)^{1/(m-1)}} \tag{9}$$
$$1\le i\le c,\quad 1\le k\le N$$

The clusters centers v_i are calculated as follows:

$$v_i=\frac{\sum_{k=1}^{N}(\mu_{ik})^m.z_k}{\sum_{k=1}^{N}(\mu_{ik})^m},\quad 1\le i\le c. \tag{10}$$

The flowchart of Gustafson-Kessel algorithm can be summarised as indicated in Fig. 1:

1. Given the data set
2. Choose the number of cluster
3. Select the weithning exponent and the termination criterion
4. Initialize the partition matrix randomly
5. Repeat for t=1,2,…
6. Compute the clusters centers
7. Compute the fuzzy covariance matrix
8. Compute the distance
9. Update the partition matrix
10. Stop if the termination criterion is satisfied

2. Dynamic Hysteresis Modeling Process

The GK algorithm is applied to a normalized gathered data of dynamic magnetic hysteresis cycles of a C core transformer made of 0.33mm laminations of cold rolled SiFe at 50 Hz (Fig. 1) comes from a research report [12]. Fig. 3 shows the clustering results for four clusters by the major hysteresis cycle data set, the 'o' markers are the cluster centers. From these results it can be seen that GK algorithm can adapt the distance norm to the underlying distribution of the data. The ellipsoidal shape of the clusters can be determined from the eigenstructure of the resulting covariance matrices Fi. The GK algorithm is not so sensitive to the normalization of data and results can be significantly changed by different initial state. Simulation result of major hysteresis cycle for four clusters is illustrated in Fig. 4. It's clearly that the measured cycle is closely reproduced by GK approach.

