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Output Data Analysis of Simulation: A Review

Byeong Yun Chang[1]

시뮬레이션 출력 자료 분석에 관한 연구

장병윤

ABSTRACT

Simulation is the imitation of the operation of a real-world process or system over time. It concerns the study of the operating characteristics of real systems. Typically, a simulation project consists of several steps such as data collection, coding, model verification, model validation, experimental design, output data analysis, and implementation. Among these steps of a simulation study this paper focus on statistical analysis methods of simulation output data. Specially, we explain how to develop confidence interval estimators for mean μ in terminating and non-terminating simulation cases. We, then, explore the estimation techniques for f(μ), where the function f(•) is a nonlinear that is continuously differentiable in a neighborhood of μ with f'(μ) ≠0.

Key words : Simulation, Output Data Analysis, Confidence Interval, Nonlinear Function

요 약

시뮬레이션은 시간에 따라 변화하는 실제 프로세스나 시스템에 대한 운영을 모방한다. 이 방법론은 실제 시스템의 운영적 특성들을 연구하는데 관심을 가진다. 일반적으로 시뮬레이션 프로젝트는 자료수집, 코딩, 모델 타당성 및 유효성 검증, 실험계획, 출력 자료 분석, 적용과 같은 여러 가지 단계들을 거친다. 본 논문에서는 이와 같은 여러 단계들 중 시뮬레이션 출력 자료의 통계적 분석에 대한 연구들을 살펴 보기로 한다. 특별히 본 연구에서는 종료형(terminating) 시뮬레이션과 비종료형(non-terminating) 시뮬레이션 각각의 경우에 대한 평균 μ를 구간 추정하는 방법을 설명한다. 그런 다음 이 연구결과를 확장한 일반적인 평균 μ의 보다 더 일반화된 형태인 $f(\mu)$를 추정하는 방법에 대해 설명한다. 여기서 함수 $f(\bullet)$은 비선형 함수로서 $f'(\mu)\neq 0$인 μ의 이웃에서 미분 가능한 함수 이다.

주요어 : 시뮬레이션, 출력 자료 분석, 신뢰구간, 비선형 함수

1. Introduction

Simulation is the imitation of the operation of a real world process to understand the behavior of the process or evaluate various operational policies. A simulation study consists of several steps such as data collection, coding, model verification, model validation, experimental design, output data analysis, and implementation. Among these steps this paper considers output data analysis from a computer simulation.

In general, the objective of simulation output analysis is to estimate some unknown characteristic or parameter of the system being studied. The analyst often wants not only an estimate of this parameter value, but also some measure of the estimator's precision. Confidence intervals are widely used for this purpose.

Let $y_{11}, y_{12}, \cdots, y_{1n}$ be a realization of the random variables $Y_1, Y_2, \cdots, Y_n$ resulting from a simulation run of length n observation (the j^{th} random number used in the i^{th} run is denoted y_{ij}). If we run the simulation

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[1] 아주대학교 경영학부
주 저 자 : 장병윤
교신저자 : 장병윤
E-mail; bychang@ajou.ac.kr

with a different set of random numbers, we will then obtain a different realization $y_{21}, y_{22}, \cdots, y_{2n}$ of the random variables $Y_1, Y_2, \cdots, Y_n$. In general, suppose that we make l independent replications (runs) of the simulation (i.e., different random numbers are used for each replication) of length n, the the following results in the observations are obtained.

$$\begin{array}{l} y_{11}, \cdots y_{1j}, \cdots y_{1n} \\ y_{21}, \cdots y_{2j}, \cdots y_{2n} \\ \quad\vdots \\ y_{l1}, \cdots y_{lj}, \cdots y_{\ln} \end{array}$$

The observations from a particular replication (row) are clearly not independent. However, note that the data $y_{1j}, y_{2j}, \cdots, y_{lj}$ (from the j^{th} column) are independent and identically distributed (IID) observations of the random variable Y_j, for $j = 1, 2, ..., n$. This independence across runs is the key to certain relatively simple output-data analysis methods. Then, from the observations y_{ij} $(i = 1, 2, ..., l; j = 1, 2, ..., n)$, we can estimate some unknown parameter of the system and find a confidence interval.