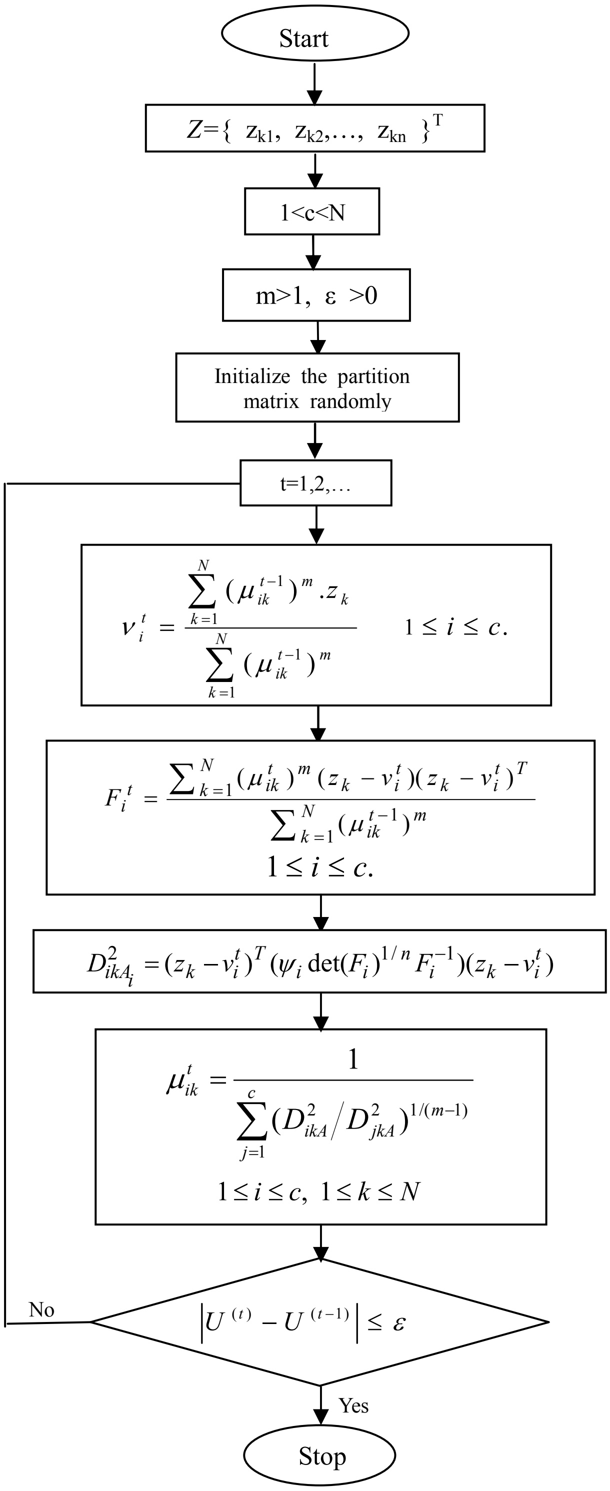

Fig. 1. Flowchart of the Gustafson-Kessel approach

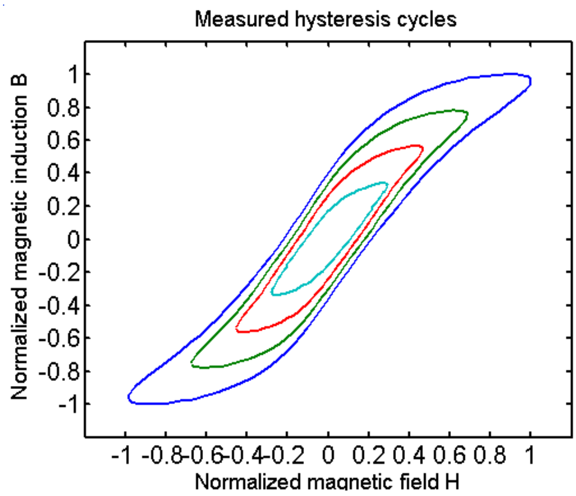

Fig. 2. Measured hysteresis loops

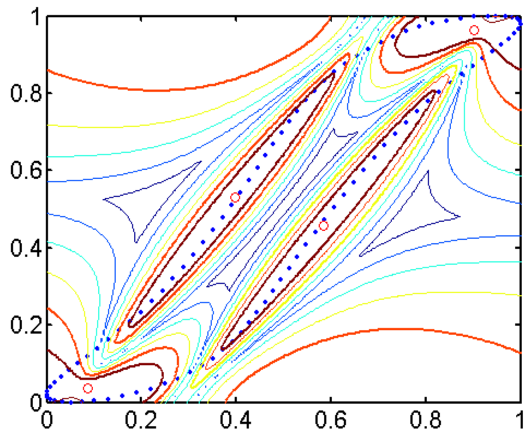

Fig. 3. The results of Gustafson-Kessel algorithm by the major hysteresis cycle data set.

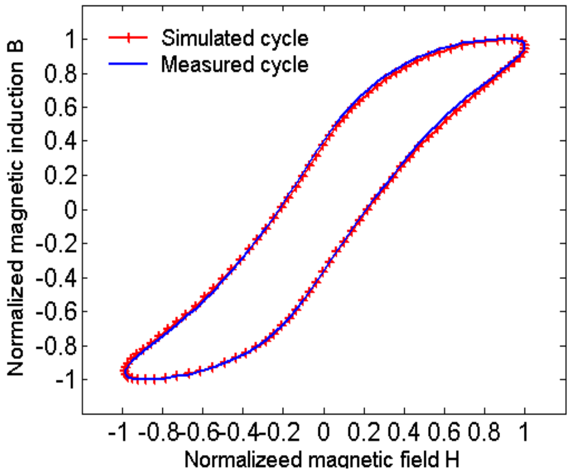

Fig. 4. Major hysteresis cycle identification for four clusters.

3. Dynamic Hysteresis Approximation and Evaluation Process

The goal is to predict dynamic magnetic hysteresis on the basis of four given cycles. The data set employed is 1x101 pairs for training and 3x101 pairs for checking. The performance model is measured by the root mean squared error (RMSE), variance accounting for (VAF) introduced by Babuska et Al [14] and a calculating time (Tcal) given by:

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(y_k - \hat{y}_k)^2} \tag{11}$$

$$VAF = 100\%\left[1 - \frac{\text{var}(y-\hat{y})}{\text{var}(y)}\right] \tag{12}$$

The best value of VAF is 100% when the two signals are equal, if different; the VAF is lower whereas Tcal is the time machine required for fuzzy model construction from data.

Results of model identification quality (VAF, RMSE and computing time) from various stages of identifications using toolbox proposed in [17, 18] in a micro-computer Pentium 4, 2.06 GHz with 256 MB of RAM and an operating system Windows 2000 Professional is shown in Table 1. The best model with respect to the RMSE and VAF criterion is for two clusters (in red, Table 1) whereas for the Tcal is for three clusters (in magenta, Table 1).

Table 1. Model performance of dynamic magnetic hysteresis by GK approach

Parameters	*c*=2	*c*=3	*c*=4
VAF	99.9978	99.9975	99.7737
RMSE	0.0034	0.0059	0.0355
Tcal (s)	0.172	0.1410	0.1570

The Fuzzy antecedent and consequent rules which describe the behaviour of the optimal local models and the centres of clusters are illustrated in Table 2 and Table 3 respectively.