Throughout this paper, we will be referencing ideas and thoughts from [1-7]. In Section 2, we briefly review the concept of simulation and steps of a simulation study. We, then, categorize the simulations as terminating and non-terminating simulations and investigate simulation output analysis in both cases in Section 3. In addition, we discuss the estimation techniques for $f(\mu)$ in a stationary stochastic process, where the function $f(\cdot)$ is a nonlinear that is continuously differentiable in a neighborhood of μ with $f'(\mu) \neq 0$. In Section 4, we present conclusions and future research areas for the interested reader.

2. Simulation

Simulation, according to Shannon [8], is "the process of designing a model of a real system and conducting experiments with this model for the purpose either of understanding the behavior of the system or of evaluating various strategies for the operation of the system." With simulation people usually study real processes or systems to understand, evaluate, and improve. Therefore, to understand, evaluate, and improve a real process or system they have goals of the simulation study. These goals can be evaluated with output performance measures such as total production, average waiting time in queue, maximum waiting time in queue, time-average number of entities waiting in the queue, average and maximum total time in system and utilization of a resource [1].

In general, a simulation model consists of the following elements [1,2].

- Entities: Most simulations have "players" called entities. Entities are the dynamic objects in the simulation. They usually are created, move around for a while, and then are disposed of as they leave. Entities cause changes in the state of the simulation.
- Attributes: All entities have a common characteristic called an attribute. However, one entity is differing from another with a specific value of the attribute.
- Resources: Resources represent anything that entities seize in the simulation. They have a restricted (or constrained) capacity. Common examples of resources include workers, machines, nodes in a communication network, traffic intersections, etc.
- Variables: A variable holds a piece of information that reflects some characteristic of a process or system, regardless of how many or what kinds of entities might be around. There are many different variables in a simulation model.
- Queues: Queues are places in the simulation where entities wait for an unspecified period of time. Entities can be waiting on resources to be available or for a given system condition to occur.
- Statistical Accumulators: Statistical accumulators collects statistics on certain state (such as the state of a resources), or the value of global variables, or certain performance statistics based on attributes of the entity.
- Events: An event is something that happens at an instant of simulated time that might change attributes, variables, statistical accumulators such as arrival,

departure, and end events.

- Simulation Clock: The simulation clock holds the current value of time in the simulation.

In a simulation study the random input causes random output. Therefore, in a simulation with random input, statistical analysis is needed. Unfortunately, since simulations do not produce independent and identically distributed normal data, we need different types of techniques to deal with simulation output data than "classical" statistical techniques [3].

In studying processes or system with simulation the following steps are generally helpful. For the detailed explanations refer to references [1,2,4].

- Understanding the system
- Have clear goals for the study
- Modeling and Coding
- Verification
- Design the experiments
- Run the experiment
- Analyze the results
- Presentation and Documentation

3. Simulation Output Analysis

In the step of output analysis in a simulation study the analyst is often trying to estimate some unknown characteristic or parameter of the process or system being studied. The analyst often wants not only an estimate of this parameter value, but also some measure of the estimator's precision. Confidence intervals are widely used for this purpose.

3.1 Classification

- Terminating simulations: In the terminating simulations, the length of the simulation run is explicitly determined naturally. In addition, since different runs in the terminating simulations use independent random numbers and the same initialization rule, the analyst can apply "classical" statistical techniques with multiple runs. As examples of terminating simulations we can consider fast-food restaurants or banks open from 9AM to 6PM.
- Non-terminating simulation: In non-terminating simulations, the analyst is interested in the long-run behavior of the process or the system. If the simulation runs long enough, it approaches a "steady state," independent of the simulation's initial conditions. In this case, however, "in the long run" does not naturally determine the length of the simulation run. To analyze output data from non-terminating simulations the analyst needs to understand more complex statistical techniques than "classical" statistical techniques. As an example of a non-terminating simulation we can consider a continuously running production line in which some long-run performance measure is interested. The non-terminating simulations are also called steady-state simulations.

3.2 Terminating Simulation Output Analysis

Consider the observations of l independent replications of the terminating simulation of length n. For now, suppose that the analyst is interested in estimating the performance measure

$$\mu \equiv E[Y].$$

Let us denote the sample mean from the i^{th} replication, $i = 1, 2, ..., l$, by

$$X_i \equiv \frac{1}{n}\sum_{j=1}^{n} Y_{i,j}$$

With $X_i's$ $100(1-\alpha)\%$ two-sided confidence interval for μ can be constructed as follows.

$$\mu \in \overline{X_l} \pm t_{l-1,\alpha/2}\sqrt{S/l} \quad (1)$$

whee $t_{d,\gamma}$ represents the $1-\gamma$ quantile of the t -distribution with d degrees of freedom. In Eq. (1),

$$S^2 \equiv \frac{1}{l-1}\sum_{i=1}^{l}(X_i - \overline{X_l}),$$

where

$$\overline{X_l} \equiv \sum_{i=1}^{l} X_i / l$$

is grand sample mean of all the replicate means. Since the sample replication means $X_1, X_2, \cdots, X_n$ are IID random variables and if n is large enough, the confidence interval (1) can be constructed by the central limit theorem.

3.3 Steady-State Simulation Output Analysis

Consider an experiment in which we wish to estimate the mean μ of a stationary (steady-state) process, $Y_j, j \geq 1$, e.g., the mean waiting time in a stationary $M/M/1$ queueing system. Assume that the initialization bias problem is ameliorated.

In this setting, to construct $100(1-\alpha)\%$ two-sided confidence interval for μ it is common to provide an estimate of $Var\left[\overline{Y_n}\right]$, or, almost equivalently, the variance parameter $\sigma^2 \equiv \lim_{n\to\infty} \sigma_n^2$, where

$$\overline{Y_n} \equiv \frac{1}{n}\sum_{j=1}^{n} Y_j, \text{ and } \sigma_n^2 \equiv n\,Var[\overline{Y}]$$

(provided that σ_2^n exists and is positive and finite). There are a number of estimators for σ^2, as described in standard references such as Law [4]. One of the most popular techniques in practice is the nonoverlapping batch means (NBM) method which is explained as follows.

The nonoverlapping batch means method is popular among experimenters because of its simplicity and effectiveness. In the batch means approach, the sample $Y_1, Y_2, \cdots, Y_n$ is divided into sub-groups of samples, and each sub-group is reduced to a single average value — a batch mean. These batch means are then used to estimate σ^2.

Suppose that one forms b nonoverlapping batches, each of size m (assuming that $n = mb$):

Batch 1: $Y_1, Y_2, \cdots, Y_m$
Batch 2: $Y_{m+1}, Y_{m+2}, \cdots, Y_{2m}$
Batch b: $Y_{(b-1)m+1}, Y_{(b-1)m+2}, \cdots, Y_n$

For $i = 1, 2, ..., b$ and $j = 1, 2, ..., m$, let

$$\overline{Y_{i,j}} \equiv \frac{1}{j}\sum_{k=1}^{j} Y_{(i-1)m+k}.$$

The NBMs are the averages $\overline{Y_{i,m}}$, $i = 1, 2, ..., b$, and form a stationary process themselves.

If we choose the batch size m large enough, it is reasonable to treat the $\overline{Y_{i,m}}$ as if they are IID normal random variables with mean μ. Then, for sufficiently large m the variance of the batch means can be estimated by their sample variance,

$$\widehat{Var}\left[\overline{Y_{1,m}}\right] = \frac{1}{b}\sum_{i=1}^{b}\left(\overline{Y_{1,m}} - \overline{Y_n}\right)^2$$
$$= \frac{1}{b-1}\sum_{i=1}^{b}\left(\overline{Y_{1,m}^2} - b\,\overline{Y_n^2}\right),$$

and the NBM estimator of σ_n^2 is given by

$$\widehat{V_B} \equiv \widehat{\sigma_n^2} = m\,\widehat{Var}\left[\overline{Y_{1,m}}\right] = \frac{m}{b-1}\sum_{i=1}^{b}(\overline{Y_{i,m}} - \overline{Y_n})^2,$$

where we use the approximation $m\,Var\left[\overline{Y_m}\right] \doteq n\,Var\left[\overline{Y_n}\right] \doteq \sigma^2$ for sufficiently large m. That is, we estimate $\sigma^2 \doteq \sigma_m^2$ by m times the sample variance of the batch means.