Table 2. Fuzzy model rules obtained with GK approach for two clusters

Clusters	Antecedent rules	Consequent rules
1	if *y(k-1)* is A_{11} and *y(k-2)* is A_{12} and *u(k)* is A_{13} and *u(k-i)* is A_{14}	*y(k)=2. y(k-1)-1.01. y(k-2)+2.26.10*$^{-2}$*. u(k)-2.45.10*$^{-2}$*. u(k-1)+1.98.10*$^{-5}$
2	if *y(k-1)* is A_{21} and *y(k-2)* is A_{22} and *u(k)* is A_{23} and *u(k-i)* is A_{24}	*y(k)=1.92. y(k-1)-9.12. y(k-2)+ 6.7.10*$^{-2}$*. u(k)-7.91.10*$^{-2}$*. u(k-1)+6.28.10*$^{-5}$

Table 3. Clusters centers of GK hysteresis model

$v_{y(k-1)}$	$v_{y(k-2)}$	$v_{u(k)}$	$v_{u(k-1)}$
5.49.10^{-3}	5.89.10^{-3}	-1.76.10^{-3}	-1.28.10^{-3}
4.52.10^{-2}	4.44.10^{-2}	5.10^{-2}	5.10^{-3}

4. Cluster Validity of Dynamic Magnetic Hysteresis Fuzzy Model

In the course of every partitioning problem the number of clusters must be given by the user before the calculation, but it is rarely known a priori, in this case it must be searched also with using validity indices [15, 16]. Different scalar validity measures have been proposed in the literature, none of them is perfect by itself thus it is suitable to use some indices simultaneously.

4.1 Partition coefficient (PC)

Measures the ammount of overlapping between clusters. It use only the membership values of the fuzzy partition of data defined by Bezdek [13] as follows:

$$PC(c) = \frac{1}{N}\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^2 \tag{13}$$

where μ_{ik} is the membership of data point *i* in cluster *k*. The main drawback of this indice is the lack of direct connection to the data itself. The optimal number of clusters can be found by the maximum value.

4.2 Classification entropy (CE)

Measures only the fuzzyness of the cluster partition, which is similar to the Partition Coefficient.

$$CE(c) = -\frac{1}{N}\sum_{i=1}^{c}\sum_{k=1}^{N}\mu_{ik}\log(\mu_{ik}) \tag{14}$$

4.3 Partition index (SC):

Express the ratio of the sum of compactness and separation of the clusters. Each cluster is measured with the cluster validation method. It is normalized by dividing it by the fuzzy cardinality of the cluster. To receive the partition index, the sum of the value for each individual cluster is used.

$$SC(c) = \sum_{i=1}^{c}\frac{\sum_{k=1}^{N}(\mu_{ik})^m \left\|X_k - v_i\right\|^2}{\sum_i \sum_{j=1}^{c}\left\|v_j - v_i\right\|^2} \tag{15}$$

SC is useful when comparing different partitions with the same number of clusters. A lower value of SC means a better partition.

4.4 Separation index (S)

In contrast with the partition index (SC), the separation index uses a minimum-distance separation for partition validity.

$$S(c)=\frac{\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^2\left\|x_k-v_i\right\|^2}{N.\min_{i,j}\left\|v_j-v_i\right\|^2} \qquad (16)$$

4.4 Xie and beni's index (XB)

Express the ratio of the total variation within the clusters and the separation of the clusters. The optimal number of clusters should minimize the value of the index.

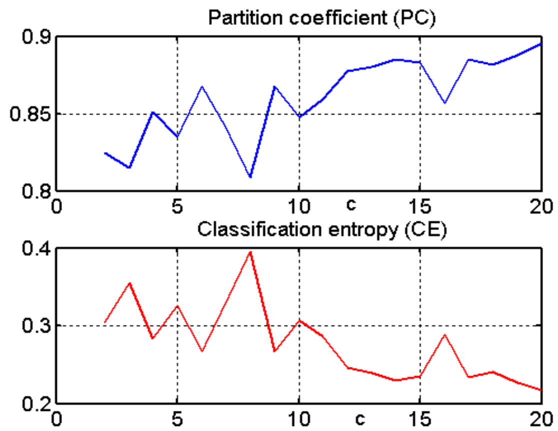

Fig. 5. Value of PC and CE coefficient

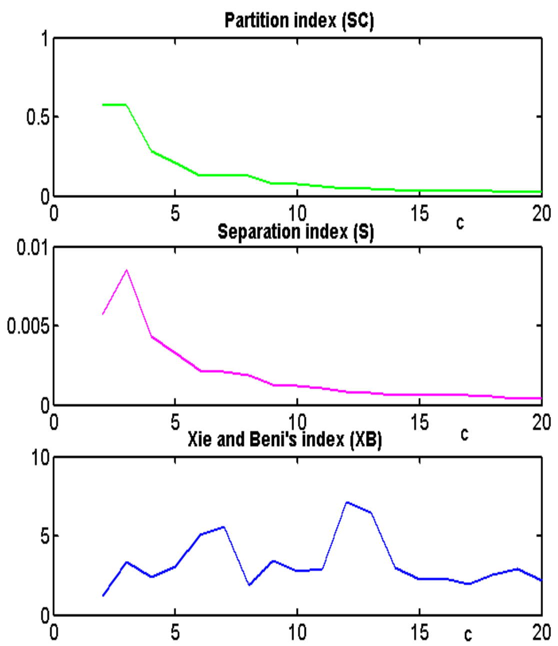

Fig. 6. Value of SC, S and XB indices

$$XB(c)=\frac{\sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m\left\|x_k-v_i\right\|^2}{N.\min_{i,k}\left\|x_k-v_i\right\|^2} \qquad (17)$$

It is essential to optimize the number of fuzzy clusters. This can be accomplished by validation analysis using fuzzy cluster validity indices. In this paper, P, C, CE, SC, XB and the separation index are adopted. Fig. 5 and Fig. 6 shows the results of clusters validity indices. All the clusters validity indices with respect to RMSE and VAF shows that two clusters can be considered as the optimal number of rules but three clusters is the optimal number of rules with respect to Tcal (Table 1).

Table 4. Training and checking error

	RMSE	VAF
Cycle 1	0.0081	99.987
Cycle 2	0.0224	99.8679
Cycle 3	0.0224	99.7441
Cycle 4	0.0085	99.8937

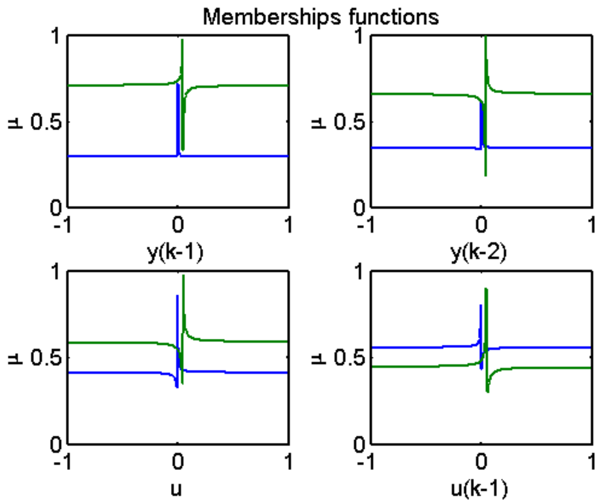

Fig. 7. Memberships functions of fuzzy model

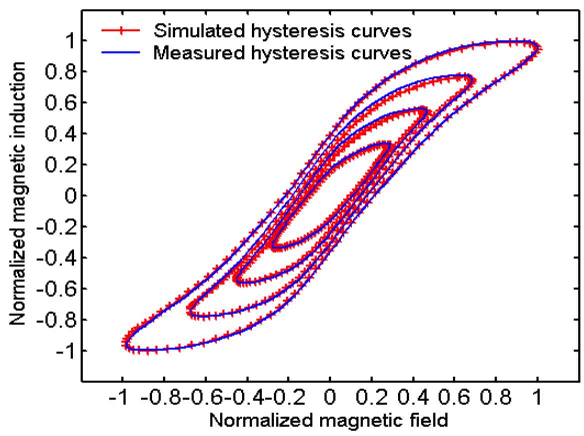

Fig. 8. Dynamic Hysteresis cycle measured and simulated with GK algorithm

The GK clustering algorithm is used to construct fuzzy model with two fuzzy rules. Four cycles are considered, the major hysteresis cycle is used for training and the other minor cycles for testing. Results of the training and validation error of the obtained fuzzy model are shown in Table 4. The corresponding fuzzy membership's functions are illustrated in Fig. 7. Fig. 8 illustrates the comparison of fuzzy model output and measured outputs.

5. Conclusion

This paper studies the fuzzy modeling of soft magnetic behavior. An original dynamic magnetic hysteresis model is proposed based on Gustafson-Kessel approach. Several indices are used to optimize the fuzzy model rules and improve its precision. Results show that in both training and testing, the model produced reliable outputs with relatively small errors. However, the developed model shows that the clustering technique is suitable for identification of highly soft magnetic materials.

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