As the batch size $m \to \infty$,

$$\widehat{V_B} \xrightarrow{D} \frac{\sigma^2 \chi_{b-1}^2}{b-1} \tag{2}$$

Based on (2), we can form an asymptotic $100(1-\alpha)\%$ two-sided confidence interval for μ as follows.

$$\mu \in \overline{Y_n} \pm t_{b-1,\alpha/2}\sqrt{\widehat{V_B}/n}\,.$$

The main problem with the application of the batch means method for fixed sample size is the choice of the batch size m. If m is too small, the batch means $\overline{Y_{i,m}}$ can be highly correlated and the resulting confidence interval may have coverage below the nominal value $1-\alpha$. Alternatively, a large batch size can result in very few batches and potential problems with the high variability of the confidence interval half-width.

3.4 Estimation Technique for $f(\mu)$

In this section, we study confidence intervals for $f(\mu)$, where the function $f(\bullet)$ is nonlinear of steady-state mean μ. We collect n observations and form b nonoverlapping batches, each of size m as in subsection 3.3:

As point estimators for $f(\mu)$, we can consider

(1) Classical estimator:
$\hat{f}_c \equiv f(\overline{Y_n})$

(2) Batch means estimator:
$\overline{f_B} \equiv \frac{1}{b}\sum_{i=1}^{b} f(\overline{Y_{i,m}})$

(3) Jackknife estimator:
$\overline{f_J} \equiv \frac{1}{b}\sum_{i=1}^{b} \tilde{f}_i,$
where
$\tilde{f}_i \equiv bf(\overline{Y_n}) - (b-1)f(\overline{Y_{-1,m}})$
and
$\overline{Y_{-1,m}} \equiv \frac{1}{b-1}\sum_{j\neq i} \overline{Y_{j,m}}.$

As point estimators for $Var[\hat{f}(\mu)]$, we can consider

(a) The NBM variance estimator:
$S_B^2 \equiv \frac{1}{b(b-1)}\sum_{i=1}^{b}[f(\overline{Y_{i,m}}) - \overline{f_B}]^2.$

(b) The Jackknife variance estimator:
$S_J^2 \equiv \frac{1}{b(b-1)}\sum_{i=1}^{b}[\tilde{f}_i - \overline{f_J}]^2.$

Then the batch menas and Jackknie variance estimators obey

$$\frac{\hat{f}(\mu)-f(\mu)}{\sqrt{\widehat{Var}[\hat{f}(\mu)]}} \xrightarrow{D} t_{b-1}, \quad as\, m \rightarrow \infty \tag{3}$$

where $\hat{f}(\mu)$ can be $\widehat{f_C}$, $\overline{f_B}$, or $\overline{f_J}$, and $\widehat{Var}[\hat{f}(\mu)]$ can be S_B^2 or S_J^2.

Based on (3), an asymptotic $100(1-\alpha)\%$ two-sided confidence interval for $f(\mu)$ are given by

$$f(\mu) \in \hat{f}(\mu) \pm t_{b-1,\alpha/2}\sqrt{\widehat{Var}[\hat{f}(\mu)]}$$

where $\hat{f}(\mu)$ can be $\widehat{f_C}$, $\overline{f_B}$, or $\overline{f_J}$, and $\widehat{Var}[\hat{f}(\mu)]$ can be S_B^2 or S_J^2.

4. Conclusions

This paper discussed output data analysis of simulation output. The proper output data analysis is the one of the most important aspects of a simulation study. As discussed, simulation output is not following data properties in "classical statistics" or IID normal. Therefore, to deal with simulation output, we need some special techniques such as multiple replications and batch means. Besides of the techniques and issues in this paper, there are numerous interesting topics in simulation output analysis such as initialization problems, comparison of systems, etc. For these topics you can refer to [4,6,7,9] and references therein.

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장 병 윤 (bychang@ajou.ac.kr)

1995 성균관대학교 산업공학과 학사
2000 Georgia Tech. Operations Research 석사
2002 Georgia Tech. Applied Statistics 석사
2004 Georgia Tech. Industrial and Systems Engineering 박사
2004～2006 Georgia Tech. Post Doc.
2006～2009 KT 네트워크 연구소 선임 연구원
2009～현재 아주대학교 경영대학 경영학부 조교수

관심분야 : 정보통신경영, BPM, OR/OM, SCM, Simulation, Applied Statistics